



AN ADVANCED NONLINEAR SIGNAL MODEL TO ANALYZE PULSATION-DERIVED PHOTOPLETHYSMOGRAM SIGNALS

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Abstract- This paper proposes an advanced nonlinear signal model to handle the intrinsic nonlinearity of the pulsation of photoplethysmogram (PPG) signals and so permit their assessment. This model consists of three-different sinusoidal source signals and it successfully reproduces the primary harmonic, the second harmonic, the respiratory and the other spectra stemming from the interaction of the three source signals. We demonstrate the potential of the model in various experiments on PPG signals. It is discovered that the model has a great performance in that it characterizes PPG signals captured under various measurement conditions if nonlinearity is small. It is also demonstrated that the model successfully characterizes the post-alcohol-intake state and the post-physical-exercise state using the behavior of the nonlinear factors, which should be very useful for alcohol-intake detection. The remaining issues of the model are also addressed.

Index terms: PPG, photoplethysmogram, Fourier transformation, nonlinear signal, harmonic spectral intensity, specific spectral intensity ratio zone.

I. INTRODUCTION

Japan is becoming an aging society; the percentage of people over 65 years old already exceeds 23 %. This is driving demands for the reliable processing of biomedical and sensor data gathered by non-invasive monitoring; various sensors will be used to monitor health status.

Although the non-invasive monitoring of bio-medical information is, generally speaking, inferior to invasive monitoring in terms of sensitivity, resolution, and reproducibility, non-invasive monitoring is definitely useful in the analysis of bio-medical signals from the viewpoint of bio-informatics [1-3]. Of particular note, many researchers have focused on photoplethysmography (PPG) for monitoring (non-invasively) the pulsation rate and the oxygen partial pressure of arterial vessels [4]. In PPG signal analysis, fast Fourier transformation is frequently used to analyze the pulsation spectrum because of its ease [4]. As is well known, Fourier transformation is a powerful approach to signal analysis but the source signal must contain only linear waves.

It is already known that pulsation-generated PPG signals are nonlinear [5-8]. The pulsation is created by a deterministic dynamic system, not a random signal source. Thus we can anticipate that being able to treat this nonlinearity will yield key advances in PPG signal analysis [4]. For example, a recent non-invasive study of pulsation signal behavior addressed the diagnosis of atherosclerosis [9]. The results confirm that the nonlinear analysis of PPG pulsation offers new ideas for advancing informatics.

Many mathematical methods have already been applied to the analysis of the nonlinear PPG signals [10]. Hilbert transformation is one of the more powerful mathematical methods. As Hilbert transformation allows nonlinear and non-stationary sources to be analyzed, it is expected to be useful in analyzing the pulsation-generated PPG signals. By slightly modifying the concept of Hilbert transformation, we show how the impact of the instantaneous frequency and non-stationary behavior of source signals on the resulting spectra can be handled in modeling PPG signals.

Starting with the knowledge that PPG pulsation is intrinsically nonlinear, we propose a nonlinear signal model based on Hamilton's principle [11]. The final target of the analysis method proposed here is the detection of alcohol intake [8, 12-17]. At first, we focus on how well the model can reproduce the entire PPG signal spectra including the sub-Hertz low-frequency range. Although nonlinear signals should not be analyzed by Fourier transformation, it is utilized here

for the purpose of spectra comparison. We also address some remaining issues of the model from the point of view of application to alcohol-intake detection.

II. NONLINEAR SIGNAL MODEL

Shimizu and Omura proposed an advanced PPG-based analysis technique for alcohol-intake detection [8, 12]. Though they fundamentally characterized the alterations in the PPG signal that could be expected with alcohol intake, they could not address the drawback of subject variability. Izawa and Omura proposed an algorithm of specific ‘spectral intensity ratio zone (SIRZ)’ to eliminate this shortcoming in the analysis of PPG signals [11-15, 18]. However, their algorithm is not a complete solution as it fails to counter the variability of the source signal. Our idea is to characterize the aspects raised by the intrinsic nonlinearity of PPG signals. Based on Hamilton’s principle and the idea of the Hilbert spectrum [10], we propose and validate the following nonlinear signal model for PPG signal analysis:

$$f_{PPG}(t) = f_0(t) + f_1(t) + f_2(t) + f_1(t)f_2(t) + f_0(t)f_1(t) + f_0(t)f_2(t) + f_0(t)f_1(t)f_2(t), \quad (1)$$

