Frequency Dependence of Hysteresis Curves in "Non-Conducting" Magnetic Materials

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Abstract **- The problem of modelling the frequency dependence of hysteresis in magnetic materials is approached in a new way. The dc magnetization curve, or hysteresis loop, is assumed to be the equilibrium position for the bulk magnetization. All microscopic processes which occur under the action of a time-varying field can be averaged to give a time dependent displacement from the equilibrium, ΔM. In this paper, we examine the case where eddy current effects do not play a significant role, that is to say the model applies to "non-conducting" media. It is shown that ΔM obeys a damped simple harmonic motion equation. This means that the time dependence of the displacement magnetization ΔM is the Laplace transform of the field waveform. This enables the time dependence of the magnetization to be modelled, once the dc hysteresis curve is known, with only two additional materials parameters of relaxation time and natural frequency. The model emerges as a natural extension of the theory of hysteresis.**

INTRODUCTION

The magnetization curves, or hysteresis loops of ferromagnetic materials change as a function of the frequency and waveform of the applied magnetic field. Most measurements of hysteresis are performed under dc, or quasi dc, conditions. In most applications the material is subjected to an ac field. The differences between the dc and ac hysteresis curves depend on a number of factors including the electrical conductivity and permeability of the material, the rate at which the magnetic moments can rotate into the field, the frequency of the applied field and its waveform, whether sinusoidal, triangular or square wave.

This paper introduces a new approach to modelling the changes in magnetic hysteresis curves as a function of frequency. The key development has been the identification of differential equations representing the time dependence of hysteresis curves in two principal cases. These are the effects of eddy currents in electrically conducting magnetic media and the effects of magnetic relaxation in non-conducting media. In this paper, we will discuss only the latter.

In previous work, a phenomenological model of hysteresis has been developed [1,2] based on the consideration of energy loss due to, among other factors, domain wall pinning. The advantage of this model over others is that it is based on a consideration of the underlying physics of hysteresis, and has a small number of physically interpretable parameters. To date, the theory has only dealt with time independent hysteresis. The equations can, however, be extended to account for time dependent effects.

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Non-Conducting Media

In "non-conducting" media, we assume that the effects of eddy currents can be ignored. This approximation works well for high frequency ferrites [3]. In this work, the time dependence of the magnetization is treated as a second order linear differential equation, in which the equilibrium position of the magnetization at a field strength H, is simply the dc hysteresis curve.

The change in magnetization in the low field region, where hysteresis occurs, is determined primarily by domain wall motion. The domain wall motion can itself be described by a second order linear differential equation, as discussed by Döring [4] and later by Chikazumi [5]. This applies on the micromagnetic scale of a single domain wall; however, the concept can be scaled to describe the macroscopic magnetization changes, since the change in bulk magnetization with time, $\frac{dM}{dt}$ is simply the average over the entire material of the individual domain wall movements.

If we consider these averaged domain wall movements, it is clearly apparent that they cease once the magnetization M(t) at a given time t has reached the dc magnetization curve, which we will denote $M_\infty(H)$.

$$\lim_{t\to\infty} M(t) = M_\infty(H) \tag{1}$$

Equally clearly $M_\infty(H)$ is a function of the magnetic field H, which must be described by a time independent hysteresis function. The value of $M_\infty(H)$ is uniquely defined by the magnetic field history of the specimen and is obtained by calculating the value of the bulk magnetization that would be achieved when all transients in the magnetization process have been completed. This means that $M_\infty(H)$ is represented by the value of bulk magnetization on the quasi-dc hysteresis loop for a given sequence of field reversals and the prevailing magnetic field strength. In other words, $M_\infty(H)$ is path dependent but is time independent. It can be modelled using the equations given in earlier papers for describing dc hysteresis loops [1], or by other descriptions of time independent hysteresis loops, such as Preisach models. Given this result, and the damped harmonic nature of the change in magnetization, the displacement magnetization

$$\Delta M(t,H) = M(t) - M_\infty(H) \tag{2}$$

obeys a differential equation of the form,

$$\frac{d^2}{dt^2}\Delta M(t,H)+2\lambda\frac{d}{dt}\Delta M(t,H)+\omega_n{}^2\Delta M(t,H) = 0 \quad (3)$$

and since the time derivatives of $M_\infty(H)$ are zero, this gives

$$\frac{d^2}{dt^2}M(t) + 2\lambda\frac{d}{dt}M(t) + \omega_n{}^2M(t) = \omega_n{}^2M_\infty(H) \quad (4)$$

where ω_n is the natural frequency, which can be calculated from ferromagnetic resonance [6] and λ is a decay constant. The natural frequency ω_n represents the frequency at which the magnetic moments inside the material can oscillate in the absence of any external damping forces. This is equivalent to the Larmor precession frequency of the electron spins under the action of the anisotropy "field," as discussed by Landau and Lifschitz [7]. The damping coefficient λ in this equation is also equivalent to that defined in Landau-Lifschitz equation of motion.

