Secondary Standard for PM and AM Noise at 5, 10, and 100 MHz

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Abstract—This paper describes a practical implementation of a portable secondary standard for phase modulation (PM) and amplitude modulation (AM) noise at 5, 10, and 100 MHz. The accuracy of the standard for both PM and AM noise is ± 0.14 dB, and the temperature coefficient is less than 0.02 dB/K. The noise floor S_{ϕ} (10 kHz) of the standard for PM noise measurements is less than -190 dBc relative to $1 \text{ rad}^2/\text{Hz}$ at 5, 10, and 100 MHz. The noise floor for AM measurements depends on the configuration. A calibrated level of PM and AM noise of approximately -130 ± 0.2 dB relative to $1 \text{ rad}^2/\text{Hz}$ (for Fourier frequencies from approximately 1 Hz to 10% of the carrier frequency) is used to evaluate the accuracy versus Fourier frequency. Similar PM/AM noise standards are under test at 10 GHz. This new standard can also be used as an alternative to the normal method of calibrating the conversion sensitivity of the PM/AM detector for PM/AM measurements. Some types of time-domain measurement equipment can also be calibrated.

I. INTRODUCTION

IN the past, it has been difficult to evaluate the accuracy and noise floor of phase modulation (PM) and amplitude modulation (AM) noise measurement equipment because no artifact standards were available. Comparisons of measurement systems in different laboratories using commercially available oscillators as transfer standards were typically limited to a repeatability of roughly ± 3 dB due to the temporal variability of the oscillator noise. This paper describes a practical implementation of a portable secondary standard and associated measurement techniques for evaluating the noise floor and accuracy versus Fourier frequency of phase modulation (PM) and amplitude modulation (AM) noise measurement systems [1]. Evaluations of these new PM/AM noise standards at 5, 10, and 100 MHz yield an accuracy of less than 0.2 dB, a temperature coefficient of less than 0.02 dB/K, and a stability of less than 0.4 dB over 1 year. This new PM/AM noise standard can also be used as an alternative to the normal method of calibrating the conversion sensitivity of the PM/AM detector for PM/AM measurements. Some types of time-domain measurement systems can also be calibrated [2]–[5].

This new PM/AM noise standard produces two high-

level signals that have exceptionally low residual differential PM and AM noise between them. At 10 MHz, for example, the differential phase noise between the two channels S_{ϕ} (10 kHz) = -194 dB relative to $1 \text{ rad}^2/\text{Hz}$. This configuration is used for measuring the noise floor of both PM and AM measurement systems.

A calibrated level of passband-limited Gaussian noise can be added to one of the signals to create equal levels of PM and AM noise. The level of added noise is roughly 40 dB above the noise floor of most measurement systems and 60 dB above the residual noise between the two signals. This calibrated noise level is typically constant in magnitude to ± 0.2 dB for Fourier frequencies from dc to 10% of the carrier frequency. This configuration is used to evaluate the accuracy of PM/AM noise-measuring equipment as a function of Fourier frequency.

When the AM noise is measured on a single channel, the differential technique applied to $S_{\phi}(f)$ does not help to extend the range. The AM noise of the signal contains both the original AM noise of the source after the leveling circuit and the calibrated AM noise. Thus it might be expected that the spectrum of $S_a(f)$ at the output would rise much faster than the PM noise as f approaches 0. Fortunately, most sources have much lower AM noise than PM noise close to the carrier, so this potential problem is usually eliminated.

When the AM noise is measured using two detectors in a differential mode, most of the AM noise of the source cancels. This technique, as compared to the single-detector methods, significantly lowers the contribution of AM noise of the source to the noise floor for AM measurements.

