Dual-band PWM Audio Coding for Ultrasound Artefact Reduction

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***Abstract*— A novel approach to PWM coding is introduced based on generating two complementary PWM streams out of the in-band signal's spectrum. The 2 streams are then recombined in a suitable way, so that out-of-phase cancellation of the carrier frequency harmonics is achieved. The approach suppresses the strong out-of-band frequencies of the carrier signal without introducing distortion of the in-band coded signal. Such method can achieve superior reduction compared to the out-of-band artefact suppression induced by traditional analog low-pass filters employed in typical Class-D audio amplifiers or other switching power delivery systems, hence allowing designs with reduced filter requirements or even filterless implementations.**

Keywords—Pulse Width Modulation; Class-D audio amplifier; audio coding; PWM ultrasound;

I. Introduction

Over the past decades, 1-bit audio modulations have emerged as attractive practical alternatives to multibit pulse code modulation (PCM). Single-bit modulations such as Pulse Density Modulation (PDM) and Pulse Width Modulation (PWM) are extensively used in telecommunications and power systems while they are employed in a wide variety of applications such as audio amplifiers (Class-D amplifiers [1,2]), signal coding and transmission [3], electric motor controllers (i.e. electric cars) [4], drivers for lasers engravers, servomechanisms, switched-mode power supplies [5] and other.

In PWM, information is encoded in the variation of the pulse duration, thus constituting a sequence of square pulses with varying duty cycle, where the variation depends on the instant amplitude of the modulating signal. PWM has been thoroughly analyzed and described in the frequency domain [6] and it is well known that its spectrum – similarly to FM – contains high amplitude harmonics of the carrier-wave frequency f_S as well as sidebands around each harmonic, which extend well beyond the signal's original bandwidth. Such unwanted spectral energy in the carrier harmonics and sidebands are a source of problems in many of the above-mentioned applications. For example, a high-power Class-D amplifier PWM stream delivered directly on a loudspeaker can cause overheating or even permanent damage of the coil, while the ultrasound harmonics can introduce non-linear distortions and cause electromagnetic interference. To prevent such problems, Class-D amplifiers incorporate a low-pass filtering stage that suppresses ultrasonic content. However, such analog filters are usually implemented with first or second order LC/RLC stages introducing linear and non-linear distortions and at the same time increase size, weight and cost (commonly such filter occupies 70% of the PCB and adds 30% to the cost [7]), strongly affecting the applicability of Class-D amplifier.

During the past decades, several techniques have been developed for reducing PWM high frequency artefacts, aiming mainly on the effect of carrier frequency harmonics due to their high energy that can cause problems to the load of a PWM switching power stage. For example, Selective Harmonic Elimination PWM (SHE PWM) has been proposed for controlling the switching times of the power stage MOSFET transistors [8]. SHE PWM aims at eliminating a set of carrier harmonics selected through the optimization of an objective function designed for a particular application. However, solving such a problem is computationally demanding and often leads to suboptimal solutions, limiting SHE PWM to signals with specific characteristics (i.e. half-wave symmetry). SHE PWM has been successfully applied in switching converters and still constitutes an active field of research [9]. Another recently proposed improvement in the field of audio, is a hysteresis PWM-based method for filterless Class-D amplifier implementation through 3-state Bang-Bang control [7]. Bang-Bang control utilizes feedback for confining the triangular carrier within limits determined by the value of input signal and leads to possibly filterless, low cost, low power and power critical applications with a tradeoff in fidelity.

In this work, a novel signal processing method is described that is based on the spectral analysis of any PWM stream derived from typical multibit PCM data corresponding to real audio data (as is the case of all-digital Class-D amplifiers) and suppresses PWM carrier frequency harmonics, reducing the total energy of high frequency modulation artefacts. This is achieved by deriving 2 complementary PWM streams through appropriate splitting of the PWM spectrum in two sub-bands, which are later superimposed to form the Dual-Band PWM (DBPWM acronym) output stream. A convention applicable to the field of audio will be followed, however the results can be generalized for any PWM signal with in-band spectrum containing two or more frequencies. This paper is structured as follows: in Section II, the mathematical process for creating the Dual-Band PWM stream from a PCM signal is presented, followed by an analysis of the DBPWM spectrum in terms of the PCM and respective PWM signals' spectra. Section III

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presents the performance results of the proposed method, measured on four different input signals compared with results from application of RLC filtering on the original PWM signal.

