Charge-Injection Devices: Part II—Multiple-Pixel Solutions with Row Injection

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Abstract—Equations derived in Part I for 1-pixel CID's are generalized to row-injected CID's of $n \geq 1$ pixel connected to the sense line at readout. When $n > 1$, specifying the fullnesses of the pixels, as in Part I, provides fewer equations than unknowns in most cases of interest. Instead, simulating successive injections during known irradiation solves problems of transients with known initial states and also steady states reached after many injections. The two choices for each pixel, that charge is injected from it or not, lead to 2^n solutions at each injection. One is the design solution, with charge injected only from the selected pixel. In saturation some problems require additional effort to ascertain which solution is valid. As in Part I, the solution for the injected charge is modified for subtractive effect. Comparing the computations with measurements of response and of maximum output versus injection voltage shows at least qualitative agreement. Other effects calculated to illustrate the theory are output at very high uniform irradiation, variation of output with CID and circuit properties, variation with n up to very large values, transients including lag, and saturation behavior.

NOMENCLATURE

(Values given in parenthesis are the constant numerical values assumed in the computations.)

A_c	Column-gate area ($1355 \mu\text{m}^2$).
A_r	Row-gate area ($1123 \mu\text{m}^2$).
$C_{ic}(C_{ir})$	Capacitance per unit area of the column (row) gate insulator ($31 \text{ nF}/\text{cm}^2$).
C_T	Total capacitance between sense line and substrate.
C'_T	The part of C_T external to a particular pixel.
C''_T	Sense-line-to-substrate capacitance external to all pixels (ordinarily 9.2 pF).
f	Fullness of a pixel just before the injection pulse. In a full pixel, $f = 1$.
FDI	Full during injection.
LHS	Left-hand side.
n	Number of pixels connected to the sense line (ordinarily 16).
n	Substrate type.

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N_D	Net donor density ($7.5 \times 10^{14} \text{ cm}^{-3}$).
q	Electronic charge ($-1.602 \times 10^{-19} \text{ C}$).
R	Radiative input in photons/pixels/integration time.
RHS	Right-hand side.
s	$= A_c C_{ic} / (A_c C_{ic} + A_r C_{ir})$.
SRI	Sequential row injection.
t_i	Integration time (ordinarily $136 \mu\text{s}$).
t_{inj}	Duration of injection pulse ($1/3 \mu\text{s}$).
t_p	Pixel time (ordinarily $4.0 \mu\text{s}$).
t_1	Time from first sample to injection pulse ($0.5 \mu\text{s}$).
t_2	Time from injection pulse to second sample ($5/6 \mu\text{s}$).
u	$= \phi + qN_D(A_c + A_r)W / (A_c C_{ic} + A_r C_{ir})$, evaluated just before the injection pulse unless otherwise specified.
$u(1)$	Full-pixel value of u .
$u_d(u_a)$	The value of u during (immediately after) the injection pulse.
$u_1(u_2)$	The value of u at first (second) sample.
UCI	Uniform-in-space, constant-in-time irradiation.
V_c	Voltage to which column-gate is reset, referred to flat-band voltage.
V_i	Injection voltage (ordinarily 0.5 V).
V_{Qa}	$= q(A_c N_{ac} + A_r N_{ar}) / (A_c C_{ic} + A_r C_{ir})$.
V_{Qb}	Analogous to V_{Qa} , but with N_{bc} and N_{br} .
V_r	Row-gate voltage, referred to flat-band voltage V_{FB} .
V_R	$= -\eta_Q q R t_p / [t_i (A_c C_{ic} + A_r C_{ir})]$.
W	Depletion depth.
W_a	The value of W immediately after the injection pulse.
ΔV_+	Output exclusive of subtractive effect.
$\Delta V_+(1)$	The all-pixels-full value of ΔV_+ .
ΔV_{+-}	Output taking subtractive effect into account.
ϵ_s	Substrate dielectric constant ($1.583 \text{ pF}/\text{cm}$ (as in InSb)).
η_Q	Quantum efficiency (0.35 carrier/photon).
ϕ	Surface potential.
$\phi(1)$	Full-pixel surface potential (-0.168 V).

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Fig. 1. A column of a row-injected CID, with $n = 3$ pixels. Each pixel has its column gate dc-electrically connected to the sense line. C_T'' is the sense-line-to-ground capacitance external to the pixels.

I. INTRODUCTION

PART I [1]¹ treated charge-injection devices (CID's), each with a single pixel from which charge is injected, while no other pixels connected to the sense line gain or lose any charge during the interval between first and second samples. The description is consistent and is in good agreement with suitable measurements on 1-pixel CID's (to be submitted for publication). It is not necessarily accurate when $n > 1$ pixels are connected to the sense line. When many unselected pixels each gain even a little new charge, their effect, which is subtractive, can be important. Among CID's of practical interest, the restriction of Part I is to linear CID's with single-gate pixels. Area CID's have multiple two-gate pixels connected to the sense line, while one gate per pixel is sufficient for a linear CID. Two gates per pixel make it possible to select both the row and column in a two-dimensional array with fewer than one wire per pixel connected to the CID from outside.

