Collaborative Adaptive Exponential Linear-in-the-parameters Nonlinear Filters

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***Abstract*—An adaptive exponential functional link artificial neural network (AEFLANN) based active noise control (ANC) system trained using a collaborative learning scheme has been designed in this paper. In the proposed approach, separate learning mechanism is used for updating the weights of the linear portion of the AEFLANN and its non-linear section. The outputs of the linear and non-linear sections are suitably combined and the update mechanism involves the update of weights of linear and non-linear portions, the combination parameter and the adaptive exponential factor. Simulation study shows enhanced noise cancellation in comparison with other non-linear ANC schemes compared.**

***Index Terms*—Active noise control, functional link artificial neural network, noise cancellation, non-linear filter.**

I. Introduction

Active noise control (ANC) has recently emerged as an effective method of noise mitigation in the low frequency zone. The fundamental principle of ANC is the concept of destructive superposition [1]–[3]. One of the most successful applications of ANC is the ANC headphone [4] and the technology of ANC has been recently introduced into digital hearing aids [5]. ANC systems can be feed-forward or feedback depending on the configuration of the noise cancellation scheme. A basic feed-forward ANC scheme consists of two microphones (reference microphone and error microphone), a loudspeaker and a control mechanism. The control mechanism is usually achieved using an adaptive finite impulse response (FIR) filter trained using a filtered-x least mean square (FxLMS) algorithm [1], [6], [7].

A few non-linear ANC schemes, which uses an adaptive non-linear filter as the controller, has been recently reported in literature to achieve noise cancellations in scenarios in which non-linearities exist in the ANC system [7]–[11]. The popular among them are the adaptive Volterra filter trained using a Volterra FxLMS (VFxLMS) and the functional link artificial neural network (FLANN) updated using a filtered-s least mean square (FsLMS) algorithm [12]–[16]. Both the adaptive non-linear filters fall under the category of linear-in-the-parameters non-linear filters.

An adaptive exponential FLANN (AEFLANN) has been recently proposed by Patel *et al.* as a new linear-in-the-parameters non-linear filter [17]. The authors have tested the efficacy of the filter for non-linear system identification and non-linear ANC. The enhanced modeling accuracy as well as noise mitigation capacity of the AEFLANN based filters have also been reported. Comminiello *et al.* has recently proposed a collaborative functional link adaptive filter model for non-linear acoustic echo cancellation [18]. In the collaborative learning approach reported in [18]–[20], different adaptive algorithms are used for updating the weights of the linear and non-linear components of a FLANN, followed by a suitable adaptive combination of the outputs of the linear and non-linear portions. In an attempt to improve the noise mitigation capability of AEFLANN based non-linear filters, a collaborative learning approach has been developed in this paper. An update rule for updating the weights of the linear and non-linear sections, the shrinkage factor in a collaborative learning scheme and the adaptive exponential parameter are derived.

The rest of the paper is organized as follows: The collaborative learning scheme based AEFLANN is introduced in Section 2 and the corresponding update rules are developed. A simulation study is carried out in Section 3 to test the noise cancellation capacity of the proposed scheme and the concluding remarks are drawn in Section 4.

II. AEFLANN based collaborative ANC system

In the proposed ANC scheme, the controller employed is an AEFLANN, which is a linear-in-the-parameters non-linear filter. Let $x(n)$ be the reference signal, which is sensed by the reference microphone and $e(n)$ be the signal at the output of the error microphone. The tap delayed reference signal vector