II. PWM SPECTRUM ANALYSIS

A. Single-band PWM

Starting from a k-bit PCM signal $x(\tau) \in [0,x_{max}]$ and duration T_0, represented by an $1\times n$ vector where n the number of PCM samples in time with sampling frequency f_s. The PCM-to-PWM transform is an $n\rightarrow N$ mapping:

$$y(\tau)=\sum_{m=0}^{n-1}\{u[\tau-(2m+1-x(m))(2^k-1)] \\ -u[\tau-(2m+1+x(m))(2^k-1)]\} \quad (1)$$

where $N=2\cdot(2^k-1)\cdot n=m_p\cdot n$, u the unit function and m the index of the PCM sample [6]. The resulting PWM signal has sampling frequency $F_p=N\cdot f_s$. The PCM and PWM spectra can be calculated using FFT as:

$$F\{x(\tau)\} = X(\omega), \quad F\{y(\tau)\} = Y(\omega) \quad (2)$$

so that $X(\omega)$, $Y(\omega)$ are $1\times {}^{N}/_{2}$ column vectors (here complex conjugate symmetry is implied but not shown for simplicity). $x(\tau)$, $X(\omega)$ are band limited to the audio spectrum, thus:

$$X(\omega) = \begin{cases} F\{x(\tau)\}, & \omega<\omega_{aud} \\ 0, & \omega>\omega_{aud} \end{cases} \quad (3)$$

where $\omega_{aud}=2\pi\cdot {}^{f_s}/_{2}$ the highest frequency considered in the audio band, corresponding to bin ${}^{n}/_{2}$-1 of $X(\omega)$. However, the PWM spectrum extends beyond the audio band in the ultrasound and can be described through an audio (Y_s) and an ultrasound (Y_u) component:

$$Y(\omega) = Y_s(\omega) + Y_u(\omega) \quad (4)$$

where $Y_s(\omega)$ and $Y_u(\omega)$ $1\times {}^{N}/_{2}$ vectors and $Y_s(\omega)=0$ for $\omega>\omega_{aud}$, $Y_u(\omega)=0$ for $\omega<\omega_{aud}$. Hence, $Y_s(\omega)$ can be expressed in terms of the spectrum of the PCM signal $X(\omega)$ and the distortions $D(\omega)$ introduced in the audio spectrum by the PCM-to-PWM transformation:

$$Y_s(\omega)=X(\omega) + D(\omega) \quad (5)$$

PWM in-band distortions have been thoroughly analyzed and effectively treated in [6], thus in the present work $D(\omega)$ will be employed without further analysis. Finally, the total PWM spectrum can be expressed in terms of the initial PCM spectrum as follows:

$$Y(\omega)=X(\omega) + D(\omega) + Y_u(\omega) \quad (6)$$

In the following section, reduction of the ultrasonic power in PWM signals is addressed through a dual-band signal processing technique.

B. Dual-Band PWM

Starting from the PCM signal in the frequency domain $F\{x(\tau)\}=X(\omega)$, the spectrum can be split in two sub-bands, low (L) and high (H):

$$\mathbf{S_L}X(\omega)=X_L(\omega) \text{ and } \mathbf{S_H}X(\omega)=X_H(\omega) \quad (7)$$

$\mathbf{S_L}$, $\mathbf{S_H}$ are ${}^{n}/_{2}\times{}^{n}/_{2}$ diagonal matrixes of the form:

$$\mathbf{S_L}=\begin{bmatrix}\mathbf{I}_l & \mathbf{0}_{l\times h}\\ \mathbf{0}_{h\times l} & \mathbf{0}_h\end{bmatrix},\ \mathbf{S_H}=\begin{bmatrix}\mathbf{0}_l & \mathbf{0}_{l\times h}\\ \mathbf{0}_{h\times l} & \mathbf{-I}_h\end{bmatrix} \quad (8)$$

where $\mathbf{I}$ the unit matrix, $\mathbf{0}$ the null matrix, l (low) the index of the highest frequency in the L sub-band and $h={}^{n}/_{2}$-l. From the above follows that $\mathbf{S_L}$-$\mathbf{S_H}$=$\mathbf{I}_n$ and $X_L(\omega)$-$X_H(\omega)$=$X(\omega)$, thus there is no spectral information loss and the original signal can be perfectly reconstructed. From these sub-band spectra, the corresponding time signals $x_L(\tau), x_H(\tau)$ can be extracted using inverse Fourier transform:

$$F^{-1}\{X_i(\omega)\} = x_i(\tau) \quad (9)$$

The resulting time signals are then converted into PWM streams y_L, y_H according to (1):

$$x_i(\tau) \xrightarrow[\text{conv}]{\text{PWM}} y_i(\tau) \quad (10)$$

Finally, the two sub-band PWM signals are subtracted from each other, forming the DBPWM output signal:

$$y_{db}(\tau) = y_L(\tau) - y_H(\tau) \quad (11)$$

C. Spectral Analysis of the DBPWM signal

The spectrum of $y_{db}(\tau)$ can be written using (4) and (11) as:

$$\begin{aligned} Y_{db}(\omega) &= F\{\, y_{db}(\tau)\} \\ &= Y_L(\omega) - Y_H(\omega) \\ &= Y_{db,s}(\omega) - Y_{db,u}(\omega) \end{aligned} \quad (12)$$

From (5) we get:

$$Y_{db,s}(\omega) = X_{db}(\omega) + D_{db}(\omega) \quad (13)$$

and expressing X_{db} in terms of the low and high audio components:

$$X_{db}(\omega) = X_L(\omega) - X_H(\omega) = X(\omega) \quad (14)$$

Accordingly, the noise in the audio spectrum takes the form:

$$D_L(\omega)\text{-}D_H(\omega) = D_{db}(\omega) \quad (15)$$

Finally,

$$Y_{L,u}(\omega) - Y_{H,u}(\omega) = Y_{db,u}(\omega) \quad (16)$$

where $Y_{db,u}$ is the total ultrasonic spectrum of the dual-band PWM signal. Finally, the overall dual-band PWM signal can be written in terms of the original PCM spectrum X(ω) as:

$$Y_{db}(\omega) = X(\omega) + D_{db}(\omega) + Y_{db,u}(\omega) \quad (17)$$

For optimal ultrasound reduction (see Sec.III) an appropriate splitting frequency must be selected. It was found that, under conditions usually met in audio applications, the splitting criterion is the equal energy distribution between the two sub-bands, thus:

$$E_L = E_H \Rightarrow \sum_{0}^{l} X^2(\omega) = \sum_{l+1}^{n/2} X^2(\omega) \quad (18)$$

where E_L, E_H the energy in the low and high sub-bands respectively. Proof of (18) is beyond the scope of this work.

III. Simulations and Results

The results were derived via simulations using realistic audio PCM signals. DBPWM was applied on such 6 different test signals (sinewaves, noise, speech and music): a) Gaussian White Noise (WGN), b) Discrete Sinewaves (DS), c) Female Speech (FS), d) Instrumental Music (IM).

The FS and IM samples were taken from 8-bit audio files with f_S=44.1kHz sampling frequency, 3 second duration and were processed off-line using MATLAB [11]. GWN and DS that were directly generated in MATLAB. For all cases, sub-band splitting was based on equal sub-band signal energies, see (18). The effectiveness of the method with respect to ultrasound artefact suppression is measured through a total ultrasonic energy reduction factor E_r, which is here defined as the ratio of ultrasonic energy present in the traditional single-band PWM and dual-band PWM coded signals and expressed in dB as:

$$E_r = 10 \cdot \log\left(\sum_{\omega_{aud}+1}^{\omega_{max}} Y^2(\omega) \Big/ \sum_{\omega_{aud}+1}^{\omega_{max}} Y_{db}{}^2(\omega) \right)$$

where $\omega_{max} = 2\pi \cdot {}^{f_p}/_{2}$. The results of DBPWM will be also compared to traditional ultrasound suppression method that is using a low-pass filter on the PWM coded signal. For such case, 1st order (6dB/oct roll-off, 30kHz cutoff frequency) and 2nd order (12db/oct roll-off 30kHz cutoff frequency) Butterworth filters were designed in MATLAB, simulating RC and RLC filters employed in existing Class-D amplifiers [10]. The filtering results and comparison are presented in TABLE I.

