Stability analysis of a novel four-wheeled motorcycle in straight running

Francesco Bucchi, Francesco Cerù and Francesco Frendo

Abstract This paper deals with the stability analysis of a novel four wheeled tilting motorcycle. This new kind of vehicle is equipped with two front steering wheels and two rear wheels; the front and rear suspension mechanisms allow the scooter to roll freely as a classic motorcycle.

An analytical model of the vehicle with locked suspensions, rigid and lenticular tires was developed in order to study the stability of the system in straight running and to carry out a modal analysis focusing on the main normal modes. The frame and all the vehicle components were considered as rigid bodies, the driver was considered fixed to the frame and the tires behavior was described by a linear function of the roll and side-slip angles. The action of the driver on the steering was not taken into account, considering the vehicle running in straight line without any control system.

The stability analysis shows that the eigenmodes of the four wheeler have several common features with the capsize, weave, wobble and rear wobble modes which characterize the dynamics of two and three wheeled motorcy-

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cles. A sensitivity analysis showing the influence of the front trail on stability and the vehicle eigenmodes is presented. A graphical representation of the eigevenctors is also proposed which easily represents the modal shapes.

Keywords Vehicle dynamics, 4-wheeler, stability analysis, eigenmodes, capsize, weave, wobble.

1 Introduction

This study deals with the stability analysis of a novel four-wheeled tilting vehicle equipped with two front steering wheels and two rear wheels. This new kind of vehicle was designed with the aim of improving passive safety and conceived to develop a new kind of urban mobility, suitable also for people that usually drive cars and do not consider two-wheelers as a safe and comfortable alternative. Indeed, this new vehicle can be regarded as a further development of the well-known three wheeled motorcycles.

Nowadays the need of developing new forms of urban mobility is of great significance; it is common experience that traffic is chaotic and, frequently, pollution caused by vehicles gets over the critical levels in many cities and for this reason the European Commission (EC) is funding research in this field. The development of a new type of vehicle with environmentally low impact propulsion system and with a novel layout represents an interesting answer to this need.

The Department of Civil and Industrial Engineering of the University of Pisa is involved in the RESOLVE research project (Range of Electric Solution for L-category Vehicles) funded by the EC in the Horizon 2020 framework programme. The aim is to develop two prototypes of novel four-wheeled tilting vehicles (Fig. 1) belonging to the L-category vehicles, L2e and L6e category respctively [EU Regulation(2013)]. These ELVs (Electric L-category Vehicles) will be characterized by low level of energy consumption, high safety requirements and low cost. To achieve that, they will be equipped with modular and scalable electric powertrains and battery architectures.

Since the layout of vehicle is completely new, an in depth analysis of the dynamic properties was mandatory, starting from the stability analysis. The study of the stability of motorcycles is a common subject in the literature [Sharp(1971),Sharp et al.(2004),Sharp and Limebeer(2001),Sharp(2001),Frendo(2006), Nishimi et al.(1985),Limebeer et al.(2001),Cossalter and Lot(2002),Cossalter et al.(2004), Schwab et al.(2005),Cossalter and Lot(2008),Sharp and Jones(1977),Cossalter et al.(2003)]. The first stability analyses were carried out by [Sharp(1971)] who considered the vehicle composed by two rigid bodies, i.e. the front system and the main frame, joined by a revolute joint at the steering head. The front system was composed by the handlebar, the fork and the front wheel. The main frame was composed by the frame itself, the rider, the engine, the gearbox, the swingarm and the rear wheel. A linear steering damper was interposed between the two parts in order to consider the friction effects. Each wheel was considered as rigid disk and the contact regions with ground was represented by a point. The
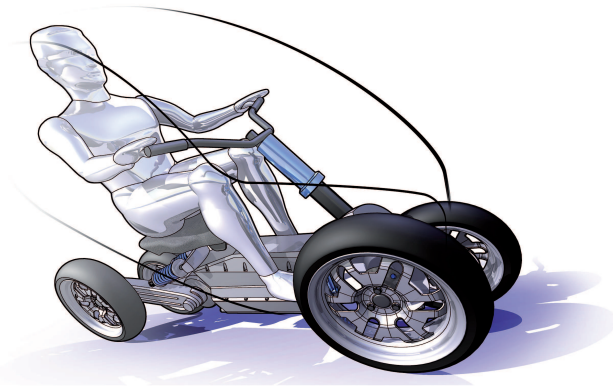
Fig. 1: RESOLVE 4-wheeler tilting vehicle concept.

wheels had not longitudinal slip and rolled on a horizontal flat road surface without rolling resistance. The aerodynamic effects and the pitch movement were not taken into account. In order to investigate the stability in straight running, the 4 dofs dynamic system was linearized and small perturbations from the equilibrium configuration were considered. The lateral tire forces were considered as a linear function of the side-slip and camber angles and a first order differential equation was used to model the tire relaxation effect. The eigenvalues were computed as functions of forward vehicle speed. Three main modes were identified as a function of the longitudinal speed, namely the capsize, the weave and the wobble. Sharp also studied the principal modal shapes when the vehicle was in a corner [Sharp(2001)].

