Tuning in to Noise

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TUNING IN TO NOISE

Since its introduction more than ten years ago, stochastic resonance has become widely recognized as a paradigm for noise-induced effects in driven nonlinear dynamic systems.

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Why, these balls bound; there's noise in it.
—W. Shakespeare, "All's Well That Ends Well."

Two sweeping generalizations can be made about most natural systems: They are intrinsically nonlinear and they operate in noisy environments. Examples abound, ranging from weather systems to oscillating chemical reactions to the movements of an eel. The most complex example is arguably the human central nervous system, flooded as it is with the "noise" of modern life.

For any such system subjected to a periodic modulating signal so weak as to be normally undetectable, one of a class of noise-induced cooperative phenomena can often be set up, leading to a resonance between the weak deterministic signal and the stochastic noise. Such a resonance, in effect, matches characteristic deterministic and stochastic time scales, making the signal apparent.

One of these cooperative phenomena, stochastic resonance, has been propounded as a possible explanation for the ice ages and has been demonstrated in a broad variety of physical systems including lasers, SQUIDs and tunnel diodes. Young as it is, SR—developed by Roberto Benzi and his coworkers a mere 15 years ago[1]—is already undergoing a renaissance of sorts.

The basic SR mechanism can be intuitively understood by considering a simple system—a bistable dynamical system that can switch between two stable states. Let's suppose that the dynamics can be characterized by a potential function (see the box on page 43). The system can then be visualized as a marble in a two-egg carton. A gentle rocking of the carton will cause the marble to roll back and forth within one of the egg wells; only under a much stronger disturbance will it surmount the wall and enter the other well. In the absence of any external forcing, friction will cause the system's output (the marble's position) to settle near the bottom of a well.

We observe a more complex output when an external forcing—composed of a deterministic "signal" (here assumed to be time-periodic) and stochastic "noise" (usually assumed to be Gaussian)—is applied. The external forcing may be interpreted as a periodic rocking of the potential (figure 1), while it is simultaneously jiggled randomly by the noise. If the deterministic rocking is too weak to cause the system to scale the potential barrier in the absence of noise we call it "subthreshold." The addition of even small amounts of noise, however, can give a finite switching probability to the response; that is, some potential barrier crossings will occur. For moderate noise, the switchings will acquire a degree of coherence with the underlying signal; the switching probability is briefly maximized whenever the signal is at its own maximum.

The barrier-crossing rate thus depends critically on the noise intensity. If the noise intensity is very low, the probability of any switching occurring at all is tiny. On the other hand, intense noise can induce switching even during an "unfavorable" interval when the signal is close to its minimum; the signal will be swamped. In between, one expects to find a range of noise intensities that induce switching events in near-synchrony with the signal. Intuitively, one expects this situation to correspond to some form of resonant behavior in the dynamics. It does.

SR classically defined

The "cooperation" between the signal and noise introduces coherence into the system, as shown in figure 2. This coherence is conveniently quantified by the power spectral density (PSD), $S(\omega)$, of the system response (figure 2a). For convenience, we assume that the signal has a sinusoidal time dependence $A \sin \omega_0 t$. For a symmetric potential function, the PSD consists of a Lorentzian-like noise background on which peaks are superimposed that correspond to the odd harmonics of the periodic signal. The amplitude, $S(\omega_0)$, of the fundamental rises with increasing noise strength, reaching a maximum value corresponding to the maximum cooperation between the signal and noise (figures 2c and 2d). A similar peak occurs in the output signal-to-noise ratio (SNR, figure 2b), defined below. Past this critical noise strength, the switchings gradually lose coherence with the signal frequency and the dynamics become noise-dominated.

The earliest definition of SR was the maximum of the output signal strength, $S(\omega_0)$, as a function of noise (although the higher harmonics in the PSD also demonstrate the effect). Under near-adiabatic conditions (discussed below), the critical noise strength corresponds to an approximate matching of the signal frequency to one-half the Kramers rate (defined as the characteristic escape rate from a stable state of the potential, in the absence of the signal).

The signal is still subthreshold. The above-described maximum in the noise-induced response means that the system will show a tendency to exhibit the periodic effects of the weak signal, as in figures 2c and 2d.

