# Interception of Comets and Asteroids on Collision Course with Earth 

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#### Abstract

I delineate the utility, performance, and range of applicability of rocket interceptors designed to disrupt (deflect or pulverize) comets or asteroids on collision course with Earth. I discuss the relationship among several quantities of practical interest in the interception problem, the most important of which are 1) the mass in orbit or initial mass of the interceptor, which will usually dominate the cost of the system, and 2) the blowoff fraction, the fraction of the assailant object's mass expelled to impart transverse momentum, which also provides a measure of the probability that the object will fracture. I calculate optimum interception strategies for both kinetic-energy deflection and nuclear-explosive deflection, assuming a fairly general relationship between the energy deposited and the blowoff mass. In the nuclear-explosive case, I calculate the interceptor mass and cratering effect for detonations above and below the surface as well as directly on the surface of the assailant. Because different assailants could possess a wide range of densities and material properties, the principal value of this work is to show the relationships among the salient parameters.


|  | $\quad$ Nomenclature |
| :--- | :--- |
| $d$ | $=$ diameter of assailant (comet or asteroid) |
| $g$ | = Earth gravitational constant |
| $I_{s p}$ | = specific impulse |
| $M_{a}$ | = mass of assailant |
| $M_{e}$ | = mass of crater ejecta |
| $M_{f}$ | $=$ final mass of interceptor |
| $M_{i}$ | $=$ initial mass of interceptor in orbit |
| $Q$ | $=\ln \left(M_{i} / M_{f}\right)$ |
| $R_{i}$ | $=$ range when assailant is intercepted |
| $R_{l}$ | $=$ range when interceptor is launched |
| $V$ | $=$ interceptor velocity |
| $v$ | $=$ closing speed of assailant |
| $v_{\perp}$ | $=$ assailant transverse velocity component |
| $\alpha$ | $=$ crater constant |
| $\beta$ | $=$ crater exponent |
| $\Delta t$ | $=$ time elapsed from launch to intercept |
| $\delta$ | $=$ energy fraction |
|  | $\sqrt{2 \times \text { ejecta kinetic energy/interceptor kinetic energy }}$ |
| $\epsilon$ | $=$ assailant deflection distance |
| $\rho$ | $=$ assailant density |

## Introduction

SINCE Alvarez et al. ${ }^{1}$ announced evidence for asteroid impact as the putative cause of the cretaceous-tertiary extinction, there has been a heightened awareness that our fair planet is and always has been in a state of merciless cosmic bombardment. Not all of this cannonade has been deleterious; for example, the event Alvarez suggests may have cleared the way for the rise of Homo sapiens. But being a selfish subspecies, we would rather hold on to our domination of the Earth and deny a chance to any more well-adapted creature for as long as we can. Less facetious is the possibility of a strike from an interplanetary body with radius on the order of 100 m . If it were an asteroid, such an assailant would likely have a relative

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velocity of about $25 \mathrm{~km} \cdot \mathrm{~s}^{-1}$, which would give it a kinetic energy of about 1000 Mton . In a populated area, the damage would be catastrophic. If it were a comet, the relative velocity would be more like $50 \mathrm{~km} \cdot \mathrm{~s}^{-1}$, and the energy would quadruple. The Tunguska event ${ }^{2,3}$ (1908) offers sobering evidence that such potentially catastrophic collisions are not so infrequent that they can be ignored. That impact was about 10 Mton and could be expected every few hundred years. Recent estimates ${ }^{4}$ indicate that a 20 -kton (Hiroshima-size) event should occur every year. This ought to be conspicuous, but apparently objects of this size tend to break up while penetrating the atmosphere, ${ }^{5}$ dissipating much of their energy as smaller frágments. That larger cataclysms are not generally recorded in the archives of natural disasters seems somewhat of a mystery. Perhaps it can be attributed to the fact that, until the 20th century, very little of the Earth's surface was populated. ${ }^{6}$ Nevertheless, it has been asserted that the risk of being killed as a result of asteroid impact is somewhat greater than the risk of being killed in an airplane crash. ${ }^{7}$
The first serious study of a defense against asteroid collision $^{8}$ was conducted as an interdepartmental student project in systems engineering at the Massachusetts Institute of Technology. Remarkably, the study was conducted in the spring of 1967, a dozen years before the Alvarez discovery. The hypothesis was a predicted 1968 collision with Icarus, a kilometerscale Apollo asteroid. The prognosis was deployment of six Saturn V rockets carrying $100-\mathrm{Mton}$ warheads to deflect or pulverize the asteroid.

In 1981, NASA and the Jet Propulsion Laboratory (JPL) sponsored a workshop ${ }^{9}$ to evaluate the rate and consequences of collisions with both asteroids and comets, which I collectively call astral assailants at the risk of creating a pathetic fallacy. The workshop also considered requirements for a mission capable of deflecting an asteroid from Earth collision and concluded that it was probably within technological means. It seems unlikely, however, that an object requiring deflection would be detected over a period of time for which the technology was relevant.
In 1984, Hyde $^{10}$ further explored using nuclear explosives to counter astral assailants. In 1990, Wood et al. ${ }^{11}$ showed that defense against small assailants could be accomplished with nonnuclear interceptors, largely using the kinetic energy of the assailant itself.
The problem of preventing a collision with a comet or asteroid can be considered two domains: 1) actions to be taken if the collision can be predicted several orbital periods in advance and can be averted by imparting a small change in
velocity (most effectively at perihelion); and 2) actions to be taken when the object is less than an astronomical unit (AU) away, collision is imminent, and deflection or disruption must be accomplished as the object closes on Earth. I call the first domain of actions remote interdiction and the second domain of actions terminal interception.