Subsequently, Sharp improved his model in order to include the suspensions deformation, more accurate tire characteristics and tire radial stiffness [Sharp and Limebeer(2001)]. In [Limebeer et al.(2001)] he also investigated the effects of acceleration and deceleration on motorcycle stability. Other mathematical models of motorcycle have been studied in the last years; in [Frendo(2006)] a motorcycle model was presented, in which the influence of the tire model, of the gyroscopic effects of wheels and engine and of the front vehicle geometry on the handling parameters was studied. In [Nishimi et al.(1985)] a model where the rider has two degrees of freedom with respect to the main frame was developed: the lower body of the driver can shift laterally, while the upper is free roll. In [Cossalter et al.(2004)] a modal analysis of a sport motorcycle was presented; the tires were considered having their real tread geometry and radial stiffness, and the position of the tire-road contact point was considered as function of the camber angle and tire deformations.

Recently a new kind of scooters was introduced on the market, which are equipped with two front wheels and a front suspension mechanism that allows the free tilting of the motorcycle. Thanks to this suspension mechanism the vehicle can be driven like a common scooter. The stability analysis of a three wheeler was presented in several papers [Meijaard et al.(2007),

Sponziello et al.(2009),Bartaloni et al.(2007),Hibbard and Karnopp(1993),Rajamani and Gohl(2003), Amati te al.(2011),Sponziello te al.(2010)]. In [Sponziello et al.(2009)] the stability analysis of a three-wheeler based on the first Sharp model was presented. The scooter was made up of rigid bodies, the suspensions were locked and the wheels were considered rigid and lenticular. The wheel-road contact was represented by a point and the lateral tire forces were linear functions of side-slip and camber angles. The driver was rigidly joined to the main frame. The authors studied the stability of the vehicle in straight running and in turning. The results were compared with those of a two-wheeler with similar characteristics and showed how the weave mode for the three-wheeler was more stable. The wobble mode of the three-wheeler was very close to that of a two wheeler and the capsize was always unstable from zero to 60 m/s, while the two-wheeler had a stable capsize mode at low speed, that turned into an unstable mode over 8 m/s.

To the best of the authors knowledge, currently no analysis can be found in the literature dealing with four-wheeler titling vehicles. For this reason, in this paper a mathematical model of a novel four-wheeler is discussed with reference to the stability. This model was developed inspired to the two-wheeler Sharp model and to the-three wheeler model described by [Sponziello et al.(2009)]. A modal analysis of the modes in straight running is presented in this paper. The natural modes of the vehicle were studied in order to establish any similarity with the modes of a two or three-wheeler. Lastly, a sensitivity analysis to the front trail, which is the most influencing stability parameter, is carried out.

A new graphical representation of the eigenmodes is also introduced to easily show the shape of the different eigenmodes at different speed.

2 Mathematical model of the vehicle

2.1 Description of the vehicle

The vehicle, which is schematically shown in Fig. 2, is made up of the 23 bodies which can be split in three subsystems:

- the front tilting system (blue parts in Fig. 2);
- the main frame (black parts in Fig. 2);
- the rear tilting system (red parts in Fig. 2).

Both the front and rear systems are made up of 11 bodies, constrained in order to guarantee one degree of freedom (tilting) to each system. In particular, the front tilting mechanism is composed of two upper and two lower A-arms which are joined to the main frame by revolute joints. On both sides, a hub-carrier is joined to the horizontal arms by two spherical joints, which determine the steering axis of each front wheel. Each superior A-arm is connected to a central rocker arm by a shock absorber, guaranteeing vertical stiffness and allow the free roll of the vehicle. The steering system, which is not represented in the scheme, is composed of two steering rods which link the rotation of
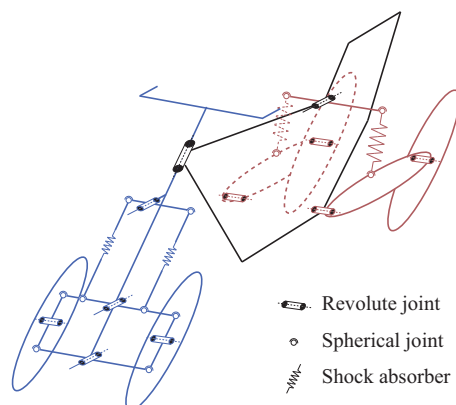
Fig. 2: Scheme of the 4-wheeler.

the handlebar to the steering motion of the wheels about the steering axes. The steering ratio between the the handlebar and the wheels is nearly unitary. The rear tilting system is achieved by two swingarms, one for each rear wheel, that are joined to the main frame by revolute joints. As for the front tilting mechanism, a shock absorber connects each swing arm with a central rocker arm, achieving the desired vertical stiffness and allowing free roll of the vehicle. The main frame, which is assumed as a unique rigid body, reproduces the mass and inertia of the frame itself, the powertrain elements, the rear swing arms, the petrol tank, the driver seat, the rear suspension mechanism and the two rear wheels and of the rider which is supposed to be rigidly fixed to it.