In these equations the forcing term which we expect in the damped harmonic motion equation, because of the applied field, is implicit in the term $\omega_n{}^2(M_\infty(H) - M(t))$. A simple physical argument can be used to prove this: once the magnetization reaches $M_\infty(H)$, it is in equilibrium and so there is no net force on the domain walls.

Rado [6] has investigated ferromagnetic resonance. The significance of this phenomenon is that it relates the resonant frequency ω_r of domain wall motion to the initial permeability of the hysteresis curve via the equation

$$\omega_r = \gamma \cdot \frac{M_s}{\mu_i-1}\cdot\left(\frac{8\pi(\mu_i-1)}{d}\,\delta\right)^{1/2} \quad (5)$$

where γ is the gyromagnetic ratio (0.22×10^6 rads•m•s^{-1}•A^{-1}). μ_i is the relative initial permeability, which is dimensionless ($\mu_i-1 = \chi_i$ the initial susceptibility), M_s is the saturation magnetization, δ is the wall thickness and d denotes the average domain size.

The actual values of ω_r can be obtained experimentally from measurements of the initial susceptibility of materials as a function of frequency. The frequencies at which the maximum of the initial susceptibility occurred in the two materials studied here were $\nu_r = 0.768\times10^6 s^{-1}$ ($\omega_r = 4.83\times10^6$ rad.s^{-1}) for the 3C80 Mn-Zn ferrite, and $\nu_r = 0.573\times10^6\ s^{-1}$ ($\omega_r = 3.60\times10^6$ rad.s^{-1}) for the 3C81 Mn-Zn ferrite [8].

The resonance frequency ω_r and the natural frequency ω_n are related by the equation

$$\omega_r = \omega_n\sqrt{1-\left(\frac{\lambda}{\lambda_{cr}}\right)^2} \quad (6)$$

where λ_{cr} is the critical value of λ, which is the value of λ separating conditions under which wall resonance, rather than wall relaxation, occur. This enables the natural frequency of the material to be determined from the initial susceptibility and the damping coefficient λ in the differential equation of motion.

The results of solving equation (4) give increasingly rounded hysteresis loops as the frequency of excitation is increased.

Values of parameters

A study has been made of manganese zinc ferrite (Philips series 3C81 ferrite material). The dc hysteresis parameters have been given previously [1], together with a comparison of the modelled and measured hysteresis curves. These parameters were $M_s = 0.4 \times 10^6$ A•m^{-1}, a = 27 A•m^{-1}, k = 30A•m^{-1}, $\alpha = 5 \times 10^{-5}$, c = 0.55.

From the initial permeability of the material under dc condition, $\mu_i \cong 2700 \pm 100$, a resonance frequency of $\upsilon_r = 550$ kHz ($\omega_r = 3.46\times10^6$ rad.s^{-1}) was calculated which agrees well with the manufacturer's specification [8].

The damping parameter λ determines the rate of response of the magnetization to an external field, and from the equation this can be expressed in terms of an equivalent relaxation time $\tau = 1/\lambda$. For this material $\tau \approx 10^{-6}$ sec. was found to give reasonable agreement with observation.

RESULTS

Model hysteresis curves were calculated based on the parameters given above. Figs. 1-3 show the hysteresis curves of the modelled ferrite material under the action of a sinusoidal magnetic field of amplitude 100 A/m (1.25 Oe), at frequencies of 1, 50, and 100 kHz. The initial permeability at 1 kHz was almost identical to the quasi static initial permeability. From these results it can be seen that there is an increase in coercivity, hysteresis loss and remanence with increasing frequency. An increase in initial permeability is predicted by the model, and this is followed by a decrease at higher frequencies, both of which are observed in practice. The changes in initial permeability with frequency for two different sets of model parameters are shown in tables 1 and 2.