This paper solves the problem of n -pixel CID's with row-injection output. Fig. 1 illustrates one column of the array. In ideal mode [1]¹, an injection pulse applied to the row gate does not inject any charge, because there is no mobile charge in the row well during readout. Therefore, charge-sharing mode [1]¹ is assumed. Column injection makes use of a transfer voltage applied throughout the read-out interval to the row gate of only one pixel. This voltage selects the pixel for readout. The equations for row injection are easier to solve than those for column injection, for two reasons. First, there is no need to deal with two separate classes of pixels at the same time, one with transfer voltage and the other without. Second, the injection voltage is applied directly to the row gate and is

¹In order to make (52) [all equation numbers refer to [1]] and the remark about [6] correct, the RHS of (29) must be replaced by $(C_T/C_T'')[1 + (1 + p')C_d/C_{ic}]^{-1}$. This replacement does not affect the value of $\eta_{1,dif}$ in the ideal mode. An expression in the text between (29) and (30) should be $V_{dc}/q < N_D W(1)/C_{ic} + \phi(1)/q$. In (51), the last term in G^{-1} should be $-p'/(1 + p')$ instead of $-p'(1 + p')$. The symbols in the identifications of the abscissa and ordinate of Fig. 9 are division symbols ($\div$), not addition symbols ($+$). The lower curve there should reach a maximum of 0.863 at abscissa = 1. In the Recurring Symbols list, $\eta_{1,dif}$ should be $\eta_{1,dif}$. At the top of Fig. 7, the diminishing deviations of the successive outputs from their steady-state value of zero should be labelled $\Delta V(f') - \Delta V(f)$, $\Delta V(f'') - \Delta V(f)$, etc., instead of $\Delta V(f')$, $\Delta(f'')$, etc. Other corrections to the theory of lag in [1] are given in Section IV-C-2 above. The authors thank S. C. H. Wang for pointing out the errors in (38) and Fig. 7.

therefore a constant. Row injection is of practical importance because it involves far fewer lossy charge transfers between row and column wells [2] than does column injection. There are innumerable schemes for reading CID's. The readout method considered here is one of very few that have given rise to known measurements suitable for comparison when $n > 1$. The simplifying assumptions of Part I are continued here, e.g., dark current is omitted.

II. EQUATIONS FOR ADDITIVE EFFECT

One seeks generalizations of Part I. The right-hand side (RHS) of [1, eq. (18)] for the output ΔV_+ is summed over all pixels.

$$-\Delta V_+ = (q/C_T'') \sum A_{cj} [N_{bc} - N_{ac} + N_D(W - W_a)]_j \quad (1)$$

The difference in the brackets is proportional to the net charge transferred on the sense line from pixel j to the (constant) sense-line to substrate capacitance C_T'' external to the pixels. Any row-gate to column-gate capacitances are additive in C_T'' . Elimination between (1) and the difference of (3) and (5) below leads to

$$\Delta V_+ = \sum p_j'' (\phi - \phi_a)/(1 + \sum p_j'') \quad (2)$$

with $p_j'' = A_{cj} C_{icj}/C_T''$. The contribution of any pixel in (1) is distributed among all the pixels in (2). At $n = 1$, $C_T'' = C_T'$ of Part I, and (2) reduces to its 1-pixel analog (see [1, eq. (23)]). Reference [3, Appendix I], gives a simple linear circuit model that leads to the factor $p''/(1 + p'')$ for $n = 1$.

In Part I, an MIS Kirchhoff equation was used for each potential well immediately before, during, and immediately after the injection pulse. Now corresponding pairs of row-injection equations for the column and row wells of the same pixel are combined and slightly generalized. They are (13) and (39), (55) and (56), and (15) and (41) of Part I. For each pixel

$$sV_c + (1 - s)V_r = V_{Qb} + u \quad \text{before} \quad (3)$$

$$s[V_c + \Delta V_+(1) - \Delta V_+] + (1 - s)(V_r + V_i) = V_{Qd} + u_d \quad \text{during} \quad (4)$$

$$s[V_c - \Delta V_+] + (1 - s)V_r = V_{Qa} + u_a \quad \text{after.} \quad (5)$$

The properties ϕ and W of the depletion region before injection have been collected into the voltage

$$u = \phi + qN_D(A_c + A_r)W/(A_c C_{ic} + A_r C_{ir}), \quad (6)$$

with u_d (during) and u_a (after) defined in analogy. The mobile charge has been collected in terms of another voltage

$$V_{Qb} = q(A_c N_{bc} + A_r N_{br})/(A_c C_{ic} + A_r C_{ir}) \quad (7)$$

with V_{Qd} and V_{Qa} defined in analogy. Here N_{bc} (N_{br}) is the surface density of carriers in the column (row) well just before injection. A measure of the asymmetry of the pixel

is

$$s = A_c C_{ic} / (A_c C_{ic} + A_r C_{ir}). \quad (8)$$

The term $(1 - s)V_i$ on the left-hand side (LHS) of (4) is present only for the selected pixel. For simplicity, the subscript identifying the pixel is omitted. When ϕ and W belong to both the same pixel and the same equation (3), (4) or (5), then

$$\phi = qN_D W^2 / 2\epsilon_s. \quad (9)$$

The output $\Delta V_+ = \Delta V_+(1)$ when all the pixels are full just before injection. Appendix I shows that all pixels are then also full during injection (FDI). Conservation of mobile surface carrier density at the end of injection implies $V_{Qd} = V_{Qa}$, as in Part I.