where $f_0(t)$ denotes the signal source yielding the primary spectrum of pulsation, $f_1(t)$ and $f_2(t)$ are the source signals producing the second harmonic spectrum, the third harmonic spectrum, and the other spectra; a total of 27 spectra. Mathematical details of the nonlinear signal model are found in the *Appendix*. Given eq. (1), the spectra targeted for analysis are schematically shown in Fig. 1; the primary spectra group consists of $f_0(t)$, $f_1(t)$ and $f_2(t)$, the second spectra group consists of $f_0(t)f_1(t)$, $f_0(t)f_2(t)$, and $f_1(t)f_2(t)$, and the third spectra group consists of $f_0(t)f_1(t)f_2(t)$. In addition, the sub-Hertz low-frequency spectra (< 0.5 Hz) consist of $f_0(t)f_1(t)$, $f_0(t)f_2(t)$, $f_1(t)f_2(t)$, and $f_0(t)f_1(t)f_2(t)$. Although it is expected that these low-frequency spectra are related to the parasympathetic nervous system (related to the respiratory spectrum) [19] and sympathetic nervous system [20], the analysis of such spectra is not so easy because they depend on the emotional state of the subject [21].

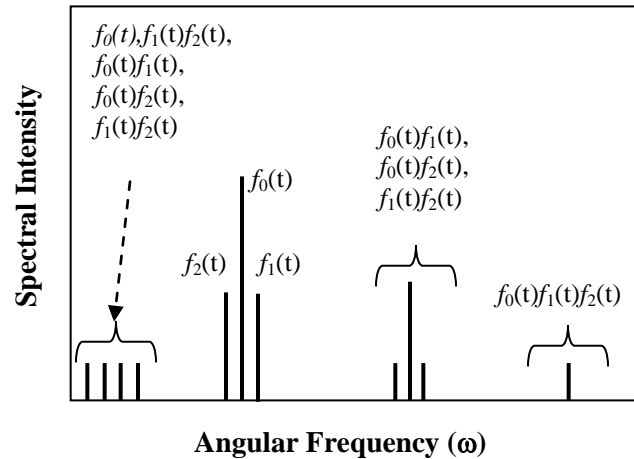


Figure 1. Schematic spectra generated by the nonlinear signal model.

III. EXPERIMENTAL SETUP

This study focuses on the sub-Hertz low-frequency spectra (0.3 ~ 0.5 Hz) of PPG signals in addition to the conventional spectra (the primary pulsation spectrum, its second harmonic and its third harmonic) in order to examine the potential of the nonlinear signal model proposed here. Accordingly, we reformed the amplification circuit so as to reliably extract all spectra from the source signal. The actual circuit diagram used here is shown in Fig. 2(a).

The procedure used in the following experiments is as follows [12]. The PPG signal is converted into the frequency domain by FFT processing after the source signal is linearly amplified by an operational amplifier (see Fig. 2(a)). The light-source LEDs are used in the continuously dc-biased state. The subject's finger is held about 7 mm from the combined light source and photo-detector (photo-transistor) (see Fig. 2(b)). The lights emitted from the source are partially absorbed, reflected around the bone, and captured by the photo-transistor [12]. The measurements are performed in a dark room to avoid the noise generated by room lights and/or natural light. FFT processing is performed at the sampling frequency of 1000 Hz. Since the magnitude of the original spectral component is so large, its square-root value is used in the analysis associated with Figs. 3 to 9. We believe that such a scale change is very useful in visualizing aspects of this experiment as is demonstrated later.

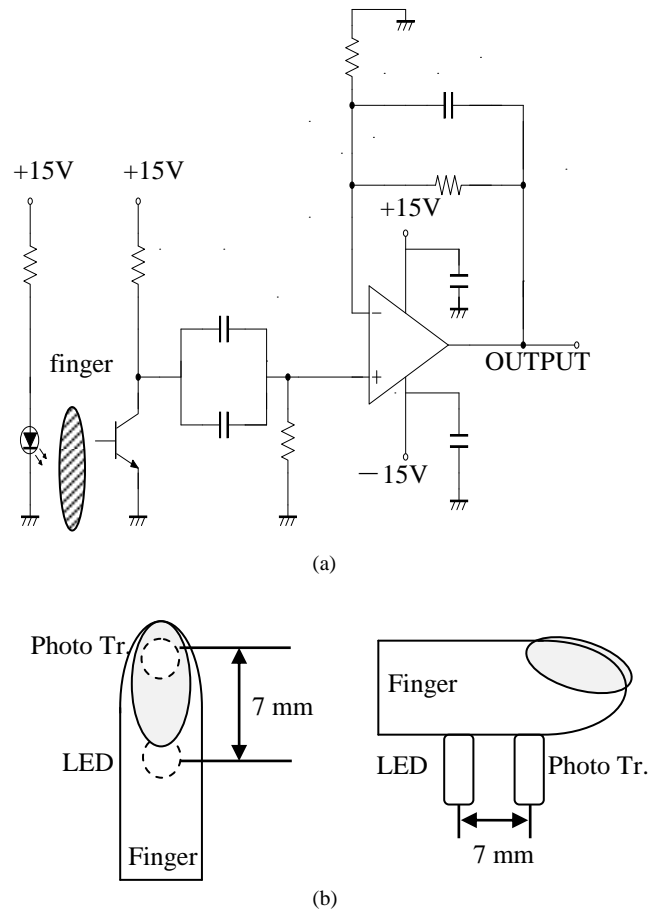


Figure 2. Signal pick-up and amplifier with specific high-pass filter. (a) Circuit diagram, (b) Signal pick-up system image.