Although detailed analysis of technical aspects regarding DBPWM implementation is beyond the scope of this work, several considerations can be mentioned:

- DBPWM, being a signal pre-processing technique, requires an FFT/IFFT-capable computational unit for calculating the splitting frequency and producing the sub-band streams.
- For real-time applications, block processing of the in-band stream is necessary in order to avoid delays at the input of the amplification unit.
- The two sub-band PWM streams are here added synchronously, implying the need for a second amplification unit. However, it is suggested that extra hardware can be avoided by adopting oversampling (x2) and time multiplexing of the amplified signals.

Gaussian White Noise

Fig.1 shows the PWM (a) and DBPWM (b) spectra for the Gaussian White Noise. The PWM signal contains only odd harmonics of f_s. In DBPWM, the amplitude of the carrier frequency and all odd harmonics was reduced by more than 50dB while there was a small amplification in the first even harmonics ($2f_S$, $4f_S$). A total reduction in ultrasonic energy was measured about E_r=4.5dB, where 1st and 2nd order filters would give $E_r^{(1)}$=5.4dB.

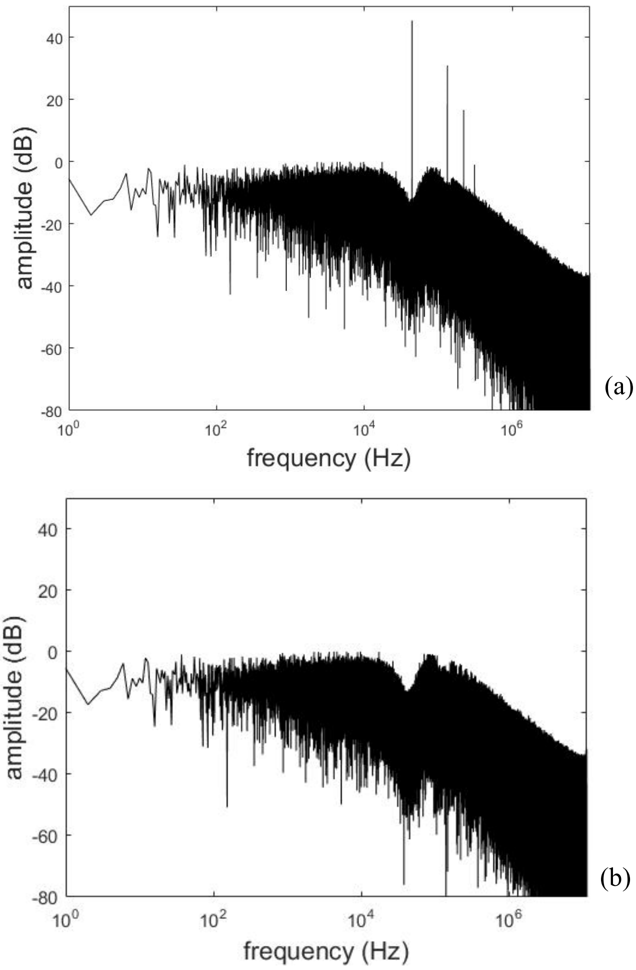

Fig.1 Gaussian White Noise: a) PWM spectrum, b) DBPWM spectrum

TABLE I. ULTRASONIC REDUCTION

Signal	*DBPWM (dB)*	*Filtering (dB)*
GWN	4.5	5.4 (1st ord.)
DS	3.4	6.1 (1st ord.)
FS	12.5	5.4 (1st ord.) 8.5 (2nd ord.) 11.4 (3rd ord.)
MU	8.5	5.8 (1st ord.) 8.5 (2nd ord.)

Discrete Sinewaves

The PWM and DBPWM spectra for the Discrete Sinewaves signal consisting of two sines, 500Hz and 600Hz are shown in Fig.2. The frequencies were selected so that the spectrum allows for clear visualization of the peaks on the carrier harmonics surrounded by the sideband spectra. All peaks on the carrier frequencies are significantly suppressed and only the sideband frequencies remain. In the low frequencies, some aliasing can be seen, however it appears 60dB lower from the signal level. DBPWM ultrasonic reduction was measured E_r=3.4dB whereas a 1st order filter would give $E_r^{(1)}$=6.1dB.