$$\boldsymbol{x}(n) = [x(n), x(n-1), ..., x(n-N+1)]^T \tag{1}$$

of length N is functionally expanded to an expanded vector given by

$$\begin{aligned}\boldsymbol{g}(n) = \{&1, x(n), e^{-a(n)|x(n)|}\sin[\pi x(n)], e^{-a(n)|x(n)|}\\ &\cos[\pi x(n)], \cdots, e^{-a(n)|x(n)|}\sin[B\pi x(n)], e^{-a(n)|x(n)|}\\ &\cos[B\pi x(n)], x(n-1), e^{-a(n)|x(n-1)|}\sin[\pi x(n-1)],\\ &e^{-a(n)|x(n-1)|}\cos[\pi x(n-1)], \cdots, e^{-a(n)|x(n-1)|}\\ &\sin[B\pi x(n-1)], e^{-a(n)|x(n-1)|}\cos[B\pi x(n-1)], \cdots,\\ &x(n-N+1), e^{-a(n)|x(n-N+1)|}\sin[\pi x(n-N+1)],\\ &e^{-a(n)|x(n-N+1)|}\cos[\pi x(n-N+1)], \cdots,\\ &e^{-a(n)|x(n-N+1)|}\sin[B\pi x(n-N+1)], e^{-a(n)|x(n-N+1)|}\\ &\cos[B\pi x(n-N+1)]\}^T.\end{aligned} \tag{2}$$

Above equation can also be written as

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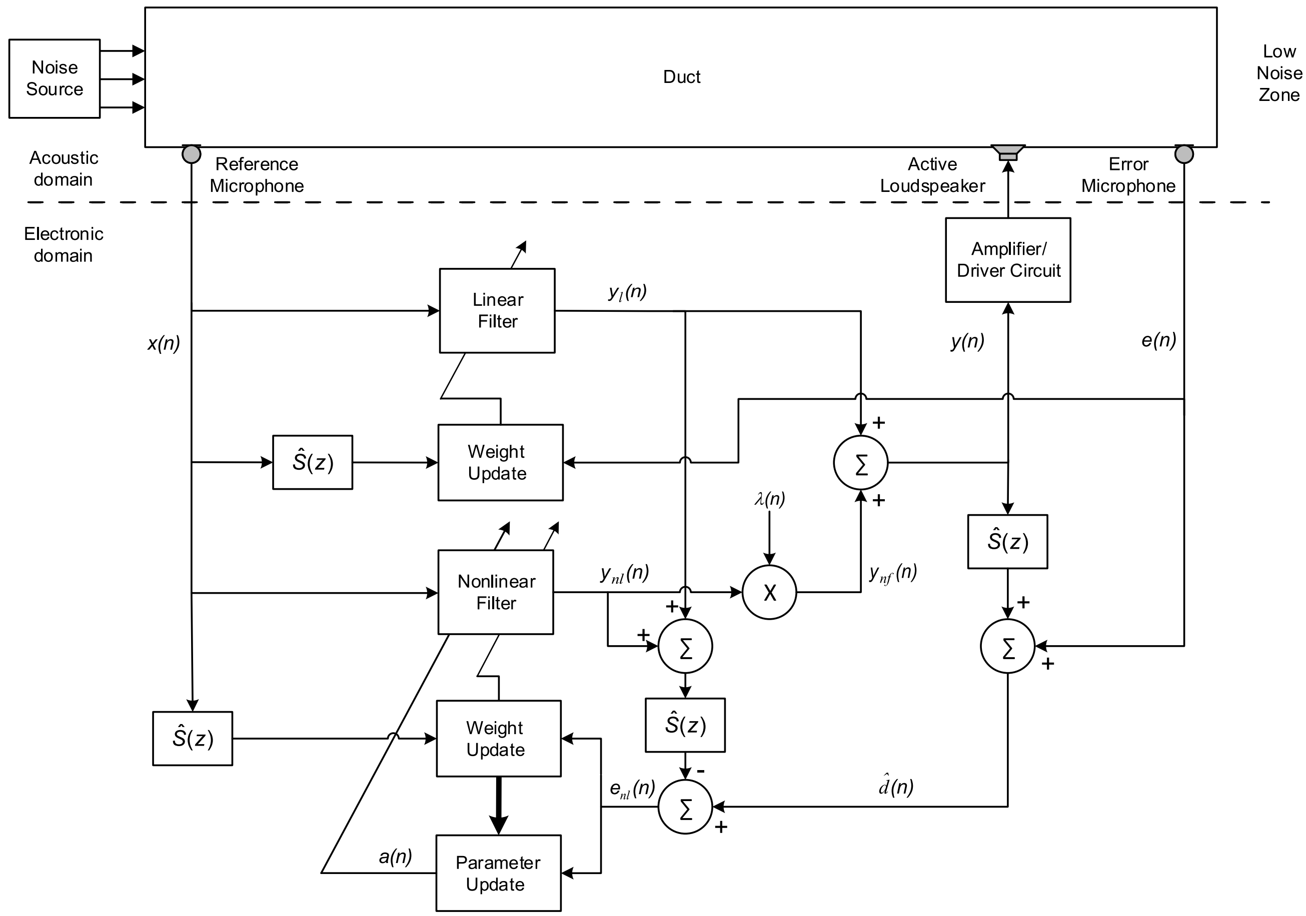