In this paper, respiratory spectrum was extracted from the FFT result by manually measuring breath frequency. Experimental conditions for alcohol intake, smoking, and physical exercise are the same as those described in [12].

PPG signals were measured using various LEDs as the optical source; the primary wavelength is in the range of ~ 935 nm for “infrared”, ~ 660 nm for “red”, ~ 470 nm for “blue”, and ~ 525 nm for “green”. However, we will stress the advantages of infra-red-LED-based photoplethysmography in the following discussion. A total of eight volunteers (22-23-year-old males) participated in the experiments; five are habitual smokers and the remainder are non-smokers.

IV. COMPARISON WITH MEASURED RESULTS

a. Parameter extraction method

As described in the *Appendix*, the model has several parameters that should be extracted self-consistently from measured PPG signals. We tried to extract them so that the model reproduced the measured results; *i.e.*, in practice, basically assuming small nonlinearity factor (ε) values, we tried to reproduce the primary spectrum and side-band spectra at the beginning. After that, parameters of other spectra are reviewed by adjusting the nonlinear exponents and nonlinearity factors. We assume that the nonlinearity is not so salient, and thus that Fourier transformation can still be used to analyze the spectra of the PPG signal. However, this point is discussed again later. We tentatively assume that both nonlinearity exponent (n and m) values are 3 to simplify the consideration. As a result, we could determine them almost uniquely.

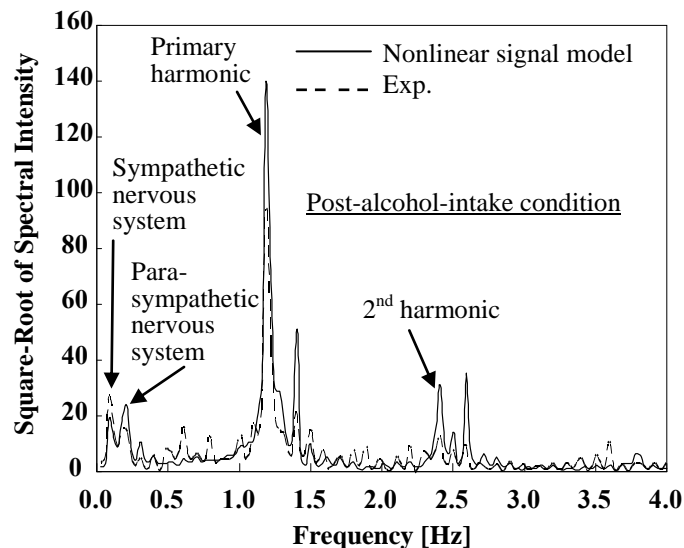


Figure 3. Spectral intensity of PPG signal (measured result) and simulation result (nonlinear signal model). The subject (smoker) took alcohol prior to the measurement, but without smoking.

b. Aspects of spectra of PPG signals and Behaviors of extracted parameters

Figure 3 shows fundamental aspects of the spectra of measured PPG signals and the simulation results obtained by Eq. (1), where the various spectra are categorized by labels; subject, smoker, consumed alcohol prior to measurements [8]. In Fig. 3, we indicate various specific spectra groups; the primary harmonic spectral group with side bands stemming from the pulsation, the 2nd spectral group with side bands stemming from the pulsation, the spectral group controlled by the parasympathetic nervous system (related to the respiratory spectra), and the spectral group related to the sympathetic nervous system. The nonlinear signal model successfully reproduced the spectra of the PPG signal in Fig. 3. These spectra aspects, reproduced by the model, are used to characterize the PPG signal in this paper. Figure 4 shows the spectra of the same subject (smoker) of Fig. 3, where the subject didn't take alcohol and didn't smoke before measurement (sober condition). The model roughly reproduced the PPG signal, but the aspects of the low-frequency spectra were not always well reproduced. This suggests that the PPG signals don't have strong nonlinearity independently of alcohol intake, or that the signal model has still some shortcomings in the sub-Hertz low-frequency range.