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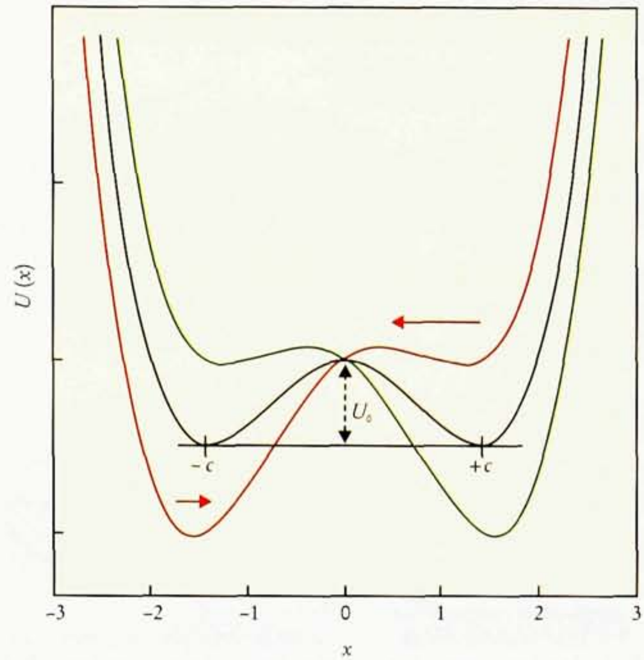


A BISTABLE POTENTIAL FUNCTION (black line) can introduce the basic features of stochastic resonance. In the absence of any forcing, the switching process between the two stable states (minima of the potential at $\pm c$) is noise-driven; the escape rate across the potential "barrier," U_0, is independent of time and state, being the same in either direction. With a finite periodic forcing term added (as in the box on page 43), the potential tilts periodically between the red and green configurations. When the periodic force is at its maximum (or minimum), the difference between the escape rates from the two states is maximum, as shown by the two unequal red arrows. FIGURE 1

This phenomenon has attracted considerable interest recently, and it has become common to compute the corresponding behavior in the output SNR, at the fundamental frequency ω_0, as the "fingerprint" of the effect. In practice the SNR, which should be contrasted with the amplitude $S(\omega_0)$ defined above, is usually computed (in decibels) as $SNR \equiv 10 \log_{10}(S/B)$ where S and B represent the values of the output PSD at the peak and the base of the signal feature respectively (figure 2a). The output SNR displays the same qualitative behavior as a function of noise (figure 2b) as the output signal strength $S(\omega_0)$, but with the resonance shifted slightly to a different noise value. (In contrast, for a *supra*-threshold periodic input signal, the output SNR decreases monotonically at high noise levels, closely following the input SNR.)

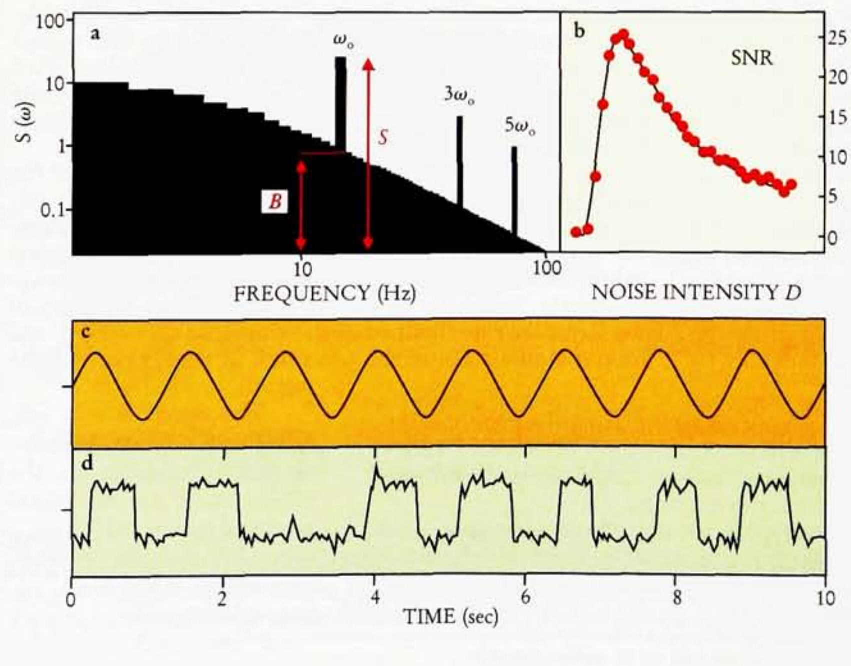
Although the usual characterization of the resonance has involved the output SNR versus the noise-strength profile, one can also, for fixed input noise, realize the resonance by changing other appropriate system parameters (notably the height of the potential barrier). This is important for practical applications, where adding noise to or reducing noise in the system might not be an option. SR is quite universal, and can occur in most *nonlinear* dynamical systems. Because SR relies on the cooperative interaction between different inputs, mediated by the system, it does not occur in linear systems.

