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# Hyper-velocity and tidal stars from binaries disrupted by a massive Galactic black hole

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**A close but newtonian encounter between a tightly bound binary and a $10^6\ M_\odot$ black hole causes one binary component to become bound to the black hole and the other to be ejected at up to 4,000 km s$^{-1}$. The discovery of even one such hyper-velocity star coming from the Galactic centre would be nearly definitive evidence for a massive black hole. The new companion of the black hole has a high orbital velocity which increases further as its orbit shrinks by tidal dissipation. The gravitational energy released by the orbit shrinkage of a such a tidal star can be comparable to its total nuclear energy release.**

The dynamics of gas in the Galactic centre (GC) suggest that it may have a $10^6\ M_\odot$ black hole[1]. Such a massive black hole would, by capturing the debris of stars that pass within its Roche radius, grow in less than $10^9$ yr to $10^8\ M_\odot$, which is the mass of black holes expected in luminous quasars[2,3]. A GC black hole more massive than $4\times10^4\ M_\odot$ is predicted to emit more non-stellar radiation than is observed from the GC, due to capture of stellar tidal debris[2]. Although this is a severe problem, it does not totally rule out a $10^6\ M_\odot$ GC black hole.

Simulations of close encounters between binaries and stellar intruders[4] have shown that in exchange collisions, in which a massive intruder replaces one binary component, the average gravitational binding energy of the binary increases with the mass, $M_I$, of the captured intruder. The increase is fivefold if $M_I$ is 100 times the mass of the binary components. I show here that this trend continues for encounters with much more massive intruders. The computations were done with the Shampine–Gordon[5] variable-order, variable-step-size integrator. The mean fractional change in the energy of the system due to integration errors was $10^{-7}$, so the high ejection velocities found in these simulations are not numerical artefacts. I used newtonian mechanics. Most encounters occurred well beyond the last stable circular orbit at three Schwarzschild radii.

In these simulations each binary component is of $1\ M_\odot$ and the black hole masses are $M_{bh}=10^4$, $10^5$, $10^6$ and $10^7\ M_\odot$. The semi-major axis of the binary is $a_0=0.01$ AU, except for two series of runs with $a_0=0.02$ and 0.1 AU for $M_{bh}=10^6\ M_\odot$.

Only 'hard' binaries, with orbital velocities larger than the mean velocity in the stellar system, survive encounters with fellow stars of approximately the same mass[6-8]. The maximum semi-major axes of hard binaries in galactic nuclei, where stellar velocities are typically 80-300 km s$^{-1}$, are in the range $a_0=$ 0.01-0.1 AU. The velocity dispersion in the GC is also in this range[9]. The average mass density within 0.25 pc of the GC exceeds $10^7\ M_\odot\ \mathrm{pc}^{-3}$ (ref. 9), so close encounters between stars are common.

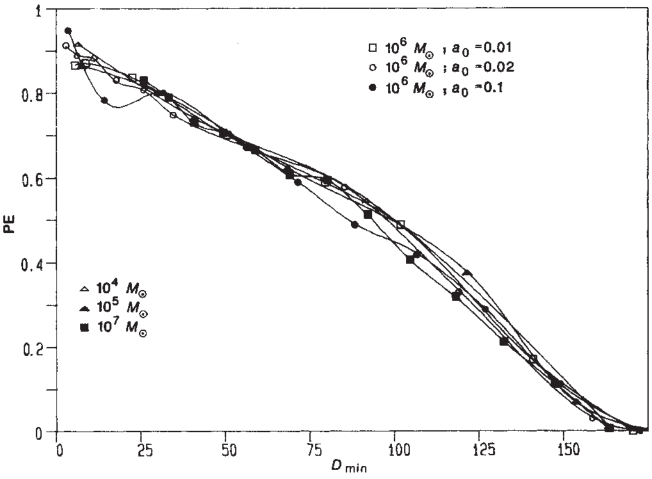


**Fig. 1** Probability of an exchange collision in which the black hole replaces one of the original binary components, plotted as a function of the dimensionless closest approach parameter $D_{min}$ of the binary to the black hole. The curves are labelled by the mass of the black hole and the semi-major axis of the binary orbit. The smallest $D_{min}$ plotted for each of the two most massive black holes is just inside the last stable circular orbit.

