Fig. 4. Relative error versus number of terms for the spectral sum of the first series in Table II.

Fig. 5. Computation time versus $1/\epsilon_c$ for the spectral sum of the first series in Table II.

Fig. 6. Relative error versus number of terms for the spectral sum of the second series in Table II.

out Shanks's transform for the first series in Table II. The results from Shanks's transform converge faster in each case. Figs. 6 and 7 show the relative error and computation time, respectively, for the second series in Table II. For a convergence factor of 10^{-5} , Shanks's transform converges in less than 100 terms whereas the first-order acceleration takes over 20000 terms. If we see the computation time for this case, Shanks's transform converges in one tenth the time taken by first-order

Fig. 7. Computation time versus $1/\epsilon_c$ for the spectral sum of the second series in Table II.

acceleration. It should be pointed out that the transform series converge very rapidly (within 20 terms) for both of the series in Table II. A reasonable choice of the smoothing parameter, u , which seems to ensure good convergence for both the spectral and transform series is about π/d . The computations for the numerical results were carried out on a VAX 6320 machine.

IV. CONCLUSION

The application of Shanks's transform improves dramatically the convergence of the one-dimensional free-space Green's functions. This is indicated by the computation time, which in some cases is reduced by a factor of 10 over the direct summation of the series. The first-order acceleration with Shanks's transform converges faster than first-order acceleration in each case. The advantage offered by the use of Shanks's transform is that no analytical work has to be done to the series. This may be significant in some cases where it is not so easy to find the Fourier transform of the series.

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Determining Adapter Efficiency by Envelope Averaging Swept Frequency Reflection Data

W. C. DAYWITT

Abstract—A simple automated network analyzer swept frequency technique for measuring adapter efficiency across its entire frequency band

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The author is with the Electromagnetic Fields Division, National Institute of Standards and Technology, 325 Broadway, Boulder, CO 80303.
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is described. Envelope averaging is used to avoid the model assumptions usually found in regression averaging techniques. Calculations show that errors arising from the theoretical assumptions leading to the technique are around 0.004 dB for a common WR 42 waveguide-to-coaxial adapter.

I. INTRODUCTION

There are numerous microwave and millimeter-wave examples where adapters are used to interconnect devices with different connector configurations, and where accurate knowledge of the adapter efficiency is required but is not readily available or easily measurable. This is an especially bothersome situation in calibration laboratories, where the unavailability of accurately characterized adapters may require a separate standard for every connector type in order to perform calibrations of devices with a variety of different connectors. In a noise calibration laboratory, for example, a coaxial noise source cannot be calibrated with a waveguide noise standard because of their different connectors. The obvious solution, attaching a waveguide-to-coaxial adapter to either the standard or the unknown, requires an adapter efficiency measurement in order to perform the calibration. Unfortunately, obtaining an accurate measurement is often difficult or impossible.

To further aggravate matters, measurement assurance often requires that an adapter be measured by more than one technique. In the noise laboratory at the National Institute of Standards and Technology (NIST), an adapter is measured by at least three different techniques. Even then it is often necessary to repeat the measurements to reduce measurement error and obtain a reliable value for the efficiency.

Of course these problems are not peculiar to noise calibrations. They are also encountered in power, attenuation, and antenna measurements as well as in many other microwave and millimeter-wave applications.

The technique described in this paper is another weapon for the arsenal of measurement assurance. Only the theory and the errors due to the theoretical approximations will be given, however, since automated network analyzer (ANA) and connector errors need to be discussed at length to present the technique in full. These topics will be taken up in a later paper.

Previously existing techniques [1]–[5] are designed for the highest possible accuracy and require considerable effort to implement. In contrast to these single-frequency techniques, the procedure to be described here is relatively simple to perform and, at least in theory, produces accurate results by taking advantage of the swept frequency aspect of modern network analyzers.