Fig. 1. Schematic diagram of an AEFLANN based collaborative ANC scheme

$$\boldsymbol{g}(n) = \{1, \boldsymbol{x}(n), \exp[-a(n)|\boldsymbol{x}(n)|]\sin[\pi\boldsymbol{x}(n)], \exp[-a(n)|\boldsymbol{x}(n)|]\cos[\pi\boldsymbol{x}(n)], \exp[-a(n)|\boldsymbol{x}(n)|]\sin[2\pi\boldsymbol{x}(n)], \exp[-a(n)|\boldsymbol{x}(n)|]\cos[2\pi\boldsymbol{x}(n)], \cdots, \exp[-a(n)|\boldsymbol{x}(n)|] \sin[B\pi\boldsymbol{x}(n)], \exp[-a(n)|\boldsymbol{x}(n)|]\cos[B\pi\boldsymbol{x}(n)]\}, \quad (3)$$

where B is the order of functional expansion and $a(n)$ is an adaptive exponential parameter, which is updated continuously. The expanded signal is of length

$$M = N(2B + 1) + 1 \quad (4)$$

and may be split into the linear portion

$$\boldsymbol{x}_l(n) = \{1, \boldsymbol{x}^T(n)\}^T \quad (5)$$

and the non-linear section

$$\boldsymbol{x}_{nl}(n) = \exp[-a(n)|\boldsymbol{x}(n)|] \cdot \boldsymbol{x}_T(n)^1 \quad (6)$$

where,

$$\boldsymbol{x}_T(n) = \{\sin[\pi\boldsymbol{x}(n)], \cos[\pi\boldsymbol{x}(n)], \sin[2\pi\boldsymbol{x}(n)], \cos[2\pi\boldsymbol{x}(n)], \cdots, \sin[B\pi\boldsymbol{x}(n)], \cos[B\pi\boldsymbol{x}(n)]\}^T. \quad (7)$$

The output of the linear and non-linear sections are given by

$$y_l(n) = \boldsymbol{w}_l(n)^T \boldsymbol{x}_l(n), \quad (8)$$
$$y_{nl}(n) = \boldsymbol{w}_{nl}(n)^T \boldsymbol{x}_{nl}(n) \quad (9)$$

where $\boldsymbol{w}_l(n)$ and $\boldsymbol{w}_{nl}(n)$ are the weights vectors of length $N+1$ and $2NB$ for linear and nonlinear sections respectively. The overall output of the controller which is fed to the control speaker is given by

$$y(n) = y_l(n) + \lambda(n)y_{nl}(n) \quad (10)$$

in (10) $\lambda(n)$ is the shrinkage parameter. The overall error signal sensed at the error microphone is given by

$$e(n) = d(n) - s(n) * [y_l(n) + \lambda(n)y_{nl}(n)]. \quad (11)$$

where $s(n)$ is the impulse response of the electro-acoustic path from control speaker to the error microphone (referred to as the secondary path) and $d(n)$ is the disturbance signal. The

[1] where, A.[B_1, B_2, $\cdots$, B_n]=[A.B_1, A.B_2, $\cdots$, A.B_n]

