

Constraining Dark Energy with Type Ia Supernovae and Large-Scale Structure

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(Received 15 January 1999)

Measurements of the distances to type Ia supernovae have produced strong evidence that the expansion of the Universe is accelerating, implying the existence of a nearly uniform component of dark energy with negative pressure. We show that constraints to this mysterious component based upon large-scale structure nicely complement the SN Ia data, and that together they require $\Omega_X = \rho_X/\rho_{\text{crit}} \in (0.6, 0.7)$ and $w = p_X/\rho_X < -0.6$ (95% C.L.), for the favored flat Universe. Other cosmological data support this conclusion. The simplest explanation, a cosmological constant, is consistent with this, while some of the other possibilities are not.

PACS numbers: 98.80.Es, 95.35.+d, 98.65.Dx, 98.70.Vc

Two groups [1,2] have presented strong evidence that the expansion of the Universe is speeding up, rather than slowing down. It comes in the form of distance measurements to some 50 supernovae of type Ia (SNe Ia), with redshifts between 0 and 1. The results are fully consistent with the existence of a cosmological constant (vacuum energy) whose contribution to the energy density is around 70% of the critical density ($\Omega_\Lambda \sim 0.7$). Other measurements indicate that matter alone contributes $\Omega_M = 0.4 \pm 0.1$ [3]. Taken together, matter and vacuum energy account for an amount close to the critical density, consistent with measurements of the anisotropy of the cosmic microwave background (CMB) [4,5].

In spite of the apparent success of the cosmological constant explanation, other possibilities have been suggested for the “dark energy”. This is in part because of the checkered history of the cosmological constant: It was advocated by Einstein to construct a static Universe and discarded after the discovery of the expansion; it was revived by Hoyle and Bondi and Gold to solve an age crisis, later resolved by a smaller Hubble constant, and it was put forth to explain the abundance of quasars at $z \sim 2$, now known to be due to the evolution of quasars themselves. Further, all attempts to compute the value of the cosmological constant, which in modern terms corresponds to the energy associated with the quantum vacuum, have been wildly unsuccessful [6]. Finally, the presence of a cosmological constant makes the present epoch special: at earlier times matter (or radiation) dominated the energy density and at later times vacuum energy will dominate (the “why now?” problem).

The key features of an alternative form for the dark energy are the following: bulk pressure that is significantly negative, $w < -1/3$, where $w \equiv p_X/\rho_X$, and the inability to clump effectively. The first property is needed to ensure accelerated expansion and to avoid interfering with

a long matter-dominated era during which structure forms; the second property is needed so that the dark energy escapes detection in gravitationally bound systems such as clusters of galaxies. Candidates for the dark energy include [7]: a frustrated network of topological defects (such as strings or walls), here $w = -\frac{n}{3}$ (n is the dimension of the defect) [8] and an evolving scalar field, where $\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi)$ and $p = \frac{1}{2}\dot{\phi}^2 - V(\phi)$ (referred to by some as quintessence) [9,10].

The SN Ia data alone do not yet discriminate well against these different possibilities [1,11]. As shown in Fig. 1, the maximum likelihood region in the Ω_M - w plane runs roughly diagonally: less negative pressure is permitted if the fraction of critical density contributed by dark energy ($\equiv \Omega_X$) is larger. Following earlier work [7], this led us to consider other cosmological constraints: large-scale

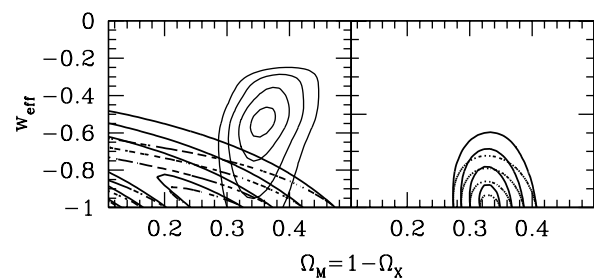


FIG. 1. Contours of likelihood, from 0.5σ to 2σ , in the Ω_M - w_{eff} plane. Left: The thin solid lines are the constraints from LSS and the CMB. The heavy lines are the SN Ia constraints (using the fit C supernovae of Ref. [1]) for constant w models (solid curves) and for a scalar-field model with an exponential potential (broken curves; quadratic and quartic potentials have very similar SN Ia constraints). Note that the SN Ia contours for dynamical scalar-field models and constant w models are slightly offset (see text). Right: The likelihood contours from all of our cosmological constraints for constant w models (solid) and dynamical scalar-field models (broken).