An example of the need to evaluate adapter efficiency is illustrated by the schematic drawing shown in Fig. 1, which depicts a situation often encountered in noise power calibrations. Here, the coaxial noise power P' is needed for a coaxial calibration, but only a waveguide noise standard of known output power P is available. The two powers are related by (noise generated in the adapter has been neglected)

$$P' = \eta P = \zeta \eta_0 P \quad (1)$$

where η is the *in situ* adapter efficiency, and η_0 is the *intrinsic* adapter efficiency, defined by

$$\eta_0 \equiv \frac{|S_{21}|^2}{1 - |S_{11}|^2} \quad (2)$$

The quantity ζ is often sufficiently close to unity to be discarded without significant error, so the *in situ* efficiency which is required for the calibration can be accurately approximated by

Fig. 1. Adapting a waveguide noise standard of known noise power P to create a coaxial noise source with an output noise power P' . η_0 and η are the adapter intrinsic and *in situ* efficiencies respectively. Γ_1 and Γ_r are the reflection coefficients in the directions shown.

the intrinsic efficiency. Both efficiencies are unity if the adapter is lossless. The adapter scattering coefficients [6] are S_{21} and S_{11} , where the transmission coefficient S_{21} contains the effects of both adapter reflection and dissipation. The denominator of (2), $1 - |S_{11}|^2$, removes the effect of these reflections, leaving only the dissipative losses in the efficiency η_0 . The present technique determines η_0 directly without measuring the separate scattering coefficients.

Preliminary studies indicate that the sinusoidal oscillations often seen in reflection measurements like those to be modeled here are due to differences between the DUT (device under test) connector at the ANA port and the connectors of the ANA calibrating standards. This knowledge could lead to a significant reduction in measurement error if further studies substantiate these findings. This paper, however, is concerned only with displaying the theoretical relationship between the adapter efficiency and the envelope average, connector errors being beyond the intended scope of the paper.

The usual method of averaging swept frequency data requires regression fitting a model of the adapter efficiency to the measured data. In contrast to this procedure, the technique described in Section II yields adapter efficiency without prior knowledge of the efficiency variation with frequency. Simulations illustrating an application of the technique to a WR 42 waveguide-to-coaxial adapter (18 GHz to 26.5 GHz) are displayed in Section II.

Errors due to the theoretical assumptions leading to the technique are presented in Section III. ANA calibration error, however, which is an incompletely investigated topic at NIST at the present time, will be discussed in a later paper. Conclusions are found in Section IV.

II. THEORY AND MEASUREMENT SIMULATIONS

The *in situ* adapter efficiency η accounts for both the adapter loss and the effect of the radiometer reflection Γ_r (Fig. 1) upon that loss. It is the quantity of interest in all connector conversion applications and is related to the intrinsic efficiency, η_0 , by the equation [7]

$$\eta = \zeta \eta_0 = \frac{(1 - |\Gamma_r|^2)|S_{21}|^2}{(1 - |\Gamma_1|^2)|1 - S_{22}\Gamma_r|^2} \quad (3)$$

where

$$\zeta \equiv \frac{(1 - |\Gamma_r|^2)(1 - |S_{11}|^2)}{(1 - |\Gamma_1|^2)|1 - S_{22}\Gamma_r|^2} \quad (4)$$

and where Γ_1 is the reflection coefficient of the adapter/radiometer combination. The approximation ($\zeta \approx 1$) mentioned in the introduction is assumed to hold and attention is now focused on the determination of the efficiency η_0 .

The first step in determining η_0 is to measure the magnitude $|\Gamma|$ of the reflection Γ shown in Fig. 2 as a function of frequency.

Fig. 2. An adapter efficiency measurement using a short circuit and a swept frequency ANA. η_0 is the intrinsic efficiency, and Γ_{sc} and Γ are reflection coefficients in the directions shown. l_s is the length of the waveguide short circuit.

A short circuit is attached to the adapter, and the ANA is calibrated with a coaxial connector input that will mate with the adapter connector. The composite adapter/short reflection coefficient is then measured. It is related to the short reflection Γ_{sc} (whose magnitude, $|\Gamma_{sc}|$, is assumed to be known) and the adapter scattering matrix, S , by the equation

$$\Gamma = S_{22} + \frac{S_{21}^2 \Gamma_{sc}}{1 - S_{11} \Gamma_{sc}}. \quad (5)$$

This can be put in the following, more useful form [7]:

$$\Gamma = A + B e^{j\Psi} \quad (6)$$

where

$$A \equiv \frac{S_{22} + \sigma S_{11}^* |\Gamma_{sc}|^2}{1 - |S_{11} \Gamma_{sc}|^2} \quad (7)$$

$$B \equiv \frac{S_{21}^2 |\Gamma_{sc}|}{1 - |S_{11} \Gamma_{sc}|^2} \quad (8)$$

and

$$\Psi = \phi + 2 \arctan \left[\frac{|S_{11} \Gamma_{sc}| \cos(\phi + \phi_{11})}{1 - |S_{11} \Gamma_{sc}| \sin(\phi + \phi_{11})} \right]. \quad (9)$$

The phases of Γ_{sc} and S_{11} are ϕ and ϕ_{11} , and $\sigma \equiv S_{21}^2 - S_{11} S_{22}$. The magnitude of (6) can be approximated by

$$|\Gamma| \doteq |B| + |A| \cos \theta \quad (10)$$

since $|A|$ is much smaller than $|B|$ [7]. The angle θ is

$$\theta \equiv \Psi + \phi_B - \phi_A \doteq \phi + \phi_B - \phi_A \quad (11)$$

where ϕ_A and ϕ_B are the angles of A and B .