The front system is connected to the main frame by the revolute joint at the steering head and by a revolute joint at the front tilting bar. The rear system is constrained to the main frame by two co-axial revolute joints at the swing arms and by a revolute joint at the rear tilting bar.

2.2 Mathematical model of the vehicle

The following simplifying hypotheses were assumed to limit the complexity of the model:

– the model is made up of rigid bodies;
– the driver is fixed to the main frame and cannot move with respect to it;
– the position of the global center of gravity of the vehicle model is assumed to be invariant in a vehicle reference frame;
– the road is flat and horizontal and the suspensions are assumed infinitely rigid;
– the kinematic pitch, due to the rotation of the front wheels along the steering axes, is neglected;
– the tires are assumed to be lenticular and rigid;
– the longitudinal tire slip is neglected.

It is worth noting that, due to the front and rear tilting mechanisms, the assumption of rigid suspensions does not allow the vehicle to bounce even if roll is possible.

The vehicle reference frame was assumed to have origin in N, which is the projection of the center of mass G on the ground and to have the z-axis always normal to the ground, the x-axis along the vehicle longitudinal direction and directed forward and the y-axis consequently obtained considering a right-handed coordinate system.

Basing on these hypotheses, a set of the state variables which fully describe the model state is the following (Fig. 3)

- longitudinal and lateral speeds (u and v respectively) of a the point N;
- yaw velocity of the main frame r;
- relative rotation between the front system and the rear frame along the steering head, namely the steering angle δ;
- tilting of the main frame with respect to the vertical plane, namely the roll angle ϕ of the vehicle.

The initial position of the point N was also assumed as the origin of the global reference frame.
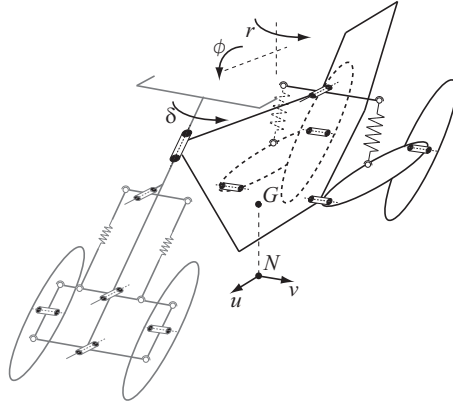

Fig. 3: State variables of the model.

The angular velocity of the main frame system ω_f is

$$\omega_\mathrm{f} = r\mathbf{k} + \dot{\phi}\mathbf{i} \tag{1}$$

and the velocity of the point N is

$$\mathbf{V}_\mathrm{N} = u\mathbf{i} + v\mathbf{j}. \tag{2}$$

Consequently the velocity of the center of mass G is

$$\mathbf{V}_\mathrm{G} = \mathbf{V}_\mathrm{N} + \omega_\mathrm{f} \times NG \tag{3}$$

and its acceleration is

$$\mathbf{a}_G = \mathbf{a}_N + \dot{\omega}_f \times NG + \omega_f \times (\omega_f \times NG) \tag{4}$$

where $\mathbf{a}_N$ is the acceleration of the point N and $\dot{\omega}_f$ is the angular acceleration of the main frame.

Starting from these values, the angular and translational velocity and acceleration of each body was computed on the basis of kinematic constraints.

Equilibrium equations. Nine dynamic equations, which are function of the degrees of freedom and of the geometrical and inertial characteristics of the bodies, can be obtained as follows:

 – three equations for the global translation equilibrium of the vehicle;
 – three equations for the global rotational equilibrium of the vehicle;
 – two equations for the equilibrium of the front system which are the rotational equilibrium along the steering head, without any driver control, and the in-plane equilibrium of the front tilting mechanism;
 – one equation for the equilibrium of the rear tilting mechanism.

