# On the complexity of congestion free routing in transportation networks

C. Menelaou, P. Kolios, S. Timotheou and C.G. Panayiotou

***Abstract*— Traffic congestion has been proven a difficult problem to tackle, particularly in big cities where the number of cars are steadily increasing while the infrastructure remains stagnant. Several approaches have been proposed to alleviate the effects of traffic congestion, however, so far congestion is still a big problem in most cities. In this work we investigate a new route reservation approach to address the problem which is motivated by air traffic control. This paper formulates the route reservation problem under different assumptions and examines the complexity of the resulting formulations. Two waiting strategies are investigated, (i) vehicles are allowed to wait at the source before they start their journey, and (ii) they are allowed to wait at every road junction. Strategy (i) though more practical to implement, results to an NP-complete problem while strategy (ii) results to a problem that can be solved in polynomial time but it is not easily implemented since the infrastructure does not have adequate space for vehicles to wait until congestion downstream is cleared. Finally, a heuristic algorithm (based on time-expanded networks) is derived as a solution to both proposed waiting strategies.**

## I. Introduction

Traffic congestion is a serious problem that occurs in all metropolitan cities and has a significant impact on the lives of citizens in terms of delay, frustration, loss of productivity, fuel cost, and environmental pollution. The problem is expected to become even worse since the number of vehicles continuously grows while the infrastructure remains the same. On the other hand some traffic researchers indicate that this phenomenon does not occur because demand exceeds network's capacity but rather due to the fact the majority of drivers prefer to follow main arterials instead of following local roads [1]. Therefore, there is the potential to better manage the traffic to reduce the effects of congestion, for example by managing traffic such that vehicles better utilize all network links (load balancing). However, achieving a better load balancing is not an easy problem since it involves accurately estimating the future state of the network while the route selection is made by drivers who are trying to optimize their own individual objectives. Furthermore, the future network state estimates depend on the earlier driver decisions which may be based on past state estimates. For example, consider a simple scenario with two paths to a single destination. When a rational driver starts her journey, she is informed that path $A$ is currently congested, thus she decides to use path $B$. If all rational drivers do the same, by the time they reach the congested area, the congestion in path $A$ will be cleared while congestion will be formed in path $B$.

Motivated by this simple problem, we propose to use a *reservation* protocol so that drivers use the least congested paths while other drivers are not allowed to interfere and increase the congestion. The idea is that the network is decomposed spatially and temporarily. Every link has a capacity that it can accommodate during a short interval. The system, using previous reservations has an estimate of the number of vehicles that are expected during any time interval. When a vehicle is about to start its journey, it sends a request to a central server. The server determines the best path (the one that would allow the vehicle to arrive at the earliest time to the destination) avoiding roads that are expected to have more vehicles than their allowed capacity. Some of the possible solutions that can be offered by the system include an alternative path (possibly longer in terms of distance) but avoiding road segments that are at their maximum capacity. Alternatively, the system may request from the vehicle to start its journey a little later from its origin (wait at source) or wait at some intermediate intersections (assuming there is space for the vehicle to stop and wait). We point out that with today's advancements in information and communication systems such an approach is feasible. Drivers can send their origin-destination pair through, for example, a navigation system which will then receive the reserved path and use it for routing. Evidently, if autonomous vehicles are used, this routing approach is even easier to implement. However, enforcing the reservations is a topic that we leave as future research.

The proposed reservation protocol ensures that each road segment operates below its critical capacity. The critical capacity is determined by the macroscopic fundamental diagram (MFD) [2] and denotes the highest permissible capacity where the network state changes from uncongested to congested. Hence, to ensure a congestion-free operation, the density of each road segment should be maintained bellow the critical capacity and in this work this is enforced by limiting the reservation availability of each road segment.

The contribution of this paper is that it proposes a new reservation protocol and formulates the problems that the system should solve in order to determine every vehicle's path under different assumptions. Furthermore, the complexity of the resulting formulations is computed and a heuristic algorithm is provided to solve them.

This paper is organized as follows: Section II includes a brief overview of related work. Section III introduces the problem formulation for different strategies while Section IV

This work was supported by the European Research Council (ERC) under the ERC Advanced Grant through the FAULT–ADAPTIVE Project.

