The Role of Fundamental Constants in the International System of Units (SI): Present and Future

Barry N. Taylor, *Fellow, IEEE*, and Peter J. Mohr

Abstract—This paper addresses the role of fundamental constants in 1) the current definitions of the SI base units; 2) practical representations of SI electric units and the consistency of those representations with the SI as deduced from the 1998 CODATA recommended values of the constants; and 3) redefinition of the kilogram and the impact on realizations of SI electric units of a newly proposed definition that fixes the value of the Planck constant h .

Index Terms—Ampere, fundamental constants, International System of Units, Josephson effect, kilogram redefinition, ohm, Planck constant, quantum Hall effect, realization of electric units, representations of electric units, SI, volt, watt, watt balance.

I. INTRODUCTION

UNDERLYING the creation of the metric system over 200 years ago at the time of the French revolution was mankind's long-sought goal of developing a "philosophically true" system of measurement "established on an invariant base, fixed in nature" to which we could turn "repeatedly to find the means of verification" [1]. In short, a system that allows its units of measurement to be realized with high accuracy at anytime and anyplace and by anyone. It is therefore no surprise that Nature's true invariants—the fundamental physical constants—play an important role in the International System of Units (SI), the modern metric system. It is the aim of this paper to briefly review that role as it now exists and to consider the possible future role of the constants in the SI. Our focus is 1) the present definitions of the SI base units; 2) practical representations of SI electric units founded on various constants and the consistency of those representations with the SI as implied by the 1998 Committee on Data for Science and Technology (CODATA) set of recommended values of the constants [2]; and 3) redefinition of the kilogram and the potential impact on realizations of SI electric units of a newly proposed definition that has the effect of fixing the value of the Planck constant h [3].

II. CURRENT ROLE OF CONSTANTS IN SI BASE UNIT DEFINITIONS

The SI is founded on seven base units: the meter (m), kilogram (kg), second (s), ampere (A), kelvin (K), mole (mol), and candela (cd) for the seven base quantities length, mass, time, electric current, thermodynamic temperature, amount of substance, and luminous intensity [4]. Of these units, the second,

meter, ampere, and mole may be viewed as depending upon fundamental constants.

A. Second

The definition of the unit of time in the SI reads [4] *The second is the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium 133 atom*. This definition has the effect of fixing the frequency of the $(F = 3, m_F = 0) \rightarrow (F = 4, m_F = 0)$ hyperfine transition in the ^{133}Cs ground state $6^2\text{S}_{1/2}$. Although this frequency is an invariant of nature, it is not usually viewed as a fundamental constant. However, in principle a theoretical expression for it can be written in terms of traditional fundamental constants such as the Rydberg constant R_∞ , the fine-structure constant α , and the speed of light in vacuum c , together with a number of less traditional constants including the magnetic moment of the ^{133}Cs nucleus and other constants that characterize the properties of this nucleus. Thus one may view the SI definition of the second as having the effect of fixing the value of that particular combination of constants. If we denote that combination by the symbol $\Delta\nu(^{133}\text{Cs})$, the definition of the unit of time in the SI could read *One second is a time interval such that the ground-state hyperfine splitting transition frequency of the cesium 133 atom $\Delta\nu(^{133}\text{Cs})$ is exactly 9 192 631 770 hertz*, which emphasizes the relationship between the second and the invariant of nature on which it is based.

B. Meter

The SI unit of length, the meter, is a more straightforward case than the second. The definition of the meter reads [4] *The meter is the length of the path traveled by light in vacuum during a time interval of $1/299\,792\,458$ of a second*. Based on the relationship between speed v , distance d , and time interval Δt , $v = d/\Delta t$, it is evident that this definition has the effect of fixing the speed of light in vacuum c to be exactly $299\,792\,458$ m/s. Thus the definition could read *One meter is a distance such that the speed of light in vacuum c is exactly 299 792 458 meters per second*, which makes the connection between the meter and the speed of light explicit.

C. Ampere

The ampere is also more straightforward than the second. The definition of the SI unit of current reads [4] *The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross section, and placed 1 meter apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} newton per*

Manuscript received May 14, 2000; revised October 24, 2000.