The 1-pixel, FDI, CID's of Part I have no analog of photogeneration in unfilled pixels during the injection pulse. Compensation of the computations for its effect on later injections is described in Section IV-A.

Charge may be injected from the selected pixel, and even from those unselected pixels that are not quite full before injection because there is an increment $s[\Delta V_+(1) - \Delta V_+]/(1 - s)$ of the effective injection voltage in the "during" equations (4). The latter case is an example of inter-pixel crosstalk. The FDI condition is $u_d = u(1)$. On the other hand, if a pixel is sufficiently empty that no charge is injected from it, then $V_{Qb} = V_{Qa}$. Each pixel is either FDI or not, for a total of n additional equations. With (3)–(5) for each pixel, and (2), there are $4n + 1$ independent equations. With V_c and V_r known, there are in the most general case $5n + 1$ unknown variables, V_{Qb} , V_{Qa} , u , u_d , and u_a for each pixel, and ΔV_+ .

Suitable definitions of the n fullnesses f , analogous to those for $n = 1$ [1], would provide the missing n equations. Subtracting (4) from (3) gives an expression proportional to the charge injected from a selected pixel

$$V_{Qa} - V_{Qb} = s[\Delta V_+(1) - \Delta V_+] + (1 - s)V_i - [u_d - u]. \quad (10)$$

For an unselected pixel, $(1 - s)V_i$ is omitted from (10). When all pixels are full, i.e., all $f = 1$, (10) shows that no charge is injected from the unselected pixels. The 1-pixel definition [1] of f_{sel} is

$$(V_{Qa} - V_{Qb})_{\text{sel}} = f_{\text{sel}}(1 - s)V_i. \quad (11)$$

If $f_{\text{sel}} = 1$, (11) is consistent with (10) for the selected pixel only when all $f_{\text{unsel}} = 1$. Equation (11) relies on the steady-state property of the injected pixel: the charge injected is just that amount acquired since the previous injection. This principle does not sufficiently describe unselected pixels.

When $f_{\text{sel}} = 0$, subtracting (5) from (3) yields

$$(u - u_a)_{\text{sel}} = s\Delta V_+. \quad (12)$$

For $n = 1$, but not necessarily for $n > 1$, $\Delta V_+ = 0$, so that (12) implies $\phi = \phi_a$, as is assumed in Part I.

III. SOLUTIONS OF ADDITIVE EFFECT WITH KNOWN FULLNESSES

When all pixels are initially full, one subtracts (5) from (4) for each pixel. For the selected pixel

$$s\Delta V_+(1) + (1 - s)V_i = u_d - u_a. \quad (13)$$

Appendix I shows all $u_d = u(1)$ in this all-pixels-full problem. One solves (2) and (13) iteratively for the ϕ_a with all $\phi = \phi(1)$. Fig. 2 shows the resulting $\Delta V_+(1)$ as a function of V_i for various n .

The case $f_{\text{sel}} < 1$, all $f_{\text{unsel}} = 1$ allows use of (11), and can now be solved as follows. For this problem, all $u_{\text{unsel}} = u(1)$, so that $u_{\text{unsel}} - u_d \geq 0$, and also $\Delta V_+(1) - \Delta V_+ > 0$. Therefore, from (10) for any unselected pixel, $V_{Qa} - V_{Qb} > 0$, charge is injected, and $u_{d,\text{unsel}} = u(1)$. Equation (13) is now a quadratic equation in W_a for each pixel. From (4) for the selected pixel, using (11) for $V_{Qa,\text{sel}}$ and (3) for $V_{Qb,\text{sel}}$, one gets an expression for ΔV_+ as a function of the unknown ϕ_{sel} . Equation (2) is an expression for ΔV_+ as a function of ϕ_{sel} . Equating the two expressions provides a quadratic equation in W_{sel} .

These are believed to be the only cases completely solvable with specified fullnesses. However, as usual $\Delta V_+ = 0$ when all pixels have so little mobile charge that none is injected.

IV. SOLUTIONS WITH KNOWN INITIAL MOBILE CHARGES OR SURFACE POTENTIALS

A. Additive Effect

The approach of this Section specifies the irradiation of each pixel and simulates time-dependent operation of the CID. If the preceding injection has been solved for V'_{Qa} (prime denotes the same pixel at the preceding injection), then with the known irradiation one can find V_{Qb} for the new injection. Alternatively, if the injection-pulse generator is turned on when all the pixels are full, then all $u = u(1)$ until the end of the first injection. For the present purpose, knowing either V_{Qb} or u is equivalent because they are related by (3). Knowing either is enough additional information for solution.

Ordinarily all the pixels connected to the same sense line are read out successively, i.e., by sequential row injection (SRI). The numerical computations here assume SRI. An output timing sequence used with SRI at the Naval Research Laboratory is specified in the nomenclature list and illustrated in Fig. 3. The pixel time t_p is the repetition period of injections of different pixels. The integration time t_i is the repetition period of injections of the same pixel. The pixels are numbered in the order that they are injected. If $t_i \geq nt_p$, as SRI requires, then a dead time $t_i - nt_p$ falls between each injection of pixel # n and the next injection of pixel #1. A low-contrast infrared scene is well represented by uniform-in-space and constant-in-time irradiation (UCI) at rate R photons per integration time on each pixel. The photocharge produced in a sufficiently empty pixel during time t_p is $-q\eta_Q R t_p / t_i = (A_c C_{ic} + A_r C_{ir}) V_R$. Here, η_Q is the quantum efficiency. The

Fig. 2. Full-pixels additive-effect output $\Delta V_+(1)$ of the CID of the nomenclature list versus injection voltage V_i . The increase of sense-line to ground capacitance as n increases leads to decreasing $\Delta V_+(1)$.