C. Menelaou, P. Kolios, S. Timotheou, and C.G. Panayiotou are with the KIOS Research Center for Intelligent Systems and Networks and the Department of Electrical and Computer Engineering, University of Cyprus, {cmenel02, pkolios, timotheou.stelios, christosp}@ucy.ac.cy

provides a complexity analysis for the proposed problems. A solution to both investigated formulations is provided in Section V which is based on time-expanded networks. The simulation setup with the performance results are presented in Section VI. Finally Section VII concludes this work.

## II. Related Work

Time slot reservation has been extensively practiced in Air Traffic Management and Control Systems (ATM/ATC) [3]. ATM/ATC systems improve the airport's efficiency by enabling the reservation of time slot for both landing and take-off; ensuring, in that respect, support for higher demands [4] [5].

In land transport, on the other hand, there has been a growing number of approaches that solve the vehicle routing problem assuming the network state is known. Some approaches consider that the network state is predicted based on static or stochastic models [6]. These approaches assume that the travel time on each road segment changes dynamically depending on the network state [7] [8]. The objective is not to mitigate congestion but to schedule each vehicle as an independent agent through shortest-travel-time paths [9]. Therefore, these approaches are susceptible to enter into the capacity drop region where congestion mitigation becomes a difficult problem [10].

Complexity analysis of time-depended route planing was commonly investigated in recent literature. Batz and Sanders [11] shows that the time-depended shortest path problem with generalized objective function and time bend points is an NP-complete problem. Ahuja in [12] illustrates that the dynamic shortest path problem where travel times change dynamically is an NP-complete problem too.

As discussed in the Introduction, this work investigates a route reservation protocol that allows congestion-free routing. To ensure that this is the case some waiting may need to take place at the origin of a route or at intermediate junctions. The former case results to an NP-complete problem while the latter case attains a polynomial solution.

## III. Problem Formulation

In this work, the road network is modeled as a graph $G = (V, E)$ with vertices $V$ being the road junctions and edges $E$ the road segments. Traffic is allowed in each road segment $(i, j) \in E, \{i, j\} \in V$ if the vehicle density for the particular segment is at or below the critical capacity ($K_{ij}$). Under this regime, free-flow conditions are experienced and thus vehicles can travel at the maximum speed limit. By $\bar{c}_{ij}$ we denote the number of time units necessary to traverse a segment $(i, j) \in E$ when traveling at maximum speed. Assuming that a vehicle enters a road at discrete time $t$, the vehicle is expected to traverse the particular segment during all time instants $t, t+1, ...t + c_{ij}$.

To ensure that traffic volume is restricted below critical capacity, reservations for each time unit are allowed only if the accumulated expected traffic $r_{ij}(t),\ (i, j) \in E$ is less than the critical capacity for the whole traveling duration. Then, the reservation state of a particular road segment $x_{ij}(t)$ is defined as follows:

$$x_{ij}(t) = \begin{cases} 1, \text{ if } r_{ij}(\tau) < k_{ij}, \forall\, \tau = t, \ldots, t + \bar{c}_{ij} \\ 0, \text{ otherwise} \end{cases}$$

where, $x_{ij} = 1$ denotes the uncongested state and $x_{ij} = 0$ the congested state.

Evidently, when the state of a particular road segment is congested, a vehicle should be instructed to either wait for the road to become non-congested or be rerouted through a different path. This decision is made based on the alternative solution that achieves the shortest arrival time to the destination. In either case, when waiting is inevitable, two possible reservation strategies arise, a) vehicles could wait at intermediate road junctions (WAIS) [1], or b) vehicles could wait only at the origin (WOAS)(delayed departure).

### A. Wait at intermediate road junctions (WAIS)

When waiting at all intermediate road junctions is allowed, the cost (time) of traversing a road segment can be expressed as follows:

$$c_{ij}(t) = \begin{cases} \bar{c}_{ij} + w_{ij}(t), \text{ if } x_{ij}(t) = 0 \\ \bar{c}_{ij}, \text{ if } x_{ij}(t) = 1 \end{cases}$$

where, $w_{ij}(t)$ denotes the smallest number of time units that a vehicle has to wait at $i$ before $(i, j)$ becomes non-congested and thus a vehicle can start traversing the link.