II. DESIGN OF PM/AM NOISE STANDARD

Fig. 1 shows a simplified block diagram of the PM/AM noise standard model PMAM 115. A frequency source with very low PM and AM noise is regulated in amplitude and divided into the reference and signal outputs using a passive power splitter or directional coupler. (Active power splitters generally add more PM noise than do passive splitters.) A switch turns the carrier signal "on" and "off." The amplitude and phase of these outputs track one another with great fidelity. The residual differential PM noise between the two outputs is typically much smaller than the resolution of available measurement systems [6], [7].

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that can be set without violating (2). Typically, $S_{\phi}(f)$ is constant for $0 < f < \nu_0/10 \leq 1$ GHz, and the integrated phase noise is in the range from 10^{-2} to 10^{-4} rad². We must also check that all of the amplifiers used in the evaluation are operated in the linear gain region. The error in the calibration for the circuit in Fig. 1 is typically 0.05 dB due to random errors and ± 0.05 dB due to uncertainties in the linearity of the receiver, 0.04 dB due to uncertainties in determining the bandwidth, yielding an overall root-sum-of-squares (RSS) uncertainty of ± 0.14 dB.

B. Measurement of Errors in PM Noise Measurement Systems

Fig. 2 shows the preferred method for determining the accuracy of a PM noise measurement system using the new PM/AM noise standard [5]–[7]. As explained above, the PM noise of the driving source cancels out to a very high degree. This approach requires that the phase noise measurement system be able to adjust the phase between the two signals at the mixer to 90° so that the output of the mixer is close to 0 V. Not all commercial units have this provision, and some may require the use of a phase shifter in the phase noise standard. The noise floor of the measurement system is determined with noise modulation off, and the calibrated PM noise of the standard is measured with the noise modulation on. Differences between the measurement of $S_{\phi}(f)$ and the measurement determined using the method in Section IV-A represent errors in the PM noise measurement system under test. The contribution of the noise floor of the PM noise measurement generally needs to be considered only near the carrier (that is, as f approaches 0 Hz) [7].

A configuration similar to Fig. 2 is useful for measuring the PM noise added by elements such as frequency multipliers and amplifiers. The element to be tested is inserted in one or both of the signal paths from the PM/AM noise standard to the PM measuring system. PM measurements with the calibrated noise modulation on are used to calibrate the system sensitivity for converting phase fluctuations to voltage fluctuations versus Fourier frequency. PM measurements with the noise modulation off yield the PM noise (noise floor of the system plus the added noise of the element under test) relative to the calibrated PM noise.

Another method of calibration is shown in Fig. 3, where a second oscillator is used. The PM noise of both oscillators and the calibrated PM noise are then detected in the measurement. This approach can also lead to accurate evaluations of the measurement system errors versus Fourier frequency if the oscillator PM noise is small enough [7]. The AM/PM noise standard accommodates this approach by providing oscillators with very low residual AM and PM noise and the ability to turn the calibrated PM noise off to check the noise floor of the measurement. As mentioned above, the measurement sequence is to measure the residual PM noise in the measurement with the calibrated PM noise off, and then to

Fig. 2. Preferred calibration method for two oscillator-type phase noise measurement systems.

Fig. 3. Calibration method for two oscillator phase noise measurement systems.

measure the PM noise with the calibrated PM noise on. From these two measurements, we can determine the contributions of the uncanceled PM noise of the oscillators and the measurement system and their contribution to the measurement of the PM noise standard. Unlike the measurement technique outlined in Fig. 2, it is not generally possible to verify the noise floor of the measurement system because the phase noise of the reference oscillators always contributes to the measured noise. Also, the measurement cannot be carried out at small values of f since the PM noise of the reference oscillators dominates over the added noise. This problem can be alleviated somewhat by increasing the added noise and decreasing the bandwidth to satisfy (2).