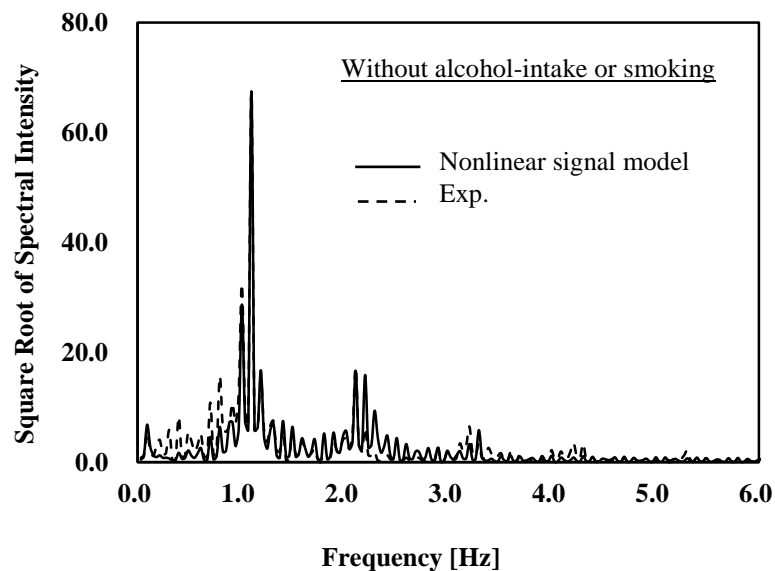


Figure 4. Spectral intensity of PPG signal (measured result) and simulation result (nonlinear signal model). The subject (smoker) didn't take alcohol or smoke prior to the measurement.

Figure 5 shows the spectra of measured PPG and simulation results, where the same subject smoked prior to measurements [13]. The model doesn't well reproduce features of the sub-Hertz low-frequency spectra. The model roughly characterizes some aspects of the measured result with regard to the primary spectral group and the 2nd harmonic group except the sub-Hertz spectra. However, the details seen in the measured results are not reproduced because smoking is apt to generate many noisy spectra as described in Ref. [13]. Figure 5 strongly suggests that the behavior of the sub-Hertz low-frequency spectra must be carefully investigated, or that an additional signal component is needed to improve the reproducibility of the measured PPG signal.

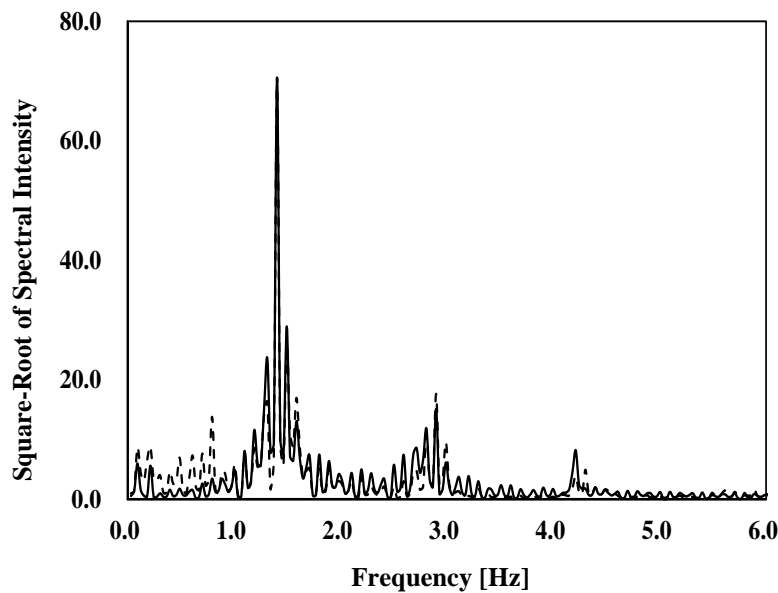


Figure 5. Spectral intensity of PPG signal (measured result) and simulation result (nonlinear signal model). The subject smoked prior to the measurement, but without alcohol intake.

Here, we consider aspects of the parameters extracted from the source signal assuming n and $m = 3$; the primary spectrum (ω_0 , angular frequency), the side-band spectra (ω_1 and ω_2 , angular frequencies), the nonlinearity factor (ε) values, and the amplitudes of two side-band component (A_1 and A_2), see Tables I to IV. Table I shows the time evolution of the parameters for the sober condition, Table II for the smoking condition, Table III for the alcohol-intake condition, and Table IV for the physical exercise condition. We applied such analysis to a total of 5 subjects. In the following, we introduce the data of a certain subject (the same subject yielding Figs. 3 to 5) for the purpose of discussion.

Table I. Parameters extracted from experimental results (sober condition).