The equilibrium equations were obtained by a hybrid approach using both the Newton-Euler and the Lagrangian forms. More in detail, the global translation equilibrium can be expressed by the equation:

$$\mathbf{F}_{11} + \mathbf{F}_{12} + \mathbf{F}_{21} + \mathbf{F}_{22} - m\mathbf{g} + \mathbf{F}_a = m\mathbf{a}_G \tag{5}$$

where $\mathbf{F}_{ij}$, with the first subscript i identifies the axle ($i = 1$ is front, $i = 2$ is rear) and the second subscript j identifies the side ($j = 1$ is left, $j = 2$ is right), are the resultant tire to road forces applied at the four wheels $\mathbf{F}_a$ is the aerodynamic drag force, m is the total mass of the vehicle including the rider. Similarly the rotational equilibrium equation is expressed by

$$GA_{11} \times \mathbf{F}_{11} + GA_{12} \times \mathbf{F}_{12} + GA_{21} \times \mathbf{F}_{21} +$$

$$+ GA_{22} \times \mathbf{F}_{22} + GC_p \times \mathbf{F}_a - \sum_k^N (GG^k \times m^k a_G^k + \dot{k}_G^k) = 0 \tag{6}$$

where GA_{ij} are the the vectors connecting the center of mass G to each wheel to road contact point A_{ij}, GC_p is the vector connecting the center of mass to the aerodynamic drag force center of pressure, GG^k is the vector connecting the vehicle center of mass to each part center of mass G^k, $\dot{k}_G^k$ is the moment due to the inertia forces of each body with respect to the vehicle center of mass G and $n = 23$ is the number of the bodies.

The equations of motion for the front and rear tilting mechanisms were obtained by the application of Lagrange equation

$$\frac{\mathrm{d}}{\mathrm{d}t} \frac{\mathrm{d}T}{\mathrm{d}\dot{\mathbf{q}}} - \frac{\mathrm{d}T}{\mathrm{d}\mathbf{q}} = \mathbf{Q} \tag{7}$$

in which T is the total kinetic energy of the mechanism, $\mathbf{q}$ is the vector of the Lagrangian coordinates and $\mathbf{Q}$ the vector of the generalized component of active forces, i.e. the gravity force and the wheel to road forces.

Tires constitutive equations. For each wheel, the tire lateral force F_{yij} was described by a linear differential equation for each lateral force Fyij

$$F_{\mathrm{yij}} = C_\alpha \alpha_{\mathrm{yij}} + C_\gamma \gamma_{\mathrm{yij}} \tag{8}$$

where C_α and C_γ are the tire slip stiffness and the camber stiffness respectively and α_{yij} ad γ_{yij} are the slip angle and camber angle of each tire. The longitudinal components of rear tires forces F_{x21} and F_{x22} were assumed as inputs for the model and are defined subsequently.

Congruence equations. The set of equations was completed with the congruence equations, which define the slip and camber angles as function of the main vehicle parameters:

$$\gamma_{11} = \gamma_{12} = \arcsin\left(\cos\left(\tau + \theta\right)\cos\left(\phi\right)\sin\left(\delta\right) + \cos\left(\delta\right)\sin\left(\phi\right)\right) \tag{9}$$

$$\gamma_{21} = \gamma_{22} = \phi \tag{10}$$

where τ is the angle between the steering head axis and the axis normal to the tilting kinematism plane and θ is a constant angle related to the suspension geometry.

The tire slip angles are obtained starting from the state variables u, v and r as follows

$$\alpha_{11} \simeq \alpha_{12} \simeq -\delta - \frac{v + rl}{u} \tag{11}$$

$$\alpha_{21} \simeq \alpha_{22} \simeq -\frac{v - rl}{u} \tag{12}$$

were l is the vehicle wheelbase.

Driver model In this paper a straight line maneuver with free handlebar is considered. For this reason a control logic was defined only to keep the speed constant and no torque control was applied to the handlebar. A reference speed u_{d} was defined for each maneuver and the actual longitudinal speed u was measured during the simulation. A proportional controller was set-up to determine the rear traction forces F_{x21} and F_{x22} as follows

$$F_{\mathrm{x21}} = F_{\mathrm{x22}} = k\left(u_{\mathrm{d}} - u\right) \tag{13}$$

where k is the proportionality constant and it was determined by some preliminary simulations.

2.3 Stability analysis

The mathematical model of the four-wheeler is ruled by 21 non-linear equations (9 equilibrium equations, 4 constitutive equations, 8 congruence equations) with 21 variables: longitudinal speed u, lateral speed v, yaw rate r, roll angle ϕ, steering angle δ, four vertical forces F_{zij}, four lateral forces F_{yij}, four slip angles α_{ij} and four camber angles γ_{ij}.

After suitable substitutions the system was linearized and written in the state space form

$$\mathrm{D}\mathbf{x} = A(u)\mathbf{x} \tag{14}$$

where $A(u)$ is the dynamic matrix and $\mathbf{x}$ is the vector of the state variables. In this form the linear system is made up of 11 equations and the state vector $\mathbf{x}$ is made up of the following state variables

$$\mathbf{x} = (F_{y11}, F_{y12}, F_{y21}, F_{y22}, u, v, \dot{\delta}, \dot{\phi}, r, \delta, \phi) \tag{15}$$

The dynamic matrix of the system $A(u)$ is function of the forward speed u and, consequently, the eigenvalues and eigenvectors are function of the speed too.