The configuration of Fig. 3 can be used to measure the phase noise of an external oscillator. PM measurements with the calibrated noise modulation on are used to calibrate the system sensitivity for converting phase fluctuations to voltage fluctuations versus Fourier frequency. PM measurements with the noise modulation off yield the PM noise (noise floor of the system plus the added noise of the oscillator under test) relative to the calibrated PM noise. The PM/AM noise standard can also calibrate the

Fig. 4. Single oscillator phase noise calibration method using a frequency discriminator (see [5]).

accuracy of single-oscillator (frequency discriminator) phase noise measurement systems by the configuration shown in Fig. 4. The PM noise of both the oscillator and the single-oscillator measurement system is detected in the measurement. This approach can lead to accurate measurements if the oscillator PM noise is small enough [7]. The AM/PM noise standard accommodates this approach by providing oscillators that have very low residual AM and PM noise and the ability to turn the calibrated PM noise on and off. The procedure is to measure the residual PM noise in the measurement with the calibrated PM noise off, and then to measure the PM noise with the calibrated PM noise on. Unlike the technique of Fig. 2, it is not generally possible to verify the noise floor of the measurement system for these calibration methods because the phase noise of the reference oscillator always contributes to the measured noise. Another limitation is that the measurement cannot be carried out at low values of f since the PM noise of the reference oscillator generally dominates the added noise. This problem can be alleviated somewhat by increasing the added noise and decreasing the bandwidth to satisfy (2).

C. Calibration of AM Noise in PM/AM Noise Standard

The AM noise in the standard is determined using the procedure described in Section III-A and (1) since the PM/AM noise standard,

$$S_a(f) = S_\phi(f) = \frac{\text{PSDV}_N(\nu_0 - f) + \text{PSDV}_N(\nu_0 + f)}{2V_0^2} \quad (3)$$

D. Measurement of Errors in AM Noise Measurement Equipment

Amplitude noise can be measured by several different methods. Each method is characterized by a detector that converts the amplitude of the RF carrier signal to a dc voltage. A working definition of $S_a(f)$ is

$$S_a(f) = \frac{\delta V^2(f)}{V_0^2} \frac{1}{BW} \quad (4)$$

where $\delta V^2(f)$ is the mean-squared fluctuation of the carrier RF amplitude measured at Fourier frequency f from the average frequency of the carrier in a noise bandwidth BW , and V_0 is the average amplitude of the carrier [5]. The most uncertain part of the process is determining the sensitivity of the detector k_a for converting the fractional changes in the carrier amplitude to a dc voltage fluctuation since k_a is a function of V_0 , f , and the carrier frequency ν_0 .

1) *Calibration of Simple AM Noise Measurement Systems:* The signal input of the AM noise measurement system to be measured is connected to the modulated output of the PM/AM noise standard, as shown in Fig. 5. The spectral density of AM noise is calculated as

$$S_a(f) = \frac{V_n^2(f)}{BW} \left(\frac{1}{(k_a(f))^2} \right) \quad (5)$$

where $V_n^2(f)/BW$ is the PSD voltage noise measured by the spectrum analyzer, and $k_a(f)$ is the sensitivity of the detector for converting fractional changes in amplitude to voltage fluctuations. (k_a also includes any gain associated with the detection.)

When the noise modulation is on, the measured noise as computed with the aid of (5) should agree with the results of the calibrated level as given by (3). The differences between the measured results and that determined from (4) yield the error of the AM measurement system under test as a function of Fourier frequency. This measurement can also be used to determine $k_a(f)$ except at very low values of f where the AM noise of the source might mask the added noise. This problem can be alleviated somewhat by increasing the level of the added noise and decreasing the bandwidth to satisfy (2).

The variation of $k_a(f)$ with f is extremely difficult to determine by traditional methods since few sources can be accurately amplitude modulated over a broad range. This issue becomes even more serious as the carrier frequency and the offset frequency from the carrier increase.

2) *Determining the AM Noise Floor of an AM Noise Measuring System:* The noise floor for AM measurements can be estimated by turning the noise modulation off, measuring $V_n(f)$, and computing $S_a(f)$ using (5). The noise floor determined in this manner includes the true noise floor of the AM noise measurement system being calibrated plus the AM noise of the internal source in the PM/AM noise standard. In most cases, the noise floor of the standard is lower than the detectors being tested.