Table 1. Comparison of measured values of μ_{in} with theoretical predictions

Manganese zinc ferrite 3C80

Frequency ν(kHz)	Measured μ_{in}	Theoretical μ_{in}
10	2100 (±10%)	2028
100	2100	2028
200	2200	2022
500	2500	2161
1000	2500	1732
2000	1500	1040
3000	600	723
4000	300	551
5000	200	444

Values obtained with the following model parameters: B_s=0.5 Tesla, a=27A•m^{-1}, k=30A•m^{-1}, $\alpha=5\times10^{-5}$, c=0.41, $\lambda=2.22\times10^{-8}s^{-1}$ ($\tau=9\times10^{-9}$s), ω_r=4.83x10^{6}rad.s^{-1}

Table 2. Comparison of measured values of μ_{in} with theoretical predictions

Manganese-zinc ferrite 3C81

Frequency ν(kHz)	Measured μ_{in}	Theoretical μ_{in}
10	2700 (±10%)	2708
50	2700	2708
100	2700	2708
200	3000	2700
500	3300	2886
1000	2900	2313
2000	1600	1400
3000	900	965
4000	550	736
5000	400	593

Values obtained with the following model parameters: B_s=0.5 Tesla, a=27A•m^{-1}, k=30A•m^{-1}, α=5x10^{-5}, c=0.55, λ=2.22x10^{8}s^{-1} (τ=9x10^{-9}sec), ω_r=3.60x10^{6} rad.s^{-1}

CONCLUSIONS

A model for describing the frequency dependence of hysteresis curves in non-conducting materials has been presented. The model is based on the second order linear differential equation of motion of domain walls, which is averaged to describe the behavior of the whole material. The result is a differential equation describing the displacement magnetization $\Delta M = M(t) - M_\infty(H)$ where $M_\infty(H)$ is the locus of points on the dc hysteresis curve.

The main contribution of the present work consists of the incorporation of hysteretic effects into the equations of motion given first by Landau and Lifschitz [7] for the description of time dependent magnetization in magnetic materials.

The frequency dependent hysteresis curves therefore consist of two independent contributions to the magnetization. These are the dc hysteresis curve, which represents the locus of equilibrium magnetization as a function of field, and the displacement magnetization, which obeys the damped harmonic motion equation.

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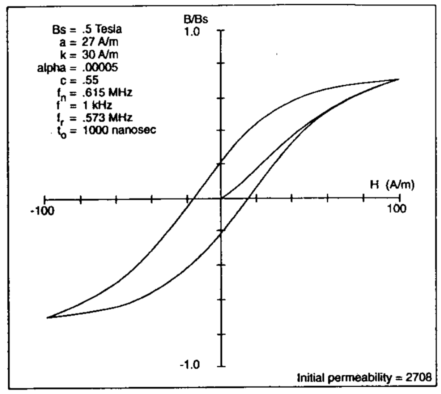

Fig. 1. Model hysteresis loop for 3C81 ferrite at 1 kHz. This curve is identical to the dc hysteresis curve on this scale.

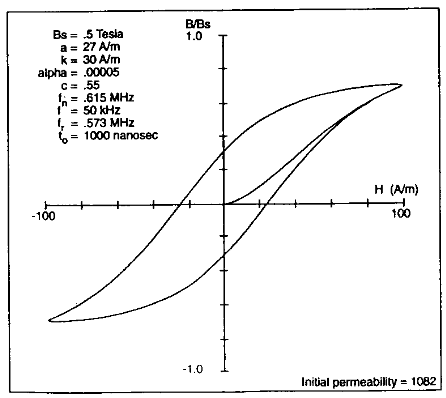

Fig. 2. Model hysteresis loop for 3C81 ferrite at 50 kHz.

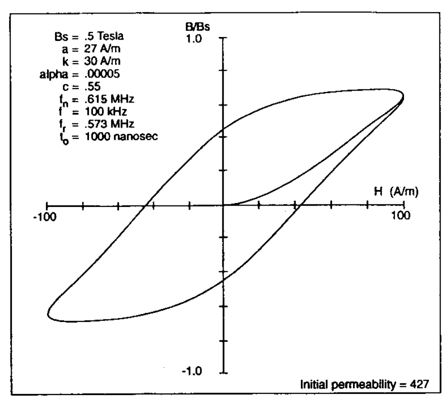

Fig. 3. Model hysteresis loop for 3C81 ferrite at 100 kHz.