Theories underlying the basic SR effect tend to be perturbative in nature, with $Ac/D \ll 1$. Here, A is the signal strength, c is the separation of the potential minima and D measures the intensity of the noise (assumed to be zero-mean, white Gaussian noise).

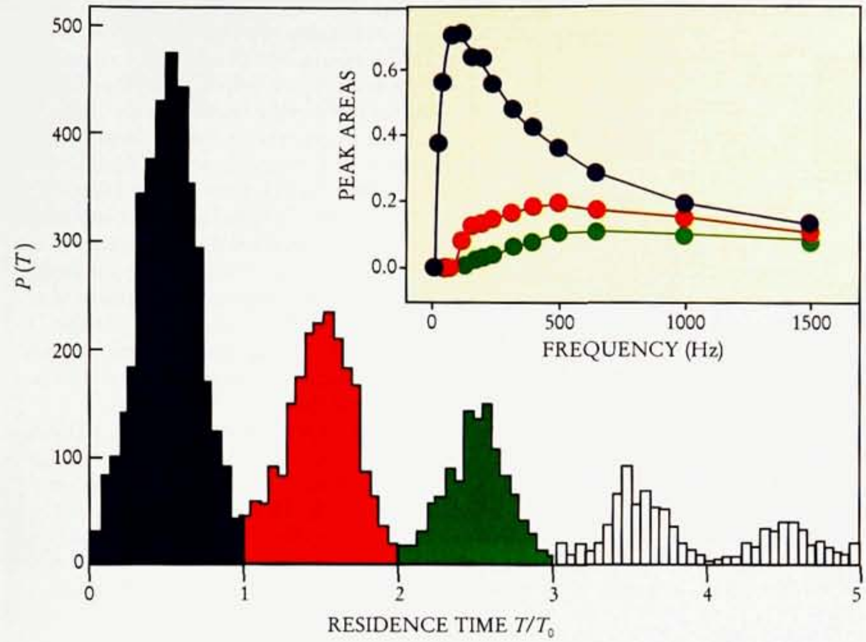
The adiabatic theory[2] is one of the earliest, and still the most commonly used, of these theories. It assumes the signal frequency ω_0 to be much smaller than the characteristic rate of equilibration in one of the (stable) states of the potential. In this theory, one introduces a Kramers rate (modified to include the effects of the signal) that explicitly uses the adiabatic and perturbative approximations, and then solves the rate equations underlying the dynamics. One can then compute the output PSD and thence the SNR at the fundamental:

$$\mathrm{SNR} \propto \left(\frac{Ac}{D}\right)^2 \exp(-U_0/D) \qquad (1)$$

The SNR has a maximum at a critical value of the noise intensity, $2D = U_0$. This result does not describe the SNR



CHARACTERISTICS OF SR, shown schematically for an overdamped standard quartic oscillator subject to white Gaussian noise and a weak periodic signal. **a:** Power spectral density, $S(\omega)$, of the output consists of odd harmonics of the driving frequency superimposed on a Lorentzian-like noise. Note the logarithmic scales. **b:** Output signal-to-noise ratio (SNR), in dB, passes through a global maximum at a critical value of the noise intensity, as discussed in the text. Intra-well motion (at very low noise) has been filtered out. **c:** The input periodic signal. **d:** The maximum coherence in the two-state output of the system, corresponding to the peak of the SNR. FIGURE 2.



RESIDENCE TIME DENSITY FUNCTION $P(T)$ consists of peaks located at odd multiples of one-half the driving period, $T_0/2$, with an exponentially decaying envelope. Individual peak areas pass through maxima either as functions of noise intensity or of frequency (inset). The critical frequency in each case corresponds to the synchronization condition (see equation 2). **FIGURE 3.**

behavior at very low noise. In that limit, the system is confined to one well and the motion is approximately linear; there is a sharp rise of the SNR with noise.

The basic SR effect is not limited to systems in which bistability occurs between two stable fixed points. Other forms of bistability (or multistability) abound in nature, and these systems can also display the same basic noise-enhanced response.[3–5] Some variant of this effect has been found in underdamped systems, in systems subject to chaotic rather than noisy backgrounds, in systems subject to state-dependent noise and in systems operating under a host of other scenarios.