In order to show the trend of the eigenvalues as a function of the forward speed, the range of speed between 0 and 60 m/s was considered and it was discretized every 0.1 m/s. For each speed step the dynamic matrix $A(u)$ was computed and the eigenvalues and eigenvectors were obtained. It was necessary to implement a post-processing procedure in order to isolate the most interesting eigenvalues. It is worth noting that the vector of the state variables is not homogeneous, the first four components are lateral forces, the following five are linear or rotational velocities and the last two are angles, which may be obtained by integration starting from the steering rate and the roll rate. For this reason, since the information related to the vehicle dynamics are all enclosed in the angular rate components $\dot{\delta}$, $\dot{\phi}$ and r, the study of the eigenmodes was reduced to these components of the eigenvectors. With reference to the Fig. 3, the roll rate is positive when the vehicle tilts on its right side, the steering rate is positive when the handlebar turns leftward and the yaw rate is positive when the main frame turns leftward. Using these reduced eigenvectors it was possible to clearly identify all the modes of the four-wheeler.

3 Results and discussion

3.1 Eigenmodes analysis at different longitudinal speeds

The results of the analyses described in the previous section are shown Fig. 4–5 in terms of real and imaginary parts of the eigenvalues, respectively, as a function of the longitudinal speed u. The trend of the eigenvalues is similar to the trends already found for a two and a three-wheeler [Sponziello et al.(2009)]. Two eigenvalues are real along the whole investigated speed range, namely

capsize and rear wobble. The weave mode is non-oscillatory below 5 m/s and then becomes oscillatory with frequency below 2 Hz. Finally the wobble is oscillatory above about 2 m/s and its frequency is about 5 Hz for $u > 5$ m/s.
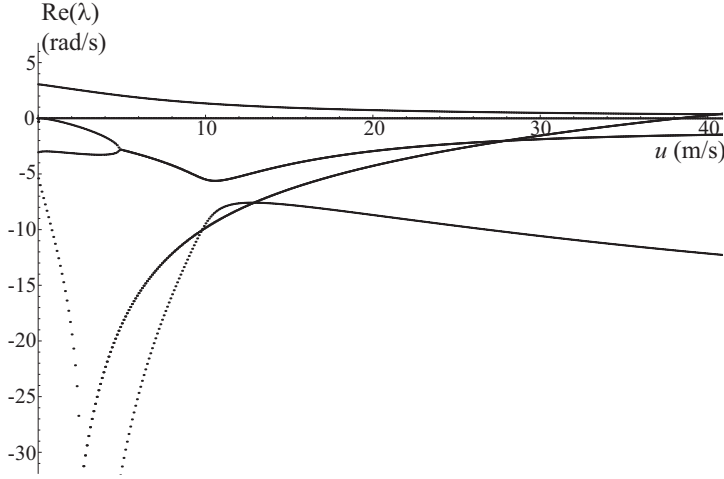

Fig. 4: Real part of the four-wheeler eigenvalues as a function of the longitudinal speed.
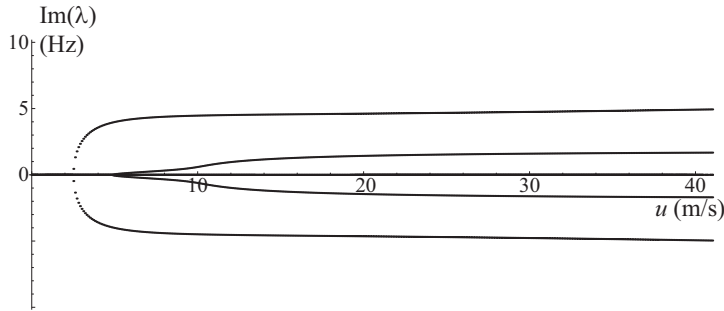

Fig. 5: Imaginary part of the four-wheeler eigenvalues as a function of the longitudinal speed.

In order to investigate how the modal shapes are influenced by the longitudinal speed, an analysis of the eigenvectors was performed, considering the relevant components r, $\dot{\phi}$ and $\dot{\delta}$. The results of the analysis are shown in Fig. 6–9 were the real part of each eigenvalue versus longitudinal speed is shown and, at four selected speed, i.e. 4 m/s, 10 m/s, 20 m/s, 30 m/s, a map of the eigenvector components is shown. Each map is represented as a cartesian

coordinate system having the real and the imaginary part of each eigenvector component on the horizontal and vertical axes, respectively. Two red circles were also drawn on each coordinate system, the smaller having unitary radius and the second having 5 rad/s radius. The eigenvector components were represented by a circle (r), a diamond ($\dot{\phi}$) and a small cross ($\dot{\delta}$). If the eigenvector component amplitude exceeded 5 rad/s, the eigenvector symbol was draft on the external circle and an outward arrow was added as in Fig. 7 and Fig. 8. The eigenvector components were calculated in order to have the amplitude of the yaw rate r unitary and zero-phase.