### B. Wait only at the origin (WOAS)

For the second strategy which is practical and easily implementable, waiting is only allowed at the originating road junction $s$ of a vehicle's route, and thus the cost of traversing a road segment $c_{ij}(t)$ can be expressed as follows:

$$c_{ij}(t) = \begin{cases} \bar{c}_{ij}, \text{ if } x_{ij}(t) = 1 \\ \infty, \text{ if } x_{ij}(t) = 0 \text{ and } i \neq s \\ \bar{c}_{ij} + W_{ij}(t), \text{ if } x_{ij}(t) = 0 \text{ and } i = s \end{cases}$$

where $W_{ij}(t)$ is the least amount of time that a vehicle should wait at the origin $s$ in order to travel from $s$ to the destination $e$ through non-congested road segments. As shown in the results that follow, computing $W_{ij}(t)$ optimally is an NP-complete problem.

In any case, as new reservation requests are issued by soon-to-be-departing vehicles, for both strategies decisions should be made on which route to take and where should vehicles wait in order to arrive at their destination on the earliest possible time. When decisions are made, vehicles are responsible for following the assigned route within the scheduled time constrains from the origin to the destination. Under this regime, the minimum travel-time problem $\Pi$ arises, as defined below.

[1]This waiting strategy is not a practical approach but it is investigated as a lower bound on the achievable delay. Assuming unlimited parking slots, and that there is enough space for other vehicles to pass through.

*Problem* $\Pi$*:* $\Pi$ takes as input the origin-destination pair $s - e$, the request time-stamp $t_0$, and using the current route reservations, computes the shortest-arrival-time route form $s$ to $e$ starting at $t_0$. Let $p_k$ denote the $k$-th path from source $s$ to destination $e$. $p_k = (v_0^k, v_1^k),\ (v_1^k, v_2^k),\ (v_2^k, v_3^k), ....(v_{n_k-1}^k, v_{n_k}^k)$, where $v_j^k \in V$ is the $j$-th visited node in the $k$-th path, with $v_0^k = s$, $v_{n_k}^k = e$ and $n_k$ is the legth of the path. Also, let $d_j^k$ denote the earliest arrival time at junction $v_j^k$. Then,

$$\begin{aligned} d_0^k &= t_0 \\ d_1^k &= d_0^k + c_{v_0^k, v_1^k}(d_0^k) \\ &\cdots \\ d_{n_k}^k &= d_{n_k-1}^k + c_{v_{n_k-1}^k, v_{n_k}^k}(d_{n_k-1}^k) \end{aligned}$$

Decision problem $\Pi$ finds an uncongested route for a vehicle to reach its destination in the earliest arrival time and is mathematically expressed as follows:

$$(\Pi)\ D^* = \min_{p_k} d_{n_k} \tag{1}$$

## IV. Complexity Analysis of $\Pi$

At a first glance, $\Pi$ looks similar to the time dependent route planning [13]. However, the difference to the latter problem is that congested road segments (with infinite cost) may arise due to the possible reservations made. In this section we show that, when waiting is allowed only at the origin, $\Pi$ becomes an NP-complete problem while when waiting is allowed at all intermediate junctions, the problem has polynomial-time computation.

The complexity analysis of $\Pi$ is based on the restriction method [14] (a similar derivation is made in [11]) where the examined proof reduces the Number Partitioning Problem ($\Pi'$) to $\Pi$ [14].

*Lemma 1:* Given a set $A$ which consists of $n$ integer numbers $A = \{a_1,\, a_2,\, ...a_n\}$ , $a_i \in \mathbb{Z}^+$ and given an integer number $b \in \mathbb{Z}^+$, $\Pi'$ tries to find a subset $A'$ where $A' \subseteq A$, satisfying the following equation:

$$\sum_{i=1}^{n} a_i y_i = b, \text{ where } y_i = \{0, 1\} \tag{2}$$

which is an NP-complete problem.

*Theorem 1:* If waiting is not allowed at any of the road junctions $i \in V$ then, $\Pi$ is an NP-complete problem in the case where at least one segment attains to congested reservation state, i.e. $x_{ij}(t) = 0, (i,j) \in E, \forall\, t$.

