Row-Column Beam Steering Control of Reflectarray Antennas: Benefits and Drawbacks

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Abstract— **This letter proposes a method for controlling single-polarized phase-only reconfigurable reflectarray antennas by rows and columns instead of element by element, which enables an important simplification of the control circuitry. First, the fundaments of the method are presented and then the implications on analog and digital implementations are discussed. Finally, applications beyond beam steering are also addressed. It is shown that this is a promising method for pure beam steering applications in combination with analog control or with phase quantization with at least two bits. Remarkably, this last result contradicts the existing literature.**

Index Terms—**Beam steering, phase quantization, reflectarray, row-column**

I. Introduction

RECONFIGURABLE reflectarray antennas (RA) constitute a good low complexity alternative to traditional phased arrays, thanks to their spatial feeding mechanism. However, the complexity of the beam scanning control circuitry becomes challenging for large RAs [1]. Several methods were proposed to simplify it such as sequential loading [1] or element gathering [2]. A high reduction of control lines and circuitry is possible by the separate illumination proposed in [3]. It is based on addressing the elements by rows and columns, which leads to a potential reduction of biasing lines from NxM to N+M, being N and M the number of RA rows and columns. However, the method is not accurately assessed in [3], especially the role of the spherical illumination and the effects of 1-bit quantization.

Preliminary work from the author proposed a RA cell enabling analog row-column beam scanning control [4]. In this letter, a thorough analysis of the benefits, drawbacks and limitations of analog and digital implementations of the row-column scheme is conducted for the first time. The study is focused on single-polarized phase-only RA solutions and addresses pure beam scanning applications as well as multi-beam or shaped beam solutions. Remarkably, it demonstrates that the proposed method is only suitable for pure beam steering applications with analog control. Digital control with quantization of at least two bits may be also feasible but at the expense of a lower bias line reduction with respect to analog schemes. The letter also proposes a quantitative indicator of the feasibility of the row-column control in each targeted application based on the singular value decomposition.

This manuscript was submitted for review on 14/09/2017. It received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No 645047 (SANSA); the Spanish Ministry of Economy and Competitiveness under project TEC2014-59255-C3-1-R; and from the Catalan Government (2014SGR1567).

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II. Row-Column Reflectarray Control Scheme

Focusing on a single linear polarization, the requirement for controlling a rectangular RA with N rows and M columns by a row-column scheme, is that the targeted matrix of co-polar scattering coefficients over the RA, must be expressed as the multiplication of a row and a column vector; hence

$$\boldsymbol{S_{RA}} = \boldsymbol{rc}; \qquad (1)$$

where $\boldsymbol{S_{RA}}$ is the N x M desired matrix whose components $\boldsymbol{s_{RA,ij}}$ are the co-polar reflection coefficients of the element at row i and column j in the polarization of interest (i.e. s_{xx} or s_{yy}); and $\boldsymbol{r}$ and $\boldsymbol{c}$ are a N x 1 and 1 x M vectors, respectively. In the case of phase-only RAs, as addressed in the remainder of the paper, the multiplication in (1) translates to a phase addition. In this way, as depicted in Fig. 1, all elements in the same row (column) share the same control and the phases set by the row and column controls are added at cell level. This scheme enables a potential reduction of biasing lines and control circuitry (e.g. digital-to-analog converters) from NxM to N+M.

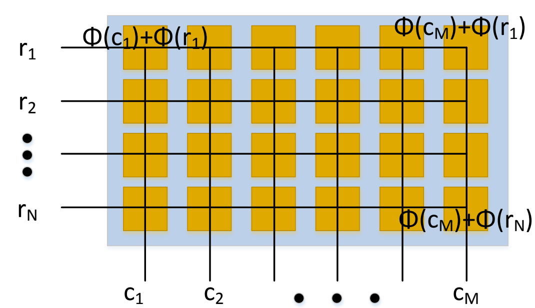

Fig. 1. Row-column reflectarray control scheme

However, basic algebra shows that the decomposition in (1) is only possible if

$$\text{rank}(\boldsymbol{S_{RA}})=1, \qquad (2)$$

which is defined here as the separability condition, and which limits the application of the row-column control. In practice, this separability condition means that the phase shift to be synthesized by a unit cell must be expressed as the sum of two terms, one depending only on the row index of the unit cell and the other on the column index.