The authors are with the National Institute of Standards and Technology, Gaithersburg, MD 20899-8401 USA (e-mail: barry.taylor@nist.gov; mohr@nist.gov).

Publisher Item Identifier S 0018-9456(01)02691-2.

meter of length. The expression from electromagnetic theory for the force F per length l between two straight parallel conductors a distance d apart in vacuum, of infinite length and negligible cross section, and carrying currents I_1 and I_2 , is $F/l = \mu_0 I_1 I_2 / 2\pi d$. This equation and the definition of the ampere in combination imply that the magnetic constant μ_0 is an exact quantity given by $\mu_0 = 4\pi \times 10^{-7} \text{ N A}^{-2}$. [Because the electric constant ϵ_0 is related to μ_0 by $\epsilon_0 = 1/\mu_0 c^2$, it too is an exact quantity, as is the characteristic impedance of vacuum $Z_0 = (\mu_0/\epsilon_0)^{1/2} = \mu_0 c$.] Thus one could define the SI unit of current as *One ampere is an electric current such that the magnetic constant μ_0 is exactly $4\pi \times 10^{-7}$ newton per ampere squared*, which focuses on the true purpose of the definition.

D. Mole

In chemistry especially, it is helpful to characterize a body by the attribute, or quantity, called *amount of substance*. This quantity recognizes that a body may be viewed as a collection of a number N of specified entities. In most practical situations where the quantity is used, N is unknown and quite large—typically between 10^{10} and 10^{26} . The definition of the SI unit of amount of substance, the mole, is in two parts and reads [4] 1. *The mole is the amount of substance of a system which contains as many elementary entities as there are atoms in 0.012 kilogram of carbon 12; its symbol is “mol.”* 2. *When the mole is used, the elementary entities must be specified and may be atoms, molecules, ions, electrons, other particles, or specified groups of such particles.* (In this definition, it is understood that unbound atoms of ^{12}C , at rest and in their ground state, are referred to.) It immediately follows from this definition that the mass of one mole of ^{12}C atoms is exactly 0.012 kg, or equivalently, that the molar mass of ^{12}C , denoted by $M(^{12}\text{C})$, is exactly 0.012 kg/mol. The number of entities per mole is called the Avogadro constant $N_A \approx 6.022 \times 10^{23} \text{ mol}^{-1}$, and the numerical value of N_A , $\{N_A\}_{\text{mol}^{-1}} = N_A \text{ mol}$, is the number of ^{12}C atoms in 1 mol of ^{12}C or the number of entities in 1 mol of any other entity. Thus in analogy with the other alternative definitions given above, the first part of the definition of the mole might read instead *One mole is an amount of substance such that the molar mass of the carbon 12 atom $M(^{12}\text{C})$ is exactly 0.012 kilogram per mole*. Again, a definition of this form highlights the relationship between the unit and the fundamental constant that defines it.

III. CURRENT ROLE OF CONSTANTS IN SI ELECTRIC UNIT REPRESENTATIONS

A. Conventional 1990 Electric Units

As is now well known, the quantized Josephson voltage $U_J(n)$ across a Josephson effect (JE) device irradiated with microwave radiation of frequency f and biased on the n th constant-voltage current step is related to the step number n and f by $U_J(n) = nf/K_J$, where $K_J = 2e/h$ is the Josephson constant and e is the elementary charge [2]. Similarly, the resistance $R_H(i)$ of the i th quantized Hall resistance (QHR) plateau of a quantum Hall effect (QHE) device carrying a current I is related to the Hall voltage $U_H(i)$ across the device by $R_H(i) = U_H(i)/I = R_K/i$, where $R_K = h/e^2 = \mu_0 c / 2\alpha$

is the von Klitzing constant and $\alpha = e^2/4\pi\epsilon_0\hbar c$ is the fine-structure constant with $\hbar = h/2\pi$ [2].