Fig. 3. Timing of the readout cycle. First sample, injection pulse, and second sample are shown. After second sample but before first sample, the sense line is reset. These numerical values were used in the measurements of Figs. 5 and 6 on a CID of 16 pixels. When n is very large, the numerical values can be severely constrained by the requirement $t_1 + t_{inj} + t_2 < t_p \leq t_i/n$ (Section IV-A).

number Rt_p/t_i of photons has been expressed in terms of the voltage V_R .

$$V_{Qb} = V'_{Qa} - V_R(1 - t_{inj}/t_p). \quad (14)$$

Using (3), (5), and (14)

$$u = u'_a + s\Delta V'_+ + V_R(1 - t_{inj}/t_p) \quad (15)$$

but $u \leq u(1)$ is required, with a corresponding bound on V_{Qb} in (14). Equation (15) is quadratic in W when the properties of the preceding injection are known. If the new injection is of pixel #1, then $t_i - (n-1)t_p$ should replace t_p in (14) and (15). The LHS of (14) and the RHS of (15) are incremented by $V_R t_{inj}/t_p$ for pixels not FDI, to correct for photogeneration during the injection pulse. A more exact treatment might distinguish pixels that become full during the injection pulse.

The sense line remains electrically floating throughout the readout processes considered previously. During the time interval spanned by (14) and (15), the column-gate voltages are reset to the constant V_c from $V_c - \Delta V_+$ as in (5). This interval is often longer than those considered previously, but still $\ll t_i$, so that neglect of dark current is a reasonable approximation if operation of the non-equilibrium device is itself reasonable. The bounds on u and V_{Qb} correct for any charge injected at reset.

After sufficiently many injections at constant or slowly varying input, a CID reaches a steady state that depends on the input but not on the initial condition. In steady state the selected pixel is FDI if it is irradiated. Then (13) holds for the selected pixel with $u_d = u(1)$ and with $\Delta V_+(1)$ known from Section III. Equation (13) is again quadratic in W_a . For FDI unselected pixels, one omits $(1-s)V_i$ from (13).

If a pixel is not FDI, then no charge is injected, and

$V_{Qa} = V_{Qb}$. For the $n_e \leq n$ such pixels, one has to solve the n_e simultaneous equations

$$u_a + s\Delta V_+ - u = 0, \quad (16)$$

from (3) and (5). With (2), these are also quadratic equations in W_a when the u of all pixels are known. If $n_e \geq 2$, then each (16) represents a hyperboloid of two sheets in the n_e -dimensional rectangular coordinate system of the W_a . In the easily visualized case $n_e = 2$, only one solution of (16) and (2) has both $W_a > 0$ as required. The geometry suggests a converging method of successive approximations, which is also found satisfactory for larger n_e . More general methods of solving simultaneous quadratic equations [4] and commercial programs may be applicable, especially because the constraint $W_a > W(1)$ is so simple.

Because of the two alternative conditions, FDI or not, for each pixel, there are 2^n different solutions at each injection of an n -pixel CID. The category of soluble problems depends on one's ability to decide which solution is valid. Section IV-C-2, Fig. 8, discusses a pertinent example. There are 2^n different classes of solutions, each class defined by certain pixels being FDI, and the others not. When in a solution a pixel is barely FDI, so that no charge is injected, then that solution belongs marginally both to the class in which that pixel is FDI and to the class in which that pixel is not FDI. Otherwise the solution belongs strongly to a class.

If the relative distribution of irradiation in space and time and the selected pixel are all fixed, then at most $n+1$ different classes of solutions become valid successively as the level of irradiation is increased. This sequence of $n+1$ classes begins at no-pixels-FDI, and ends at all-pixels-FDI if all pixels are irradiated and injected. Like other image-detecting arrays, CID's are designed for reading individual pixels, i.e., they are intended for only the selected pixel to be FDI. The corresponding class of solutions will be called the design solutions. Fig. 4 shows design and all-pixels-FDI solutions for the CID of the nomenclature list. The solutions valid at intermediate values of R have not been computed.

B. Subtractive Effect

Output (1) and (2) can be modified to include the subtractive effect [1], [6]. One starts with before (3), modified to refer to the moment of first sample, and after (5), modified to refer to the moment of second sample. That was done for $n = 1$ in Part I, Appendix II. In place of (2)

$$\Delta V_{+-} = \Sigma p_j''(\phi_1 - \phi_2)j/(1 + \Sigma p_j''). \quad (17)$$

where ϕ_1, ϕ_2 are the surface potentials of a pixel at first and second samples, respectively. Analogous to [1, eq. (B1)], $\phi_1 - \phi_2 = (\phi_1 - \phi) + (\phi - \phi_a) + (\phi_a - \phi_2)$. The terms $(\phi_1 - \phi) + (\phi_a - \phi_2)$ are purely subtractive.