The discussion above considered a single linear polarization. Dual linear or circular polarization schemes are out of the scope of this paper. However, it must be noted that in such cases, condition (1) still applies independently to each linear polarization.

III. Analog Beam Steering Control

The RA phase distribution required in order to form a pencil beam towards (θ_o,φ_o) is

$$\phi_{RA,ij} = kd_{ij} - kx_{ij}cos\varphi_o sin\theta_o - ky_{ij}sin\varphi_o sin\theta_o \quad (3)$$

where d_{ij} is the distance from the feed to each RA unit cell, and (x_{ij},y_{ij}) is the relative position of each unit cell. It is easy to check that the corresponding $\boldsymbol{S_{RA}} = e^{j\boldsymbol{\phi_{RA}}}$ does not fulfil the separability condition (2). Indeed, the two last terms of (3) are variable phases that only depend on the target direction and on the *x* and *y* coordinates of the unit cell, respectively; thus supporting separability. However, the first term is a fixed phase related to the spherical illumination, which cannot be split in two independent terms depending only on the row and column indexes. Remarkably, this term was neglected in the formulation of [3]. The row-column control can be still used by building three phase shifting structures independently implementing the three separated terms of (3) and being added at cell level. Since the first term is fixed for each unit cell in the RA, it does not need any control line, thus the scheme in Fig. 1 can be still applied. A unit cell following this scheme was proposed in [4]. Its evaluation revealed two new sources of phase errors: (i) due to the imperfect independency of the three phasing structures; and (ii) due to the unavailability of compensating the different incidence angles element by element since the controls are shared by a whole row (column). However, simulated radiation patterns showed that the main impact of these errors is a slight increase of the sidelobe levels, which may be acceptable for a large number of applications, thus confirming the feasibility of the method.

A. Rank one approximation

The main challenge of the previous method is the implementation of three independent phasing structures in the area of a unit cell [4]. Therefore, this section analyzes the approximation of the phase distribution in (3) by a rank one matrix. This solution permits a direct application of the scheme in Fig. 1, thus requiring only two variable phasing structures [the first term in (3) is not compensated].

The best rank one approximation in the sense of MMSE (Minimum mean squared error) is given by the singular value decomposition (SVD). That is, performing the SVD, the targeted $\boldsymbol{S_{RA}}$ can be expressed as

$$\boldsymbol{S_{RA}}=\mathbf{USV^T}. \quad (4)$$

In turn, the rank one approximation writes

$$\boldsymbol{S'_{RA}} = s_{11}\boldsymbol{u_1}\boldsymbol{v_1^T}, \quad (5)$$

where s_{11} is the highest singular value and $\boldsymbol{u_1}$ and $\boldsymbol{v_1}$ are the singular vectors associated to it, thus the first columns of matrices **U** and **V**, respectively. Therefore, the desired $\boldsymbol{r}$ and $\boldsymbol{c}$ vectors in (1) are approximated by $\boldsymbol{u_1}$and $\boldsymbol{v_1^T}$ normalized to amplitude one. The quality of this approximation can be assessed by the ratio of the highest singular value to the sum of all singular values:

$$K = \frac{s_{11}}{\sum s_{ii}}. \quad (6)$$

The closer *K* is to 1, the better is the approximation in the MMSE sense.

For phase distributions of the type of (3), the values of *K* only depend on the first term of (3), i.e. on the RA optics. This is exemplified in Table I, which shows the *K* values of an ideal square RA with 29x29 elements and edge illumination of -10 dB, for two different focal length to diameter ratios (F/D), and for a center-fed and an offset configuration. In addition, Table I also shows the simulated gain reduction and the sidelobe level (SLL) increase experienced with the rank one approximation with respect to the traditional direct application of (3). Two conclusions can be extracted: (i) large F/D ratios produce *K* values close to one enabling the use of the rank one approximation; (ii) *K* values close to one assure little impact on the radiation patterns when using the rank one approximation, while the effects become more unpredictable for reduced *K*. The reason for this last is that the radiation patterns are not only sensitive to the MMSE but to the distribution of errors across the RA [5].

TABLE I. Evaluation of the Rank One Approximation

Configuration	*K*	Gain reduction (dB)	SLL increase (dB)
a) F/D=1, offset=0	0.91	0	0.2
b) F/D=0.5, offset=0	0.67	0.6	9.5
c) F/D=1, offset=0.5D	0.76	0.2	1.8
d) F/D=0.5, offset=0.5D	0.45	1.9	6.8

Fig. 2. compares the radiation patterns obtained for the four configurations considered in Table I when applying the phase distribution of (3) or their rank one approximation as in (5).