If all of the Earth-threatening asteroids were known, the orbits could be calculated and the process of deflection could be carried out in a leisurely manner. Remote interdiction would be the option of choice. But $99 \%$ have not yet been discovered. ${ }^{12}$ Furthermore, there are an enormous number of unknown long-period comets for which a thorough search is completely impractical.

Asteroids in the $100-\mathrm{m}$ size range are exceedingly difficult to detect unless they are very close. Comets in this size range are more conspicuous owing to their coma, but they move a lot faster and can be in retrograde orbits or out of the plane of the ecliptic. In either case, it seems likely we will have little time to respond to a potential collision. It therefore appears that terminal interception-disruption or deflection at relatively close range-is the most important issue.

In this paper, I consider the dynamics of the terminal intercept problem. I explore the possibility of using kinetic-energy deflection as well as nuclear explosives. Nuclear explosives can be employed in three different modes depending on their location at detonation: 1) buried below the assailant's surface by penetrating vehicle, 2) detonated at the assailant's surface, or 3 ) detonated some distance above the surface.

## Interception and Deflection Scenario

Figure 1 shows the interception scenario. In Fig. 1a, the asteroid or comet is headed toward Earth at a velocity $v$. The interceptor traveling at a diametric velocity $V$ is about to engage the assailant object. The assailant has a mass $M_{a}$, and the interceptor, because it has long since exhausted its fuel, has its final mass $M_{f}$. We cannot hope to deflect the assailant like a billiard ball because $M_{a} \gg M_{f}$. So the interceptor must supply energy to blow off a portion of the assailant's surface, as shown in Fig. 1b. The blowoff material is very massive compared with the interceptor, $M_{a} \gg M_{e} \gg M_{f}$. One might think that a conventional high explosive would suffice, but the energy it would supply would be relatively insignificant. Standard high explosive releases $10^{3}$ calories $=4.184 \times 10^{10} \mathrm{erg}$ per gram. An asteroid moving at $25 \mathrm{~km} \cdot \mathrm{~s}^{-1}$ has a specific energy of $3.125 \times 10^{12} \mathrm{erg}$ per gram-about 75 times the specific energy of high explosive. If the interceptor is moving at the same speed in the opposite direction ( $V=v=25 \mathrm{~km} \cdot \mathrm{~s}^{-1}$ ), the interceptor would impact with a specific energy 300 times that of high explosive. There is a whole lot of kinetic energy available; a chemical energy release would be in the noise. However, even this tremendous kinetic energy would be completely swamped by a nuclear explosive. The yield-to-weight ratio of nuclear explosives is generally measured in kilotons per kilogram, that is, tons per gram. A typical specific energy is a million times that of chemical high explosive or about four


Fig. 1 Interception scenario: a) interceptor about to engage the assailant, and b) interceptor supplies energy to blow off a portion of the assailant's surface and imparts a transverse velocity.
orders of magnitude higher than the kinetic energy of the interceptor collision.

## Kinetic-Energy Deflection

The final velocity of an interceptor missile relative to the Earth, or the orbit in which it is stationed, is given by the rocket equation,

$$
\begin{equation*}
V=g I_{s p} \ln \frac{M_{i}}{M_{f}} \tag{1}
\end{equation*}
$$

In general, the time required to reach this relative velocity will be short compared with the total flight time. The time elapsed from launch to intercept is

$$
\begin{equation*}
\Delta t=\frac{R_{l}}{v+V} \tag{2}
\end{equation*}
$$

So the range at which the assailant is intercepted will be given by

$$
\begin{equation*}
R_{i}=R_{l}\left(1-\frac{v}{v+V}\right) \tag{3}
\end{equation*}
$$

If the impact gives the assailant a transverse velocity component $v_{\perp}$, then the threatening assailant will miss its target point by a distance

$$
\begin{equation*}
\epsilon=R_{l} \frac{v_{\perp}}{\stackrel{v}{v}}\left(\frac{V}{v+V}\right) \tag{4}
\end{equation*}
$$

where I have neglected the effect of the Earth's gravitational focusing and used a linear approximation to Keplerian motion. To obtain the transverse velocity component, we would use the kinetic energy of the interceptor to blast a crater on the side of the assailant. The momentum of the ejecta would be balanced by the transverse momentum imparted to the assailant. From Glasstone's empirical fits, ${ }^{13}$ the mass of material in the crater produced by a large explosion is

$$
\begin{equation*}
M_{e}=\alpha^{2} E^{\beta} \tag{5}
\end{equation*}
$$

where $\alpha$ and $\beta$ depend on the location of the explosion, the soil composition and density, gravity, and a myriad of other parameters. Clearly the crater constant $\alpha$ and the crater exponent $\beta$ will vary depending on whether we are considering an assailant composed of nickel iron, stony nickel iron, stone, chondrite, ice, or dirty snow. For almost every situation, however, we find $\beta \simeq 0.9$.
The kinetic energy available when the interceptor collides with the astral assailant is

$$
\begin{equation*}
E=1 / 2 M_{f}(V+v)^{2} \tag{6}
\end{equation*}
$$

Only a fraction of the interceptor's kinetic energy is converted to kinetic energy of the ejected or blowoff material. Let this fraction be equal to $1 / 2 \delta^{2}$, or

$$
\begin{equation*}
\delta=\sqrt{2 \frac{\text { ejecta kinetic energy }}{\text { interceptor kinetic energy }}} \tag{7}
\end{equation*}
$$