I simulated encounters having a wide range of impact parameters. At each impact parameter, 250 runs were made using different binary orientations randomly chosen with respect to the approaching intruder, whlich has a velocity at infinity of $V=250$ km s$^{-1}$. At this velocity, the total energy of the three-body system (binary plus black hole) is positive even for $a_0=$ 0.01 AU (by design, so the encounters are flybys). It is thus energetically possible for the encounter to dissociate the binary, but in almost every case the encounter left behind either the original binary or one in which the black hole had replaced one star. Because $V$ is much less than either the velocity of the binary with respect to the black hole at closest approach or the recoil velocity of the ejected binary component, the results should not be sensitive to the chosen value of $V$.

The binary is 1,500 times its semi-major axis from the black hole at the beginning of each computation, far enough away for the tidal field of the black hole to be small but close enough so the binary comes to pericentre with the black hole after relatively few integration steps. I treat the stars as point masses, but they are broken apart if they pass within their Roche radius of the black hole. While tidal breakup is not a problem for neutron stars and white dwarfs, it is serious for main-sequence stars because of their relatively low densities. A $1\ M_\odot$ main-sequence star is torn apart by a $10^6\ M_\odot$ black hole if it passes within $R_{min}=150\ R_\odot=0.70$ AU of the black hole (W. Benz and J. G. Hills, in preparation). Because lower main-sequence stars are denser than the Sun, they can pass significantly closer to the black hole before tidal breakup. A main-sequence star of $0.1\ M_\odot$ has one-fourth the Roche radius of a $1\ M_\odot$ star.

For a $1\ M_\odot$ main-sequence star, the Roche radius exceeds the Schwarzschild radius if the black hole is less massive than $3\times10^8\ M_\odot$ (ref. 2), so such a star is much more likely to be tidally torn apart by the black hole than to be swallowed whole.

Figure 1 shows the probability PE of an exchange collision as a function of the dimensionless closest approach parameter $D_{min}=(R_{min}/a_0)[2\ M_{bh}/10^6(M_1+M_2)]^{-1/3}$, where $R_{min}$ is the closest approach of a binary with semi-major axis $a_0$ and component masses $M_1$ and $M_2$ to a black hole of mass $M_{bh}$. The factor in the square bracket compensates for the increase in the tidal interaction radius of the black hole as $M_{bh}$ increases. Exchange collisions occur in nearly 90% of the closest encounters. We note that PE is nearly the same at a given value of

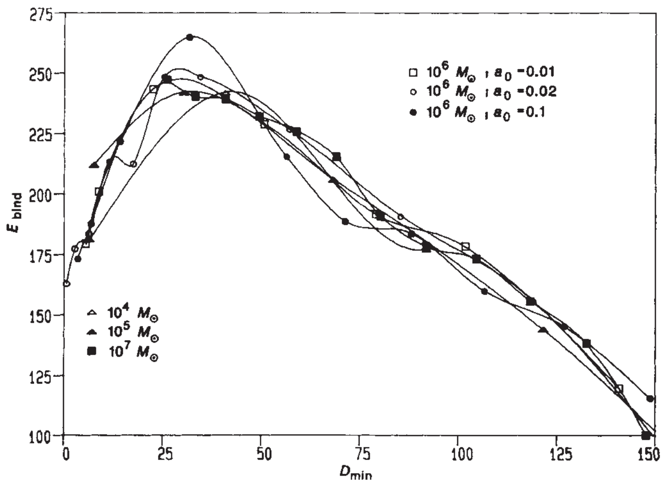


**Fig. 2** The increase in the dimensionless binding-energy parameter of the binary as the result of an exchange collision with a black hole.

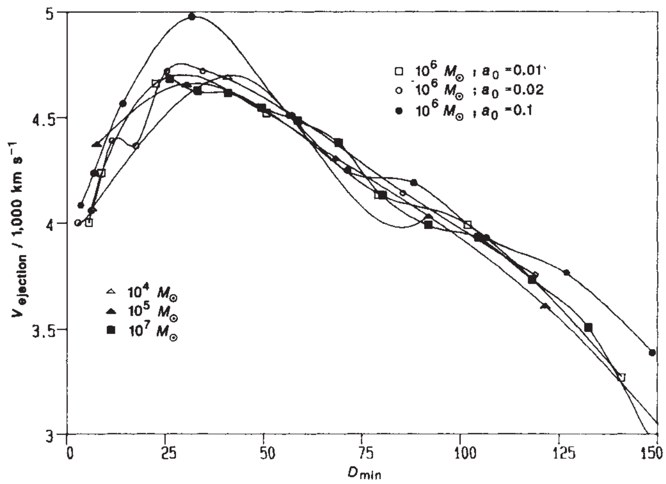


**Fig. 3** The ejection velocity parameter of the unbound binary component after an exchange collision with the black hole.