structure, anisotropy of the CMB, the age of the Universe, gravitational lensing, and measurements of the Hubble constant and of the matter density. As we shall show, some of the additional constraints, especially large-scale structure, complement the SN Ia constraint, and serve to sharpen the limits to Ω_M and w ; others primarily illustrate the consistency of these measurements with the SN Ia result. In the end, we find $\Omega_X \in (0.6, 0.7)$ and $w < -0.6$ (95% C.L.).

Method.—Our underlying cosmological paradigm is a flat, cold dark matter model [12] with a dark-energy component ($\Omega_M + \Omega_X = 1$), though as we will discuss later our results are more general. We restrict ourselves to flat models both because they are preferred by the CMB anisotropy data and because a flat Universe is strongly favored by inflation. We restrict ourselves to cold dark matter models because of the success of the cold dark matter paradigm and the lack of a viable alternative. For our space of models we construct marginalized likelihood functions based upon SNe Ia, large-scale structure, and other cosmological measurements, as described below.

Our model parameter space includes the usual cosmological parameters (Ω_M , $\Omega_B h^2$, and h) and the amplitude and spectral index of the spectrum of Gaussian curvature fluctuations (σ_8 and n). For the dark-energy component, we choose to focus on the dynamical scalar-field models, because the frustrated defect models are at best marginally consistent with the SN Ia data alone [1,11].

In the dynamical scalar-field models the equation of state w can vary with time. However, for most of our purposes, only one additional free parameter needs to be specified, an “effective” equation of state. We choose \hat{w}_{eff} to be that value w which, if the Universe had w constant, would reproduce the conformal age today. We choose this definition because the CMB anisotropy spectrum and the Cosmic Background Explorer (COBE) normalization of the matter power spectrum remain constant (to within 5%–10%) for different scalar field models with the same \hat{w}_{eff} [13]. For the models under consideration \hat{w}_{eff} is closely approximated by [14]

$$w_{\text{eff}} \equiv \int da \Omega_\phi(a) w(a) / \int da \Omega_\phi(a), \quad (1)$$

where $\Omega_\phi(a)$ is the epoch-dependent fraction of critical density contribution by the scalar field. Since it is simpler to compute, we have used w_{eff} throughout. Our results also apply to constant w models (e.g., frustrated defects), by taking $w = w_{\text{eff}}$.

While w_{eff} neatly parametrizes the scalar-field models from the standpoint of large-scale structure and the CMB anisotropy, it does not do as well when it comes to the SN Ia data. Recall that w_{eff} as defined in Eq. (1) receives a contribution from a wide range of redshifts. The SN Ia data, however, are sensitive mostly to $z \sim 1/2$. Since w becomes less negative with time in the models we are considering, the SN Ia data “see” a less negative w than

the CMB by a model dependent amount. We shall return to this point later.

We normalize our models to the COBE 4-year data [15] using the method of Ref. [16]. Beyond the COBE measurements, the small-scale anisotropy of the microwave background tells us that the Universe is close to being spatially flat (position of the first acoustic peak) and that Ω_M is less than one and/or the baryon density is high (height of the first acoustic peak). We have not included a detailed fit to the current data (see, e.g., Ref. [4]), but rather impose flatness. The additional facts that might be gleaned from present CMB measurements, $\Omega_M < 1$ and high baryon density, are in fact much more strongly imposed by the large-scale structure data (in the absence of a significant amount of hot dark matter) and the Burles-Tytler deuterium measurement.

To allow for the possibility that light does not faithfully trace mass (biasing), we require that the power-spectrum [17] fit the *shape* of the redshift-survey data as compiled by Ref. [18]. On smaller scales we require that all of our models reproduce the observed abundance of rich clusters of galaxies. This is accomplished by requiring $\sigma_8 = (0.55 \pm 0.1)\Omega_M^{-0.5}$, where σ_8 is the rms mass fluctuation in spheres of $8h^{-1}$ Mpc computed in linear theory [19]. The baryon density is fixed at the central value indicated by the Burles-Tytler deuterium measurements, $\Omega_B h^2 = 0.019 \pm 0.001$ [20]. We marginalize over the spectral index and Hubble constant, assuming Gaussian priors with $n = 0.95 \pm 0.05$, which encompasses most inflationary models, and $h = 0.65 \pm 0.05$, which is consistent with current measurements.

