Mitigation of Alien Crosstalk for Downstream DSL Impaired by Multiple Interferers

Diego Gomes, Eduardo Medeiros, Aldebaro Klautau and Evaldo Pelaes

***Abstract*—Alien crosstalk is one of the major impairments for copper-based transmissions. This letter outlines a method for mitigating alien crosstalk for DSL downstream transmissions impaired by multiple interference sources. The method requires a reference channel, and includes a post processing stage in which induced correlation is applied to prepare the interference at the target channel to be reasonably removed by a prediction based mitigation step. The results show that the proposed method outperforms published alien crosstalk mitigation methods as the number of interference sources increase in G.fast scenarios.**

I. INTRODUCTION

Digital subscriber line (DSL) systems convey information through copper twisted pair channels, and are largely used because they benefit from the legacy telephone loop plant [1]. In its most recent version, G.fast, it achieves bit rates of up to 1 Gbps over short loops, using bandwidth of up to 212 MHz [2]. The architecture of the G.fast is called *fiber-to-the-distribution-point*, in which a fiber extends from the Central Office to the Distribution Point (DP), and from there the lines are plugged in the customer premises. However, the copper transmission is often impaired by crosstalk [3], which is the electromagnetic coupling between the close twisted pairs through which the signal transmission is carried out. When crosstalk occurs among lines that are coordinated by a single device at one of the ends of the cable, it is called *in-domain crosstalk* [4]. The *out-of-domain crosstalk* or alien crosstalk (AXT), which originates from a line that is not coordinated, represents a challenge for DSL systems. This is owing to the growth in the number of DSL deployments of different companies, which increases the number of *alien* signals. In general, the coordinated transceivers' information about out-of-domain crosstalk is limited to statistics, such as the correlation matrix of this interference, which can then be exploited to mitigate the effects of alien crosstalk [4]–[6].

Three AXT mitigation methods for two-sided coordinated systems are examined in [4]. The first achieves AXT mitigation through whitening, pre and post-processing. The last two are based on a *decision feedback equalizer* structure, in which the first processed users provide information to the AXT mitigation of the ones to follow through an interference predictor. Another AXT mitigation method for upstream direction is outlined in [5], which uses whitening to mitigate AXT and decodes the received symbols through a *successive interference cancellation* structure.

Despite the good performance achieved by the methods employed in [4] and [5] under the conditions set out for these works, they present a sharp reduction in their effectiveness when the number of *alien lines* (AL) becomes larger than the number of coordinated lines. In specific terms, as observed in [7] and evaluated in [6], when the number of AXT sources generating the interference is larger than the number of lines providing information (i.e. the reference channels), the interference mitigation tends to be poor. Additionally, the methods in [4] and [5] are not suitable for regular downstream transmission because the receivers are not collocated. In fact, the lack of coordination between the receivers in downstream, makes the AXT mitigation a challenge, because neither standard whitening nor interference prediction can be applied. Seeking to overcome these limitations, this work presents an effective AXT mitigation method for downstream transmissions impaired by multiple interference sources, in which a minimum coordination at the receivers is created through the use of one reference channel per information transmitter. This method needs both precoding and post-coding, and includes a stage of signal conditioning at receiver, that can enable the interference to be suitably removed by only using one reference channel through an AXT prediction-based procedures in the next stage.

This work is organized as follows. Sec. II outlines the transmission model, in Sec. III there is a detailed examination of the method and in Sec. IV our method is compared with others in G.fast scenarios.

II. SYSTEM MODEL

In this work we consider a downstream DSL transmission in frequency domain using *Discrete Multitone Modulation*, in which data is conveyed by the *transmission channel* (TC). This transmission is impaired by some alien crosstalk sources, that leads to an interference term at the received signals. Additionally, the TC is served by a *reference channel* (RC) (as in Fig.1), which can be either another twisted pair or an alternative transmission mode, such as the *common mode* [7]. For example, an extra line can be found in quad cables. The common mode can be accessed through a transceiver hardware modification to obtain the signal from the center tap of its transformers. In this paper it is assumed that all the coordinated channels are synchronized, which is a plausible constraint according to [2]. For the sake of simplicity and to focus on the description of the novel method, we will assume