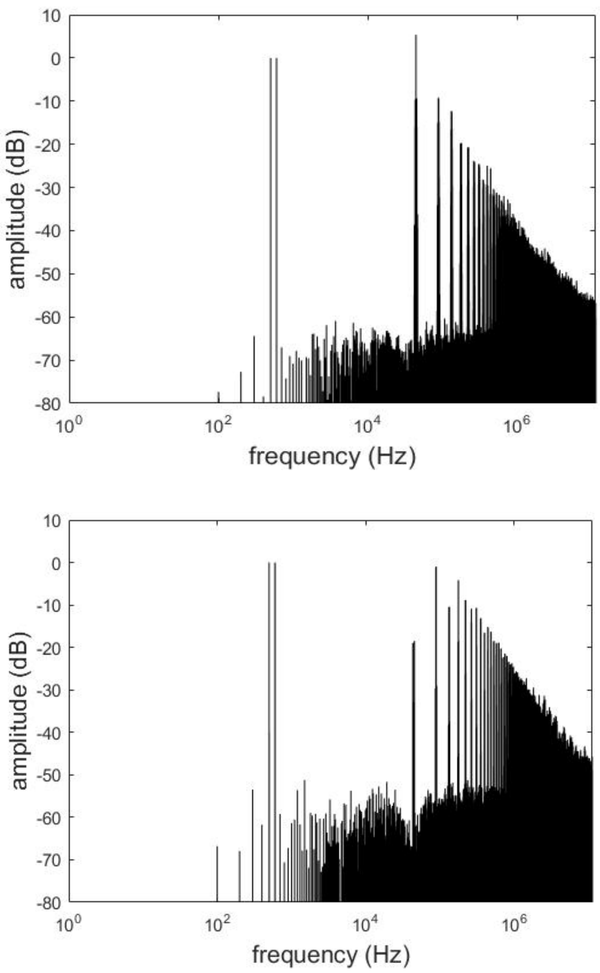

Fig.2 Discrete Sinewaves: a) PWM spectrum, b) DBPWM spectrum

Female Speech

The female speech spectrum, shown in Fig.3, was found to contain both even and odd harmonics of the carrier frequency, although even harmonics have significantly lower amplitude. Again, odd harmonics were suppressed through dual-band cancellation by up to 50dB while even harmonics were not significantly affected. A total reduction of E_r=12.5dB in the ultrasound spectrum was measured, compared to $E_r^{(1)}$=5.4dB, $E_r^{(2)}$=8.5dB and $E_r^{(3)}$=11.4dB for 1st, 2nd and 3rd order filtering respectively.