One further approximation is needed, a relationship between $|B|$ and η_0 . Taking the magnitude of (8) leads to

$$|B| = \frac{|S_{21}|^2 |\Gamma_{sc}|}{1 - |S_{11} \Gamma_{sc}|^2} = \frac{|S_{21}|^2}{1 - |S_{11}|^2} |\Gamma_{sc}| \left(\frac{1 - |S_{11}|^2}{1 - |S_{11} \Gamma_{sc}|^2} \right) \doteq \eta_0 |\Gamma_{sc}|. \quad (12)$$

The last approximation follows from the definition of η_0 , and the fact that $1 - |S_{11}|^2$ and $1 - |S_{11} \Gamma_{sc}|^2$ are very nearly equal since $|\Gamma_{sc}|$ is close to 1.

The magnitudes $|B|$ and $|A|$ in (10) are slowly varying functions of frequency while the corresponding angles ϕ_B and ϕ_A in (11) are approximately linear in frequency. The angle θ is, therefore, approximately linear in frequency because the short circuit angle ϕ is linear in frequency. Equation (10) then describes a slowly varying function $|B|$ with a small additive sinusoidal term $|A| \cos \theta$ (the added term is small because $|B|$ is much larger than $|A|$ [7]). These ideas are easily visualized with the simulated reflection of the WR 42 waveguide-to-coaxial

Fig. 3. The magnitude $|\Gamma|$ of the reflection shown in Fig. 2 as a function of frequency from 0 to 26.5 GHz. The $|B|$ curve is to be extracted by envelope averaging. An l_s of 6 cm was used to calculate the curves in the figure.

adapter shown in Fig. 3. Sweeping the ANA in frequency from 0 GHz to 26.5 GHz produces the $|\Gamma|$ shown (l_s in Fig. 2 was arbitrarily chosen to be 6 cm), where the waveguide cutoff frequency f_c (14.05 GHz) is indicated, as is the lowest frequency (18 GHz) in the WR 42 band.

The ANA "sees" only the coaxial portion of the adapter below the cutoff frequency in Fig. 3, where there is a gradual falloff in $|\Gamma|$ from 0 GHz to f_c due to the line loss from the coaxial portion of the adapter. Above f_c , where waveguide propagation takes place, the ANA sees all the way through the adapter to the end of the waveguide short circuit, producing the sinusoidal portion of the plot.

A curve representing $|B|$ is superimposed on the $|\Gamma|$ plot in Fig. 3 to show the relationship between $|A|$, $|B|$, and $|\Gamma|$. Transmission through the adapter is represented by a nonvanishing $|B|$. Therefore, $|B|$ is 0 below cutoff at f_c and gradually increases above f_c in the transition region as waves begin to propagate through the waveguide portion of the adapter. The quantity $|B|$ continues to rise until it is vertically half way between the peaks and valleys of the sinusoidally varying $|\Gamma|$ curve in the WR 42 frequency band. This "half way" feature is the characteristic that permits the envelope averaging technique to work. In a coaxial adapter, of course, the "half way" feature holds at all frequencies.

If the sinusoidal variations associated with $|A| \cos \theta$ in (10) are rapid enough, then the envelopes drawn along the peaks and valleys of $|\Gamma|$ in Fig. 3 can be averaged to obtain $|B|$ since

$$|B| = \langle |\Gamma| - |A| \cos \theta \rangle = \langle |\Gamma| \rangle - \langle |A| \cos \theta \rangle. \quad (13)$$

The symbol $\langle \rangle$ represents the envelope average, to be described next.