As is also now well known, in order to achieve international uniformity in measurements of voltage and resistance, as well as of related electric quantities, on 1 January 1990 the International Committee for Weights and Measures (CIPM) introduced new, practical representations of the volt V and ohm Ω for worldwide use based on the JE and QHE and conventional (i.e., adopted) values of K_J and R_K [5], [6]. These assigned exact values, denoted respectively by K_{J-90} and R_{K-90} , are $K_{J-90} \equiv 483\,597.9 \text{ GHz/V}$ and $R_{K-90} \equiv 25\,812.807 \Omega$. They were deduced by the Consultative Committee for Electricity (CCE, now the Consultative Committee for Electricity and Magnetism, CCEM) of the CIPM from an analysis of all the relevant data available by 15 June 1988 [6]. These data included measurements of K_J and R_K as well as other fundamental constants. The goal was to select conventional values of K_J and R_K (within certain constraints) that were as close to their SI values as possible so that the new volt and ohm representations, and representations of other electric units derived from them, would closely approximate the corresponding SI units.

For our purposes here, we interpret the CIPM's adoption of K_{J-90} and R_{K-90} as establishing conventional, practical units of voltage and resistance V_{90} and Ω_{90} defined by $K_J \equiv 483\,597.9 \text{ GHz/V}_{90}$ and $R_K \equiv 25\,812.807 \Omega_{90}$. (V_{90} and Ω_{90} are printed in italic type in recognition of the fact that they are physical quantities.) The conventional units V_{90} and Ω_{90} are related to the SI units V and Ω by

$$V_{90} = \frac{K_{J-90}}{K_J} \text{ V} = \frac{h}{e} \frac{K_{J-90}}{2} \text{ V} \quad (1)$$

$$\Omega_{90} = \frac{R_K}{R_{K-90}} \Omega = \frac{1}{\alpha} \frac{\mu_0 c}{2R_{K-90}} \Omega, \quad (2)$$

which follow from the various expressions given above. (For ease of understanding, we have written the far right-hand side of these equations in the form of constants that are not exactly known times constants that are exactly known, i.e., that have no uncertainty.) Other conventional electric units follow immediately from V_{90} and Ω_{90} , for example, $A_{90} = V_{90}/\Omega_{90}$, $C_{90} = A_{90} \text{ s}$, $W_{90} = A_{90} V_{90}$, $F_{90} = C_{90}/V_{90}$, and $H_{90} = \Omega_{90} \text{ s}$, which are the conventional units of current, charge, power, capacitance, and inductance, respectively. The relevant relations between these units and their SI counterparts, the ampere (A), coulomb (C), watt (W), farad (F), and henry (H), are

$$A_{90} = \frac{K_{J-90} R_{K-90}}{K_J R_K} \text{ A} = e \frac{K_{J-90} R_{K-90}}{2} \text{ A} \quad (3)$$

$$C_{90} = \frac{K_{J-90} R_{K-90}}{K_J R_K} \text{ C} = e \frac{K_{J-90} R_{K-90}}{2} \text{ C} \quad (4)$$

$$W_{90} = \frac{K_{J-90}^2 R_{K-90}}{K_J^2 R_K} \text{ W} = h \frac{K_{J-90}^2 R_{K-90}}{4} \text{ W} \quad (5)$$

$$F_{90} = \frac{R_{K-90}}{R_K} \text{ F} = \alpha \frac{2R_{K-90}}{\mu_0 c} \text{ F} \quad (6)$$

$$H_{90} = \frac{R_K}{R_{K-90}} \text{ H} = \frac{1}{\alpha} \frac{\mu_0 c}{2R_{K-90}} \text{ H}. \quad (7)$$

We have not explicitly included the conventional tesla T_{90} because there are several possible ways of defining it. For ex-

ample, if one bases it on the magnetic flux density B generated by a solenoid of known dimensions carrying a current known in terms of A_{90} , then $T_{90} \propto A_{90} \propto e$. If it is based on B determined by measuring the force on a conductor of known dimensions carrying a current known in terms of A_{90} , then $T_{90} \propto 1/A_{90} \propto 1/e$. And if it is based on B determined by measuring the nuclear magnetic resonance (NMR) frequency of a spherical H_2O sample with the shielded proton gyromagnetic ratio γ'_p determined from its expression involving α and h (Ref. [2, Eq. (179)]), then $T_{90} \propto (h/\alpha^5)^{1/2}$.