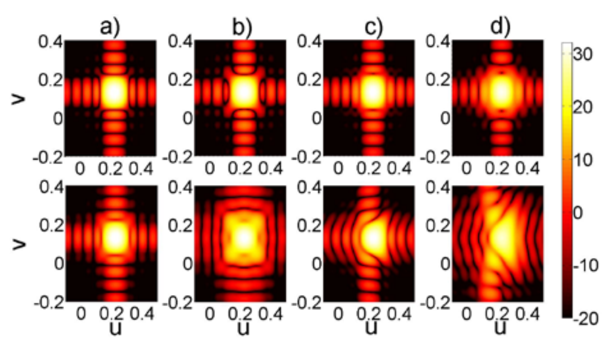

Fig. 2. Radiation pattern [gain (dB)] comparison for the four configurations of Table I when pointing a beam towards (θ,φ)=(15°, 30°); i.e. (u,v)=(0.22, 0.13). First row: ideal phase distribution as in (3); second row: rank one approximation as in (5).

The rise of SLL and the deformation of the main beam become evident for F/D = 0.5 configurations. The radiation patterns are shown in the angular coordinates $u=sin(\theta)cos(\varphi)$ and $v=sin(\theta)sin(\varphi)$.

IV. Digital Beam-Steering Control

In reference [3], a drastic reduction of the cell complexity is proposed by combining the row-column scheme with 1-bit quantization. In this case, the phase addition at cell level can be implemented as a simple XOR function. However, a SVD analysis using the RA configuration tagged as *a)* in previous section showed not only that the quantized version of $\boldsymbol{S_{RA}}$ [i.e. $\boldsymbol{S_{RA}} = e^{jQ(\boldsymbol{\phi_{RA}})}$, being $\boldsymbol{\phi_{RA}}$ the phase distribution in (3)] does not fulfil the separabilitiy condition, but that K values can be as low as 0.24 or 0.36 for 1-bit and 2-bit quantization, respectively. In these cases, K depends on the pointing direction; for example, the above values were obtained for (θ,φ)=(20°, 30°). The main reason is that the quantization of the addition of two terms is not equal to the addition of the two terms quantized.

Therefore, a good rank one approximation in the MMSE sense does not exist in the digital case. However, since the relation between MMSE and the distortion of the radiation pattern is not direct [5], it is important to evaluate the effects of the digital row-column control on the radiation patterns. Three digital row-column schemes are discussed next:

Scheme A: Following the analog implementation in Section III, the spherical illumination [first term of (3)] is compensated by a fixed phasing structure at each cell, and quantized versions of the other two terms are added at cell level, so

$$\phi^{Q}_{RA,ij} = kd_{ij} - Q(kx_{ij}cos\varphi_o sin\theta_o) - Q(ky_{ij}sin\varphi_o sin\theta_o) \quad (7)$$

where Q is the quantization function.

Scheme B: First, the phase distribution in (3) is quantized. Then the rank one approximation to the corresponding $\boldsymbol{S_{RA}} = e^{jQ(\boldsymbol{\phi_{RA}})}$ defined by (5) is applied.

Scheme C: The $\boldsymbol{r}$ and $\boldsymbol{c}$ vectors building the desired rank one matrix in (1) are directly optimized using the iterative random search (RS) algorithm [6] depicted in Fig. 3.

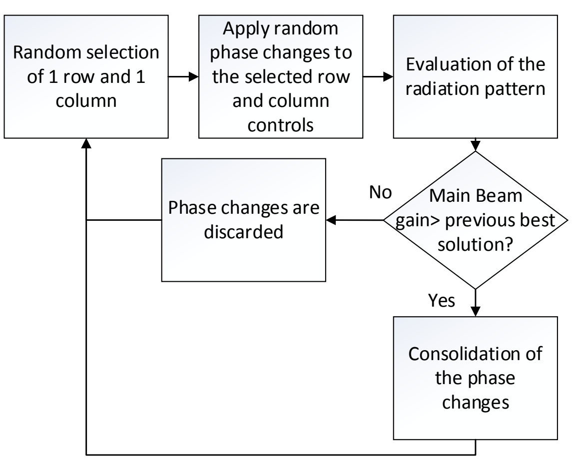

Fig. 3. Flow chart of the random search iteration.