In cases where the AM noise of the internal source is so high that it is difficult to evaluate the AM noise floor of a measurement system as described in Section III-A, a differential measurement technique can be used, as shown in Fig. 6. This technique takes full advantage of the very low differential AM noise between the reference and modulated outputs of the PM/AM noise standards by using two similar detectors. The RF input of one detector to the reference output and the RF input of the other detector are connected to the modulated output. The dc outputs of the

Fig. 5. Simple calibration method for AM noise measurement systems.

Fig. 6. Differential AM calibration method to determine the noise floor of AM detectors and measurement systems.

two detectors are preferably adjusted to be the same within approximately 1% using attenuators on the input of the detectors. The gain following the detectors may also be adjusted; however, the cancellation of the residual AM noise in the source may not be as complete as when input attenuators are used. The outputs are then subtracted in a conventional device such as a differential amplifier (see Fig. 6). Some spectrum analyzers also can process the difference between the two input signals. With either of these methods, the AM noise of the internal source is substantially reduced, perhaps as much as 20–40 dB. (At very high Fourier frequencies, it may be necessary to adjust the phase slightly on the incoming signal to one of the AM detectors to optimize the cancellation.) The noise floor measured in this manner is that of both detectors (and their associated amplifiers). If they were identical, we would subtract 3 dB from the measured result to obtain the noise floor of a single unit. A more accurate value of the AM noise floor of the individual detectors (and amplifiers) requires that measurements be made on three detectors with all combinations (detectors 1–2, 1–3, and 2–3) [4], [6]. The determination of $S_a(f)$ of unit 1 is

$$S_a^1(f) = \frac{1}{2}(S_a^{1-2}(f) + S_a^{1-3}(f) - S_a^{2-3}(f)) \quad (6)$$

since the random noise in the detectors is uncorrelated [4], [6].

The variation of $k_a(f)$ with f of a differential AM noise

Fig. 7. One method of determining the sensitivity for converting small changes in amplitude to small changes in voltage.

measurement system can be determined by turning the added differential noise on, measuring $V_n(f)$, and comparing the results obtained from (3) with these from (6).

Using the PM/AM noise standard in the manner prescribed above calibrates all of the measurement errors, even at very low values of f where the AM noise of the internal source might swamp the noise floor of the measurement system under test or the added differential noise if we used the technique described in Section III-A.

A typical method of determining $k_a(f)$ prior to measuring AM noise is shown in Fig. 7. First, the modulated output of the PM/AM noise standard is connected to a detector and the detected dc level is noted. Second, the output of an auxiliary source that can be amplitude modulated at a known level is connected to the detector and the RF level, impedance (50 Ω), and the frequency is adjusted to that chosen for the PM/AM noise standard. The auxiliary source is then AM modulated at a convenient frequency and level. A common choice is 1% modulation at 1 kHz. The level of the modulated signal from the port labeled "OUTPUT POWER" in Fig. 7 is then recorded as "AM REF LEVEL." The detector sensitivity multiplied by the gain of the amplifier is then found from

$$k_a^2(f) = \frac{(\text{AM REF LEVEL})^2}{\frac{1}{2} \left(\frac{\% \text{ AM}}{100} \right)^2} \quad (7)$$

In general, the AM REF LEVEL, and hence $k_a(f)$, is a function of ν_0 , f , and V_0 .