   Figure 6 shows the capsize mode which is non-oscillatory since its associate eigenvalues is always real; in the case of the analyzed four-wheeler vehicle the capsize eigenvalue is greater than zero, that means that the mode is unstable in the whole investigated speed range. Concerning the eigenvector, the roll rate $\dot{\phi}$ is always in counter-phase with respect to the yaw rate r; with reference to the vehicle scheme in Fig. 3, this means that if the vehicle yaw is leftward also the roll rate will be leftward. The $\dot{\phi}$ component is the largest at any speed value, even if its amplitude decreases as the longitudinal speed rises. The steering rate $\dot{\delta}$ is almost negligible expecially at high speed.
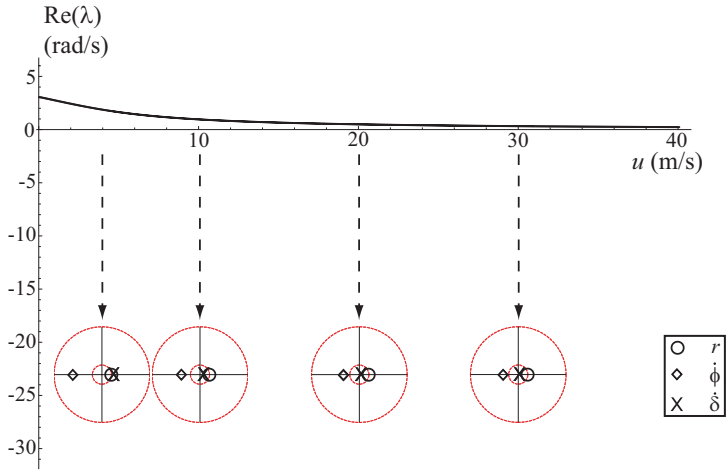

Fig. 6: Capsize eigenvalue and eigenvector representation versus longitudinal speed.

   Figure 7 shows the weave mode, which is characterized by two different real eigenvalues at low longitudinal speed (non-oscillatory behavior) and two complex-conjugate eigenvalues (oscillatory behavior) at higher longitudinal speed; the real part of the eigenvalues is always negative. Concerning the eigenvectors, at 4 m/s, one eigenvector is mainly characterized by roll rate and steering rate, which are in counter phase ; the other eigenvector has almost equivalent contributions of r, $\dot{\phi}$ and $\dot{\delta}$. At longitudinal speed above 5 m/s the

weave mode becomes oscillatory and its eigenvector is characterized by almost equivalents amounts of r, $\dot{\phi}$ and $\dot{\delta}$, with $\dot{\phi}$ and $\dot{\delta}$ in quasi-counter phase with respect to r. This means that when the yaw speed is leftward, also the roll speed is leftward while the steering speed is rightward, which is the typical behavior of the capsize mode. The wobble mode is represented in Fig. 8, is not
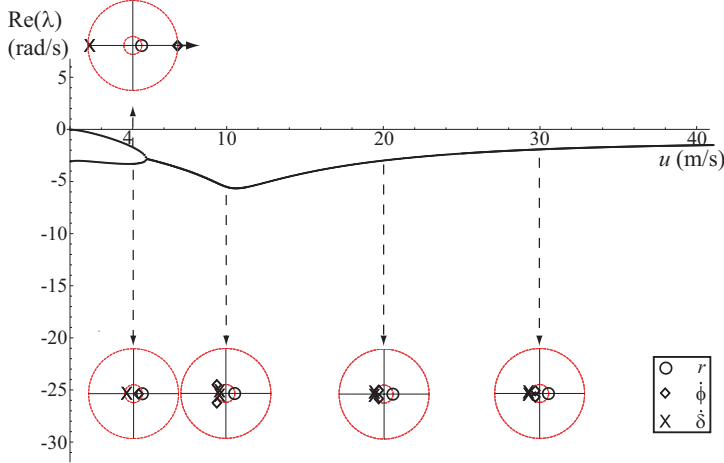

Fig. 7: Weave eigenvalue and eigenvector representation versus longitudinal speed.

influenced by speed in the examined range. The wobble eigenvalue is stable in almost all the considered longitudinal speed range and becomes unstable at about 38 m/s; this shall be taken in due consideration in case of vehicle operating at high speed. At very low speed, two different real eigenvalues were found which becomes two complex-conjugate eigenvalues at about 5 m/s. The main component of the wobble eigenvector is the steering rate which is in quasi-counter-phase with respect to yaw and roll rate in the whole investigated longitudinal speed range.