A rash of SR theories,[3–6] in addition to the adiabatic approach, has erupted in the physics literature. Subject to their appropriate constraints, they all yield the same qualitative behavior in the output SNR for subthreshold signal inputs. Unfortunately, space constraints do not allow us to mention in detail all of the fine work and most recent developments in this field. A complete bibliography may be found on the World Wide Web at http://www.pg.infn.it/sr/.

A *bona fide* resonance

The signal-to-noise ratio behavior discussed above is the most widely used definition of SR. The "resonance" in the SNR, however, is not well quantified. The maximum in the SNR-versus-noise profile is only an approximate match between the deterministic and stochastic time scales that underlie the dynamics. Moreover, the SNR does not display a resonance as a function of the forcing frequency ω_0.

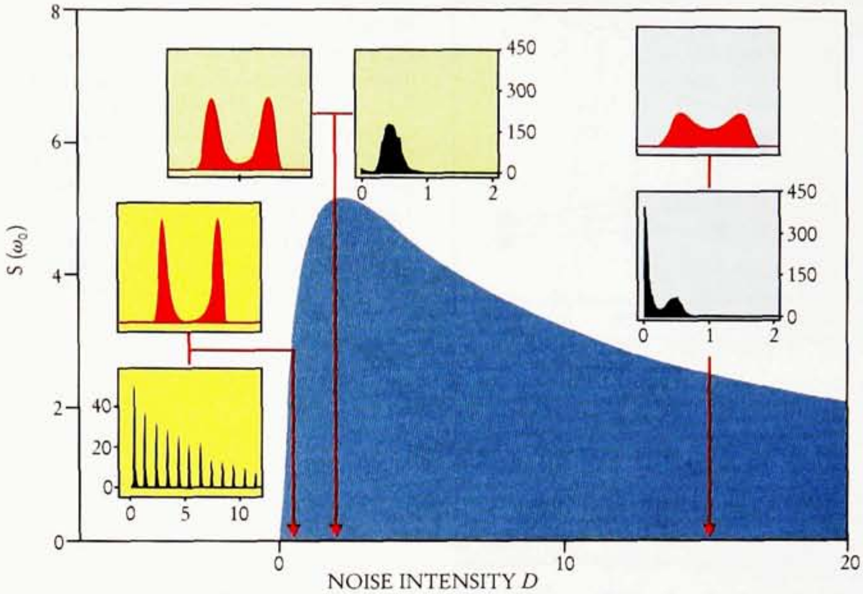
An alternative statistical description of the response is provided by mapping the continuous stochastic process into the discrete process corresponding to the sequence of residence times. A residence time, T, is defined as the time the system spends in a stable state between consecutive switches. The histogram $P(T)$ (see figure 3) of these residence times is called a residence times density function (RTDF) and consists of a sequence of peaks centered at $T_n = (n - \frac{1}{2})\, T_0$ (where $T_0 \equiv 2\pi/\omega_0$ is the forcing period and n is a positive integer). These peaks have exponentially decreasing amplitudes, and are superimposed on an exponentially decaying background.[2,6] The sequence of peaks implies a sort of phase locking of the dynamics to the signal; if an escape does not occur at the first maximum of the signal cycle, then the next opportunity (with the greatest probability) to escape occurs one cycle later, and so on. Although the mere occurrence of these multimodal histograms is not a signature of SR, they *are* a manifestation of the synchronization between the switching mechanism (the noise) and the external signal. The heights $P(T_n)$ of successive peaks are known[3,6] to pass through maxima as functions of the noise intensity D. Given that the unperturbed Kramers rate μ_K (introduced in the preceding section) is a function of noise, the condition for a resonance in the peak areas, at the critical noise intensity D_c, can be expressed as a frequency-synchronization condition:[7]

$$T_0^{-1} = (n - \tfrac{1}{2})\, \mu_K (D_c) \qquad (2)$$

We now have an alternative definition of SR: namely the $n = 1$ case, for which the peak at $T_1 = T_0/2$ achieves its maximum area (the green insets in figure 4) and which corresponds to the maximum synchronization ($T_0 = 2\mu_K^{-1}$). The area under the first peak in the residence time density function is, in fact, a direct measure of the probability that the system switchings are driven by the periodic force. The resonant condition corresponds to the leading peak being located at $T_1 = \mu_K^{-1}$, and having the greatest area or height. Despite the intuitive depiction in figure 4, no precise connection has yet been demonstrated between the RTDF and the PSD, except in very simple systems wherein all correlations between successive escape events can be neglected. The new formalism, however, permits us to exactly define SR as a *bona fide* resonance that matches the two characteristic time scales: stochastic (the Kramers rate in the absence of the signal) and deterministic (the signal frequency). The resonance may be realized by varying either the noise intensity or the signal frequency,[7] as in the inset to figure 3.