There are three other cosmological constraints that we did not impose: the age of the Universe, $t_0 = (14 \pm 2)$ Gyr [21]; direct measurements of the matter density, $\Omega_M = 0.4 \pm 0.1$, and the frequency of multiply imaged quasars. While important, these constraints serve to prove consistency, rather than to provide significant complementary information. For example, the SN Ia data together with our Hubble constant constraint lead to an almost identical age constraint [1,2]. The lensing constraint, recently studied in detail for dynamical scalar-field models [22], excludes the region of large Ω_X and very negative w (at 95% C.L., below the line $w_{\text{eff}} = -0.55 - 1.8\Omega_M$), which is disfavored by the SN Ia data. The matter density determined by direct measurements, $\Omega_M = 0.4 \pm 0.1$ [3], is consistent with that imposed by the large-scale structure (LSS) and Hubble constant constraints. [Even if one imposes the most stringent constraint to the matter density, that based upon the cluster baryon fraction and big bang nucleosynthesis (BBN) baryon density, $\Omega_M h^{1/2} = 0.3 \pm 0.05$, our likelihood contours do not change appreciably.]

Results.—As can be seen in Fig. 1, our LSS and CMB constraints neatly complement the SN Ia data. LSS tightly constrains Ω_M , but is less restrictive along the w_{eff} axis. This is easy to understand: in order to fit the power spectrum data, a COBE-normalized CDM model must have

“shape parameter” $\Gamma = \Omega_M h \sim 0.25$ (with a slight dependence on n). Together with the constraint $h = 0.65 \pm 0.05$ this leads to $\Omega_M \sim 0.35$. This constraint from LSS and CMB is consistent with the direct determinations of Ω_M [3]. As discussed in Ref. [7], the σ_8 constraint can discriminate against w_{eff} ; however, allowing the spectral index to differ significantly from unity diminishes its power to do so.

Note that the SN Ia likelihood contours for the dynamical scalar-field model and the constant- w models *are not* the same while the LSS contours are identical. With the fit C supernovae of Ref. [1] and the dynamical scalar-field models considered here (quadratic, quartic, and exponential scalar potentials), the contours are displaced by about 0.1 in w_{eff} : the 95% C.L. upper limit to w_{eff} for the constant w models is -0.62 , while for the quartic, quadratic, and exponential potentials for $V(\phi)$ it is -0.75 , -0.76 , and -0.73 , respectively. The reason for this shift is simple: the w dependence of LSS is almost completely contained in the distance to the last-scattering surface and w_{eff} is constructed to hold that constant. On the other hand, the w dependence of the SN Ia results is more heavily weighted by the recent value of w ; said another way, there is a different effective w for the SN Ia data. *This fact could ultimately prove to be very important in discriminating between different models.*

Additionally, there is a class of dynamical scalar-field models that have attracted much interest recently [10,23]. For these potentials (here we consider $V(\phi) = c/\phi^p$ and $V(\phi) = c[e^{1/\phi} - 1]$), and a wide range of initial conditions the scalar field settles into a “tracking solution” that depends only upon one parameter (here c) and the evolution of the cosmic scale factor, suggesting that they might help to address the “why now?” problem.

For our purposes, the most interesting fact is that each tracker potential picks out a curve in Ω_M - w_{eff} space. Typically the lower values of Ω_M go with the most negative values of w_{eff} and vice versa (see Fig. 2). This fact puts the tracker solutions in jeopardy, as shown in the same figure. For the tracker models shown here ($p = 2, 4$ and exponential), the 95% C.L. intervals for the SN Ia and LSS data barely overlap. The situation is even worse for larger values of p . A similar problem was noted in Ref. [24].