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weights of the linear portion are updated with an objective to minimize the cost function given by

$$\xi(n) = \mathrm{E}[e^2(n)] \approx e^2(n), \tag{12}$$

where E[·] is the expectation operator. The weight update rule is given by

$$\boldsymbol{w}_l(n+1) = \boldsymbol{w}_l(n) + \frac{\mu_l}{\boldsymbol{x}_l'^T \boldsymbol{x}_l' + \delta} \boldsymbol{x}_l' e(n) \tag{13}$$

where μ_l is the step size of the linear portion, $\boldsymbol{x}_l'$ is $\boldsymbol{x}_l$ filtered through a model of the secondary path $\hat{s}(n)$ and δ is a small constant to avoid divide by zero error. In a collaborative learning approach, the weights of the non-linear section are updated with an objective to minimize the cost function

$$\xi_{nl}(n) = \mathrm{E}[e_{nl}^2(n)] \approx e_{nl}^2(n), \tag{14}$$

where e_{nl} is the local error (contribution due to non-linear section). Using a gradient descent approach, we can write the update rule for the non-linear section as

$$\boldsymbol{w}_{nl}(n+1) = \boldsymbol{w}_{nl}(n) + \frac{\mu_{nl}}{\boldsymbol{x}_{nl}'^T \boldsymbol{x}_{nl}' + \delta} \boldsymbol{x}_{nl}' e(n), \tag{15}$$

where μ_{nl} is the step size of the non-linear section and $\boldsymbol{x}_{nl}'$ is $\boldsymbol{x}_{nl}$ filtered through a model of the secondary path. In addition to the weights of the linear and non-linear portions, we need to update the shrinkage parameter as well as the adaptive exponent factor. The shrinkage parameter is given by

$$\lambda(n) = \frac{1}{1 + \exp[-\alpha(n)]}, \tag{16}$$

which is updated as

$$\alpha(n+1) = \alpha(n) + \frac{\mu_\alpha}{p(n)} e(n) y_{nl}'(n) \lambda(n)[1 - \lambda(n)] \tag{17}$$

where

$$p(n) = \gamma p(n-1) + (1-\gamma) y_{nl}'^2(n) \tag{18}$$

where γ is the smoothing factor, $y_{nl}'(n)$ is $y_{nl}(n)$ filtered through a model of the secondary path, μ_α is the step-size and $p(0) = 1$ [18]. In a similar fashion, the adaptive exponential factor $a(n)$ is updated as

$$a(n+1) = a(n) - \frac{\mu_a}{2} \Delta_a(n) \tag{19}$$

where $\Delta_a(n)$ is an instantaneous estimate of the gradient of ξ_{nl} with respect to the parameter $a(n)$ and μ_a is the step size. We have

$$\begin{aligned} \Delta_a(n) &= \frac{\partial \xi_{nl}(n)}{\partial a(n)} \\ &= 2 e_{nl}(n) \frac{\partial \{\boldsymbol{g}^T(n) \boldsymbol{w}_{nl}(n)\}}{\partial a} \\ &= -2 e_{nl}(n) \boldsymbol{z}^T(n) \boldsymbol{w}_{nl}(n) \end{aligned} \tag{20}$$

where

$$\boldsymbol{z}(n) = \frac{\partial \boldsymbol{g}(n)}{\partial a(n)} = -|\boldsymbol{x}'(n)| \exp[-a(n)|\boldsymbol{x}'(n)|] \cdot \{\sin[\pi \boldsymbol{x}'(n)], \cos[\pi \boldsymbol{x}'(n)], \sin[2\pi \boldsymbol{x}'(n)], \cos[2\pi \boldsymbol{x}'(n)], \cdots, \sin[B\pi \boldsymbol{x}'(n)], \cos[B\pi \boldsymbol{x}'(n)]\}^T \tag{21}$$

where $\boldsymbol{x}'(n)$ is $\boldsymbol{x}(n)$ filtered through a model of the secondary path and $\boldsymbol{z}(n)$ is a $2NB \times 1$ vector. Using (19) and (20), we get

$$a(n+1) = a(n) + \mu_a e_{nl}(n) \boldsymbol{z}^T(n) \boldsymbol{w}_{nl}(n). \tag{22}$$

Thus (13), (15), (17) and (22) together form the collaborative learning strategy for an AEFLANN based ANC system. The schematic diagram of the proposed approach is shown in Fig. 1.