Time	ω_0	ω_1	ω_2	ε_1	ε_2	A_1	A_2
[min]	[rad]	[rad]	[rad]				
0	6.91	6.97	6.72	0.027	0.067	0.54	0.65
15	6.91	6.92	6.79	0.007	0.073	0.40	0.60
30	6.91	7.16	6.85	0.063	0.039	0.95	0.50
45	7.54	7.57	7.45	0.020	0.060	0.40	0.60
60	7.54	7.67	7.48	0.042	0.040	0.77	0.60
75	7.54	7.56	7.48	0.016	0.040	0.40	0.60
90	6.91	6.94	6.85	0.010	0.037	0.70	0.60
105	6.91	6.92	6.84	0.004	0.037	0.75	0.65
120	6.91	6.97	6.84	0.029	0.044	0.50	0.55

Table II. Parameters extracted from experimental results (smoking condition).

Time	ω_0	ω_1	ω_2	ε_1	ε_2	A_1	A_2
[min]	[rad]	[rad]	[rad]				
0	8.80	8.97	8.67	0.066	0.086	0.80	0.65
15	8.17	8.48	7.98	0.177	0.171	0.50	0.45
30	8.17	8.36	8.01	0.088	0.099	0.60	0.65
45	7.54	7.67	7.41	0.052	0.079	0.62	0.60
60	7.54	7.67	7.32	0.059	0.082	0.55	0.70
75	7.54	7.63	7.48	0.048	0.043	0.50	0.55
90	8.17	8.45	8.11	0.099	0.043	0.80	0.60
105	6.91	6.95	6.82	0.011	0.050	0.80	0.65
120	6.91	6.95	6.77	0.011	0.060	0.80	0.60

In Tables I and II, it is seen that most of the nonlinearity factor (ε) values are smaller than 0.1 for both conditions. However, smoking condition tends to yield large values within 30min after experiment initiation. This suggests that the nonlinearity of the PPG signal for the smoking condition is more significant than that for the sober condition within the first 30minutes of the experiment. In contrast, nonlinearity factor (ε) is under 0.1 for the sober condition, which suggests that the vascular motion is calm. The Lagrangean equation (A-2) suggests that the side-

band functions $f_1(t)$ and $f_2(t)$ rule the primary configuration of the PPG source signal. This means that a large value of ε_1 yields a triangle-like wave shape, and that a large value of ε_2 yields a convex-like shape. In other words, we can guess that enhancement of the systolic process results in an increase in the value of ε_1 and that enhancement of the diastolic process results in an increase in the value of ε_2 . Therefore, it is considered that the negative impact of smoking appears in the systolic and diastolic processes [18]. These characteristics (of parameters) were also observed in other subjects (not shown here).

Table III. Parameters extracted from experimental results (alcohol-intake condition).

Time [min]	ω_0 [rad]	ω_1 [rad]	ω_2 [rad]	ε_1	ε_2	A_1	A_2
0	7.54	7.85	7.48	0.136	0.060	0.60	0.40
15	8.17	8.48	8.11	0.126	0.043	0.70	0.60
30	7.54	7.85	7.48	0.148	0.060	0.55	0.40
45	7.54	7.67	6.79	0.054	0.547	0.60	0.50
60	6.91	7.41	6.85	0.187	0.044	0.65	0.50
75	6.91	7.29	6.85	0.129	0.073	0.70	0.30
90	6.91	7.41	6.85	0.174	0.049	0.70	0.45
105	6.91	7.48	6.88	0.211	0.024	0.65	0.45
120	8.17	8.20	8.11	0.022	0.037	0.40	0.70

In Table III, it is seen that most nonlinearity factor ε_1 values are larger than 0.1 for the alcohol-intake condition. This suggests, as expected, that alcohol intake makes the nonlinearity of the PPG signal more significant than the sober condition. However, it should be noted that ε_1 takes a value larger than 0.1 much more frequently than ε_2 takes such a value which strongly suggests that the cardiovascular response contributing to the systolic process is unusually stimulated by alcohol intake.

In Table IV, physical exercise condition, it is seen that ε_2 values exceed 0.1 soon after experiment initiation, while ε_1 remains under 0.1. For the physical exercise condition, other subjects yielded similar results. This means that the cardiac response for the physical exercise condition is quite different from that for the alcohol intake condition. As is mentioned in the previous paragraph, the data of Table III reveals that alcohol intake leads to dysfunction in the systolic process, which

is an important result. In a similar manner, the data of Table IV reveals that physical exercise enhances the dysfunction of the diastolic process at an early stage. Such consideration is also examined in the following sections. As a result, the PPG signal may not be easily reproduced by means of a set of simple nonlinear signal functions [22] for the physical exercise condition.

Table IV. Parameters extracted from experimental results (physical exercise condition).