The ϕ_1 of (17) is calculated from u'_a (for the previous injection). Reasoning as in (14) and (15), u_1 at first sample before injection satisfies

$$u_1 = u'_a + s\Delta V'_+ + V_R(t_p - t_{inj} - t_1)/t_p, \quad (18)$$

Fig. 4. Additive-effect output ΔV_+ in two near-antipodal classes of solutions versus level R of uniform constant irradiation (UCI). The CID is that of the nomenclature list. There is steady state and sequential row-injection (SRI). Solutions are shown as continuous curves within their ranges of validity and dashed elsewhere. For the design solutions, ΔV_+ reaches hard saturation at $\Delta V_+ \approx \Delta V_+(1)$, where it remains at all larger R . For the all-pixels-FDI solutions, $\Delta V_+ \approx \Delta V_+(1)$ in its range of validity, but at smaller R , ΔV_+ deviates to smaller values and shows strong negative pixel dispersion, i.e., ΔV_+ is a decreasing function of the ordinal number of the selected pixel. Strong pixel dispersion is common for the theoretical outputs outside their ranges of validity.

except that $u_1 \leq u(1)$. To include the case of saturation, it is essential to reason forward in time as in (18), rather than backward, e.g., by deriving u_1 from u , because a full pixel remains full after additional irradiation, but given that a pixel is full, one cannot calculate from the intervening irradiation what its condition was earlier. When u_a has been calculated, u_2 at second sample after the same injection satisfies

$$u_2 = u_a + V_R t_2 / t_p \quad (19)$$

except that $u_2 \leq u(1)$. From u_1 and u_2 one calculates ϕ_1 and ϕ_2 , respectively.

Photogeneration during the injection pulse is accounted for by incrementing the RHS of both (18) and (19) by $V_R t_{inj} / t_p$, as with additive effect. The resulting change in ΔV_{+-} is ordinarily very small.

C. Machine Computations of ΔV_{+-} and Comparison with Measurements

The CID properties in the nomenclature list are those of a real row-injected CID. The measurements were published only in the Proceedings of the 1981 IRIS Detector Meeting.

Measurement of C_T in steady state with parallel column-injection (PCI) and with very small R yielded $C_T = 18.6$ pF. The multiple-pixel problem of PCI has not yet been solved, i.e., the W 's (which enter in the capacitances) are not known as functions of R . The value $C_T'' = 9.2$ pF has been estimated for present use by means of a 1-pixel column-injection approximation. Measurement of C_T' in UCI with very large R/V_i , i.e., with all the pixels full, would provide an easily computed unambiguous value of C_T'' .

The values of V_i used in the row injection measurements are not known. Only the output of the injection-pulse generator, before voltage division in a constant ratio, was measured. $V_i = 0.5$ V was optimum in column injection of the same CID, and therefore is used here as likely to approximate the value used in row injection, too.

1) *Steady State*: Fig. 5 shows the computed steady-state output ΔV_{+-} , for the CID of the nomenclature list

Fig. 5. Calculated (curves) and measured (circles) steady-state output ΔV_{+-} including subtractive effect versus level R of UCI, with SRI. A blackbody source was used for the high power needed, and was filtered to the wavelength of the laser used in Fig. 6 for simplicity of interpretation. The design solutions are shown for each of three pairs of conditions V_i , C_T'' . Comparison with Fig. 4 reveals the theoretical subtractive effect $\Delta V_+ - \Delta V_{+-}$ at $V_i = 0.5$, $C_T'' = 9.2$. Because $\Delta V_+ \approx \Delta V_+(1)$ in saturation (Fig. 4) but $\Delta V_+ - \Delta V_{+-}$ continues to increase far into saturation, ΔV_{+-} has a maximum versus R , as the measurements also do.

and a closely related one, using SRI, and compares with measurements. The measurements lie higher than the computed values and show super-linearity of ΔV_{+-} versus R . In the last respect the computation cannot be fit to the measurements even by varying numerical inputs. The agreement here is at least as good as in a 1-pixel CID with similar readout technique and at similar timing and V_i .

The unchopped background irradiation is kept very small, $R = 0.054 \times 10^6$ photons, by cooling the spectral filter. Therefore this background is ignored in the comparison with measurements in favor of presenting theoretical curves simply of ΔV_{+-} versus R . Background could be included by replacing ΔV_{+-} (blackbody) by ΔV_{+-} (blackbody + background) - ΔV_{+-} (background only).

The effect of small variations of other independent numerical input values has been explored in the conditions of Fig. 5, with $V_i = 0.5$ V, $C_T'' = 9.2$ pF, and $R = 0.05 \times 10^6$ photons. For each property x , the dimensionless ratio $\partial \ln \Delta V_{+-} / \partial \ln x$ has been computed, with no other change. The results for the various properties are

ϵ_s	-0.31
N_D	-0.17
$\phi(1)$	0.14
C_{ic}	0.45
C_{ir}	-0.27
A_c	0.19
A_r	-0.49.