The reason for this strange definition is that it greatly simplifies the algebra. I will call the parameter $\delta$ the energy fraction. Then the transverse velocity imparted to the assailant is

$$
\begin{align*}
v_{\perp} & =\delta \frac{\sqrt{M_{e} E}}{M_{a}}=\frac{\delta}{M_{a}} \sqrt{\frac{M_{e} M_{f}(V+v)^{2}}{2}} \\
& =\frac{\alpha \delta}{M_{a}}\left[\frac{M_{f}(V+v)^{2}}{2}\right] \frac{\beta+1}{2} \tag{8}
\end{align*}
$$



Fig. 2 Dimensionless plot of kinetic-energy deflection.

We can combine Eqs. (4), (5), and (8) to obtain

$$
\begin{equation*}
\epsilon=\alpha \delta R_{l} \frac{V(V+v)^{\beta}}{M_{a} v}\left(\frac{M_{f}}{2}\right)^{\frac{\beta+1}{2}} \tag{9}
\end{equation*}
$$

Equation (9) reveals the importance of the intercept velocity $V$, which is proportional to specific impulse $I_{s p}$. If $V \leqslant v$, the deflection is proportional to $V$, and if $V \gg v$, the deflection is proportional to $V^{\beta+1} \sim V^{2}$.

## Optimum Mass Ratio for Kinetic-Energy Deflection

The energy on impact is proportional to the final mass of the interceptor, and the square of its relative velocity as given in Eq. (6). The smaller its final mass, the higher its relative velocity, so there is some optimum mass ratio that produces the greatest deflection for a given initial mass. This would be the optimal interceptor design, the most bang for the buck.

Substituting Eq. (1) into Eq. (9), setting

$$
\begin{equation*}
\frac{d \epsilon}{d\left(M_{i} / M_{f}\right)}=0 \tag{10}
\end{equation*}
$$

and solving, we find the mass ratio that produces the largest value of $\epsilon$,

$$
\begin{equation*}
\frac{M_{i}}{M_{f}}=e^{Q} \tag{11}
\end{equation*}
$$

where

$$
\begin{equation*}
Q=1-\frac{v}{2 g I_{s p}}+\sqrt{1+\frac{1-\beta}{1+\beta} \frac{v}{g I_{s p}}+\left(\frac{v}{2 g I_{s p}}\right)^{2}} \tag{12}
\end{equation*}
$$

We note that this optimal mass ratio depends only on the velocity of the assailant relative to Earth $v$ and the interceptor's specific impulse $I_{s p}$. The value of $\beta$ is a constant of the assailant's composition and is very close to 0.9 , and $g \simeq 980$ $\mathrm{cm} \cdot \mathrm{s}^{-2}$ is a constant of planet Earth. In the limit of very high specific impulse, the optimum mass ratio is

$$
\begin{equation*}
\frac{M_{i}}{M_{f}}=e^{2} \tag{13}
\end{equation*}
$$

This limit can be approached but is not realistic owing to $v / g I_{s p}$ limitations. The maximum displacement of the impact location on Earth is then given by

$$
\begin{equation*}
\epsilon=\frac{\alpha \delta v^{\beta} R_{l}}{M_{a}}\left(\frac{M_{i} e^{-Q}}{2}\right)^{\frac{\beta+1}{2} g I_{s p} Q} \frac{v}{v}\left(1+\frac{g I_{s p} Q}{v}\right)^{\beta} \tag{14}
\end{equation*}
$$

Remarkably, when Eq. (12) is put into Eq. (14), the resulting exceedingly complex expression can be put in dimensionless form. Figure 2 plots the dimensionless parameter $\epsilon M_{a} /$ $\alpha \delta v^{\beta} R_{l} M_{i}^{1 / 2(\beta+1)}$ vs the dimensionless parameter $g I_{s p} / v$ for
$\beta=0.8,0.9$, and 1.0 . It shows the increasing advantage to higher specific impulse derived from Eq. (14).

A great deal of physical insight can be obtained just by studying the axis labels of the dimensionless plot. From the ordinate, we see that for the same value of $g I_{s p} / v$, which is more or less fixed by interceptor design, the asteroid deflection $\epsilon$ is proportional to the range of the assailant at launch $R_{l}$, inversely proportional to the mass of the assailant $M_{a}$, nearly proportional to the velocity of the assailant relative to Earth $v^{\beta} \approx v^{0.9}$, nearly proportional to the initial mass of the interceptor $M_{i}^{1 /(\beta+1)} \simeq M_{i}^{0.95}$, proportional to the crater constant $\alpha$, and proportional to the square root of the fraction of interceptor kinetic energy converted to blowoff kinetic energy $1 / 2 \delta^{2}$.