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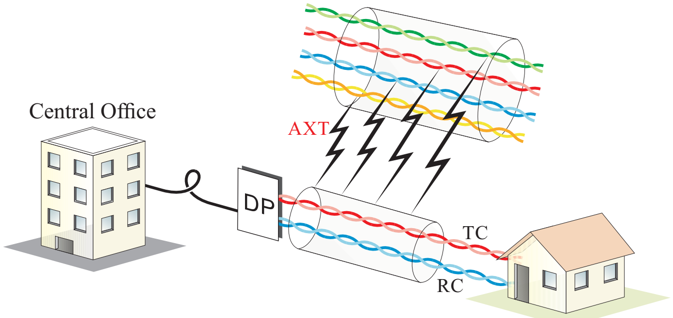

Fig. 1. System model for a DSL transmission in TC with one RC and multiple interference sources.

a single TC/RC pair here, but the algorithm can be scaled to support several TCs, which will be shown in the Sec. III-A. Then, for a synchronized transmission, we can represent the received signals in a given tone k by

$$\mathbf{y}_k = \mathbf{H}_k\mathbf{x}_k + \mathbf{q}_k + \mathbf{n}_k = \mathbf{H}_k\mathbf{x}_k + \mathbf{z}_k \quad (1)$$

where $\mathbf{y}_k = [y_{TC} \;\; y_{RC}]^T$ is a 2×1 vector with the received symbols both in TC and RC, $\mathbf{H}_k$ is a 2×2 matrix with the direct channels of the TC and RC in its diagonal, and the crosstalk channels between TC and RC in its off diagonal, $\mathbf{x}_k$ is a 2×1 vector with the transmitted symbols $[x_{TC} \;\; x_{RC}]^T$, $\mathbf{q}_k$ is a 2×1 vector with the AXT observed in the TC and the RC, $\mathbf{n}_k$ is a 2×1 vector that denotes background noise which can be modelled as additive white Gaussian noise, and $\mathbf{z}_k = \mathbf{q}_k + \mathbf{n}_k$. The subscript k is omitted in the next paragraphs since a per-tone processing is assumed.

III. AXT MITIGATION FOR MULTIPLE INTERFERENCE SOURCES (AMMIS)

This section outlines details of the method *AXT Mitigation for Multiple Interference Sources*, or AMMIS. This method needs to be executed both at the transmitter and receiver, where the combination of these stages makes it possible to mitigate the AXT and decode the transmitted symbols simultaneously. We begin by carrying out the pre-processing at the transmitter to make the equivalent channel decodable, and hence the resulting channel will be triangular. Let the square matrix $\mathbf{A}$ represent the equivalent channel, i.e., the channel observed by the transmitted symbols after the pre and post-processing. Let $\mathbf{A}'$ have the *QR* decomposition [5] $\mathbf{A}' = \mathbf{QR}$, where $\mathbf{Q}$ is a unitary complex matrix and $\mathbf{R}$ is an upper triangular matrix. Pre multiplying the transmitted symbols by $\mathbf{Q}$, $\mathbf{s} = \mathbf{Qx}$, we obtain

$$\tilde{\mathbf{y}} = \mathbf{As} + \mathbf{v} = (\mathbf{QR})'\mathbf{s} + \mathbf{v} = \mathbf{R}'\mathbf{Q}'\mathbf{Qx} + \mathbf{v} = \mathbf{R}'\mathbf{x} + \mathbf{v} \quad (2)$$

where $\mathbf{v}$ represents the resulting interference (after processing) at the receiver and the superscript $'$ indicates the conjugate transpose (Hermitian).