For the three curves from bottom to top of Fig. 5, $\eta = -(C_T'' + \sum A_{cj} C_{icj}) \Delta V_{+-} / qR$ (sum over pixels) = 11.9, 15.0, and 13.0 percent at small R . Section II helps to understand that η depends only weakly on C_T'' . Performance efficiency may be defined by replacing the capacitance in parentheses with the variable and smaller C_T or C_T' . Its definition in the literature is sometimes obscure.

Fig. 6. Calculated (dashed curve) and measured (continuous curves) variation with V_i of the maxima of $\Delta V_{+,-}$ versus R curves. Fig. 5 shows the low-input ends of such curves. For Fig. 6, however, there was equal constant irradiation on pixels #8 and #9 only, and more on the other pixels, rather than UCI. The CID is that of the nomenclature list. There is steady state with SRI. A focussed laser beam was used, in an effort to eliminate subtractive effect by irradiating only one pixel, and thus maximize the response. Because the lens was outside the dewar, the smallest achievable focus still irradiated two adjacent pixels. With one of the two irradiated pixels selected, $\Delta V_{+,-}$ was maximized by varying the irradiation level. Because only $V_{i,ex}$, a constant but unknown multiple of V_i , but not V_i , was measured, curves are shown for two values of the ratio $V_{i,ex}/V_i$.

In Fig. 6 computations and measurements of maximum $\Delta V_{+,-}$ are compared at various V_i , not with UCI, but with equal constant irradiation on pixels #8 and 9, and none on the other pixels. The computed values are independent of which two adjacent pixels are irradiated, within the accuracy of the graph. At, e.g., $V_i = 0.5$ V, $R = 3.13 \times 10^6$ photons at maximum $\Delta V_{+,-}$ is barely within the range of validity of the design solution for pixel #8 selected. However, when pixel #9 is selected, the range of validity extends to much larger R because only time $t_p - t_{inj}$ has elapsed since the irradiated unselected pixel was last injected.

The more pixels irradiated the smaller is the maximum of $\Delta V_{+,-}$, because of subtractive effect in the unselected pixels. The corresponding R shift only from 3.13×10^6 to 3.12×10^6 as the number of pixels (uniformly) irradiated increases from one to 16.

Row-gate voltage $V_r + V_{FB} = -7.0$ V in the measurements of Fig. 6. Measurements at $V_r + V_{FB} = -4.5$ V, showing $\Delta V_{+,-}$ less than half those shown, have been omitted. No dependence on $V_r + V_{FB}$ in this range is expected.

Fig. 7 shows the outputs $\Delta V_{+,-}$ computed with the design solution for various values of n . At $n = 1$, the outputs calculated by the multi-pixel method agree with those by the 1-pixel method of Part I within 300 nV.

2) *Transients*: The method of Section IV-A takes account of transient behavior. The transient property previously studied in CID's is lag, i.e., transient response after the input has reached steady state. In a 1-pixel CID, lag L is defined [1] as the sum of the (unwanted) responses to the transient input after the first response, divided by the sum of all the responses attributable to the transient input. Reference [1, eq. (38)] for readout lag in the ideal mode should be $L = p\eta_{1,dif}/(1 + p\eta_{1,dif})$, with $p = A_c C_{ic}/C_T$. Reference [2] incorrectly claims that "there is no 'readout lag' associated with SRI." Part I is also incorrect in claiming that "... are unchanged from those in [column injection]. So is the transient analysis and for-

Fig. 7. Outputs ΔV_{+} and $\Delta V_{+,-}$ versus R with UCI and SRI and various values of n . Other numerical values are those of the nomenclature list, except that the timing is modified at $n = 128$ pixels. To satisfy the SRI requirement (Section IV-A) $t_i \geq nt_p$, $t_i = 256 \mu\text{s}$ and $t_p = 2 \mu\text{s}$ are used at $n = 128$, with t_i , t_{inj} , and t_2 unmodified. The dashed line shows the input for $f = 1$ when $n = 1$. Increasing n increases capacitance and thus reduces both ΔV_{+} and $\Delta V_{+,-}$, as in Fig. 2. Smaller capacitances were used in real 128-pixel CID's [5]. The higher capacitance at injections soon after the dead time produces slight positive pixel dispersion in ΔV_{+} at $n = 16$. In $\Delta V_{+,-}$ this pixel dispersion is closely compensated by negative pixel dispersion arising from saturation of the subtractive effect in these same injections. There is no pixel dispersion at $n = 128$ because dead time $t_i - nt_p = 0$.

mula for lag." For column injection in the charge-sharing mode, $L = p\eta_{2,dif}/(1 + p\eta_{2,dif})$ [1], but for row injection

$$L = p\eta_{2,r,dif}/(A_r C_{ir}/A_c C_{ic} + p\eta_{2,r,dif}). \quad (20)$$

These η 's are the readout efficiencies [1] pertaining to the three cases, respectively. Both $\eta_{2,dif}$ and $\eta_{2,r,dif}$ are given by the same far RHS of [1, eq. (53)].

The transient sequences of additive output ΔV_{+} in the 1-pixel row-injected CID of Part I (not the CID of the nomenclature list) have now been used to compute L for four different transient situations. The total change of input was small to keep the sequence of transient responses near a geometric sequence [1], yet not too small, to maintain accuracy. The computer code was specialized to $n = 1$, also for accuracy. The resulting L 's are shown in Table I, with corresponding L 's calculated from (20) for comparison. Discrepancies are within 1 percent. Reference [1, eq. (38)] corrected above is equivalent to $\alpha_r = \eta_r \alpha_i$ of [4, Appendix I], derived from a simple linear circuit model. Measurements of L far exceed the values of Table I because of the relatively large $C_T/A_c C_{ic}$ used here and/or because of device lag. The present theory omits device lag, but [3] describes it.