Stochastic resonance and neuroscience

An exciting possible role for SR has recently been found in the neurosciences. Neurons are often subject to both random and deterministic signals. In a neurophysiological experiment, the observed response is usually a sequence of narrow pulses or spikes that correspond to firing events. It is generally accepted that sensory information is en-



CONNECTION BETWEEN ORIGINAL and new formulations of stochastic resonance. The main figure is similar to figure 2b, but at the driving frequency ω_0. Each pair of inserts shows the position probability density function, $P(x)$, in red and the residence time density function, $P(T)$, versus T/T_0 in black. At low noise (yellow insets) the RTDF has many peaks, located at $T/T_0 = n - 1/2$, indicating that interwell hopping can occur in any of several cycles of the signal. At the maximum of $S(\omega_0)$ (green insets), there is a dominant peak in the RTDF at $T_0/2$, indicating the near-synchrony of switching events with the signal. This peak is much higher than at low noise, and is a true resonance. At high noise (blue insets), the switchings are noise-dominated. **FIGURE 4**

coded in the interspike intervals, which can contain significant random components. The classical SR effect has been recently observed in some biological experiments with injected noise.[3,8] We must note, however, that neurophysiologically relevant signals are often aperiodic; for these signals, the output SNR can be ill-defined, uninformative, or irrelevant, and alternative measurements of the response must be considered.

There has been some recent work in this area.[9] Jay Levin and John Miller have shown the existence of SR and noise-enhanced signal transduction in a cricket *cercal* neuron subject to a nonsinusoidal input signal in a white Gaussian noise background. The "resonance" in this case appears in an experimentally measured input–output trans-information function that has been studied theoretically by Michael deWeese and William Bialek and by us. Other measures of the response of nonlinear model neurons to nonsinusoidal inputs have been recently proposed by Jim Collins, Laszlo Kiss and their coworkers.

In the neurophysiological literature, however, the distribution $P(T)$ of the interspike time intervals T_i is ubiquitous. The inverse of the mean value $\langle T \rangle$ of such a histogram leads directly to a mean firing rate that is often used to characterize neurons in noisy environments. These observations led André Longtin and his coworkers[10] to advance a bistable dynamical single-neuron model, with a potential as in the box on page 43 for the Hopfield neuron. This model provides a simple explanation of the residence-time histograms obtained experimentally[11] in sensory neurons subjected to time-periodic signals in Gaussian noise backgrounds. The dynamical variable in this model is the neuronal cell membrane voltage, and firing corresponds to crossing the potential barrier, which plays the role of a firing threshold. The model elucidated the sequence of "reset" intervals between firings, in addition to explaining many of the experimentally observed properties of the RTDF and the mean firing rate. Further, the model could fit the experimental data by changing any of the stimulus, the noise intensity or the potential barrier height (in this case, the firing threshold). However, the model provides only a coarse-grained picture of neural dynamics in the presence of noise. Real neurons, when bistable, are more likely to follow excitable dynamics—described by a bifurcation between a fixed point and limit cycle—in which SR was first quantified by Longtin.[4]

In the simplest cases, neural firing may be modeled by "integrate–fire dynamics." The firing threshold is modeled as a fixed barrier that is approached by a random walk under a stimulus that can have both random and deterministic components. When the threshold is reached, a delta spike is emitted (corresponding to a firing event) and the state point (the cell membrane voltage) is reset to its initial value. With subthreshold time-periodic signals in white Gaussian noise backgrounds, one can obtain a slightly modified synchronization signature of stochastic resonance as well as a resonance in the mean (noise-dependent) firing rate as a function of the stimulus frequency.

There are some important caveats. There is no known mechanism for dynamically changing the noise in real neurons. However, there have been experiments in which the neuronal response has been optimized by changing such factors as ambient temperature or neurotransmitter concentrations, but neither of these changes has been precisely connected to a variable noise strength. Hence, although background randomness seems to play a significant role in the dynamics of certain neurons in the central nervous system, the SR effect has not yet been shown to be fundamental in the processing of information. Nevertheless, in response to deterministic signals embedded in *external* Gaussian noise, all the neural models listed in this section display a version of SR. It must be remembered, though, that the experimentally obtained RTDFs[11] cannot exist without noise. Clearly, therefore, neuroscience is an area in which theoretical predictions, based on accepted neurophysiological models, can be used to guide future experiments.