It is worth noting that the introduction of the conventional values K_{J-90} and R_{K-90} can also be interpreted as introducing exact, “conventional” fundamental constants. For example, we can define the conventional elementary charge e_{90} , the conventional fine-structure constant α_{90} , and the conventional Planck constant h_{90} by $e_{90} = 2/K_{J-90}R_{K-90} = 1.602\,176\,49\dots \times 10^{-19}$ C, $\alpha_{90} = \mu_0 c/2R_{K-90} = 1/137.035\,996\,7\dots$, and $h_{90} = 4/K_{J-90}^2 R_{K-90} = 6.626\,068\,85\dots \times 10^{-34}$ J s. Hence we have, for example, $A_{90}/A = e/e_{90}$, $\Omega_{90}/\Omega = \alpha_{90}/\alpha$, and $W_{90}/W = h/h_{90}$.

B. Consistency of Conventional 1990 Electric Units with the SI

The 1998 CODATA self-consistent set of recommended values of the basic constants and conversion factors of physics and chemistry is a result of the 1998 least-squares adjustment of the values of the constants carried out by the authors under the auspices of the CODATA Task Group on Fundamental Constants [2]. The 1998 set, which replaces its immediate predecessor recommended by CODATA in 1986, is based on all of the relevant data available through 31 December 1998. Obtainable in electronic form at <http://physics.nist.gov/constants>, as well as in printed form [2], the new set of values is a significant advance over its 1986 counterpart: Because of the many outstanding experimental and theoretical advances made during the 13-year period between the 1986 and 1998 adjustments, the standard uncertainties (i.e., estimated standard deviations) of the new recommended values are in most cases 1/5 to 1/12, and in some cases 1/160, times the standard uncertainties of the 1986 values. Using the 1998 recommended values of e , h , and α , we find from (1) to (7)

$$V_{90} = [1 + 0.4(3.9) \times 10^{-8}] \text{ V} \quad (8)$$

$$\Omega_{90} = [1 + 2.22(37) \times 10^{-8}] \Omega \quad (9)$$

$$A_{90} = [1 - 1.8(3.9) \times 10^{-8}] \text{ A} \quad (10)$$

$$C_{90} = [1 - 1.8(3.9) \times 10^{-8}] \text{ C} \quad (11)$$

$$W_{90} = [1 - 1.4(7.8) \times 10^{-8}] \text{ W} \quad (12)$$

$$F_{90} = [1 - 2.22(37) \times 10^{-8}] \text{ F} \quad (13)$$

$$H_{90} = [1 + 2.22(37) \times 10^{-8}] \text{ H}. \quad (14)$$

For the relationship between K_J and K_{J-90} , and between R_K and R_{K-90} , we have

$$K_J = K_{J-90}[1 - 0.4(3.9) \times 10^{-8}] \quad (15)$$

$$R_K = R_{K-90}[1 + 2.22(37) \times 10^{-8}]. \quad (16)$$

We see from (8) and (9) that the practical unit of voltage V_{90} exceeds V by the fractional amount $0.4(3.9) \times 10^{-8}$, and that the practical unit of resistance Ω_{90} exceeds Ω by the fractional amount $2.22(37) \times 10^{-8}$. This means that measured voltages U traceable to the Josephson effect and K_{J-90} , and measured resistances R traceable to the quantum Hall effect and R_{K-90} , are *too small* relative to the SI by these same fractional amounts, respectively. Although these deviations from the SI are inconsequential for the vast majority of measurements and are well within the original uncertainties of 40×10^{-8} for V_{90}/V and 20×10^{-8} for Ω_{90}/Ω assigned by the CCE [5], [6], corrections to account for them may need to be made in those rare cases where consistency with the SI is critical. Analogous statements apply, of course, to the other five practical electric units.