At the receiver the method begins with the *conditioning signal* stage, in which the received signals will be applied to sucessive matrix multiplications. In this stage, the proposed method requires an interference correlation matrix $\mathbf{C}_z = E[\mathbf{zz}']$ [4] to be estimated during a training phase, where $E[.]$ denotes statistical expectation. From $\mathbf{C}_z$ we obtain the whitening matrix $\mathbf{W} = \mathbf{G}_z^{-1}$, where $\mathbf{G}_z$ is a lower triangular matrix, obtained from the Cholesky decomposition [8] $\mathbf{C}_z = \mathbf{G}_z\mathbf{G}_z'$. In *show-time* phase, the whitening matrix is used at the receiver in the expression

$$\hat{\mathbf{y}} = \mathbf{Wy} = \mathbf{WHs} + \mathbf{Wz}. \quad (3)$$

This stage makes the correlation matrix of the interference term, $\mathbf{Wz}$, a 2×2 identity matrix $\mathbf{I}$, because $E[\mathbf{Wzz}'\mathbf{W}'] = \mathbf{G}_z^{-1}\mathbf{C}_z\mathbf{G}_z^{-\prime} = \mathbf{I}$. At this stage the correlation matrix of the reminiscent interference has a canonical form, in which a similar process can be used to the one used to make it diagonal, and impose on it a desired behavior. Then, to make the correlation matrix to have the form $\mathbf{C}_u = \mathbf{G}_u\mathbf{G}_u'$, we left multiply $\hat{\mathbf{y}}$ by $\mathbf{G}_u$, to get

$$\tilde{\mathbf{y}} = \mathbf{G}_u\hat{\mathbf{y}} = \mathbf{G}_u\mathbf{WHs} + \mathbf{G}_u\mathbf{Wz}, \quad (4)$$

where $\mathbf{v} = \mathbf{G}_u\mathbf{Wz}$, and $E[\mathbf{vv}'] = \mathbf{C}_u$. Now we define $\mathbf{G}_u\mathbf{WH} = \mathbf{A}$, and according to (2), the channel observed by the transmitted symbols is given by the $\mathbf{R}'$ matrix, which makes the end-to-end transmission

$$\begin{aligned} \tilde{\mathbf{y}} &= \mathbf{R}'\mathbf{x} + \mathbf{v} \\ \begin{bmatrix} \tilde{y}_{TC} \\ \tilde{y}_{RC} \end{bmatrix} &= \begin{bmatrix} R(1,1)^* & \\ R(1,2)^* & R(2,2)^* \end{bmatrix} \begin{bmatrix} x_{TC} \\ x_{RC} \end{bmatrix} + \begin{bmatrix} v_{tc} \\ v_{rc} \end{bmatrix} \end{aligned} \quad (5)$$

where the superscript * denotes complex conjugation, and $R(i,j)$ denotes the element in row i and column j of $\mathbf{R}$ matrix.

The conditioning signal step, in which the behavior of the correlation matrix of the interference was induced (4), was carried out to ensure the interference in the TC could be suitably predicted based on the interference observed in the RC, that comprises the next stage called *AXT removal*. The taps of this predictor can be found through the Cholesky decomposition of the correlation matrix of the reminiscent interference [9]. However, to get a predictor through this strategy, the data referring to the first line of the correlation matrix are used to predict the data related to the second line. This means, we must generate a correlation matrix of the reminiscent interference in which the position of v_{tc} and v_{rc} are changed, $\mathbf{b} = [v_{rc} \;\; v_{tc}]^T$. Finally, we compute the correlation matrix $\mathbf{C}_b = E[\mathbf{bb}']$, which can be decomposed into $\mathbf{G}_b\mathbf{D}_b\mathbf{G}_b'$, where $\mathbf{G}_b$ is a monic matrix [4] and $\mathbf{D}_b$ is a diagonal matrix. After this, the predictor is given by $\dot{v}_{tc} = G_b(2,1)v_{rc}$, where $G_b(2,1)$ is the element in the second line of the first column of the $\mathbf{G}_b$, and $\dot{v}_{tc}$ is the prediction of v_{tc}.