Fig. 8 shows growth from steady state of all pixels empty to steady state with UCI. As in Part I, empty pixels does not mean that there is no mobile charge in the pixels, but rather that, with zero input and starting from comparative fullness of all pixels, the CID has reached zero output. No charge is injected at injection #1 of Fig. 8, but the design solution is marginally valid, and is strongly valid at later injections, after irradiation.

A complementary problem is decay to a steady state of smaller fullnesses. Such a problem has been identified that is not described by either the design solution or the all-pixels-full solution. All pixels initially full for definiteness, UCI with $R = 1.9 \times 10^6$ photons, and SRI beginning with pixel #1 are assumed. No charge is injected from unselected pixels at injection #1 (see (10)). It follows that $u = u(1)$ for any unselected pixel not previously se-

TABLE I
ANALYTICAL VERSUS SIMULATIONAL VALUES OF ONE-PIXEL ADDITIVE-EFFECT SMALL-AMPLITUDE LAG
(Section IV-C-2)

	L from Numerical ΔV_+	From analog of Part I (38)
Decay from full well to steady state	0.00595	0.00595
Growth from empty well to steady state, as in Fig. 8	0.00676	0.00671
Short-pulse negative input at full well	0.00597	0.00595
Short-pulse positive input at empty well, as in Fig. 9	0.00677	0.00671

Fig. 8. Growth of the outputs ΔV_+ and ΔV_- (lower curves) from the steady state of zero fullness in all pixels to steady state at UCI $R = 1.2 \times 10^6$ photons per pixel per integration time. The CID is that of the nomenclature list, and there is SRI. With all wells empty, irradiation was initiated at the end of injection pulse #1, applied to pixel #1. Subtractive effect is initially dominant. Readout lag is too small to be discernible in the graph, but the numerical values are similar to those of Table I. It has not been ascertained which classes of solutions are needed for transient decay of pixels to a less full steady state, but the design and all-pixels-FDI classes are shown in Section IV-C-2 to be inadequate when all pixels are initially full with UCI at R too small to sustain the fullness. The steady-state outputs are drawn in the figure.

Fig. 9. Responses of the CID of the nomenclature list, in steady state with SRI and all fullnesses zero, to short-pulse inputs. A short-pulse input of $50t_p/t_i \times 10^6$ photons is delivered uniformly after second sample following injection #16, of pixel #16, but before first sample preceding injection #17, of pixel #1. Because there is no irradiation in the subtractive interval, $\Delta V_+ = \Delta V_-$. With the same initial steady state, the same input is delivered to pixel #9 only, but not any other pixels, just before first sample preceding injection #73, of the same pixel. One can see that lag L for the uniform input exceeds those of Table I, but L for the point input does not.

lected. Therefore, subtracting (3) from (4) for any such pixel, $\Delta V_+(1) - \Delta V_+ = -[u(1) - u_d]/s \leq 0$. However, according to the machine-computed ΔV_+ for the design solution, $\Delta V_+ < \Delta V_+(1)$ at #2-4. Thus the design solution is inconsistent with the initial conditions from the previous injection. Machine computation with the all-pixels-FDI solution predicts $\Delta V_+ < 0.0006$ for all injections after the first, less than the steady-state $\Delta V_+ = 0.0026$ (Fig. 8) from the design solution. This result is not plausible, even without comparison to the equations and initial conditions. Presumably a solution from at least one of the other classes, not yet computed, satisfies these criteria.

Fig. 9 shows the responses to short-pulse inputs with an initial steady state of empty pixels.

V. CONCLUSIONS

The original 1-pixel theory has been reformulated for many pixels and solved numerically, almost exactly. Applicability to multiple-pixel CID's is supported by the consistency of the theory and comparison with measurements. Row-injection is only a small subset of possible multiple-pixel CID problems, but others may be relatively straightforward with the present calculations as an example. With these accurate results, approximations can be evaluated. The theory includes CID's of current inter-

est [2], [4] and can provide information useful to designers. Comparison is made with all the known applicable measurements. Discrepancies are attributed to uncertainties in some numerical inputs and to effects beyond the scope of the theory. The increasingly adverse influences of line capacitance and of subtractive effect as n becomes large are shown. Transient problems are simulated numerically, and the readout lag analysis of Part I is accurately verified. Irradiation levels for pixel fullness, for a transition between classes of solutions, and for complete saturation corresponding to output $\Delta V_{+-} = 0$, all in the saturation range, are analytically derived in agreement with numerical computation. Additional properties such as variable capacitances, efficiencies and differential efficiencies, and crosstalk can be calculated.