E. Calibrations of Time-Domain Measurement Systems

Measurement of frequency stability in the time domain is another common method used to characterize the frequency stability of oscillators and other components [2]–[5]. In the past, a convenient method to calibrate the accuracy of such systems had not been available because the measured time-domain frequency stability depended on the measurement bandwidth, the averaging time, and the type of phase noise. The phase noise standard provides a differential level of PM noise that is constant in amplitude (white PM) for measurement bandwidths from 1 Hz to 10% of the carrier. Under these conditions, the fractional frequency stability between the reference and modulated outputs as characterized by the square root of the Allan variance [5], [8] is given by

$$\sigma_y(\tau) = \frac{3f_h S_\phi(f)}{2\pi\nu_0\tau_0} \quad (8)$$

where f_h is the measurement bandwidth and τ is the measurement time. When a time-domain frequency stability measurement system [4] is connected between the two outputs, $\sigma_y(\tau)$ should vary as τ^{-1} for measurement times from approximately 0.1 μ s to 0.5 s. Comparing the results with (8) yields the measurement bandwidth since $S\phi(f)$ is known. The noise floor of time-domain measurement systems can be evaluated by turning the added PM noise off. Time-domain calibrations can also be done by comparing the modulated output to an external reference. In this case, the time-domain stability may deviate from the τ^{-1} behavior for the longer measurement times due to noise in the external and the internal reference [5].

V. RESULTS

The first PM/AM noise standard model PMAM 115 operates at 5, 10, and 100 MHz. Switches select the proper carrier frequency and matching passband filters. The noise source is flat within a few tenths of 1 dB from less than 0.1 MHz to about 1 GHz and is stable with temperature to 0.02 dB/K. The differential noise added to the carrier ν_0 is exceptionally flat due to the symmetry of (1). The results obtained at 100 MHz are shown in Fig. 8.

The PM noise of the 100 MHz reference oscillator of the PM/AM standard is shown in curve A of Fig. 9. The added differential PM/AM noise between the reference and modulated outputs at 100 MHz is shown in curve B. The noise floor of the PM/AM standard at 100 MHz using the method of Fig. 2 with the noise modulation off is shown in curve C. The cancellation of the 100 MHz reference oscillator phase noise by using the method of Fig. 2 (the difference between curves A and B) reaches approximately 85 dB at 1 Hz. The large suppression of the reference oscillator PM noise makes it possible to use the PM/AM noise standard over a much larger range of Fourier frequencies about the carrier than any technique that is sensitive to the PM noise of the reference, such as that shown in Figs. 3 and 4. At 100 MHz, the difference is having an accurate PM standard for Fourier frequency offsets from 1 Hz to 10 MHz versus 300 Hz to 10 MHz. The peaks at 60, 120, and 180 Hz in curve B are due to spurious signals originating from the power lines.

Curve A of Fig. 10 shows the PM noise in the PM/AM noise standard at 100 MHz determined using a high-dynamic-range scanning receiver in the calibration mode described in Section III-A above. The noise is constant to ± 0.05 dB for Fourier frequencies from dc to 5 MHz and gradually decreases 0.3 dB from 5 to 10 MHz. Curve B of Fig. 10 shows the measurement of the PM noise level using a totally separate and independently calibrated phase noise measurement system PNMS [7], [11]. Curve C of Fig. 10 shows the AM noise determined using the simple measurement technique of Section III-C. The divergence of curve C from curves A and B for Fourier frequencies below 100 Hz is easily shown, by the AM noise floor measurements, to be due to noise in the diode detector used for these measurements. The fluctuations in curve B

Fig. 8. Calibration of V_N added to the carrier V_0 versus Fourier frequency f at 100 MHz.

Fig. 9. Measurement results for PM/AM standard model PMAM 115 at a carrier frequency of 100 MHz. Curve A shows the PM noise of the 100 MHz reference oscillator. Curve B shows the calibrated level of PM and AM noise. Curve C shows the typical noise floor for PM noise in the PM/AM noise standard at 100 MHz. Somewhat better results are obtained at 5 and 10 MHz.