*Proof:* A special case of $G(V,E)$ is constructed as illustrated in Fig. 1. Let $(i,e)$ be the single edge on this graph that attains to congested reservation state. Every other edge on the graph is assumed to be on non-congested state and thus the cost of traversing a road segment is equal to $\bar{c}_{ij}$. On the other hand, the cost function ($c_{ie}(t)$) of $(i,e)$ can be expressed as follows:

$$c_{ie}(t) = \begin{cases} \bar{c}_{ie}, \text{ for } t = b \\ \infty, \text{ for } t \neq b \end{cases}$$

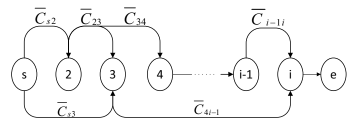


Fig. 1: Special case of $G(V,E)$

$\Pi$ has a feasible solution only if there is a non-congested path $p$ from the origin to the destination (i.e. $s - e$) and the arrival time at node $i$ is exactly equal to $b$ (i.e., $d_i = b$). Let, $p$ contain the subpath $p'$ from nodes $s$ to $i$ where the total travel time of $p'$ (i.e., $c_{p'}(t)$) is defined as follows:

$$c_{p'}(t) = \sum_{i=1}^{n} \bar{c}_{ij} y_i, \text{ where,} \tag{3}$$

$$y_i = \begin{cases} 1,\ (i,j) \text{ is contained in } p' \\ 0,\ (i,j) \text{ is not contained in } p' \end{cases}$$

The above claim, indicates that the chosen $y_i$ for $p'$ as shown in eq. (3) provide a solution to the number partitioning problem. This is because the total travel time is the summation of all edge costs, $c_{ij}(t)$ of the selected nodes on the path ($p'$). This show that in general case $\Pi$ is an NP-complete problem and this completes the proof. ■

*Theorem 2:* If waiting is allowed at any road junction, then $\Pi$ can be solved in polynomial time.

*Proof:* Theorem 2 relaxes the equality constraint expressed in eq. (2) to an inequality since now there is no need for a solution that equals exactly to $b$, i.e., $d_i \leqslant b$. Therefore, $\Pi$ can find a feasible solution in the case where the total travel time cost of $p'$ is equal to:

$$c_{p'}(t) \leqslant b, \text{ where}, c_{p'}(t) = \sum_{i=1}^{n} \bar{c}_{ij} y_i \text{ and } y_i \ = \ \{0, 1\} \tag{4}$$

Equation (4) indicates that there are many feasible solutions that solve $\Pi$. For example, a possible solution selects the unconstraint shortest path from $s$ to $i$ and after that if $d_i \leqslant b$ waiting can occur at any intermediate junction in order to achieve $d_i = b$. Doing so ensures that a feasible path $p$ is always obtained in polynomial time. ■

### A. Special Cases 1

In the case where only one segment has congested reservation states and waiting is allowed only at the source, then $\Pi$ can be solved in polynomial time. As shown in theorem 2 a solution can easily be found with a feasible path from node $s$ to $i$ where $d_i \leqslant b$ using eq. (4). In that case a vehicle can wait for the remaining interval at $s$ in order to achieve $d_i = b$.

### B. Special Case 2

Suppose now, the alternative case as shown in Fig. 2 where two segments with congested reservation states are present (edge $(i, i+1)$ and edge $(j, e)$). As in the previous case, waiting is also allowed only at the source. The cost functions for these edges are defined as follows:

$$c_{ii+1}(t) = \begin{cases} \bar{c}_{ii+1}, \text{ for } t = b_1 \\ \infty, \text{ for } t \neq b_1 \end{cases} \quad c_{je}(t) = \begin{cases} \bar{c}_{je}, \text{ for } t = b_2 \\ \infty, \text{ for } t \neq b_2 \end{cases}$$

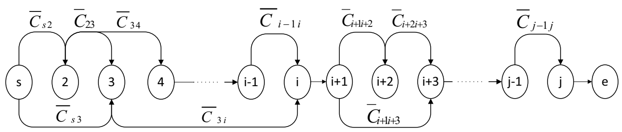

Fig. 2: Special case of $G(V, E)$ (two congested links)

As before $\Pi$ has a solution only if there is a uncongested path $p$ from the origin to the destination (i.e. $s - e$) and by the same token, the earliest arrival time at nodes $i$ and $j$ must be $b_1$ and $b_2$ respectively. Similar to the proof of theorem 1, let $p$ contain sub-paths $p'$ and $p''$. $p'$ is the path from nodes $s$ to $i$ and $p''$ is the path form nodes $i+1$ to $j$. The total travel time costs of both paths ($c_{p'}(t)$ and $c_{p''}(t)$) is defined in equation (3).