Equation (14) can be rearranged to give the required initial mass or mass in orbit of the interceptor,

$$
\begin{equation*}
M_{i}=2 e Q\left[\frac{M_{a} v \epsilon}{\alpha \delta R_{l} g I_{s p} Q}\left(\frac{1}{v+\dot{g} I_{s p} Q}\right)^{\beta}\right] \frac{2}{\beta+1} \tag{15}
\end{equation*}
$$

The mass given by Eq. (15) will generally be the largest single factor in the cost of a defensive system of this sort. To appreciate the magnitude of the problem, it is now necessary to put in a few numbers. The best chemical fuels might have a specific impulse as high as 500 s , which I will use to make the point. The density of potential astral assailants varies greatly, from less than $1 \mathrm{gm} \cdot \mathrm{cm}^{-3}$ for a snowball comet to a little over $1 \mathrm{gm} \cdot \mathrm{cm}^{-3}$ for a dirty ice comet to about $3 \mathrm{gm} \cdot \mathrm{cm}^{-3}$ for a chondrite to about $8 \mathrm{gm} \cdot \mathrm{cm}^{-3}$ for a nickel-iron asteroid. An agreeable average is $3.4 \mathrm{gm} \cdot \mathrm{cm}^{-3}$. The velocity of the assailant relative to Earth could range from $5 \mathrm{~km} \cdot \mathrm{~s}^{-1}$ for an asteroid in nearly coincident orbit with Earth to $70 \mathrm{~km} \cdot \mathrm{~s}^{-1}$ for a long-period comet in retrograde orbit near the plane of the ecliptic. I will take $25 \mathrm{~km} \cdot \mathrm{~s}^{-1}$ for this example.

Because the material properties of asteroids and comets vary so widely, an estimate of the crater constant and crater exponent is somewhat arbitrary. Here I will make an estimate for impact cratering of medium hard rock. Glasstone uses $\beta \approx 0.9$ and $\alpha \approx 8.4 \times 10^{-4} \mathrm{gm}^{1 / 2(1-\beta)} \cdot \mathrm{cm}^{-\beta} \cdot \mathrm{s}^{\beta}$ for an explosive buried at the optimal depth for maximum ejection of dry soil. For a surface burst, Glasstone takes $\alpha \simeq 1.6 \times 10^{-4}$ $\mathrm{gm}^{1 / 2(1-\beta)} \cdot \mathrm{cm}^{-\beta} \cdot \mathrm{s}^{\beta}$. The correct value of $\alpha$ for the impact crater is somewhere between a surface burst and an optimally buried explosion. For the purpose of estimating the crater size for kinetic-energy deflection, I will take $\alpha \simeq 2 \times 10^{-4}$ $\mathrm{gm}^{1 / 2(1-\beta)} \cdot \mathrm{cm}^{-\beta} \cdot \mathrm{s}^{\beta}$. Kreyenhagen and Schuster ${ }^{14}$ have noted that impacts in the $20 \mathrm{~km} \cdot \mathrm{~s}^{-1}$ range couple $50-80 \%$ of their energy to the ground, whereas surface bursts couple only $1-10 \%$. I will assume about $60 \%$ coupling and about half that goes to the blowoff. Thus about $30 \%$ of the interceptor's kinetic energy is converted to kinetic energy of the blowoff, corresponding to $\delta \simeq 0.775$.


Fig. 3 Initial masses of optimally designed interceptors using kineticenergy deflection for ocean diversion ( $1 \mathbf{~ M m}$ ).

Figure 3 shows the initial mass of the interceptor required to deflect the astral assailant by 1 Mm , as a function of the assailant's diameter $d$ and its range when the assailant is launched $R_{l}$. I have assumed an assailant density of $\rho=3.4$ $\mathrm{gm} \cdot \mathrm{cm}^{-3}$, an assailant velocity of $v=25 \mathrm{~km} \cdot \mathrm{~s}^{-1}$, a crater exponent of $\beta=0.9$, a crater constant of $\alpha=2 \times 10^{-4}$ $\mathrm{gm}^{1 / 2(1-\beta)} \cdot \mathrm{cm}^{-\beta} \cdot \mathrm{s}^{\beta}$, and an energy fraction of $\delta \simeq 0.775$. A $1-\mathrm{Mm}$ deflection is typical of the course change required to divert the assailant from impact in a populated area to a nearby ocean. To interpret the figure for a $10-\mathrm{Mm}$ deflection, which would be conservative for missing the planet entirely $\left(R_{\odot}=6.378 \mathrm{Mm}\right)$, multiply the masses by a factor of 10 ( $M_{i} \propto \epsilon^{2 /(\beta+1)}$, so a factor of 10 in $\epsilon$ corresponds to a factor of 11.3 in $M_{i}$ ).