IV. POSSIBLE FUTURE ROLE OF CONSTANTS IN THE SI

A. Redefinition of the Kilogram Based on the Planck Constant

It is generally agreed that if the SI unit of mass, the kilogram, which is embodied in the platinum-iridium international prototype of the kilogram [4], can be related to an invariant of nature with a relative uncertainty of $\leq 1 \times 10^{-8}$, then one should give serious consideration to redefining it [7], [8]. Motivated by promising advances in determining the Planck constant h using a moving-coil watt balance, we recently proposed a possible new definition of the kilogram that fixes h [3]. It reads *The kilogram is the mass of a body at rest whose equivalent energy equals the energy of a collection of photons whose frequencies sum to $135\,639\,277 \times 10^{42}$ hertz.*

The fixed value of h that results from this definition follows from the well-known relations $E = mc^2$ and $E = h\nu$. Thus

$$\begin{aligned} h &= \frac{(1 \text{ kg})(299\,792\,458 \text{ m s}^{-1})^2}{135\,639\,277 \times 10^{42} \text{ Hz}} \\ &= 6.626\,068\,76\dots \times 10^{-34} \text{ J s}, \end{aligned} \quad (17)$$

where in this paper we have chosen the value of the sum frequency so as to yield very nearly the 1998 CODATA recommended value of h . Adopting this definition would eliminate the last material artifact from the SI and, as discussed in [3], allow watt balances to be readily used to directly calibrate unknown standards of mass. This follows from the fundamental equation of the moving-coil watt balance, which may be written as $m_s = h\beta$, where m_s is the mass of the standard of mass used to balance the force F on the coil in the “force” part of the experiment, and β represents the four key quantities that are measured during the course of the experiment: the current I in the coil when the force $F = m_s g$ is determined, the local value of the acceleration due to gravity g , the velocity of the coil v in the “voltage” (i.e., moving coil) part of the experiment, and the resulting voltage U induced across the terminals of the coil. The appearance of the Planck constant in this equation is due to the fact that, although g and v are measured in their respective SI units, the current I and induced voltage U are measured in terms of the JE and QHE.

It is clear that in crafting the above definition of the kilogram, we followed the tradition established by the definitions

of the other SI base units. If we follow the form of the alternate definitions of the second, meter, ampere, and mole given above, we would write instead *One kilogram is a mass such that the Planck constant h is exactly $6.626\,068\,76 \times 10^{-34}$ joule second*, thereby making explicit the relationship between the kilogram and the Planck constant h . {A form of the first definition of the kilogram that more explicitly indicates the fixed numerical value of h that it implies would be *The kilogram is the mass of a body at rest whose equivalent energy equals the energy of a collection of photons whose frequencies sum to $[(299\,792\,458)^2/662\,606\,876] \times 10^{42}$ hertz.*}

B. Impact of Redefinition on Realizations of Electric Units

Redefining the kilogram so that the value of h is fixed would have a significant, positive impact on our ability to realize SI electric units. This becomes immediately apparent by examining Eqs. (1) to (7) and recognizing that since $e = (2\alpha h/\mu_0 c)^{1/2}$, a redefinition that makes h an exactly known constant means that the relative standard uncertainty u_r of e would be half that of α . [The relative standard uncertainty $u_r(y)$ of a quantity y is defined as $u_r(y) = u(y)/|y|$, if $y \neq 0$, where $u(y)$ is the standard uncertainty of y .] Currently, $u_r(\alpha) = 3.7 \times 10^{-9}$, and work presently underway should reduce this to no more than $u_r(\alpha) = 1 \times 10^{-9}$ within the next few years [2]. This implies that an “ideal” Josephson effect voltage standard (i.e., one with no experimental uncertainty) could be used to realize the volt with the impressively small relative standard uncertainty $u_r = 5 \times 10^{-10}$. A similar “ideal” quantum Hall effect resistance standard could realize the ohm with $u_r = 1 \times 10^{-9}$. The two standards together could realize the ampere (as well as the coulomb) with $u_r = 5 \times 10^{-10}$, and in principle the watt with no uncertainty since its realization would depend only on exactly known constants. For the realization of the farad and henry, the uncertainty would be $u_r = 1 \times 10^{-9}$ (assuming that the impedance of a resistance could be compared to that of a capacitance and of an inductance with no uncertainty). Realizing the ampere using a single electron tunneling (SET) device via the relation $I = ef$ [9], where I is the current through the device and f is the frequency of the applied voltage pulses, would be equivalent to realizing the ampere via the JE and QHE in the sense that the uncertainty would be equal to $u_r(e)$. Similarly, the uncertainty of realizing the farad by using a SET device to charge a capacitor and then measuring the voltage across it in terms of a Josephson voltage [10] would be the same as that obtained by using the JE and QHE together and would be equal to $u_r(\alpha)$.