Time [min]	ω_0 [rad]	ω_1 [rad]	ω_2 [rad]	ε_1	ε_2	A_1	A_2
0	11.31	11.34	10.93	0.020	0.283	0.60	0.75
15	9.42	9.49	9.17	0.033	0.197	0.60	0.60
30	8.80	8.86	8.48	0.037	0.196	0.50	0.70
45	8.17	8.20	7.95	0.014	0.138	0.60	0.65
60	7.54	7.57	7.32	0.015	0.127	0.55	0.65
75	8.17	8.26	8.16	0.044	0.010	0.60	0.50
90	7.54	7.57	7.45	0.020	0.051	0.40	0.70
105	7.54	7.57	7.41	0.023	0.079	0.35	0.60
120	7.54	7.57	7.48	0.020	0.060	0.40	0.40

c. Time evolution of specific spectrum

It has been demonstrated that the nonlinear signal model basically reproduces important aspects of the PPG signal, and that it successfully produces parameters important in considering the nonlinearity of the PPG signal. Here, in order to advance the consideration, we define two spectral intensity ratios in order to elucidate another potential of the signal model. R_{12} is defined as the square root of the ratio of the primary spectral intensity to the 2nd harmonic spectral intensity. This ratio is also used later in order to suppress subject variability [8, 12-15, 23].

Measured and simulation results of the time evolution of R_{12} are shown in Fig. 6, where the subject (smoker) took alcohol prior to the measurement. The data is from the same subject yielding Figs. 3 to 5. As Izawa *et al.* revealed that R_{12} is a possible measure of the impact of smoking [13], it is used here, too. The nonlinear signal model well reproduces the measured results in Fig. 6. Figure 7 shows the time evolution of R_{12} for the subject that smoked. The nonlinear signal model also reproduced the measured results. These results basically suggest that the proposed model has high potential when the analyses employ the specific ratio of spectral intensities. In other words, the model catches important and substantial aspects of the PPG signal

even though the raw spectral data output by the model have some shortcomings as seen in Figs. 4 and 5.

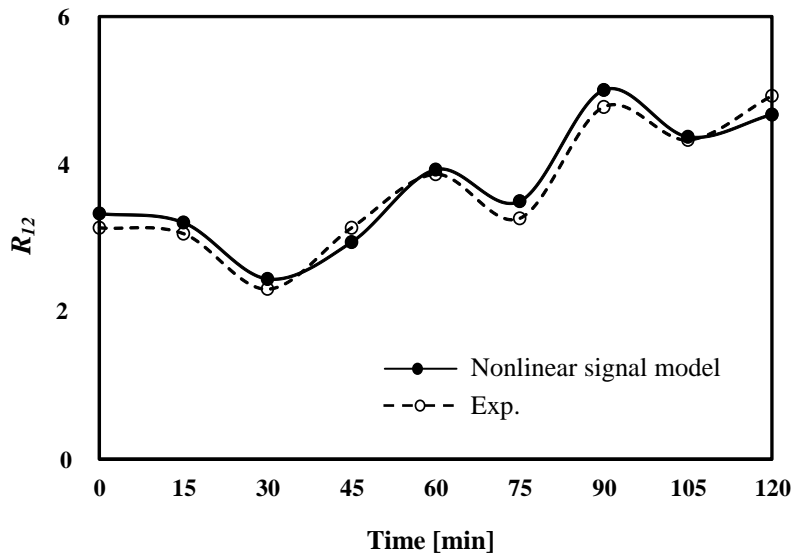


Figure 6. Time evolution of spectral intensity ratio (R_{1b}) for the subject (smoker) after taking alcohol without smoking.

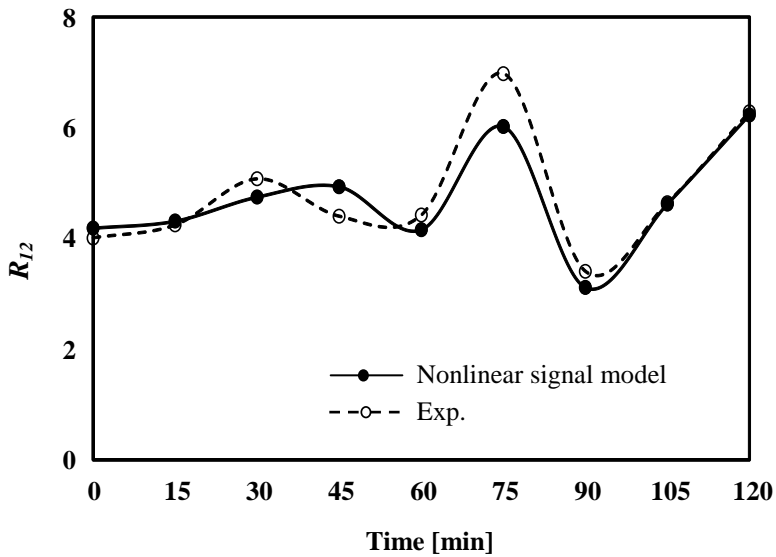


Figure 7. Time evolution of spectral intensity ratio (R_{12}) for the subject (smoker) after having smoked without alcohol intake.