In our method the RC is only used to support information for TC, and only pilot symbols are transmitted in this channel. Then, we begin the decoding by subtracting the known part of the received signal at the RC as

$$\begin{aligned} \breve{y}_{RC} &= \tilde{y}_{RC} - R(2,2)^* x_{RC} \\ &= R(1,2)^* x_{TC} + v_{RC} \end{aligned} \quad (6)$$

From this we derive the prediction of the v_{TC} by the multiplication $G_b(2,1)\breve{y}_{RC} = G_b(2,1)R(1,2)^* x_{TC} + \dot{v}_{tc}$, and we

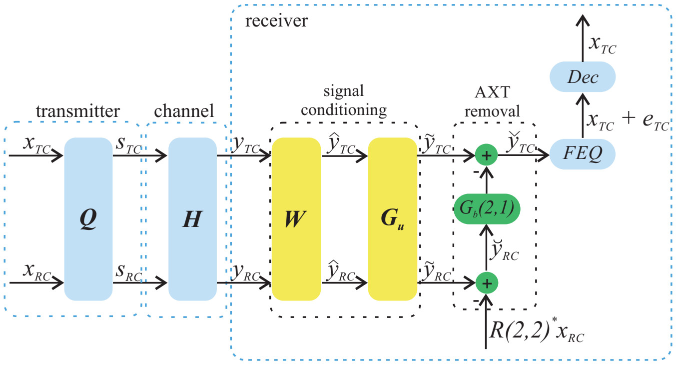

Fig. 2. The schematic representation of the AMMIS method.

subtract $\tilde{y}_{TC}$ from this sum, which yields

$$\begin{aligned} \breve{y}_{TC} &= \tilde{y}_{TC} - G_b(2,1)R(1,2)^* x_{TC} - \dot{v}_{tc} \\ &= R(1,1)^* x_{TC} + v_{TC} - G_b(2,1)R(1,2)^* x_{TC} - \dot{v}_{TC} \\ &= [R(1,1)^* - G_b(2,1)R(1,2)^*]x_{TC} + e_{TC} \end{aligned} \tag{7}$$

where e_{TC} represents the error in the prediction of the v_{TC}. Now, the AXT is mitigated in the TC and the decoding of the x_{TC} can be performed by adjusting the *frequency equalizer* (FEQ) from the standard $1/R(1,1)^*$ to

$$FEQ = \frac{1}{R(1,1)^* - G_b(2,1)R(1,2)^*}. \tag{8}$$

Following this, the signal can be submitted to the decoder (Dec). From (7), it can be observed that the channel gain observed by the x_{TC} is $\tau = R(1,1)^* - G_b(2,1)R(1,2)^*$, which allows us to represent the *signal-to-interference-plus-noise ratio* (SINR) to the TC according to [10], by tone, as

$$SINR_{TC} = \frac{\rho|\tau|^2}{E[e_{TC}e'_{TC}]} \tag{9}$$

where ρ is the transmission power of the x_{TC}. The whole AMMIS processing is shown in Fig. 2, where *Dec* denotes the decoding operation.

A. The Effect of Vectoring on AMMIS

To achieve high data rates, the G.fast standard adopts *vectoring* [11]. This makes it logical to evaluate the performance of AMMIS when it is applied to a group of coordinated lines, or vectored group (VG). From this perspective, our model can be expanded to L coordinated TC/RC pairs as

$$\ddot{\mathbf{y}} = \ddot{\mathbf{H}}\mathbf{P}\ddot{\mathbf{s}} + \ddot{\mathbf{z}} \tag{10}$$

where $\ddot{\mathbf{s}} = [\mathbf{s}_1 \ ... \ \mathbf{s}_L]^T$, in which $\mathbf{s}_l$ is a column vector that contains the transmitted symbols in TC and RC of the lth TC/RC pair, $\ddot{\mathbf{H}}$ is a $2L \times 2L$ matrix which contains all the direct channel of the TCs and RCs in its diagonal and the crosstalk channels among them in out-of-diagonal elements, $\ddot{\mathbf{z}} = [\mathbf{z}_1 \ ... \ \mathbf{z}_L]^T$, and $\mathbf{P} = (1/\beta)\ddot{\mathbf{H}}^{-1} diag(\ddot{\mathbf{H}})$ is the precoder applied to remove the crosstalk among the coordinated channels, in which $diag(.)$ represents the diagonal of a matrix and β is a factor used to control the transmission power of the transmitted symbols [11]. In this situation, the received symbols become

$$\ddot{\mathbf{y}} = \frac{1}{\beta}\text{diag}(\ddot{\mathbf{H}})\ddot{\mathbf{s}} + \ddot{\mathbf{z}} \tag{11}$$