As theorem 2 shows, there are many feasible path from node $s$ to $i$ with $d_i \leqslant b_1$. Thus, a vehicle can wait the remaining time at $s$ in such a way to achieve $d_i = b_1$.

Subpath $p''$ reduces to the problem of Theorem 1. This is due to the fact that the total travel time of $p''$ is the summation of particular costs $c_{ij}(t)$ of the selected nodes on this subpath. Therefore the selected $y_i$ can express a solution to the number partitioning problem eq. (2).

## V. Proposed Time Expanded Algorithm

A solution to the two investigated strategies (WAIS and WOAS) is the Time Expanded Shortest Path algorithm (TESP) which is based on time-expanded graphs that are commonly studied in dynamic networks [15]. The proposed TESP is a heuristic solution that solves the minimum arrival time problem using both waiting strategies (as described in the previous section).

TESP executes in two phases. In phase 1 (prepossessing phase) TESP constructs the Time Expanded graph $G_T(V', E')$ using multiple replicas of $G(V, E)$. In phase 2, it returns the minimum destination arrival time route.

### A. Phase 1: Construction of $G_T$

$G_T(V', E')$ is constructed using replicas (clones) of $G(V, E)$ (e.g. Fig. 3). Every replica represents a future instance of $G(V, E)$ for $D_{max}$ consecutive time instances. $D_{max}$ is set to be the upper bound on the delay that a vehicle needs to wait at the origin in order to follow the unconstraint shortest path [13] (Algorithm 1).

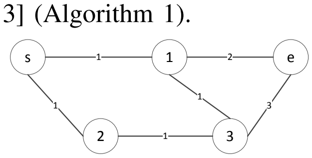

Fig. 3: Example of $G(V, E)$

In the time-expanded graph, each replica is identified by $i_k, i \in V, k = 1, \ldots, D_{max}$. This indexing method is used in order to declare which node of $G(V, E)$ represents every node on $G_T(V', E')$. Both strategies can use the same $D_{max}$ as an upper bound since, a solution to WOAS is also a solution to WAIS.

The time-expanded network includes all node replicas, a dummy source node ($s'$) and a dummy sink node ($e'$).

**Data**: $G(V, E)$, $r_{ij}(t)$, $s$, $e$, $t_0$
**Result**: $D_{max}$
Set $D_{max} = 0$;
Return $s - e$ unconstraint shortest path $p$ (initial time is $t_0 = D_{max}$) ;
**while** *$p$ is congested for all consecutive time units* **do**
  $D_{max} = D_{max} + 1$;
  Return $s - e$ unconstraint shortest path $p$ (initial time is $t_0 = D_{max}$) ;
**end**

**Algorithm 1:** Calculation of $D_{max}$

The dummy source node connects to all replicas of the originating source node while all replicas of the destination node connect to the dummy sink node. The cost $c_{s',s_1}(t)$ starts with a zero value at the first replica and increments by one for all subsequent replicas. Doing so ensures that the delay in waiting at the source is included in the arrival time calculation. The cost to the dummy sink is simply set to zero, $c_{e_n,e'}(t) = 0$. An example of a time-expanded graph is illustrated in Fig. 4 while Alg. 2 depicts the aforementioned process.

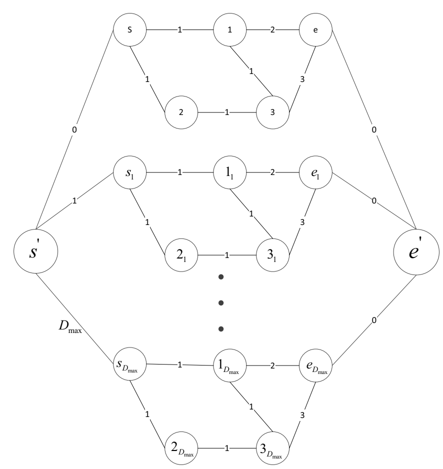

Fig. 4: Example of $G_T(V', E')$

**Data**: $G(V, E)$, $s$, $e$, $t_0$, $D_{max}$
**Result**: $G_T(V', E')$
Create $D_{max}$ replicas of $G(V, E)$;
Create nodes $s'$, $e'$;
**for** $i = 1$; $i \leqslant D_{max}$; $i++$ **do**
  $c_{s',s_i}(t) = i$; $c_{e_i,e'}(t) = 0$;
**end**

**Algorithm 2:** Costructing $G_T(V', E')$

Algorithm 3 is responsible of calculating the earliest arrival time $d_i$ on each node of $G(V, E)$, and to sort these nodes in ascending order with respect to time. This procedure is used in the following two algorithms (Algorithms 4 and 5) where depending on the strategy followed, costs need to be recalculated accordingly.