$D_{min}$ for various values of $M_{bh}$ and $a_0$, so we can use Fig. 1 to find PE for arbitrary values of $a_0$, $R_{min}$ and $M_{bh}$.

Figure 2 shows $E_{bind}$, the normalized (dimensionless) increase in the average binding energy of those binaries which suffer exchange collisions. Here

$$E_{bind} \equiv \left(\frac{\text{Increase in binding energy}}{\text{Original binding energy}}\right)\left(\frac{2\,M_{bh}}{10^6(M_1+M_2)}\right)^{-1/3}$$

We note that the results are again nearly independent of $a_0$ and $M_{bh}$ when given as a function of $D_{min}$.

Figure 3 shows the ejection velocity parameter

$$V_{ejection} \equiv V_\infty\left(\frac{2\,M_{bh}}{10^6(M_1+M_2)}\right)^{-1/6} \times\left[\left(\frac{0.01\text{ AU}}{a_0}\right)\left(\frac{M_1+M_2}{2\,M_\odot}\right)\right]^{-1/2}$$

for the ejected star in these exchange collisions. Here $V_\infty$ is the r.m.s. velocity at infinity of the ejected star as found in the numerical experiments. Figure 3 shows that $V_{ejection}$ is well-behaved, so it is easy to determine $V_\infty$ for arbitrary values of $M_{bh}$ and $a_0$. For a black hole with $M_{bh}=10^6\ M_\odot$, r.m.s. ejection velocities at infinity are about $V_\infty = 4{,}000$, 3,000 and 1,400 km s$^{-1}$ for $a_0 = 0.01$, 0.02 and 0.1 AU respectively. These velocities greatly exceed that of any known star in the Galaxy, and the two higher ones are much larger than the escape velocity from the Galaxy.

The high ejection velocities occur because $V_{break}$, the centre-of-mass velocity of the binary at the point where the tidal field of the black hole pulls it apart, is much greater than the internal orbital velocity, $V_{orb}$, of the binary components. If each binary component experiences an average change in its velocity with respect to the black hole on the order of $V_{orb}$ because of the perturbation of its companion, the corresponding change in its specific kinetic energy is $\delta\text{KE} \approx V_{orb}V_{break}$. For $M_{bh}=10^6\ M_\odot$, $V_{break} \approx 0.14c = 10^2\ V_{orb}$ for $a_0 = 0.01$ AU, so the change in the specific kinetic energy of the binary components with respect to the black hole is more than two orders of magnitude larger than their original orbital binding energy.

For a binary to have an exchange probability of 50% or more in an encounter with a $10^6\ M_\odot$ black hole requires it to pass within about $100a_0$ of the black hole, or within $R_{min}=1$ and 10 AU for $a_0 = 0.01$ and 0.1 AU respectively. For a stellar mass density at the GC of $10^7\ M_\odot$ pc$^{-3}$, a star passes within 1 AU of a $10^6\ M_\odot$ black hole about every 100 yr and within 10 AU every 10 yr[2,3]. (Here gravitational focusing is important, so the cross-section for passing within distance $R_{min}$ of the black hole is proportional to $R_{min}$ rather than to $R^2_{min}$.) If 1% of the stars involved in these close encounters are binaries with $a_0 = 0.01$ AU, and another 1% are binaries with $a_0 = 0.1$ AU, then one of the closer binaries suffers an exchange collision about every $10^4$ yr and one of the more distant binaries every $10^3$ yr. (If the binary components are 1 $M_\odot$ main-sequence stars, then about half of the encounters which are close enough to produce an exchange collision for $a_0 = 0.01$ AU lead to tidal breakup of the individual stars by the black hole, but the fraction of stars broken up is negligible in exchange collisions in which $a_0 = 0.1$ AU.)