The result of the (11) indicates that the effect of a vectorized coordinate group on AMMIS is to diagonalize $\mathbf{H}$ in (1), (3) and (4), but each $\mathbf{s}_l$ keeps its precoding applied in the AMMIS (2), and the method can be usually employed for each TC/RC pair. Additionally, the SINR is scaled by $(1/\beta)^2$.

IV. RESULTS

In this section there is a performance evaluation of AMMIS, which is compared with two other AXT mitigation methods: the one discussed in [5] (denoted *DFC*) and the third method in [4] (called here *RxPred*, chosen because it had an equivalent performance to the ones obtained with the first and second methods in [4]). Two scenarios were examined for this evaluation and the methods were compared with respect to their simulated bit rates, which were evaluated in the Matlab platform. Although DFC was not originally thought to be suitable for downstream [5] (due to the absence of coordination at receiver), the presence of the RC makes its implementation possible.

The channels of the first scenario (SCEN1) were measured with an Agilent E5071C network analyzer, using a setup composed by a 100 m long quad cable encapsulated in a bundle with three other quad cables. In this scenario, the extra twisted pair of the target quad cable was used as the RC and all the pairs of the three other quad cables caused interference, in a total of 6 alien crosstalk sources. The *Computer Simulation Technology* (CST) software was used to simulate scenarios with varying difficulty to the algorithms. From these simulations, the second scenario (SCEN2) was chosen to be discussed here because it is particularly problematic for AMMIS (and other methods), due its weak coupling channels, which yield AXT level close to background noise. SCEN2 represents DSL transmission over a 50 m long *Category 6* (Cat6) twisted pair. The CST simulator provided information about the common mode, that is used as the RC. In SCEN2, the transmission was impaired by the AXT generated by 4 *Category 5* twisted pairs.

G.fast downstream transmissions were assumed in both scenarios and the parameters of the simulations were: bandwidth of 212 MHz (SCEN1)/100 MHz (SCEN2); transmission/interference *power spectrum density* (PSD) of -76 dBm/Hz; background noise PSD of -150 dBm/Hz; and, SINR Gap of 9.75 dB.

Fig. 3 and Fig. 4 show the simulation results for SCEN1 and SCEN2, respectively. In both scenarios, we evaluated the downstream aggregated transmission rates achieved by RxPred and DFC (i. e., the sum of the rates available in the TC and the RC, because in these methods both channels convey useful data), but for AMMIS only the downstream transmission rate in the TC is used, given that its RC transmits only pilot symbols. In these figures "*No Mitigation*" indicates the data rate achieved in a transmission in the presence of AXT but without mitigation. The $\mathbf{C}_u$ matrix used by AMMIS was obtained from the interference correlation matrix generated from a simulation of a scenario with 2 coordinated lines and only 1 AL. This will induce the interference to behave like one that is only generated by one source, which allows an effective AXT mitigation in (7) with prediction that is based on one reference channel [6].

Comparing Fig. 3 and Fig. 4, it can be noted that the transmission rates achieved by RxPred and DFC, in the situation with 1 AL, are larger than that achieved by AMMIS. This can be attributed to the fact that with 1 AL and 1 RC, the standard AXT mitigation methods have a good performance [6]. However, with the increase in the number of ALs, the AMMIS outperforms the other methods in the SCEN1, whereas AMMIS only achieves a better performance in the worst situation in the SCEN2. This behavior in the SCEN2 is mainly owing to the weak AXT channels in the differential mode of the Cat6. This causes low power interference in this mode and compensates for a poorer performance in some situations with multiple AXT (up to 3 ALs), keeping the aggregated transmission rate in a high level, same with the poor transmission rate in the common mode due to the low SINR in this mode.