Finally, the rear wobble is shown in Fig. 9. This mode is non-oscillatory and it is strongly stable in the whole considered longitudinal speed range and for this reason it is usually not deeply investigated in the literature; in particular its eigenvector is not meaningful [Cossalter and Lot(2002)] and its yaw, roll rate and steering rate components significantly varies in the analyzed speed range.

3.2 Sensitivity analysis

Starting from the reference configuration of the four-wheeler considered in the previous section, the influence on the eigenvalues of the front trail a was investigated. This parameter was selected since it was found as the most influencing
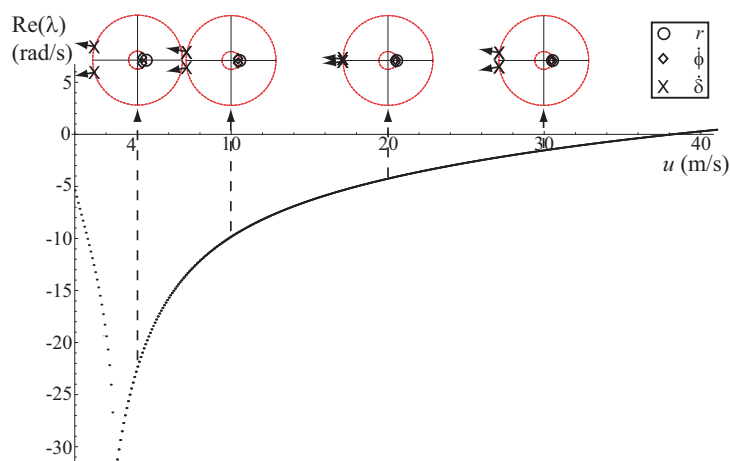
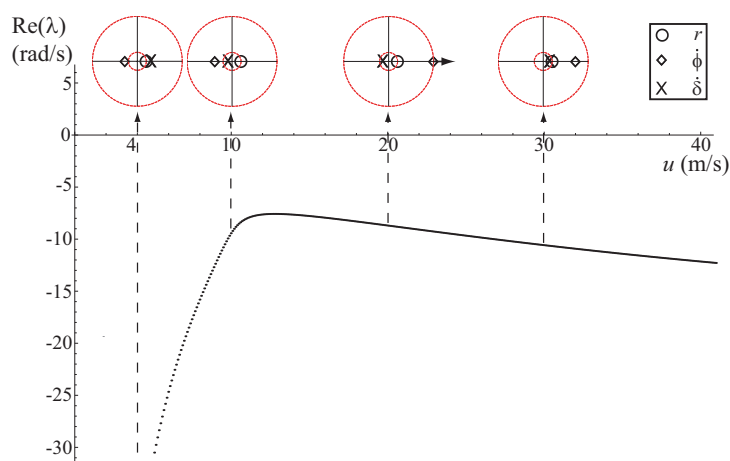Fig. 8: Wobble eigenvalue and eigenvector representation versus longitudinal speed.

Fig. 9: Rear wobble eigenvalue and eigenvector representation versus longitudinal speed.

parameter on stability. Three front trail values were investigated: a) the reference configuration, having front trail a, which gave the results presented in Fig. 4; b) a configuration with halved front trail $a/2$; c) a configuration with doubled front trail $2a$.

The results are shown in Fig. 10–12 and are related to the real part of the eigenvalues; the imaginary part is not shown here since it does not add substantial information to the analysis.

Figure 10 shows the effects of front trail variation on the non-oscillating modes, i.e. the capsize and the rear wobble. It is worth noting that the capsize

eigenvalue is always positive and it is not considerably affected by the front trail variation. On the contrary, the second wobble is strongly affected by the front trail value and it becomes more stable as the front trail value is reduced. It is worth noting that configuration c) shows a different behavior with respect to the other; indeed at longitudinal speed below 5 m/s the rear wobble eigenvalue does not fall at very low (negative) value but it tends to zero as the longitudinal speed diminishes.
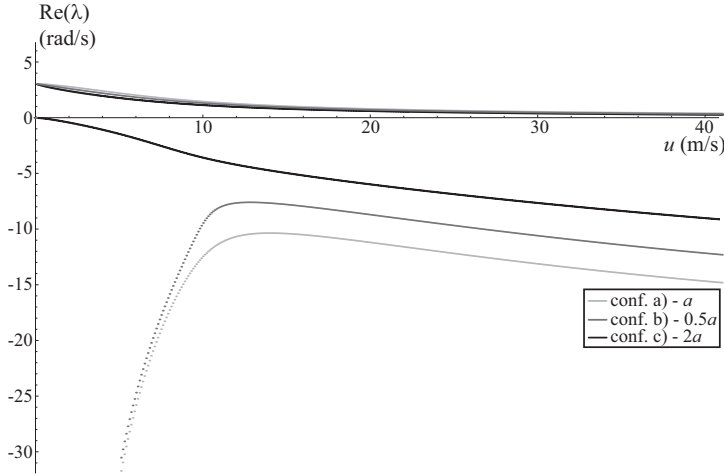

Fig. 10: Capsize and rear wobble modes - Front trail sensitivity analysis.