An ocean impact is not without damage, ${ }^{15,16}$ but, in general, the damage will be far less than if the impact were in a populated area. Roughly speaking, the height of the wave and the distance from impact are jointly proportional to the square root of the energy deposited in the water. ${ }^{17}$ A $100-\mathrm{m}$ diameter chondrite might typically impact with $100^{\prime}$ Mton of energy. Taking about $5 \%$ of that energy ${ }^{18}$ as coupled to the water, a water wave of about 3 m in height would be encountered 100 km from the impact point.

To interpret Fig. 3 for a $10-\mathrm{Mm}$ deflection, which would be conservative for missing the planet entirely ( $R_{\odot}=6.378 \mathrm{Mm}$ ), we need to multiply the masses by about a factor of 10 . [From Eq. (15), $M_{i} \propto \epsilon^{2 /(\beta+1)}$, so a factor of 10 in $\epsilon$ corresponds to a factor of 11.3 in $M_{i}$.] Figure 3 makes a clear statement about the applicability of kinetic-energy deflection. Kinetic-energy deflection is practical only for assailants considerably less than 100 m in diameter. To handle a $100-\mathrm{m}$ assailant would require a 1000 -ton interceptor even if launched when the assailant was still $1 / 10$ AU away. The mass would go to 10,000 ton if the assailant were deflected to miss the planet entirely rather than diverted to an ocean. Thus, dealing with assailants larger than 100 m requires another technology.

## Kinetic-Energy Fragmentation and Pulverization

Equation (15) gives the initial mass of an optimally designed interceptor for deflecting an astral assailant by blowing off its surface. It was derived under the assumption that the amount of mass blown off is small compared with the assailant's mass. If the ejected mass is too large, the crater will have dimensions a significant fraction of the assailant's dimension, and it is more likely that the assailant will break up. If the fragments are too large and are scattered at random, they may still be able to penetrate the Earth's atmosphere and do damage. A 2-m fragment of a nickel-iron asteroid has about the same average $\rho r$ as the atmosphere measured vertically from sea level and thus will penetrate the atmosphere losing only about half its energy. A $50-\mathrm{m}$ chondrite, however, will probably break up owing to the dynamic stress of traversing the atmosphere. Shock from the energy of its explosion may still do


Fig. 4 Blowoff fraction for ocean diversion ( $1 \mathbf{~ M m}$ ) using kinetic-energy deflection.


Fig. 5 Asymptotic decrease of blowoff fraction with specific impulse.
damage as appears to have been the case with Tunguska. It should be noted that the blast from a bomb does more damage to a city when the detonation is some altitude above the target than when the detonation is on the surface. To insure that no damage is done, it will be necessary to pulverize the assailant, that is, break it into very small pieces that are sure to dissipate all of their energy in the atmosphere.

To get a handle on the problem of whether the assailant will be deflected, fragmented, or pulverized, we need an estimate of what fraction of the assailant will be blown off in the collision. By combining Eqs. (1), (5), (11), and (15), we find that the fraction of the assailant blown off is given by

$$
\begin{align*}
f & =\frac{M_{e}}{M_{i}}=\frac{\alpha^{2}}{M_{a}}\left(\frac{M_{i}}{2 e^{Q}}\right)^{\beta}\left(g I_{s p} Q+v\right)^{2 \beta} \\
& =M_{a}^{\frac{\beta-1}{\beta+1}}\left\{\alpha\left[\frac{\epsilon v}{R_{l} \delta}\left(1+\frac{v}{g I_{s p} Q}\right)\right]^{\beta}\right\} \frac{2}{1+\beta} \tag{16}
\end{align*}
$$

where $Q$ is again given by Eq. (12). Some qualitative features of the blowoff fraction are immediately apparent.

1) The blowoff fraction is nearly independent of assailant mass, $M_{a}^{(\beta-1) /(\beta+1)} \simeq M_{a}^{-0.0526}$.
2) The blowoff fraction is nearly proportional to the crater constant, $\alpha^{2 /(1+\beta)} \simeq \alpha^{1.05}$.
3) The blowoff fraction is nearly inversely proportional to the energy coupling, $\delta^{2 \beta /(1+\beta)} \simeq \delta^{0.947}$.
4) The blowoff fraction decreases asymptotically with specific impulse.

Using the aforementioned parameters, Fig. 4 shows the blowoff fraction for ocean diversion as a function of assailant diameter for three different ranges to the assailant at interceptor launch $\left(\rho=3.4 \mathrm{gm} \cdot \mathrm{cm}^{-3}, \quad \beta=0.9, \alpha=2 \times 10^{-4}\right.$ $\left.\mathrm{gm}^{1 / 2(1-\beta)} \cdot \mathrm{cm}^{-\beta} \cdot \mathrm{s}^{\beta}, v=25 \mathrm{~km} \cdot \mathrm{~s}^{-1}, \delta \simeq 0.775\right)$. If more than $10 \%$ is blown off, the assailant will probably break up. What we learn from Fig. 4 is that, if we cannot launch the interceptor at about 1/30 AU or better, we cannot deflect the assailant without fracturing it. Under those circumstances, it is better to try to pulverize it with an array of masses, probably resembling spears for maximum penetration.