APPENDIX I

FULL PIXELS AND BEHAVIOR IN STEADY-STATE SATURATION

Equation (4) was formulated in terms of $\Delta V_+(1)$, defined as the value of ΔV_+ when $u = u(1)$ for all pixels. Here all $u = u(1)$ will be shown to imply all pixels FDI. This is the all-pixels-full situation. Section III anticipated this conclusion to compute properties of the all-pixels-full solution, such as $\Delta V_+(1)$ in Fig. 2. One may look for a solution in, say, the i th class (Section IV-A), defined by some subset of pixels being FDI, to the problem of all $u = u(1)$. The empty class (no pixels FDI) is not consistent

with all $u = u(1)$, but the all-pixels-full solution belongs marginally (Section IV-A) to all other classes. It will continue to be assumed here that to any suitably defined problem there is only one solution in any class, i.e., only one solution when there are as many equations as unknowns. Because the all-pixels-full solution belongs to every non-empty class, it is the unique solution, all pixels are FDI, and $\Delta V_+ = \Delta V_+(1)$. Analogous reasoning shows the converse, that all pixels FDI implies all $u = u(1)$.

Hypothesizing that a pixel is full before injection reduces the number of unknown variables. Therefore, one can estimate the corresponding irradiation levels without complete machine simulation. This provides a check on and increased understanding of the results of machine computation.

One can estimate the smallest R at which u reaches $u(1)$ for any pixel j . When pixel j is irradiated and selected (denoted by superscript (j)),

$$u^{(k)} = u_a^{(j)} + V_{Qa}^{(j)} - V_{Qb}^{(k)} + s\Delta V_+^{(j)}. \quad (A2)$$

by subtracting (4) and (5) for pixel j . Subtracting (3) for pixel k selected and (5) for pixel j selected

$$u^{(k)} = u_a^{(j)} + V_{Qa}^{(j)} - V_{Qb}^{(k)} + s\Delta V_+^{(j)}. \quad (A2)$$

Eliminating $u_a^{(j)}$ between (A1) and (A2)

$$u^{(k)} = u(1) + V_{Qa}^{(j)} - V_{Qb}^{(k)} - s[\Delta V_+(1) - \Delta V_+^{(j)}] - (1-s)V_i. \quad (A3)$$

If no charge is injected from the j th pixel unless it is selected, then, reasoning as in (14)

$$V_{Qa}^{(j)} - V_{Qb}^{(k)} = V_R \Delta t / t_p. \quad (A4)$$

Here Δt is the time from end of injection with pixel j selected until the beginning of injection with pixel k selected. Therefore, $u^{(k)} = u(1)$ when

$$V_R = [(1-s)V_i + \epsilon] t_p / \Delta t. \quad (A5)$$

The entire fourth term of the five on the RHS of (A3) is abbreviated as $-\epsilon$.

Machine computation on the CID of the nomenclature list, using the design solution, in steady state with UCI and SRI, shows the $\epsilon \ll (1-s)V_i$. If ϵ is neglected, the RHS of (A5) is minimum when Δt is maximum. Thus the smallest value of R for any $u_{\text{unsel}} = u(1)$ corresponds to $\Delta t = t_i - t_p - t_{\text{inj}}$, for which $j = k + 1$. When this R is minimized with respect to k , the smallest value can be called R_{sat} (for saturation) if it is less than the limit of validity of the design solution, as often happens. The computation shows that then $R_{\text{sat}} = 3.21 \times 10^6$ photons. This situation is reached when pixel $k = 15$ is selected, $u = u(1)$ for pixel $k = 15$ and $j = 16$, and $\epsilon = -3.2 \times 10^{-5}$, in agreement with (A5). Equation (A5) can be used similarly when $j = k + 2$, $k + 3$, etc., and when $j < k$. To determine whether an unselected pixel is FDI, one can compare numerically $u_d = u_a + s\Delta V_+(1)$ to $u(1)$.

For the same CID with UCI and SRI, if $\Delta t = t_p - t_{\text{inj}}$, corresponding to $u = u(1)$ for all pixels, and $\epsilon = 0$, then (A5) requires $R = 115.1 \times 10^6$ photons. At this or larger R all pixels are FDI. Machine computation with the all-

pixels-FDI solution verifies that $\epsilon = 0$ at $R \geq 115.1 \times 10^6$ photons. The same computation shows that, as R falls below that value, $u(1) - u_j$ increases from zero faster than ϵ does, for at least one pixel j and at least one choice of k . With (3) and (4), this implies $V_{Qb} > V_{Qa}$ for that k and j , i.e., charge is absorbed into pixel j during the injection pulse applied to pixel k , contrary to the assumptions. This computation and reasoning verify that the all-pixels-FDI solution is invalid at $R < 115.1 \times 10^6$ photons.

At all $R \geq R_{\text{max}}$, all pixels are full at both first and second samples, so that $\Delta V_{+-} = 0$. With the timing of Fig. 3 or the nomenclature list, this requires that the selected pixel be refilled by photogeneration between the end of injection and the next second sample, i.e., that $u_{2,\text{sel}} = u(1)$. u_2 is given by (19). Analogy with (A5) or using (19), (3) and (4) shows that V_R corresponding to R_{max} is given by (A5) with $\epsilon = \epsilon_{\text{sel}}$ for the selected pixel and $\Delta t = t_2$. For the same CID as above with UCI, $R_{\text{max}} = 506.6 \times 10^6$ photons and $\epsilon_{\text{sel}} = 0$ at $R = R_{\text{max}}$ by machine computation with the all-pixels-FDI solution, in agreement with the formula for R_{max} .

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