The value of $|B|$ as a function of frequency (Fig. 3) is recovered from the plot by finding the midpoint between the envelopes formed by the peaks and valleys of the sinusoidal variations, the succession of midpoints being equivalent to the curve $|B|$. This *envelope averaging* scheme requires a rapid enough sinusoidal variation to construct an accurate envelope. Since the number of variations is a function of the length l_s of the waveguide short circuit, the greater the length, the more rapid the oscillations. Trial and error is used to find a short circuit length that gives an acceptably small $|B|$ error.

Fig. 4. Simulated reflection coefficient of an adapter with an attached flat waveguide short circuit. The dashed curves form the envelope from which $|B|$ is to be estimated.

Fig. 5. Simulated reflection coefficient of an adapter with an attached offset waveguide short circuit of length $l_s = \lambda_g/2$.

Fig. 4 shows $|\Gamma|$ and $|B|$ from 17 to 26.5 GHz for a flat short ($l_s = 0$). The dashed curves form the envelopes that are constructed by connecting peaks and valleys. The corresponding $|B|$ curve is found by connecting the midpoints (circled dots) of the vertical lines drawn between the dashed envelope curves. The resulting error in $|B|$, i.e., the vertical difference between the true and approximate $|B|$ curves, is clearly unacceptable, indicating that a flat short causes too few oscillations to produce an accurate estimate of $|B|$. In practice, more vertical lines are drawn than the six shown in the figure.

The simulation in Fig. 5 is the result of using a short that is half of a guide wavelength long ($l_s = \lambda_g/2$) at the lower end of the WR 42 frequency band (18 GHz). The approximate $|B|$ curve that would be obtained from the midpoints is considerably closer to the true $|B|$ curve than that with the flat short, but still leaves something to be desired in accuracy (that is, the circled points still diverge unacceptably far from the true $|B|$ curve).

Fig. 6 shows a plot where $l_s = \lambda_g$ at 18 GHz. The agreement between $|B|$ and the approximate $|B|$ indicated by the circled points is now quite good, giving an error of less than 0.001 dB at the point marked with a "max" on the plot. This same conclusion is obtained with simulations in other waveguide bands. Therefore, if the offset short has a length of a guide wavelength or longer, then the fitting error is no greater than 0.001 dB.

A curve drawn through a closely packed set of points similar to those in Fig. 6 gives $|B|$ as a function of frequency across the

Fig. 6. Simulated reflection coefficient of an adapter with an attached offset waveguide short circuit of length $l_s = \lambda_g$.

frequency band. But, according to (12), $|B|$ is related to the adapter efficiency η_0 by

$$\eta_0 = \frac{|B|}{|\Gamma_{sc}|} \quad (14)$$

Therefore, normalizing the $|B|$ curve by $|\Gamma_{sc}|$ (which is assumed to be known or which can be measured) will produce the adapter efficiency η_0 as a function of frequency.

The simulations in Figs. 3 through 6 are well mirrored in actual swept frequency data plots.

III. ERRORS

There are three sources of error in the envelope averaging technique (ANA measurement and calibration errors excluded): the envelope averaging error (Fig. 6) and the two errors in (15) associated with setting η equal to η_0 and setting ζ equal to unity.

Taking the first-order differential of (3) leads to the relative error

$$\frac{\Delta\eta}{\eta} = \frac{\Delta\eta_0}{\eta_0} + \frac{\Delta\zeta}{\zeta} \quad (15)$$

where η_0 can be expressed as (see (2) and (12))

$$\eta_0 = \frac{|B|}{|\Gamma_{sc}|} \left(\frac{1 - |S_{11}\Gamma_{sc}|^2}{1 - |S_{11}|^2} \right) = \frac{|B|}{|\Gamma_{sc}|} [1 + |S_{11}|^2(1 - |\Gamma_{sc}|^2)] \quad (16)$$

The quantity $|B|$ is determined from envelope averaging the swept data as described by (13). Imperfect averaging ($\langle |A|\cos\theta \rangle \neq 0$) due to the finite length of the offset short leads to the relative error

$$\frac{\delta|B|}{|B|} = \left| \left\langle \left(\frac{A}{B} \right) \cos\theta \right\rangle \right| \quad (17)$$

where the brackets $\langle \rangle$ symbolize the envelope averaging described in Section II. The nonvanishing of the right side of (17) was seen in Fig. 6 to be less than 0.00022 (0.001 dB).