*WOAS:* As shown in section III the edge cost function changes with respect to the strategy that is going to be used. Algorithm 4 calculates the edge costs on $G_T(V', E')$ based

**Data**: $G(V,E)$, $s$
**Result**: Sorted $d_n$ list
Run unconstraint shortest path algorithm over $G(V,E)$;
$d_i$ is the earliest arrival time on each node;
Sort $d_i$ ;

**Algorithm 3:** Find $d_n$ on $G(V,E)$

on the reservation states. In case where a road segment is non-congested the cost of segment $(i_k j_k)$ is set to $\bar{c}_{ij}$. In every other case, it sets the cost to infinity and in that case the algorithm recursively recalculates the arrival time to every road junction (Alg. 4).

```
Data: G_T(V', E'), r_ij(t), d_i, D_max
Result: Recalculated weights of G_T(V', E')
for i = 1; i ⩽ (D_max * V); i++ do
    for j = 1; j ⩽ (D_max * V); j++ do
        if c̄_{i,j} != ∞ then
            d_{i_n} = i + d_i;
            if (i_n, j_n) is uncongested for all consecutive
            time units that needed for traverse then
                c_{i,j_n} = c̄_{i,j};
            else
                c_{i,j_n} = ∞;
                Rerun algorithm 3;
            end
        end
    end
end
```

**Algorithm 4:** Recalculation in the WOAS strategy

*WAIS:* In this strategy, for each $(i_k, j_k)$ that is not uncongested, WAIS finds the least waiting time that enables the traversal through the particular edge (i.e., $w_{i_k j_k}(t)$) that a vehicle needs to wait at $i_k$. For all congested road segments $w_{i_k j_k}(t)$ is added to the total traversal cost of the particular segment as shown in Algorithm 5. As before, when an arrival cost is updated, the algorithm recursively recalculates the new arrival times to every road junction and terminates when no updates take place.

### *B. Phase 2: Execution of algorithm*

In the second phase, Dijkstra's algorithm [13] is used to compute the shortest path between the dummy source and dummy sink nodes. An inverse indexing procedure is applied onto the returned path in order to determinate the path from $s$ to $e$ and to find all computed waiting intervals. This procedure also returns the earliest arrival time route for both strategies with complexity $O((D_{max} * V)^2)$. The complete TESP algorithm is illustrated in Alg. 6.

```
Data: G_T(V', E'), r_ij(t), d_i, D_max
Result: Recalculated weights of G_T(V', E')
for i = 1; i ⩽ (D_max * V); i++ do
    for j = 1; j ⩽ (D_max * V); j++ do
        if c̄_{i,j} != ∞ then
            d_{i_n} = i + d_i; w_{ij_n}(d_{i_n}) = 0 ;
            if (i_n, j_n) is uncongested for all consecutive
            time units that needed for traverse then
                c_{i,j_n} = c̄_{i,j};
            else
                w_{ij_n}(d_{i_n}); c_{i,j_n} = c̄_{i,j} + w_{ij_n}(d_{i_n});
                Rerun algorithm 3;
            end
        end
    end
end
```

**Algorithm 5:** Recalculation of WAIS strategy

```
Data: G(V, E), r_ij(t), s, e, t_0
Result: Returns the minimum-arrival-time-route
Run algorithm 1 ; Run algorithm 2 ; Run algorithm 3;
if WAIS==TRUE then
    Run algorithm 5;
else
    Run algorithm 4;
end
Run unconstraint shortest algorithm over G_TE(V', E')
and return s', e' shortest path p' ;
Apply reverse indexing to p' thus, to declare p on
G(V, E) and w_ij(t);
```

**Algorithm 6:** Time Expanded Shortest Path Algorithm

## VI. Simulation Setup And Results

WAIN and WOAS approaches were simulated in the SUMO microscopic simulator [16], using the TraCI interface [17]. Single lanes are used for each road segment that create a Manhattan-style network. In total 24 road segments and 9 junctions are set up. No overtaking was allowed to ensure that all vehicles followed first-in-first-out queues, i.e., $t + c_{ij}(t) \leqslant (t+1) + c_{ij}(t+1)\ \forall\ i \to j$.