From Fig. 3 we see that the typical ejection velocity, $V_\infty$, of the discarded binary component is ~4,000 km s$^{-1}$ for $a_0 = 0.01$ AU, and 1,400 km s$^{-1}$ for $a_0 = 0.1$ AU, if $M_{bh} = 10^6\ M_\odot$. Because it takes hyper-velocity stars with $V_\infty = 4{,}000$ km s$^{-1}$ about $2\times10^6$ yr to reach the Sun's distance of ~8 kpc from the GC, there are about 200 of them between the GC and the solar circle if one is produced every $10^4$ yr. Hyper-velocity stars with $V_\infty = 1{,}400$ km s$^{-1}$ take at least $6\times10^6$ yr to traverse this distance even if we ignore their gravitational deceleration, so there are more than 6,000 of them inside the solar circle if one is produced every $10^3$ yr.

The average space density of hyper-velocity stars decreases with the inverse square of their distance from the GC, with the closest one to the GC being within 40 pc for $V_\infty = 4{,}000$ km s$^{-1}$ and within 1 pc for $V_\infty = 1{,}400$ km s$^{-1}$. Proper motion survey fields near the GC but above the Galactic plane should be best for detecting hyper-velocity stars.

For $V_\infty = 4{,}000$ km s$^{-1}$ = 800 AU yr$^{-1}$, the maximum proper motion of a hyper-velocity star at 8 kpc is 0.1 arc s yr$^{-1}$. A solar-luminosity star at this distance would be at magnitude 19 in the absence of interstellar extinction. Many other hyper-velocity stars will be closer, brighter, and have larger proper motions. If hyper-velocity stars exist, some may be included in existing proper motion surveys. The simplest explanation for their not being reported is that they do not exist, which implies that the mass of the GC black hole is much less than $10^6\ M_\odot$. Alternatively, observers may have disbelieved the outlandish velocities implied by a facile interpretation of their proper motions and assumed these stars to be peculiar and much closer to us than they actually are. Hyper-velocity stars should also have large radial velocities which are hard to rationalize away. But radial velocities are much more expensive to determine than proper motions, so relatively few proper motion stars have known radial velocities.

Hyper-velocity stars may well be peculiar compared to stars in the solar neighborhood. Because they originate in the core of the Galaxy, their metal abundances may actually be several times greater than solar, and because they reach the solar distance from the GC in much less than the solar Kelvin time, they may not have returned to the main sequence if they were strongly perturbed by the tidal field of the black hole. A strong tidal perturbation would also cause them to be in rapid rotation (W. Benz and J. G. Hills, in preparation).

All active galactic nuclei (AGNs) eject hyper-velocity stars which become intergalactic tramps. The expansion of the universe gradually slows them down with respect to the Hubble flow until they are captured by galaxies. Hyper-velocity stars from AGNs may constitute a minor impurity of metal-rich stars in the Galactic halo.

If hyper-velocity stars are not found, it may still be possible to retain the hypothesis of a massive GC black hole by arguing that my estimate of the binary frequency in the GC is too optimistic, that most GC stars are less massive, so fainter, than the Sun, or that exchange collisions between binaries and stellar remnants (white dwarfs, neutron stars, and small black holes) have allowed these hard-to-detect objects to replace so many main-sequence stars in GC binaries that few main-sequence stars are left in them[10]. But the detection of even one hyper-velocity star coming from the GC would be nearly definitive proof of its having a massive black hole.

After an exchagne collision with a $10^6 M_\odot$ black hole, the semi-major axis of the binary is typically 50 AU for $a_0 = 0.01$ AU and 500 AU for $a_0 = 0.1$ AU. The r.m.s. orbital velocity, $V_{\rm orbital}$, of the star about the black hole is nearly the same as the ejection velocity, $V_\infty$, of its former companion, or about 4,000 and 1,400 km $s^{-1}$ respectively for these two binaries.

The velocity of an unbound star which collides with one of these bound stars exceeds the local parabolic speed which is, on average, about $\sqrt{2} V_{\rm orbital}$, so the two stars collide at an average speed of about $\sqrt{3} V_{\rm orbital} = 6{,}900$ and 2,400 km $s^{-1}$ for the two cases. The minimum collision velocity required to break up two equally massive main-sequence stars is only about 2.3 times the escape velocity from their surface, or about 1,400 km $s^{-1}$ (ref. 11). Most of the debris from the colliding stars should remain bound to the black hole and eventually be accreted by it.