Fig. 10. Measurement results for PM/AM standard model PMAM 115 at a carrier frequency of 100 MHz. Curve A shows the PM noise level of the standard determined using the technique of Section III. Curve B shows the measurements of PM noise using the totally independent NIST Phase Noise Measurement System (see [7], [11]). Curve C shows the level of AM noise of the standard at 100 MHz using the method shown in Fig. 5. The peaks at 60 and 120 Hz are due to the power line. Similar results are obtained at 5 and 10 MHz.

of Fig. 10 versus frequency are primarily due to the finite number of samples taken to estimate the spectral density [10]. The differences between curves A and B of Fig. 10 are consistent with the estimated errors in the internal phase modulator used in the calibration of the PNMS. These estimated inaccuracies are shown in Fig. 11 as a function of Fourier frequency for carrier frequencies of 5

Fig. 11. Typical errors in the internal calibration of the phase modulator in the NIST Phase Noise Measurement System (see [7], [11]).

TABLE I
SYSTEM NOISE FLOOR AND PHASE NOISE OF SOURCES

Nominal Source Phase Noise Channel ± 3 dBc/Hz Fourier Frequency								
Source Frequency	1 Hz	10 Hz	100 Hz	1 kHz	10 kHz	100 kHz	1 MHz	10 MHz
5 MHz	-121	-151	-163	-171	-174	-174	-174	
10 MHz	-115	-145	-157	-165	-168	-168	-168	
100 MHz	-70	-100	-130	-156	-170	-170	-173	-173
Maximum Residual Noise Between Channels dBc/Hz								
5 MHz	-162	-172	-182	-190	-194	≤ -175	≤ -175	
10 MHz	-161	-176	-183	-191	-197	≤ -175	≤ -175	
100 MHz	-152	-162	-172	-182	-193	≤ -175	≤ -175	≤ -175
Differential PM/AM Noise Level ± 0.14 dBc/Hz								
5 MHz	-128.3	-128.3	-128.3	-128.3	-128.3	-128.3		
10 MHz	-129.1	-129.1	-129.1	-129.1	-129.1	-129.1	-129.2	
100 MHz	-128.9	-128.9	-128.9	-128.9	-128.9	-128.9	-128.9	-129.2

MHz, 10 MHz, 300 MHz, 10 GHz, and 42.7 GHz. Table I summarizes the PM/AM calibrated noise and noise floor at 5, 10, and 100 MHz. The totally independent data from the PNMS confirm the calibration of the PM/AM standard to ± 0.5 dB.

The techniques illustrated here can be extended to high frequencies and to higher accuracies using presently available technology. Preliminary results at 10.6 GHz look promising for Fourier frequencies out to at least 500 MHz. The PM/AM noise of the first PM/AM standards has remained constant within ± 0.4 dB at all three carrier frequencies (5, 10, and 100 MHz) over the last year. Accuracies better than ± 0.1 dB are possible using components with lower VSWR, circuits with better level control, and more extensive measurements on receiver bandwidth and linearity.

VI. CONCLUSION

A new portable PM/AM noise standard (model PMAM 115) has been developed, tested, and used to evaluate the noise floor and accuracy of PM and AM noise [$S_{\phi}(f)$ and $S_a(f)$] measurement equipment. The differential PM noise

in the PM noise standard used to test the noise floor is typically much less than the noise floor of present measurement systems. The PM noise in the standard was evaluated for Fourier frequencies from 1 Hz up to 10% of the carrier frequency to an accuracy of ± 0.14 dB. The temperature coefficient of the PM and AM noise level is less than 0.02 dB/K. These calibrations were independently verified at carrier frequencies of 5, 10, and 100 MHz to ± 0.5 dB using the Phase Noise Measurement System [7], [11]. The design of the standard is such that the fractional amplitude noise $S_a(f)$ is equal to $S_{\phi}(f)$ over a very wide range of Fourier frequencies. The new secondary standard has maintained the same level of PM and AM noise to within ± 0.4 dB over the past year [1].

Although the PM/AM noise standards described above are at frequencies from 5 to 100 MHz, the principle can be applied to any other frequency from audio to the visible. Prototypes are presently working at 10.5 GHz [1].

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