Stochastic resonance in extended systems

Extended systems are currently in vogue. They are composed of coupled nonlinear-dynamic elements subjected to the usual weak periodic signals in a noisy background. This line of research has potential applications in signal processing, the physics of coupled nonlinear devices and neurophysiology in which one might describe the dynamics of populations of neurons. Although mean-field treatments of coupled many-body dynamical systems abound in the physics literature, the first studies of SR-like behavior in such systems were carried out in 1985 by Roberto Benzi and coworkers on a stochastically perturbed system of Ginzburg–Landau equations. In 1992, studies

Nonlinear Dynamic Systems

The simplest version of a one-dimensional nonlinear dynamic system that exhibits stochastic resonance is an overdamped system of the form

$$\frac{dx}{dt} = -\frac{\partial U}{\partial x} + F(t) + A\sin\omega_0 t$$

with the restoring force expressed as the gradient of some bistable or multistable potential function $U(x)$. In addition to the time-periodic signal $A\sin\omega_0 t$, we assume the presence of noise $F(t)$, which is generally Gaussian and exponentially correlated. In this article, however, we take $F(t)$ to be zero-mean white noise: $\langle F(t)\rangle = 0$ and $\langle F(t)F(t')\rangle = 2D\,\delta(t-t')$. The signal amplitude A is assumed to be subthreshold throughout, so that deterministic switching between the two or more stable states of the potential does not occur.

The state point $x(t)$ denotes, in the most commonly considered case, the displacement of a particle in a "standard quartic" potential (see figure 1),

$$U(x) = -\alpha x^2 + \beta x^4$$

with stable fixed points (extrema) at $x = \pm\sqrt{\alpha/2\beta}$, an unstable fixed point at $x = 0$, and a potential barrier height $U_0 = \alpha^2/4\beta$. This system remains dynamically stable for $\beta > 0$, and becomes monostable (although not parabolic) for $\alpha \le 0$.

Another example of a nonlinear dynamic system is the analog Hopfield neuron,

$$U(x) = \alpha x^2 - \beta \ln\cosh x$$

in which the state point $x(t)$ might denote a cell membrane voltage. Still another is the rf SQUID loop,

$$U(x) = \alpha x^2 - \beta\cos 2\pi x,$$

in which $x(t)$ denotes the magnetic flux in the loop. For these last two cases, α and β are positive (which also guarantees dynamic stability) and for $\beta = 0$ the potential is parabolic (like a harmonic oscillator), becoming bistable (or multistable, for the SQUID) when β exceeds a critical value. When models such as these are used to characterize real sytems, α and β are usually given in terms of system parameters, so that they can be altered or "tuned" in experiments or simulations.

by Peter Jung and his coworkers focused on a system of linearly coupled bistable elements described by a nonlinear master equation, in the mean-field limit.

Last year, John Lindner and his coworkers[12] studied the behavior of a bistable Duffing chain, with linear nearest-neighbor coupling, subjected to white noise $F_i(t)$ that is independent from site to site but with the identical subthreshold periodic signal at every site. The ith element obeys the dynamics

$$\frac{dx_i}{dt} = \alpha x_i - \beta x_i^3 + \varepsilon(x_{i+1} - 2x_i + x_{i-1}) + A\sin\omega_0 t + F_i(t) \quad (3)$$