Equation (16) suggests a way to beat the fracture problem. The blowoff fraction can be reduced by increasing the specific impulse. Figure 5 shows the blowoff fraction as a function of specific impulse for a $100-\mathrm{m}$ assailant with the mission launched at a range of $1 / 100 \mathrm{AU}$. With a specific impulse of 500 , over $14 \%$ of the assailant mass is blown off, whereas at a specific impulse of 5000 , less than $4 \%$ is blown off.

## Nuclear-Explosive Deflection

Much more deflection can be obtained if a nuclear explosive is used to provide the cratering energy. In this scenario, most of the weight after the rocket fuel is expended would be the nuclear explosive, which produces a yield of

$$
\begin{equation*}
E=\varphi M_{f} \tag{17}
\end{equation*}
$$

where $\varphi$ is the yield-to-weight ratio. Again, $\delta^{2} / 2$ of this energy goes into the dirt ejected from the crater, so the transverse velocity imparted to the assailant is

$$
\begin{equation*}
\nu_{\perp}=\frac{\delta}{M_{a}} \sqrt{\varphi M_{f} M_{e}}=\frac{\alpha \delta}{M_{a}}\left(\varphi M_{j}\right)^{\frac{\beta+1}{2}} \tag{18}
\end{equation*}
$$

We can combine Eqs. (4), (5), and (18) to obtain

$$
\begin{equation*}
\epsilon=\frac{\alpha \delta R_{l}}{M_{a} v} \frac{V\left(\varphi M_{f}\right)^{\frac{\beta+1}{2}}}{V+v} \tag{19}
\end{equation*}
$$

## Optimum Mass Ratio for Nuclear-Explosive Deflection

Substituting Eq. (1) into Eq. (19) and solving Eq. (10), we find the logarithm of the mass ratio that produces the largest value of $\epsilon$,

$$
\begin{equation*}
Q=-\frac{v}{2 g I_{s p}}+\frac{1}{2} \sqrt{\frac{8 v}{(1+\beta) g I_{s p}}+\left(\frac{v}{g I_{s p}}\right)^{2}} \tag{20}
\end{equation*}
$$

In the limit of very high specific impulse, the optimum mass ratio is

$$
\begin{equation*}
\frac{M_{i}}{M_{f}}=1 \tag{21}
\end{equation*}
$$

In the limit of very low specific impulse, the optimum mass ratio is

$$
\begin{equation*}
\frac{M_{i}}{M_{f}}=\exp \left(\frac{2}{1+\beta}\right) \tag{22}
\end{equation*}
$$

The maximum displacement of the impact location on Earth is then given by

$$
\begin{equation*}
\epsilon=\frac{\alpha \delta R_{l}}{M_{a} v} \frac{g I_{s p} Q\left(\varphi M_{i} e^{-Q}\right)^{\frac{\beta+1}{2}}}{g I_{s p} Q+v} \tag{23}
\end{equation*}
$$

For a surface burst, Glasstone uses $\beta=0.9$ but takes $\alpha \simeq 1.6 \times 10^{-4} \mathrm{gm}^{1 / 2(1-\beta)} \cdot \mathrm{cm}^{-\beta} \cdot \mathrm{s}^{\beta}$. He describes the medium as dry soil. Medium strength rock would be more consistent with $\alpha \simeq 10^{-4} \mathrm{gm}^{1 / 2(1-\beta)} \cdot \mathrm{cm}^{-\beta} \cdot \mathrm{s}^{\beta}$ and, in the 20-kton range, would roughly agree with Cooper. ${ }^{19}$ If about $5 \%$ of the nu-clear-explosive energy goes into the kinetic energy of the blowoff, then $\delta=1 / \sqrt{10} \simeq 0.316$.

Equation (23) can be rearranged to give the required initial mass of the interceptor,

$$
\begin{equation*}
M_{i}=\frac{e^{Q}}{\varphi}\left[\frac{M_{a} v \epsilon}{\alpha \delta R_{l}}\left(1+\frac{v}{g I_{s p} Q}\right)\right] \frac{2}{\beta+1} \tag{24}
\end{equation*}
$$

where now $Q$ is given by Eq. (20).
It is generally known that the yield of nuclear warheads can be a few kilotons per kilogram if they weigh more than about a hundred kilograms. For the purpose of these estimates, I will take the conservative of $\varphi=1 \mathrm{kton} \cdot \mathrm{kg}^{-1}$. Figure 6 is analogous to Fig. 3, using the values of $\alpha$ and $\delta$ given earlier. Ocean deflection of 1 Mm is sought, and the following values are used: $\rho=3.4 \mathrm{gm} \cdot \mathrm{cm}^{-3}, \nu=25 \mathrm{~km} \cdot \mathrm{~s}^{-1}, \beta=0.9$, $\alpha=10^{-4} \mathrm{gm}^{1 / 2(1-\beta)} \cdot \mathrm{cm}^{-\beta} \cdot \mathrm{s}^{\beta}$ and $\delta \simeq 0.316$.
A good way to compare kinetic-energy deflection with nu-clear-explosive deflection is to look at the ratio of the initial masses of the interceptors. If we divide Eq. (24) by Eq. (15), we see that all variables drop out except specific impulse $I_{s p}$, the assailant's velocity $v$, the energy fraction $\delta$, and the cratering constant $\alpha$. For a comparison of the techniques, we would keep the same values of $I_{s p}$ and $v$. We define the ratio

$$
\begin{equation*}
R_{m}=\frac{M_{i} \text { given by Eq. (24) }}{M_{i} \text { given by Eq. (15) }} \tag{25}
\end{equation*}
$$



Fig. 6 Initial masses of optimally designed interceptors using nu-clear-explosive deflection for ocean diversion ( $1 \mathbf{~ M m}$ ).