Finally, we comment on the robustness of our results. While we have restricted ourselves to flat models, as preferred by the CMB data, our constraints do not depend strongly on this assumption. This is because the LSS constraints are insensitive to the flatness assumption, and curvature, which corresponds to a $w_{\text{eff}} = -\frac{1}{3}$ component, is strongly disfavored by the SN Ia results. We have not explicitly allowed for the possibility that inflation-produced gravity waves account for a significant part of the CMB anisotropy on large-angular scales (i.e., $T/S > 0.1$), which would have the effect of decreasing the overall amplitude of the COBE normalized power spectrum. In fact, allowing for gravity waves would not change our

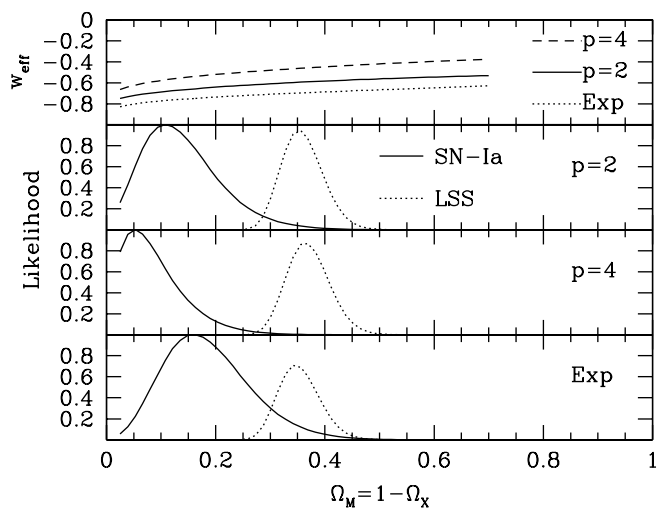


FIG. 2. Upper panel: The relationship between w_{eff} and Ω_M for a selection of tracker potentials. Lower panels: The CMB and LSS likelihoods from Fig. 1 as a function of Ω_M (dotted line) and the SN Ia likelihood (solid line, normalized to unity at the peak). As can be seen clearly, tracker models have difficulty simultaneously accommodating the SN Ia and LSS constraints.

results, as this degree of freedom is implicitly accounted for by a combination of n , the normalization freedom in the power spectrum, and the uncertainty in the COBE normalization.

Our model space does not explore the full range of possibilities, for example, that neutrinos contribute significantly to the mass density [25] or a nonpower-law or isocurvature spectrum of density perturbations [26]. Even allowing for these possibilities (or others) would not change our results significantly provided that we adopted the mass density constraint, $\Omega_M = 0.4 \pm 0.1$. As discussed earlier, it is almost as powerful as the CDM-based LSS constraint.

Conclusions.—The evidence provided by SNe Ia that the Universe is accelerating rather than slowing solves one mystery—the discrepancy between direct measurements of the matter density and measurements of the spatial curvature based upon CMB anisotropy—and introduces another—the nature of the dark energy that together with matter accounts for the critical density. SNe Ia alone do not yet strongly constrain the nature of the dark energy.

In this Letter we have shown that consideration of other important cosmological data both complement and reinforce the SN Ia results. In particular, as illustrated in Fig. 1, consideration of large-scale structure leads to a constraint that nicely complements the SN Ia constraint and strengthens the conclusions that one can draw. Other cosmological constraints—the age of the Universe, frequency of gravitational lensing, and direct measures of the matter density—provide information that is consistent with the SN Ia constraint (lensing and age) and the LSS constraint (matter density), and thereby reinforces the self-consistency of the whole picture of a flat Universe with cold dark matter and dark energy.

Finally, what have we learned about the properties of the dark-energy component? The suite of cosmological constraints that we have applied indicate that $\Omega_X \in (0.6, 0.7)$ and $w_{\text{eff}} < -0.6$ (95% C.L.), with the most likely value of w_{eff} close to -1 (see Fig. 1). The frustrated network of light cosmic string ($w_{\text{eff}} = -\frac{1}{3}$) is strongly disfavored, and a network of frustrated walls ($w_{\text{eff}} = -\frac{2}{3}$) is only slightly more acceptable. Also in the disfavored category are tracker models with $V(\phi) = c/\phi^p$ and $p = 2, 4, 6, 8, \dots$. Dynamical scalar-field models can be made acceptable provided w_{eff} is tuned to be more negative than -0.7 . The current data definitely prefer the most economical, if not the most perplexing, solution: Einstein's cosmological constant.

This work was supported by the DoE (at Chicago, Fermilab, and Lawrence Berkeley National Laboratory) and by NASA (at Fermilab by Grant No. NAG 5-7092). M. W. is supported by the NSF.

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