Fig. 4 compares the radiation patterns obtained by the three row-column schemes under 1-bit quantization, when forming a pencil beam towards (*θ*, *φ*)=(15°, 30°). The classical per element control is shown as a reference. Table II extends these results to a 2-bits quantization.

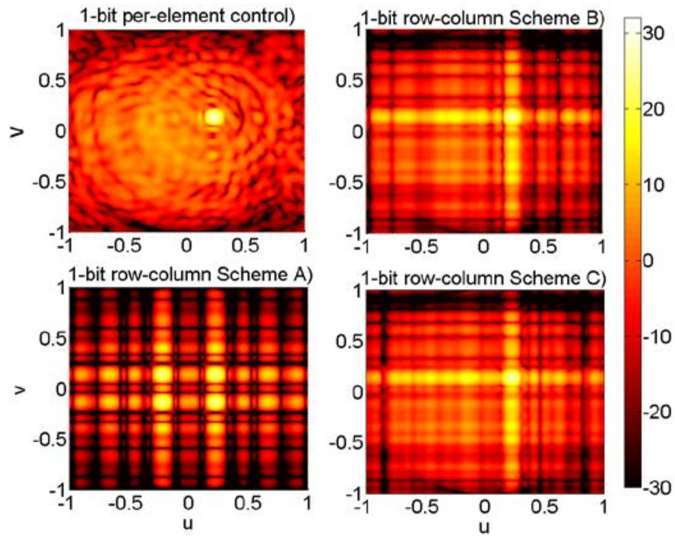

Fig. 4. Radiation patterns [gain (dB)] obtained by the three schemes combining 1-bit and row-column control when pointing to (*θ*, *φ*)=(15°, 30°); i.e. (*u*, *v*)=(0.22, 0.13). Traditional 1-bit per-element control is shown as a reference.

Table II. Radiation properties obtained by the three digital row-column schemes when pointing to (θ, φ)=(15°, 30°); Analog, 1- and 2-bits per-element control are shown as a reference.

Control scheme	Gain (dB)	SLL (dB)
Analog per element control	32.7	-23.1
1-bit per element control	28.9	-17.6
1-bit row-column Scheme A	24.9	0
1-bit row-column Scheme B	26.7	-8.9
1-bit row-column Scheme C	26.9	-8.7
2-bits per element control	31.8	-23
2-bits row-column Scheme A	30.9	-10.2
2-bits row-column Scheme C	31.4	-16.7

For 1-bit quantization, it can be observed that Scheme A results in undesired quantization lobes with the same gain as the main beam. The reason is that the first term in (3) is just the term that randomizes the phase errors when $\boldsymbol{\phi_{RA}}$ is quantized [5]. These lobes disappear when Schemes B and C are used, which provide very similar results. However, in both cases, the SLL at the principle plains of the main beam are above -10 dB, too high for most applications. The RS optimization of Scheme C was compared against the binary particle swarm optimization (BPSO) described in [7], and again, very similar results were obtained, confirming that the limitation is on the row-column solution and not on the optimization method. The BPSO results are omitted here for space constraints.

The same trend is observed for 2-bits, but in this case, the direct optimization of Scheme C produces SLL below -16 dB, which start becoming acceptable in some applications. Note however, that the 2-bits implementation suffers again from high complexity since it cannot be just obtained through a XOR function as with 1-bit. Indeed, the unit cell implementation for the 2-bits control could follow the analog design of [4], substituting each varactor by multiple switches, but such implementation would require multiple control lines per row (column) limiting the benefits of the row-column scheme. Note also that the rank one approximation is not calculated for the 2-bits case, since the SVD of a phase distribution matrix quantified to 2-bits produces singular vectors whose values are not restricted to the four phase states.

The analysis of the RA phase distributions gives a better idea of the limitations of the 1–bit and row-column combination. As shown in Fig. 5, the 1-bit row-column schemes produce rectangular blocks, which cannot emulate the circular shapes required for a proper beam focusing.