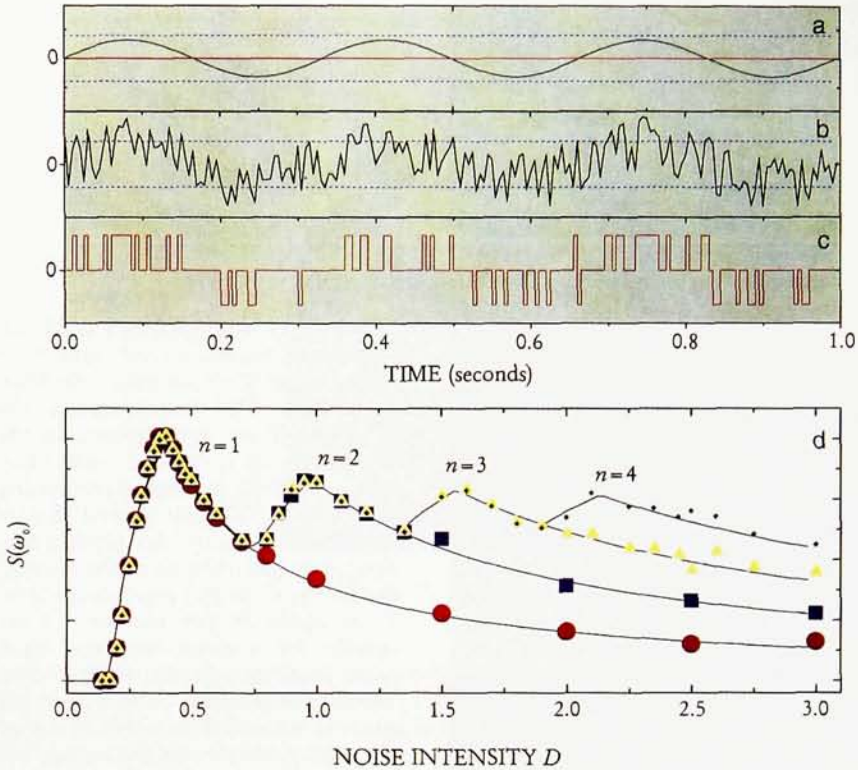
The SNR response of a single element can be enhanced by coupling it into the chain; this is now known as array-enhanced stochastic resonance. Further, the output SNR can be maximized by treating either the noise or the coupling as design parameters. The optimal noise variance scales as the number of elements in the array, N, and the optimal coupling strength scales as N^2. These scaling relations have been recently derived rigorously by Fabio Marchesoni and us. The global maximum in the output SNR across the array corresponds to a noise-induced spatiotemporal synchronization of the array to the external signal frequency, with the location on the noise axis and magnitude of the SNR maximum depending critically on system parameters such as the coupling.

This behavior is seen in the cover picture of this issue. Each panel of the mosaic represents the time evolution (vertical axis) of a two-state (red or blue) array of 513 elements (horizontal axis). The optimally tuned system is second from the top and third from the left, and shows a well-defined band structure. In this configuration, the entire array behaves like a solid rod, switching between red and blue states every $T_0/2$ seconds. Further, in this best of all possible states, the output SNR of any element in the array is at its maximum; with carefully selected system parameters, this output SNR can exceed that of an isolated element. An optimal coupling strength (column in the mosaic) can be found for any given noise level if the noise is not too low. Conversely, for a given coupling, some noise level can be found that best "tunes" the system. The upper left panel is entirely dominated by noise. (For the example on the cover, the array's elements are linearly coupled, subjected to the same time-periodic signal and to white Gaussian noise that is independent, or uncorrelated, from site to site. The ith element obeys equation 3.)

This tunability exists in a smorgasbord of nonlinear dynamic systems. Another example, recently studied by Mario Inchiosa and us,[12] is an array of globally coupled neuronlike bistable elements with coupling that is nonlinear and nonidentical but with noise that *is* identical at every site. In this case the SNR enhancement depends critically on the magnitudes and signs of the coupling coefficients. One can also define an input SNR for this array. For Gaussian input noise, the output SNR of a single element or an array is always bounded above by the input SNR. In an enticing conspiracy, however, nonlinear coupling can completely remove oscillator nonlinearities, allowing two coupled nonlinear elements to perform like a linear system. Hence the benefits of SR for signal processing applications are likely to be most pronounced when the elements are intrinsically nonlinear.

Spatiotemporal patterns have been observed in neurophysiological experiments, and synchrony of the type described above may be a feature of epileptic seizures. Accordingly, simple models such as those discussed here, although lacking neurophysiological rigor, may provide an important starting point for understanding very complex phenomena in the central nervous system.

The models can also be used to describe some features of phenomena as diverse as coupled oscillations in chemical reactions, nonlinear transmission lines and the locomotive function of the tail of the lamprey eel. Some of the groups studying noise-induced cooperative behavior in many-body systems include Jung and Gottfried Mayer-Kress (University of Illinois), Collins and coworkers (Boston University), Alexander Neiman (Saratov State University, Russia) and Lutz Schimansky-Geier (Humboldt University, Berlin), but this still remains a largely unexplored area.