However, the strong interference in the medium quality lines of the SCEN1 quickly reveals the decline in performance of RxPred and DFC. These results confirm that AMMIS can achieve better results than standard AXT mitigation, in situations in which only 1 RC is available and multiple AXT sources impair a G.fast downstream transmission. In some situations (with AL $= 0$, for example) the bit rates of the AMMIS were small even than *No Mitigation*, a penalty due its channel gain to be given by the difference $R(1,1)^* - G_b(2,1)R(1,2)^*$. A detailed analysis to determinate the parameters that impact this value will be treated in a future work.

Another advantage of AMMIS is that unlike the other methods, it is able to reduce the transmission power, because in AMMIS the RC only transmits pilot symbols, and then less power can be assigned to this channel. Additionally, this fact can also reduce the level of the interference in other systems, which tends to increase with the use of the extra channel (RC). On the other hand, this reduction in power cannot be carried out in the DFC and RxPred without a bit rate decrease, because it can affect their aggregated bit rate, which also depends on the RC.

With regard to the computational cost in showtime, AMMIS requires six additions and nine multiplications, whereas RxPred and DFC need $4+$ and $6\times$, and $3+$ and $5\times$, respectively. However, as AMMIS requires only one decoding against two of the other methods, it will require fewer operations in situations with $M \geq 16$ (M is the size of the constellation), because $O(M)$ [12].

V. CONCLUSION

We have presented an alien crosstalk mitigation method called AMMIS that is suitable for DSL downstream transmissions in which the target channel has access to the signals in a reference channel, and where the transmission is impaired by multiple alien crosstalk sources. The proposed method achieved better data rates than other mitigation methods in G.fast scenarios when the number of interference sources is relatively large. For example, its performance was better in a scenario with medium quality cables at the expense of more additions and multiplications.

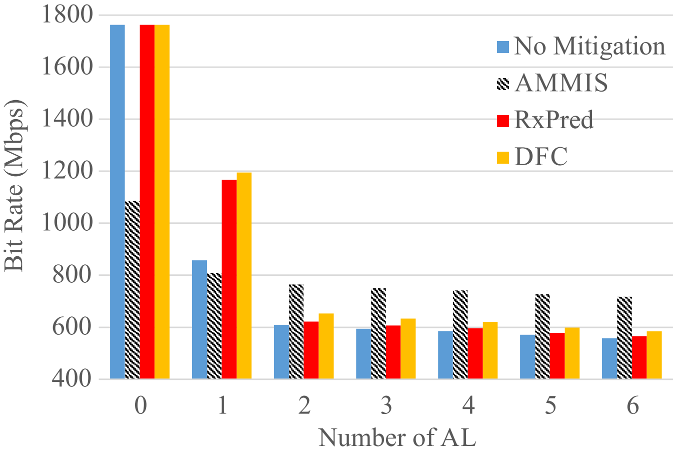

Fig. 3. Transmission rates achieved by each mitigation method with a different number of ALs in SCEN1 - 212 MHz.

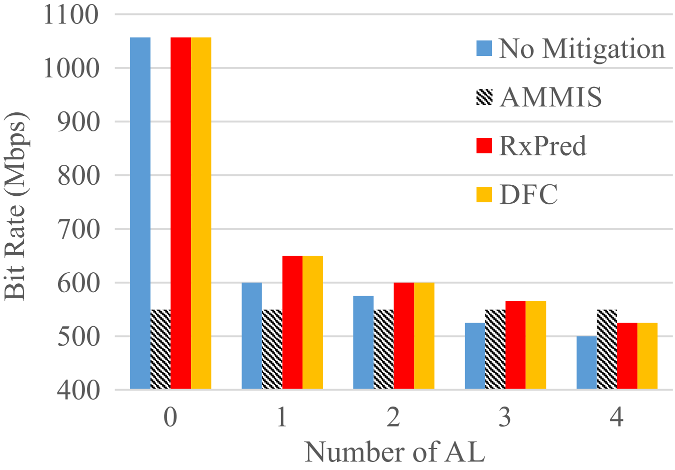

Fig. 4. Transmission rates achieved by each mitigation method with a different number of ALs in SCEN2 - 100 MHz.

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