Both strategies were simulated for different flow rates (with Poisson arrival distributions) and the origin-destination pairs were randomly selected. Vehicle characteristics are set according to the Krauss car following model [18], the simulation time was set to an hour (3600s) and the critical capacity of each road segment $(i,j)$ was set $K_{ij} = 18\%$ of the maximum density of the road, based on [19]. Different vehicles parameters were also used to test the performance of the two strategies, as shown in table I.

TABLE I: Vehicle's Parameters

| | Scenario 1 | Scenario 2 |
|---|---|---|
| acceleration | $2.5m/s^2$ | $2.0m/s^2$ |
| deceleration | $4.5m/s^2$ | $4.5m/s^2$ |
| maximum speed | $15m/s$ | $15m/s$ |
| vehicle length | $5m$ | $5m$ |
| driver reaction time | $0.5s$ | $1s$ |
| speed deviation factor | 0% | 30% |
| driver imperfection | 0% | 30% |

Monte Carlo simulations were conducted for each investigated flow rate. The results evaluated the performance of the system based on those vehicles that managed to complete their journey within the simulated hour. The proposed strategies are compared against the traditional behavior (TB) experienced by vehicles when no reservations are made.

Fig. 5 illustrates the average travel time over the two simulated scenarios. The dashed lines in Fig. 5 represents the mean value of the mean travel time for different simulated scenarios and the scattered plots represents the mean travel time of all Monte Carlo simulations. The travel time is measured as the difference between the time that a vehicle enters and exits the network.

As shown in the results of Fig. 5, on low flow rates both waiting strategies behave similarly to the traditional behavior since no congestion is experienced. On higher flow rates however, WAIS performs better since waiting is allowed at all road junctions. As shown in the figure, the average travel time of WAIS for each vehicle is lower than that of WOAS. In either case, both strategies perform much better than TB, demonstrating that both approaches can greatly reduce congestion and traveling time. Additionally, Fig. 5 indicates that irrespective of the changes in vehicle characteristics (as set up in table I), the mean travel time performance is very similar; demonstrating that the proposed solution is robust to such variations.

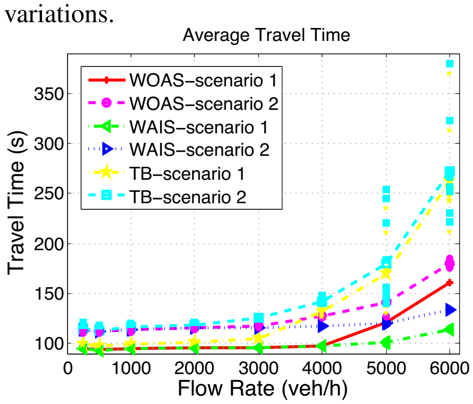


Fig. 5: Travel time comparison for WAIS, WOAS, TB

The following two figures, Fig. 6 and Fig. 7 plot the travel time distributions for flow rate of 6000 $veh/hour$ for scenario 1 and 2, respectively. Both figures demonstrate the higher resilience of WAIS to congestion compared to WOAS. Indicatively, the standard deviation for WAIS is 32.75 and for WOAS is 100.78. Hence, as the congestion of the road segments increases, the travel time of WOAS increases at a higher rate than that of WAIS. Both strategies however, clearly demonstrate that route reservation can achieve substantial improvements in road utilization and thus practical solutions could result in considerable benefits.

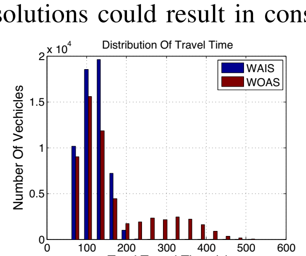


Fig. 6: Travel time distribution for scenario 1

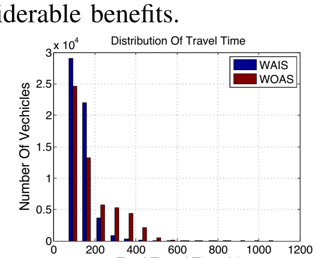


Fig. 7: Travel time distribution for scenario 2

## VII. Conclusions

This work examines the potentials of route reservation as an approach to alleviate road congestion and minimize vehicle arrival times to their destinations. In the process, two waiting strategies have been investigated. It has been demonstrated that when waiting is only allowed at the origin of a route, the problem becomes NP-complete.