SR IN SIMPLE THRESHOLD SYSTEMS, such as analog-to-digital converters, loses its peculiar resonant character and can be interpreted as a noise-induced threshold-crossing mechanism known to electronic engineers as dithering. **a:** The periodic signal is below threshold (dashed lines), and its quantization would result in a constant output (red line) with a clear loss of signal detail. **b:** The addition of noise dither of the appropriate intensity enables the signal to cross the thresholds in rough coincidence with its maximum amplitude. **c:** The quantized output then reproduces more closely the characteristics of the analog input signal. **d:** For an n-threshold system in the presence of a uniformly distributed noise dither, the power spectral density amplitude at the signal frequency, $S(\omega_0)$, displays multiple peaks as a function of the noise intensity D. See reference 13 for further details. FIGURE 5

Putting noise to work

There is optimism that new algorithms and devices, based on SR, may be realized in the near future. Here are a few possibilities.

Signal analysis. The signal-processing capabilities of an algorithm based on SR have recently received attention due to the counterintuitive notion discussed above: For subthreshold input signals, increasing the input noise can increase the output SNR. At least three issues are being looked at: detection, estimation and processing. For signal analysis, the best measure of performance consists of the so-called receiver operating characteristics, which are analogous to SNRs but are curves of detection probability. For a large class of nonlinear transducers, the detection probability passes through a maximum at the same value of input SNR that maximizes the output SNR; once again, this is stochastic resonance standing up to be noticed. These nonlinear systems, however, do not outperform the optimal linear systems when detecting periodic signals in white Gaussian noise backgrounds, although careful arraying (see preceding section) can improve their performance. The SR effect and its potential for signal processing have not yet been explored for more complicated (such as non-Gaussian and nonstationary) noise backgrounds that are often encountered in real-world signal-processing applications.

When signal detection in noisy environments *is* carried out under SR conditions, the interchangeable roles between the noise and the detector threshold can be exploited. Although adding more noise to an already noisy system is not a procedure to be trifled with, the fact that SR can be obtained by adjusting other control parameters is useful. For example, the signal might be made more detectable by lowering the detector threshold, if possible.

Electronic devices and dithering. Electronic devices have been used in SR studies for many years, and most of them have been intrinsically bistable analog devices. Recent theoretical studies by Jung, Zoltan Gingl and their coworkers have focused on a class of discrete bistable (threshold) systems, whose behavior can be reproduced by hybrid devices. One of the simplest of these devices (see figure 5) is the 1-bit analog–digital converter. The performance of an ADC can be improved by the well-known dithering effect: A carefully controlled amount of noise (dither) added to the analog signal before it enters the ADC will greatly reduce (on average) the quantization error introduced during the ADC operation. The benefits of this technique depend on the statistics of the dither (a uniformly distributed noise outperforms Gaussian dither) and on its intensity. The quantization procedure can thus be improved by tuning the noise intensity to an optimal value, in much the same way as in the SR phenomenon. For such systems, SR loses its peculiar resonant nature and becomes synonymous with dithering.[13] The output signal enhancement can thus be obtained for nonperiodic signals as well.

The generalized multibit ADC acts as a multithreshold system in the presence of noise and periodic inputs. Here the SR phenomenon displays some peculiarities that are not observed in simple bistable systems. For instance, there is a clear dependence on the noise statistics: While Gaussian noise produces a single-peak output-versus-noise curve, uniformly distributed noise produces a multipeaked characteristic curve (figure 5). As in the single-threshold case, the SR in multithreshold systems can be described without reference to any frequency matching condition, as a special case of the dithering effect.[13]

Optical and magnetic devices.[14] One of the first experimental realizations of stochastic resonance was carried out by Bruce McNamara and his coworkers in a bistable ring laser; the hopping dynamics between the clockwise and counterclockwise propagating modes were modulated by the injection of an external noisy periodic signal. More recent experimental demonstrations include the distributed feedback laser (John Ianelli and his coworkers), the optical bistability (Mark Dykman and his colleagues) and the unidirectional photorefractive ring resonator (Bradley Jost and Bahaa Saleh).

Magnetic systems have also proved to be fertile ground for SR-based devices. These include electron paramagnetic resonance systems (Gammaitoni and colleagues) and SR-based superconducting quantum interference devices (Andrew Hibbs, Richard Rouse and their coworkers). In particular, recent work has investigated how SR might be utilized in a novel way to permit more robust operation of SQUIDs in a noisy environment. Such devices, which may ultimately be incorporated into simple arrays of the type discussed in the preceding section, are expected to find utility in applications centered on the detection and quantification of extremely weak magnetic signatures. Such applications include biomagnetic and geomagnetic imaging, noninvasive testing, fundamental physics experiments and even mine detection for the military.

There may well be a new generation of nonlinear devices and applications on the horizon, in which background fluctuations need not be minimized, but rather can be used in a constructive way as an aid to performance.

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