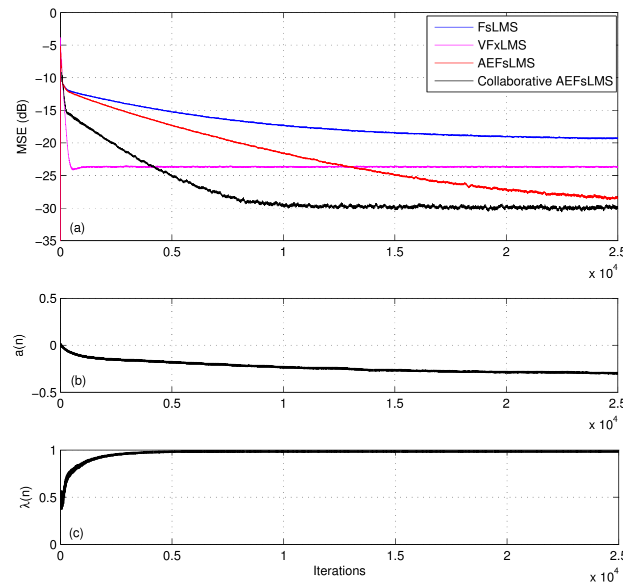

Fig. 2. Experiment 1: (a) Comparison of convergence characteristics obtained as an average of 50 independent trails followed by a smoothing using a moving average filter of length 100 (b) Variation of exponential factor and (c) variation shrinkage parameter with respect to iterations.

III. Simulation Study

In this section, we evaluate the noise mitigation capability of the proposed non-linear filter based ANC scheme. The performance comparison with ANC schemes based on a FLANN [12], adaptive Volterra filter [21] and AEFLANN [17] has been made in terms of mean square error (MSE) defined by

$$\xi = 10\log_{10}\left\{\mathrm{E}\left[e^2(n)\right]\right\}. \tag{23}$$

A measurement noise, with a signal to noise ratio of 40 dB has been considered in all the experiments.

A. Experiment 1

In this experiment, we have considered a tonal primary disturbance. The disturbance signal used is given by

$$x(n) = \sin\left(\frac{2\pi f n}{f_s}\right) \tag{24}$$

where $f_s = 6000$ Hz is the sampling frequency and f is taken as 150 Hz. The primary disturbance signal sensed at the error microphone is given by

$$d(n) = u(n-2) + 0.8u^2(n-2) - 0.4u^3(n-1), \tag{25}$$

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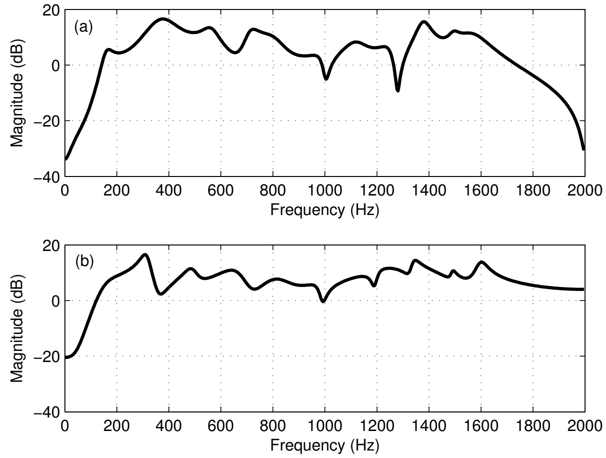

Fig. 3. Experiment 2: Magnitude of the frequency response of (a) Portion of the primary path and (b) Secondary path.