The appropriate dimensionless ratio for the comparison is

$$
\begin{equation*}
R_{m} \frac{\alpha_{n} \delta_{n}}{\alpha_{k} \delta_{k}} \tag{26}
\end{equation*}
$$

where the subscripts $n$ refer to the parameters for nuclear-explosive deflection and the subscripts $k$ refer to the parameters for kinetic-energy deflection. This is the actual ratio of initial interceptor weights for kinetic-energy vs nuclear-explosive deflection. Figure 7a shows this ratio as a function of assailant velocity $v$ for specific impulse $I_{s p}=500 \mathrm{~s}$. Figure 7 b shows the same ratio as a function of specific impulse $I_{s p}$ for assailant velocity $v=25 \mathrm{~km} \cdot \mathrm{~s}^{-1}$. Figure 7c shows the same ratio as a function of both specific impulse and assailant velocity. For the numerical examples we have chosen, we have

$$
\begin{equation*}
\frac{\alpha_{n} \delta_{n}}{\alpha_{k} \delta_{k}}=\frac{10^{-4} \times 0.316}{2 \times 10^{-4} \times 0.775}=0.204 \tag{27}
\end{equation*}
$$

So for my particular selection of parameters, we can read the mass ratios in Figs. 7a, 7b, and 7c by multiplying the number on the vertical axis by 0.204 .

From Figs. 7a, 7b, and 7c, we learn the following qualitative features.

1) The interceptor weight is about three orders of magnitude less for nuclear-explosive deflection than for kinetic-energy deflection.
2) The advantage of nuclear-explosive deflection decreases significantly with assailant velocity.
3) The advantage of nuclear-explosive deflection decreases slightly with specific impulse.

## Nuclear-Explosive Fragmentation and Pulverization

By combining Eqs. (1), (5), (11), and (24), we find that the blowoff fraction is given by

$$
\begin{align*}
f & =\frac{M_{e}}{M_{i}}=\frac{\alpha^{2}}{M_{a}}\left(\frac{\varphi M_{i}}{e^{Q}}\right)^{\beta} \\
& =M_{a}^{\frac{\beta-1}{\beta+1}}\left\{\alpha\left[\frac{\epsilon v}{R_{l} \delta}\left(1+\frac{v}{g I_{s p} Q}\right)\right]^{\beta}\right\} \frac{2}{1+\beta} \tag{28}
\end{align*}
$$

where $Q$ is given by Eq. (20). Somewhat remarkably, Eq. (28) is independent of $\varphi$ and has the same form as Eq. (16). The only differences are 1) the different form of $Q$, and 2) the value of the energy fraction $\delta$, and 3 ) the value of the cratering constant $\alpha$.

Figure 8 shows the blowoff fraction for planetary miss ( 10 Mm ) as a function of assailant diameter for two different ranges to the assailant at interceptor launch. If the interceptor is launched at a range much closer than $1 / 3 \mathrm{AU}$, the assailant will be fragmented rather than deflected.


Fig. 7 Ratio of kinetic-energy interceptor mass to nuclear-explosive interceptor mass: a) ratio vs $v$ for $I_{s p}=500 \mathrm{~s}, \mathrm{~b}$ ) ratio vs $I_{s p}$ for $v=\mathbf{2 5}$ $\mathrm{km} \cdot \mathrm{s}^{-1}$, and c) ratio vs $I_{s p}$ and $v$.


Fig. 8 Blowoff fraction for collision avoidance ( 10 Mm ) using nu-clear-explosive deflection.

## Penetrators

The biggest crater is not produced by a surface blast but by an explosive buried some distance below the surface. Clearly if it is buried too deeply, it will produce no crater at all. The optimum depth for cratering is a function of all of the usual parameters describing material properties but, quite importantly, gravity, which to a large extent, can be ignored for comets and asteroids. For dry soil on the surface of the Earth, Glasstone gives the optimum depth as $150 E^{0.3} \mathrm{ft}$, and he would obtain the crater constant and exponent as $\beta \simeq 0.9$ and $\alpha=8.4 \times 10^{-4} \mathrm{gm}^{1 / 2(1-\beta)} \cdot \mathrm{cm}^{-\beta} \cdot \mathrm{s}^{\beta}$ for use in Eq. (5). For the moment, let us say that the value of $\alpha$ is increased an order of magnitude.
Looking at Eq. (24), we might expect the initial mass to decrease an order of magnitude, but to penetrate to the optimal depth the explosive has to be fitted with a weighty billet: a cylinder of metal (probably tungsten) that will erode during penetration of the assailant's surface material. In general, this will increase the weight by about an order of magnitude or decrease the yield-to-weight $\varphi$ by about an order of magnitude. Thus, in Eq. (24), the decrease in initial interceptor mass $M_{i}$ owing to the increase in the cratering constant $\alpha$ is just about compensated by the decrease in yield-to-weight $\varphi$.