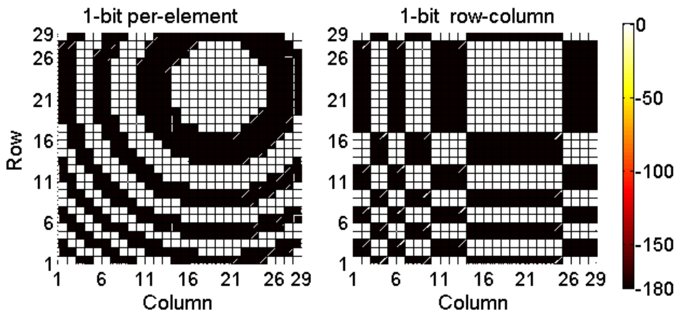

Fig. 5. Comparison of the RA phase distributions (in degrees) with 1-bit quantization when pointing to (θ,φ)=(15°, 30°).

V. Advanced Beamforming

So far, only pure beam steering has been discussed but other applications such as multi-beam or shaped beams can be also targeted by reconfigurable RAs. In these cases, the rank of the targeted $\boldsymbol{S_{RA}}$, and more specifically its associated K value, can be used to predict the feasibility of the row-column scheme. In general, it has been observed that K values are only close to one in very particular cases such as having the multiple beams or the special shapes constrained in a single u=ct or v=ct plane, where ct is a constant; or for a pure square beam. This is exemplified in Fig 6 and 7, which compare the radiation patterns obtained with the traditional per-element and the proposed row-column controls in a multi-beam and a shaped beam application, respectively. In both cases, an analog control is assumed and the row-column controls are obtained through optimization. Fig. 6 addresses a dual-beam RA pointing to $(\theta_1, \varphi_1)=(15°, 30°)$ and $(\theta_2, \varphi_2)=(30°, 130°)$; *i.e.* $(u_1, v_1)=(0.22, 0.13)$ and $(u_2, v_2)=(-0.32, 0.38)$.

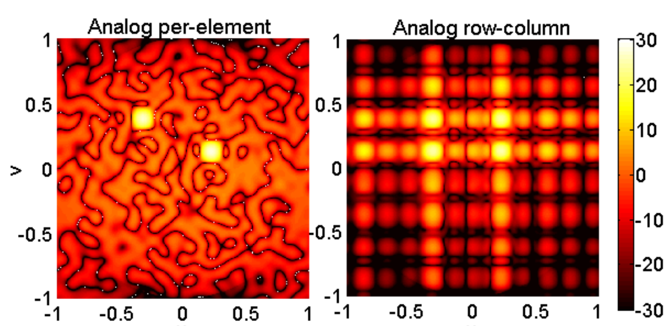

Fig. 6. Radiation patterns obtained by analog per-element and row-column controls when pointing at (θ_1,φ_1)=(15°, 30°) and (θ_2, φ_2)=(30°, 130°); i.e. (u_1,v_1)=(0.22,0.13) and (u_2,v_2)=(-0.32,0.38).

It can be observed that, in addition to the targeted beams, the row-column control produces two main undesired beams at (u_1, v_2) and (u_2, v_1). In this case, the value of K was 0.16, thus predicting the poor performance of the row-column scheme.

Fig. 7 evaluates the feasibility of producing a circular beam shape centered at $(\theta,\varphi)=(15°,60°)$. Again a K value of 0.17, predicted a poor performance of the row-column control, which here translates to the conversion of the targeted circular beam into a square beam.

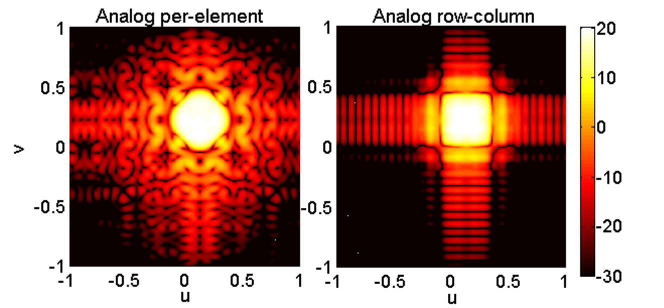

Fig. 7. Radiation patterns obtained by analog per-element and row-column controls when synthesizing a circular beam shape centered at (θ_1,φ_1)=(15°, 60°), i.e. (u_1,v_1)=(0.13,0.22).

VI. Conclusion

This letter analyzed a promising method for reducing the complexity of the control circuitry of RA based on addressing the elements by rows and columns. In contrast to the existing literature, it showed that this method only works in combination with analog controls or with phase quantization with at least two bits, though in this last case, the simplification in the number of control lines is limited. Moreover, the method is found to be generally applicable for pure beam steering applications only. A SVD analysis of the required phase distribution across the RA is proposed to predict the feasibility of the row-column control method in each case.

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