Combining (15)–(17) leads to the relative error

$$\frac{\Delta\eta}{\eta} = |S_{11}|^2(1 - |\Gamma_{sc}|^2) + 2|\Gamma_r|(1 - \eta_0)^2 + \left| \left\langle \left(\frac{A}{B} \right) \cos\theta \right\rangle \right| \quad (18)$$

These three errors from left to right are 1) the error in replacing

the bracketed factor in (16) by 1; 2) the error from the approximation $\zeta \approx 1$; and 3) the error due to imperfect envelope averaging (Fig. 6). This second term in (18) corresponds to the second term in (15) and was obtained from [7]. To summarize, (18) is the relative error in using

$$\eta \approx \frac{|B|}{|\Gamma_{sc}|} \quad (19)$$

where $|B|$ is determined by envelope averaging.

A feel for the size of the error in (18) can be obtained by using the following values: $|S_{11}| = 0.1$; $|\Gamma_{sc}| = 0.98$; $|\Gamma_r| = 0.1$; $\eta_0 = 0.955$ (0.2 dB); and $|\langle (A/B) \cos \theta \rangle| = 0.00022$. The magnitudes of the three terms in (18) are then 0.0004, 0.0004, and 0.0002, for a total of 0.001 or 0.004 dB.

IV. CONCLUSIONS

The theory and the simulations presented in Section II (Figs. 4–6) show that it is possible to separate $|B|$ from swept reflection data $|\Gamma|$ to better than 0.00022 (0.001 dB) and indicate that envelope averaging is a useful technique for measuring adapters. Combined with the theoretical approximations leading to (19), this leads to a total error calculated from (18) of 0.004 dB for fairly conservative choices for the variables in that equation.

It can be concluded from the above that the envelope averaging technique is capable of producing accurate results.

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Comparison of Approximations for Effective Parameters of Artificial Dielectrics

EDWARD F. KUESTER, MEMBER, IEEE,
AND CHRISTOPHER L. HOLLOWAY

Abstract—Formulas available in the literature for approximating the effective permittivity and permeability of a periodic array of (possibly lossy) dielectric rods of arbitrary cross section are assessed for their accuracy by comparing them with precise numerical values for special forms of the arrays. It is found that while no single formula is capable of very high precision in every case, we can construct good approximate formulas in individual cases as a result of this comparison.

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Fig. 1. Artificial dielectric composed of a periodic array of dielectric cylinders.

I. INTRODUCTION

A periodic array of (possibly lossy) dielectric cylinders (Fig. 1) forms an artificial dielectric medium whose behavior in electromagnetic fields of wavelength sufficiently large compared with the period of the array is equivalent to that of a homogeneous but anisotropic medium of some effective (or *homogenized*) permittivity and permeability as far as the average electric and magnetic fields are concerned [1]–[6]. Such artificial media have many important applications in electromagnetics because they allow materials to be made with properties not easily realized in ordinary dielectrics [7]. The authors' interest stems from an analysis of the low-frequency reflection properties of arrays of pyramid-cone absorbers used to line electromagnetic anechoic measurement chambers [8]. It is of prime importance to be able to predict the effective properties of artificial dielectrics from a knowledge of their geometry and the material properties of their constituents. It is the goal of this short paper to review some of what is known about these properties from the widely scattered literature on the subject and to determine from it simple yet accurate formulas for the properties for two specific geometries of interest.

II. REVIEW OF KNOWN APPROXIMATIONS

Consider the two-dimensional rectangular array of rods shown in Fig. 1, whose period is a in the x direction and b in the y direction. The permittivity $\epsilon(x, y)$ is ϵ_2 in the rods (referred to as phase 2) and ϵ_1 in the surrounding medium (phase 1) and similarly for μ . According to the homogenization method, this medium behaves like a uniaxially anisotropic but homogeneous material with tensor permittivity $[\epsilon^h]$ and permeability $[\mu^h]$:

$$[\epsilon^h] = \begin{bmatrix} \epsilon_x & 0 & 0 \\ 0 & \epsilon_y & 0 \\ 0 & 0 & \epsilon_z \end{bmatrix} \quad (1)$$

$$[\mu^h] = \begin{bmatrix} \mu_x & 0 & 0 \\ 0 & \mu_y & 0 \\ 0 & 0 & \mu_z \end{bmatrix} \quad (2)$$

If a and b are small compared with a wavelength or skin depth in either medium, then the elements μ_x and ϵ_z are known exactly [4], [5]:

$$\begin{aligned} \epsilon_z &= (1 - f_2)\epsilon_1 + f_2\epsilon_2 \\ \mu_z &= (1 - f_2)\mu_1 + f_2\mu_2 \end{aligned} \quad (3)$$