However, the blowoff fraction given in Eq. (28) becomes an order of magnitude larger because it does not depend on yield-to-weight $\varphi$. The conclusion is that a penetrator has no value-enhancing deflection but may be of great value if we chose to pulverize the astral assailant.

## Standoff Deflection

The fracture problem can be much mitigated by detonating the nuclear explosive some distance from the astral assailant. Rather than forming a crater, the neutrons, $x$ rays, gamma rays, and some highly ionized debris from the nuclear explosion will blow off a thin layer of the assailant's surface. This will spread the impulse over a larger area and lessen the shear stress to which the assailant is subjected. Of these four energy transfer mechanisms, by far the most effective (at reasonable heights of burst) is neutron energy deposition, suggesting that primarily fusion explosives would be most effective.

The problem of calculating the momentum transferred from a standoff detonation is sufficiently complicated that it is difficult to address analytically. Computer simulations seem the most effective approach. However, some general statements can be made. At an optimal height of burst, about $2-8 \%$ of the explosive's energy is coupled to the assailant's surface, again depending on the assailant's actual composition and the neutron spectrum and total neutron energy output of the explosive. This corresponds to an energy fraction $\delta$ of $0.2-0.4$. Most of the energy is deposited in the first 10 cm of the soil. The cratering constants can still be used as in Eq. (5), but for this surface blowoff, $\beta=1$ and $\alpha$ ranges from $10^{-6}$ to $2 \times 10^{-6} \mathrm{~cm}^{-1}$. s. If we select an assailant for which $\delta=0.3$ and $\alpha=1.5 \times 10^{-6} \mathrm{~cm}^{-1} \cdot \mathrm{~s}$, we find from Eq. (24) that the blowoff fraction will be about a factor of 35 times smaller than for the surface burst. The blowoff fraction given in Fig. 8 would be in the range of $1 \%$ for $R_{l}=1 / 10 \mathrm{AU}$ and in the range of $1 / 3 \%$ for $R_{l}=1 / 3$ AU. Similarly, from Eq. (28) we find that the initial mass of the interceptor would have to be about 40 times as large. So in Fig. 6 the mass would be multiplied by 40 , i.e., ranging from about 28 ton to about 28 kiloton. The latter would not be very practical.

## Comments, Summary, and Tentative Conclusions

The problem of terminal interception of comets and asteroids on collision course with Earth has two components: 1) detection of these relatively small assailants and 2) smashing or deflecting them should they be on an endangering path. In this paper, I have addressed the latter issue. The relationships I have derived should guide thinking on how to counter such
assailants. The main value is to show the functional relationship among the parameters. This paper is not intended to be an exhaustive study, and much research will be required to evaluate the constants in the equations I have derived. But the following observations are compelling and refractory.

1) Kinetic-energy deflection is effective for ocean diversion for assailants smaller than about 70 m , if the interceptor is launched when the assailant is further than $1 / 30 \mathrm{AU}$. At shorter range, interceptors become impractically massive and the probability of fracture increases rapidly. Ocean impact is probably unacceptable for larger assailants, and an order-ofmagnitude larger interceptor is required for missing the planet with concomitant increase in fracture probability. Higher specific impulse interceptors are more effective at increasing deflection and reducing fracture probability, mainly because they divert the assailant at a greater distance. Objects less than 10 m are better pulverized at short range.
2) Nuclear-explosive deflection is imperative for assailants greater than about 100 m detected closer than $1 / 30 \mathrm{AU}$ because of the enormous mass of the interceptor required for kinetic-energy diversion.
3) Nuclear-surface-burst deflection offers a three to four order of magnitude reduction in interceptor mass. The advantage decreases slightly with specific impulse and decreases dramatically with assailant velocity. Fragmentation is a problem for intercepts closer than about 1/30 AU.
4) Nuclear penetrators offer no advantage for deflection but are better for pulverization.
5) Nuclear standoff deflection greatly reduces fragmentation probability but involves a substantial increase in interceptor mass.

The assailant object depicted in Fig. 1 is roughly spherical in shape. In fact, comets or asteroids are generally quite spherical, a "potato" or "peanut" being the most popular descriptions. All of the deflection techniques except the standoff nuclear burst make a crater that is small compared with the characteristic dimension of the assailant. The linear momentum impulse will be imparted along a line connecting that crater and the center of mass, with corrections for local geology and topography. An aspheric object will also receive some angular momentum, depending on the location of the crater and the object's inertial tensor. The size of the impulse will depend on material properties, geology, and topography.

Thus, it will be necessary to characterize the geology and mechanical properties of the assailant when using the cratering deflection techniques. Such characterization could be accomplished by a vanguard spacecraft. Standoff deflection is much less sensitive to these details. In general, linear momentum will be imparted along the line connecting the detonation point with the center of mass. Little angular momentum will be imparted, and this will depend on relative projected areas of various assailant topographic features compared with components of the inertial tensor. Thus, beside its inherent fracturemitigation virtues, the standoff deflector demands substantially less information about the object it is deflecting.

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