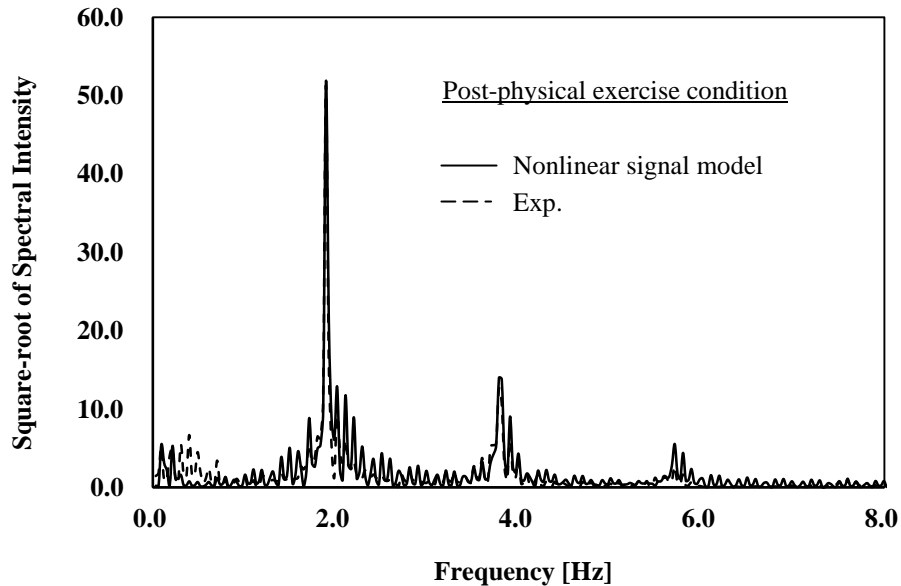


Figure 8. Spectral intensity of PPG signal (measured result) and simulation result (nonlinear signal model). The subject performed physical exercise before the measurement.

d. Consideration on the impact of physical exercise on the PPG signal

We investigated the impact of physical exercise on the PPG signals because Table IV suggests unexpected enhancement of the PPG signal nonlinearity. Spectra of the same subject (smoker) are shown in Fig. 8 for the physical exercise condition, where the reproducibility of the nonlinear signal model is the target. It is seen that the model basically reproduces the spectra of the PPG signals except for a sub-Hertz range. When a small value (~ 0.1) of $|\varepsilon|$ is assumed, the model doesn't well reproduce the spectra of the measured results. Measured and simulation results of the time evolution of R_{12} are shown in Fig. 9, where the subject (smoker) took physical exercise prior to the measurement. The data is from the same subject yielding Figs. 3 to 5. The nonlinear signal model well reproduces the measured result in Fig. 9 by assuming the nonlinearity factors larger than 0.1.

It has been discovered that physical exercise strengthens the nonlinearity of the PPG signal beyond that expected; the behavior of the nonlinearity factor is quite different from the case of alcohol intake. In addition, it has been strongly suggested that the parasympathetic nervous

system (related to the respiratory spectrum) and the sympathetic nervous system modulate the side-band spectra of the primary spectrum and the 2nd harmonic spectrum [20]. It is anticipated that these findings, extracted from the measured results, can be utilized in analyzing PPG signals from the viewpoint of the monitoring of the cardiovascular system.