where

$$u(n) = x(n) * q(n) \tag{26}$$

with $q(n)$ denoting the impulse response of the transfer function

$$Q(z) = z^{-3} - 0.3z^{-4} + 0.2z^{-5} \tag{27}$$

and $*$ representing the linear convolution operator [12]. The transfer function of the secondary path considered is given by

$$S(z) = z^{-2} + 0.5z^{-3}. \tag{28}$$

The convergence characteristics for the different algorithms compared is shown in Fig. 2(a), where we have used a moving average window of length 100. The average MSE values obtained as an average of last 1000 samples are -19.63, -23.65, -29.51 and -30.14 dBs for FsLMS, VFxLMS, AEFsLMS and collaborative AEFsLMS algorithms respectively. The variation of shrinkage factor and the adaptive exponential factor for the collaborative AEFsLMS algorithm with respect to iterations is shown in Fig. 2(b) and (c) respectively. The other simulation parameters used are: FsLMS (step size $\mu_t = 0.2$, $N = 10$, $B = 1$, $M = 30$), VFxLMS (step size $\mu_v = 0.06$, $N = 10$, order $C = 2$, $M = 65$), AEFsLMS (step size $\mu_{ae} = 0.2$, $\mu_a = 0.01$, $N = 10$, $B = 1$, $M = 30$) and for collaborative AEFsLMS ($\mu_l = 0.05$, $\mu_{nl} = 0.3$, $\mu_a = 0.2$, $\mu_\alpha = 0.4$, $\gamma = 0.99$, $N = 10$, $B = 1$, $M = 30$).

B. Experiment 2

The disturbance signal considered in this experiment is given by (24), where f is taken as 200 Hz and $f_s = 4000$ Hz. The primary path used is a cascade of a transfer function $H_p(z)$, the magnitude of its frequency response is shown in Fig.3(a) [22] and a non-linear function given by

$$d(n) = \left[\frac{2\eta_2}{1 + \exp[-\eta_1 x_p(n)]} - \eta_2\right] \tag{29}$$

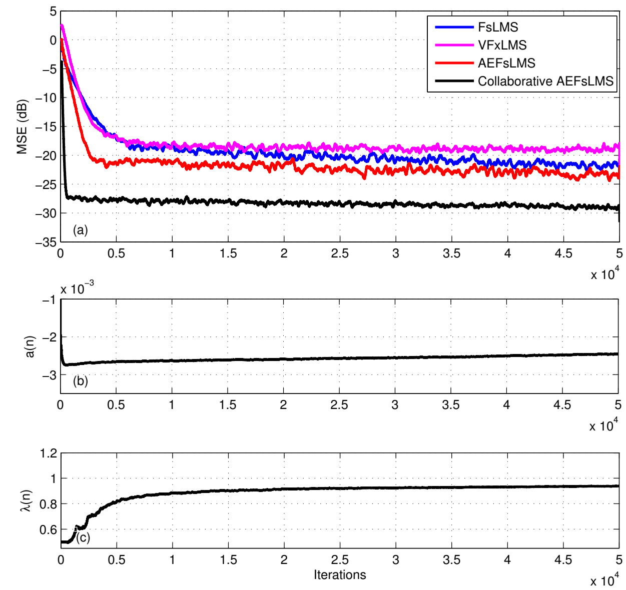

Fig. 4. Experiment 2: (a) Comparison of convergence characteristics obtained as an average of 20 independent trails followed by a smoothing using a moving average filter of length 100 (b) Variation of exponential factor and (c) variation shrinkage parameter with respect to iterations.

where $x_p(n)$ is the input to the nonlinear function, $\eta_1 = 6$ and $\eta_2 = 3.5$. The magnitude of the frequency response of the secondary path is shown in Fig.3 (b) [22]. Similar to the earlier experiments, an improved noise mitigation is achieved using the proposed collaborative learning approach. The variation of MSE, exponential factor and shrinkage parameter with respect to iterations is shown in Fig.4 (a), (b) and (c) respectively. The mean MSE values calculated over the last 1000 iterations are -21.60, -18.75, -23.57 and -29.06 dBs for FsLMS, VFxLMS, AEFsLMS and the proposed collaborative learning scheme. The other simulation parameters for the different algorithms compared are as follows: FsLMS (step size $\mu_t = 0.2$, $N = 40$, $B = 2$, $M = 200$), VFxLMS (step size $\mu_v = 0.05$, $N = 20$, order $C = 2$, $M = 230$), AEFsLMS (step size $\mu_{ae} = 0.2$, $\mu_a = 0.05$; $N = 40$, $B = 2$, $M = 200$) and for collaborative AEFsLMS ($\mu_l = 0.02$, $\mu_{nl} = 0.04$, $\mu_a = 0.01$, $\mu_\alpha = 0.01$, $\gamma = 0.99$, $N = 40$, $B = 2$, $M = 200$).