A unified time-expanded shortest path algorithm has been developed to solve both these problems and using this algorithm extensive simulations have been conducted. The presented results demonstrate the multiple gains in performance that can be achieved by route reservation compared to the traditional vehicle behavior.

## References


[1] C. Chen, Z. Jia, and P. Varaiya, "Causes and cures of highway congestion," *IEEE Control Systems Magazine*, vol. 21, no. 6, pp. 26–32, 2001.

[2] L. Immers and S. Logghe, "Traffic flow theory," *Faculty of Engineering, Department of Civil Engineering, Section Traffic and Infrastructure, Kasteelpark Arenberg*, 1976.

[3] R. De Neufville and A. Odoni, *Airport Systems. Planning, Design and Management*, 2003.

[4] D. Condorelli, "Efficient and equitable airport slot allocation," *Rivista di politica economica*, vol. 1, pp. 81–104, 2007.

[5] C. G. Panayiotou and C. G. Cassandras, "A sample path approach for solving the ground-holding policy problem in air traffic control," *Control Systems Technology, IEEE Transactions on*, vol. 9, no. 3, pp. 510–523, 2001.

[6] I. Kaysi, M. Ben-Akiva, and H. Koutsopoulos, *Integrated approach to vehicle routing and congestion prediction for real-time driver guidance*, 1993, no. 1408.

[7] D. E. Kaufman and R. L. Smith, "Fastest paths in time-dependent networks for intelligent vehicle-highway systems application∗," *Journal of Intelligent Transportation Systems*, vol. 1, no. 1, pp. 1–11, 1993.

[8] L. Xiao and H. K. Lo, "Adaptive vehicle navigation with en route stochastic traffic information," 2014.

[9] L. Du, S. Chen, and L. Han, "Coordinated online in-vehicle navigation guidance based on routing game theory," in *Transportation Research Board 94th Annual Meeting*, no. 15-3613, 2015.

[10] S. Hoogendoorn and V. Knoop, "Traffic flow theory and modelling," 2012.

[11] G. V. Batz and P. Sanders, "Time-dependent route planning with generalized objective functions," in *Algorithms–ESA 2012*. Springer, 2012, pp. 169–180.

[12] R. K. Ahuja, J. B. Orlin, S. Pallottino, and M. G. Scutella, "Dynamic shortest paths minimizing travel times and costs," *Networks*, vol. 41, no. 4, pp. 197–205, 2003.

[13] E. W. Dijkstra, "A note on two problems in connexion with graphs," *Numerische mathematik*, vol. 1, no. 1, pp. 269–271, 1959.

[14] R. G. Michael and S. J. David, "Computers and intractability: a guide to the theory of np-completeness," *WH Freeman & Co., San Francisco*, 1979.

[15] E. Pyrga, F. Schulz, D. Wagner, and C. Zaroliagis, "Efficient models for timetable information in public transportation systems," *Journal of Experimental Algorithmics (JEA)*, vol. 12, pp. 2–4, 2008.

[16] M. Behrisch, L. Bieker, J. Erdmann, and D. Krajzewicz, "Sumo-simulation of urban mobility-an overview," in *SIMUL 2011, The Third International Conference on Advances in System Simulation*, 2011, pp. 55–60.

[17] A. Wegener, M. Piórkowski, M. Raya, H. Hellbrück, S. Fischer, and J.-P. Hubaux, "Traci: an interface for coupling road traffic and network simulators," in *Proceedings of the 11th communications and networking simulation symposium*. ACM, 2008, pp. 155–163.

[18] S. Krauss, P. Wagner, and C. Gawron, "Metastable states in a microscopic model of traffic flow," *Physical Review E*, vol. 55, no. 5, p. 5597, 1997.

[19] H. Rakha and B. Crowther, "Comparison of greenshields, pipes, and van aerde car-following and traffic stream models," *Transportation Research Record: Journal of the Transportation Research Board*, vol. 1802, no. 1, pp. 248–262, 2002.