The weave versus longitudinal speed trend is shown in Fig. 11. Configurations a) and b) are very similar, it can be observed that the transition between non-oscillatory to oscillatory mode happens at 3–4 m/s and the oscillatory weave mode related to the reference configuration is more damped at low speed (below 9 m/s) while it become less damped for higher speed values. Configuration c) shows a different trend: the transition between non-oscillatory and oscillatory mode happens at about 9 m/s and the oscillatory mode is the most damped up to about 25 m/s.

Finally, the influence of the front trail on the wobble eigenvalue is shown in Fig. 12. The curves show that a different value of the front trail does not cause significant differences on wobble stability at low speed, while it significantly affects the speed at which the real part of the eigenvalue becomes positive. As it was already found for two wheelers, the greater the front trail the more stable the wobble mode.
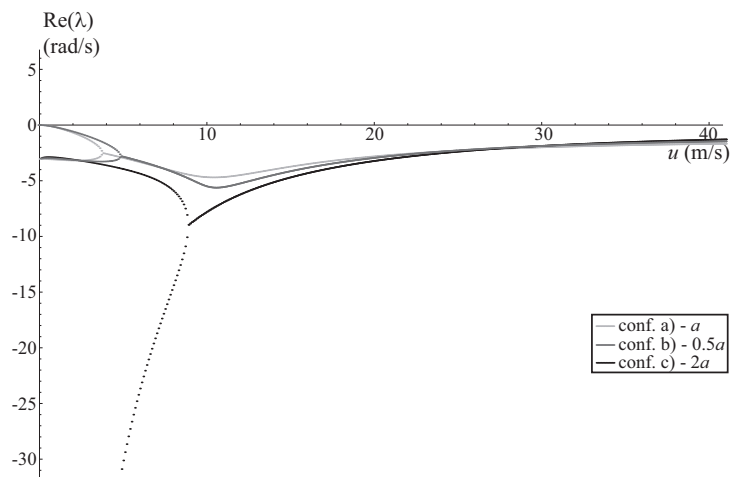
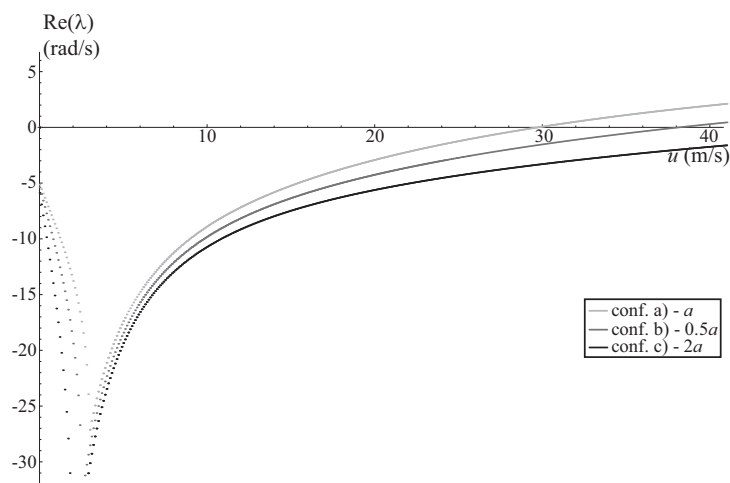Fig. 11: Weave mode - Front trail sensitivity analysis.

Fig. 12: Wobble mode - Front trail sensitivity analysis.

4 Conclusions

In the present work the stability of a novel four wheeled tilting motorcycle was presented based on a mathematical model which did not consider the suspensions travel and the tires radial stiffness. The stability analysis was discussed with reference to eigenvalues and eigenvectors in straight running.

The eigenmodes which were found for the tilting four-wheeler were the well-known capsize, weave, wobble and rear wobble which characterize also two and three-wheelers. A smart graphical representation of the eigenvectors

was also introduced to summarize in a single plot both the eigenvalue and the eigenvector information.

A sensitivity analysis to the front trail was also performed, showing the changes of the trend of the real part of the eigenvalues in straight running and the correlations between weave and rear wobble mode. It was shown that the front trail significantly affects almost all the natural modes; in particular, the weave and second wobble modes are influenced expecially at low speed, while the wobble mode shows a significanltly different trend in the whole speed range, changing also the speeed at which the eigenvelue becomes positive.

The modes which involve the suspension travel can not be analyzed by this simplified model and will be analyzed in the near future.

Acknowledgments

Compliance with Ethical Standards

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