The orbit of the bound star around the black hole is very eccentric, but tidal energy dissipation at pericentre may be able to circularize it before the star is destroyed in a stellar collision. Because of angular momentum conservation, the final semi-major axis of the circularized orbit will be about $a_f = 2R_{\rm min}$. Many tidal stars can accumulate in these close orbits because the high velocities in them make them very 'stiff', so it is very difficult for stars in other bound orbits to perturb them gravitationally.

The tidal energy dissipated in circularizing the orbits of stars captured in an exchange collision with a $10^6 M_\odot$ black hole is $\sim$0.1–1% of their rest-mass energy, or $10^{51-52}$ erg for a 1 $M_\odot$ star. (Comparable energy is released in one final supernova-class cataclysm when a tidal star in a nearly circularized orbit collides with an unbound star.) The radius of the captured tidal star increases, eventually following its Hayashi track, until its luminosity matches its rate of tidal energy dissipation. If each tidal star emits $10^{51}$ erg before its destruction in a collision and one is captured every $10^4$ yr, the total integrated luminosity of the tidal stars orbiting the GC black hole is $\sim 10^6 L_\odot$. The maximum luminosity of an individual star is its Eddington value, so at least 30 solar-mass stars are needed to produce the integrated luminosity. The integrated spectrum of these stars should mimic that of a giant with its absorption lines washed out by the extreme differential Doppler motion resulting from the high orbital velocities (0.03–0.1 $c$), of the individual tidal giants. The GC black hole may have accumulated a large number of these low-mass tidal giants in orbits with semi-major axes, $a_t$, on the order of few times $R_{\rm min}$. It is tempting to identify $a_t$ with radius of the GC radio source, $\sim$10 AU. Stellar winds and Roche overflow from the tidal giants may terminate any accretion disk beyond $a_t$.

Stellar remnants such as neutron stars, small black holes, and white dwarfs do not suffer appreciable tidal dissipation, so they tend to remain in their original post-exchange orbits. The GC black hole can accumulate a large number of these stellar remnants in orbits with semi-major axies of 50–500 AU and orbital velocities of 1,000–4,000 km $s^{-1}$. They are 'meat grinders' which help provide fuel for their master, the central black hole, by tearing material from main-sequence stars and giants that pass near them.

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# Laboratory simulation of Jupiter's Great Red Spot

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**Isolated large stable vortices have long been observed in the jovian atmosphere and more recently on Saturn. The existence of such stable vortices in strongly turbulent planetary atmospheres is a challenging problem in fluid mechanics. In a numerical simulation Marcus[1] found that a single stable vortex developed for a wide variety of conditions in a turbulent shear flow in a rotating annulus. To test this we conducted an experiment on a rotating annulus filled with fluid pumped in the radial direction. The annulus rotates rigidly (there is no differential rotation), but the action of the Coriolis force on the radially pumped fluid produces a counter-rotating jet. Coherent vortices spontaneously form in this turbulent jet, and for a wide range of rotation and pumping rates the flow evolves until only one large vortex remains.**

Figure 1 shows the annular tank used in the experiments. The tank is rotated rapidly to achieve an essentially geostrophic flow (a two-dimensional flow in which the Coriolis force is balanced by pressure gradients[2]). In such a flow the inertial force is small compared with the Coriolis force, that is, the Rossby number is small. In our system the Rossby number is $\sim$0.1, as estimated from the ratio of the characteristic time for the inertial force (the turnover time for a vortex) to the characteristic time for the Coriolis force (the rotation period of the tank).

On Jupiter the Great Red Spot and other persistent coherent vortices lie in latitudinal zones of strong shear[3]. Marcus's numerical simulation indicates that in a quasi-geostrophic flow the existence of a strong shear can lead to the formation of a single persistent coherent vortex. In our experiment there is a strong shear on the outer edge of the counter-rotating jet (see Fig. 2).

Dissipation appears to play little role in the dynamics of the jovian atmosphere[4], and the simulation by Marcus was done for an inviscid fluid. Our experiment is unusual in that the dynamical timescales of interest are typically an order of magnitude shorter than the viscous timescale. For example, the Ekman spin-down time $t_E$ is typically 20 s while the vortex turnover time is only 2 s. (Measurements of $t_E$ agree well with the calculated value, $t_E = h_0/2(\Omega\nu)^{1/2}$, where $\nu$ is the kinematic viscosity, $\Omega$ is the angular velocity of the tank, and $h_0$ is the mean depth

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