It is noteworthy that the constants h and α are already sufficiently well known that “ideal” Josephson and quantum Hall effect voltage and resistance standards can in principle be used now to realize the ohm, farad, and henry with relative standard uncertainties $u_r = 3.7 \times 10^{-9}$, the volt, ampere, and coulomb with $u_r = 3.9 \times 10^{-8}$, and the watt with $u_r = 7.8 \times 10^{-8}$.

It is important to recognize that realizations of SI electric units via the JE and QHE are perfectly legitimate; one is not required to realize an SI unit by means of its formal definition. This is perhaps obvious for the SI derived units such as the volt, ohm, etc., since derived units are defined uniquely only in terms

of the SI base units. For example, the often cited definition of the volt given by the CIPM in 1946 [4], *The volt is the potential difference between two points of a conducting wire carrying a constant current of 1 ampere, when the power dissipated between these points is equal to 1 watt*, is just one of many approaches to realizing the volt; there is no “right” way. However, it is perhaps not so obvious for the SI base units. Some may have been led to believe that the only correct way to realize the ampere is to use a current balance to measure the force between current carrying conductors. But as we saw earlier, the definition of the ampere simply fixes μ_0 , nothing more, and μ_0 enters the realization of the ampere via the JE and QHE through the expression $e = (2\alpha h/\mu_0 c)^{1/2}$. As long as the relations $K_J = 2e/h$ and $R_K = h/e^2$ are correct, these two quantum effects can be used to realize the ampere and other electric units.

Another proposed redefinition of the kilogram fixes the value of N_A rather than h [8]. However, this would not have the advantage of providing improved realizations of SI electric units. The Planck and Avogadro constants are related by

$$h = \frac{cA_r(e)M_u\alpha^2}{2R_\infty N_A}, \quad (18)$$

where $A_r(e)$ is the relative atomic mass of the electron, $M_u = 1 \times 10^{-3} \text{ kg mol}^{-1}$ exactly is the molar mass constant, and R_∞ is the Rydberg constant. Although the relative standard uncertainty of the group of constants multiplying $1/N_A$ is less than 1×10^{-8} , it is finite and hence h would not be exactly known, as it would be if a definition of the kilogram that fixes h were to be adopted. Further, values of mass assigned to standards of mass via a watt balance would have this additional component of uncertainty and these values would in principle need to be revised whenever improved values of $A_r(e)$, α , or R_∞ became available.

V. CONCLUSION

The current role of the fundamental constants in the definitions of the SI base units, in realizing SI electric units, and in realizing practical representations of SI electric units, is certainly impressive. Nonetheless, as we attempted to show with our proposed new definition of the kilogram that fixes the value of the Planck constant, this role could become even larger and more important in the future. And of course, there is always the possibility that future improved measurements of other constants, for example, the molar gas constant R , the Boltzmann constant k , or the Stefan–Boltzmann constant σ , could lead to new definitions of the kelvin and the candela based on fundamental constants. We would then be in the satisfying position of having a measurement system that was truly “established on an invariant base, fixed in nature” to which we could turn “repeatedly to find the means of verification.” [1].

ACKNOWLEDGMENT

The authors thank their NIST colleague E. R. Williams for emphasizing to them the impact of fixing the value of the Planck constant h on SI electric unit realizations.

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