C. Experiment 3

The primary disturbance considered in this experiment is same as that of first experiment. The primary path is considered to have a transfer function given by $Q(z)$ for the first 50000 iterations.The primary disturbance signal observed at the error microphone is given by

$$d(n) = u(n-2) + 0.8u^2(n-2) - 0.4u^3(n-1) + u(n-2)u^4(n-1), \tag{30}$$

for the rest of the iterations with $u(n)$ as obtained in first experiment and the secondary path considered here is same as (28). The improved performance characteristics can be observed from the Fig 5 (a) for both the linear and nonlinear portions. The variation of exponential factor and shrinkage

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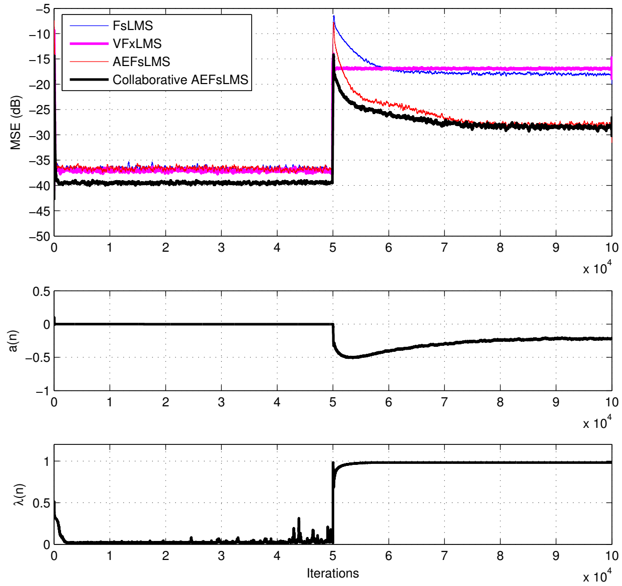

Fig. 5. Experiment 3: (a) Comparison of convergence characteristics obtained as an average of 50 independent trails followed by a smoothing using a moving average filter of length 100 (b) Variation of exponential factor and (c) variation shrinkage parameter with respect to iterations.

parameter with respect to iteration are depicted in Fig 5 (b) and (c) respectively. The mean MSE values calculated over the last 1000 iterations for the portion having nonlinear primary path are -18.01, -16.91, -27.46 and -28.82 dBs for FsLMS, VFxLMS, AEFsLMS and the collaborative learning scheme respectively. The other simulation parameters for the different algorithms compared are as follows: FsLMS (step size $\mu_t = 0.3$, $N = 10$, $B = 1$, $M = 30$), VFxLMS (step size $\mu_v = 0.06$, $N = 10$, order $C = 2$, $M = 65$), AEFsLMS (step size $\mu_{ae} = 0.3$, $\mu_a = 0.09$, $N = 10$, $B = 1$, $M = 30$) and for collaborative AEFsLMS ($\mu_l = 0.05$, $\mu_{nl} = 0.3$, $\mu_a = 0.6$, $\mu_\alpha = 0.4$, $\gamma = 0.99$, $N = 10$, $B = 1$, $M = 30$). It can be clearly noticed that the proposed learning rule is quite good in handling both linear and nonlinear scenarios.

IV. Conclusions

In this paper, a collaborative learning approach has been proposed for an AeFLANN based non-linear ANC scheme. A set of learning rules for updating the weights of the linear and non-linear sections of the controller has been developed. In addition, update rules for the shrinkage parameter and the adaptive exponential factor has been derived. Improved noise cancellation has been observed in comparison with other non-linear ANC schemes compared.

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