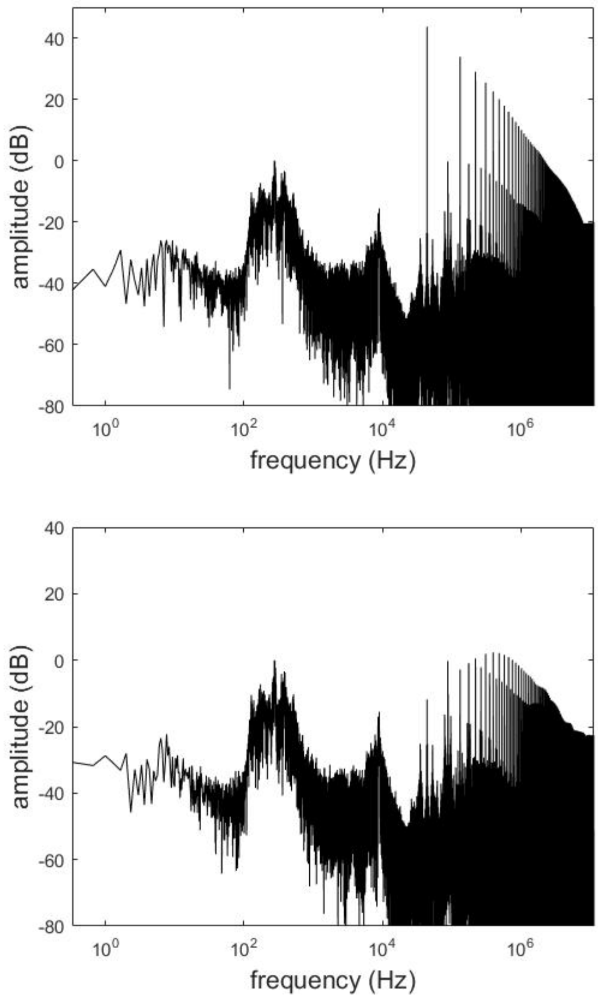

Fig.3 Female Speech: a) PWM spectrum, b) DBPWM spectrum

Music

The PWM and DBPWM spectra for the MU signal, a soft beat music sample with percussion, bass guitar, xylophone and piano, are shown in Fig.4. Similarly to speech, odd harmonics have been suppressed by up to 40dB while even harmonics have remained mainly unaffected, some have been slightly amplified. A total reduction of E_r=11.0dB in ultrasound energy was measured, corresponding approximately to filtering with a 2nd order filter ($E_r^{(2)}$=12.2dB).

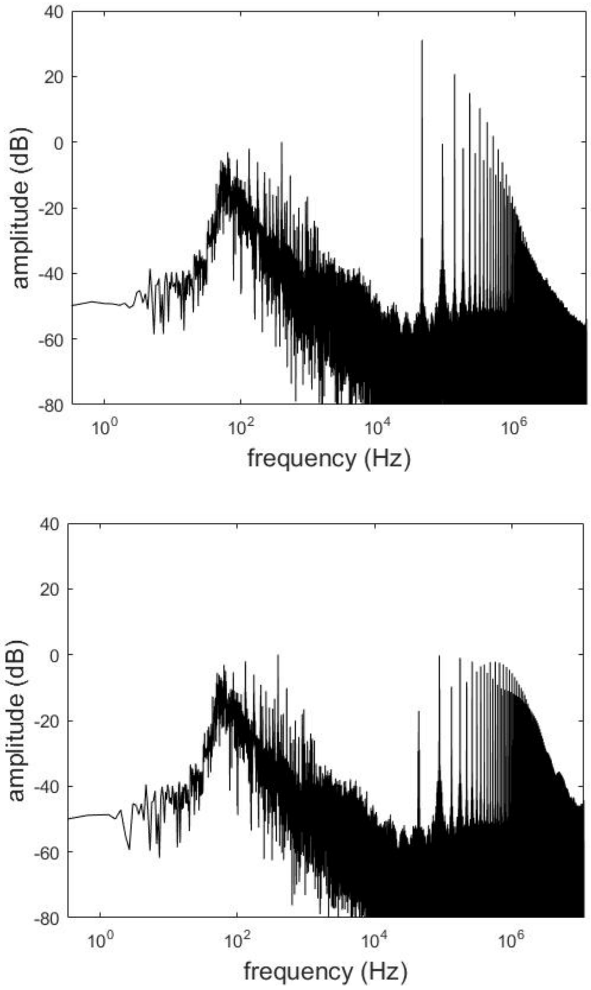

Fig.4 Music: a) PWM spectrum, b) DBPWM spectrum

IV. CONCLUSIONS

A novel methodology for suppressing PWM bitstream carrier frequency artefacts was outlined, by forming two superimposing PWM streams from an analog or PCM signal through splitting the signal's spectrum in two sub-bands (DBPWM). It was shown that DBPWM can significantly suppress the PMW carrier frequencies – especially the odd harmonics – leading to a total reduction of the energy of the high frequency artefacts appearing in PWM sequences. Given the signal-dependent nature of PWM coding distortions, the effectiveness of the method was found to be also signal-dependent, as shown by the varying results taken for different test signals, beyond the usual sinewave cases.

According to these simulated tests, the best results were achieved for the speech sample, where a total ultrasonic energy reduction of about 12.5dB was observed, followed by 8.5dB reduction for the music signal. The DBPWM was also compared to traditional low-pass filtering applied to PWM signal stream. Such comparison has shown that the ultrasound suppression results of the proposed filterless to the reduction achieved by 1st to 3rd order LP filters employed on the original PWM output signals (e.g. in Class-D audio amplifiers).

Future work on DBPWM will be to identify the relationship between characteristics of the analog/PCM signal to the expected energy reduction in the carrier frequencies, along with understanding of the DBPWM sideband artefacts with respect to the sub-band signals. Optimizations on the band-splitting criteria and the size of processing blocks (samples) will be investigated for maximum cancellation. Finally, techniques and routines for real time execution with minimal computational requirements will be also investigated.

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