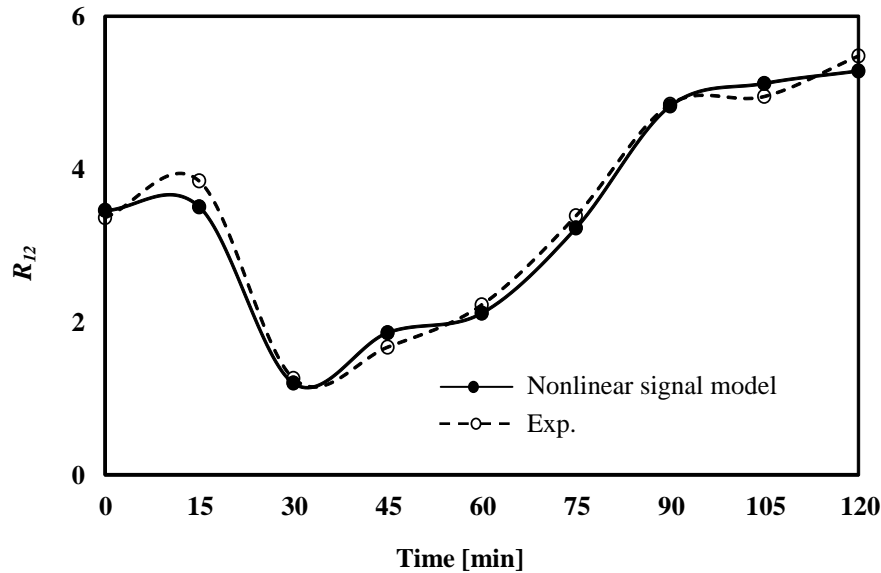


Figure 9. Time evolution of spectral intensity ratio (R_{1b}) for the subject after physical exercise.

Figure 9 shows the time evolution of the spectral intensity ratio (R_{1b}) after physical exercise. It reveals that the signal model fails to well reproduce the time evolution of the ratio. This also suggests that the nonlinearity of the PPG signal is very strong after physical exercise.

e. Comparison of nonlinearity factor among various conditions

It has been shown that nonlinearity factor ε_1 tends to take larger values for about 2 hours under the alcohol-intake condition, which is different from the other conditions. Figure 10 shows the measured average values of nonlinearity factor ε_1 over 2 hours. Data extracted from the subjects under the alcohol-intake condition always exhibit the largest values among all the conditions. However, it is seen in Fig. 11 that ε_2 tends to take large values under the physical exercise condition, which suggests that physical exercise has a specific impact on the cardiovascular motion.

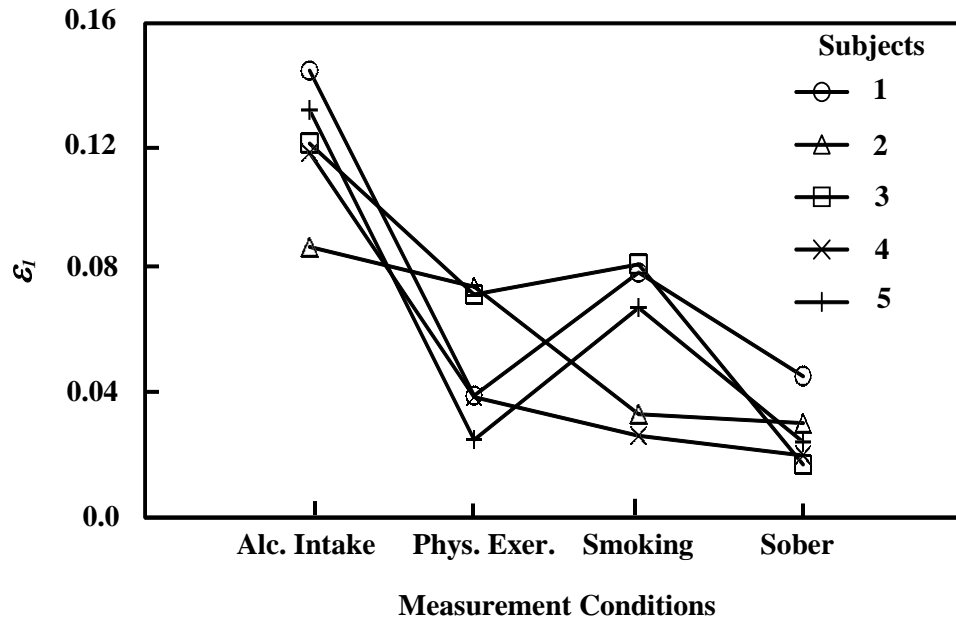


Figure 10. Average values of nonlinearity factor ε_l for five subjects under all conditions.

V. ADDITIONAL CONSIDERATION OF THE PROPOSED NONLINEAR SIGNAL MODEL

The above results demonstrate that the proposed nonlinear signal model is basically valid for the cases examined (smoking, alcohol intake and physical exercise). However, the PPG signals recorded showed that physical exercise yield signal nonlinearity with aspects different from the case of alcohol intake as discussed in 4.3 and 4.4. One approach to characterize such features of the PPG signals in detail is to numerically solve Eq. (A-2) of the *Appendix* assuming a large value of $|\varepsilon|$. Though numerical calculations do make it possible to extract parameters from experimental results, this approach is not realistic for practical use.

Another solution is to obtain an analytical solution of Eq. (A-2). One possible way is to start from the simplified Sine-Gordon equation or to solve the Korteweg-de-Vries equation. It is expected that the nonlinearity of the PPG signal should, due to its nature, partially resemble the behavior of the Toda lattice [24]. When the value of $|\varepsilon|$ is large, the behavior of the solution of

Eq. (A-2) may replicate the behavior of the Toda lattice. Accordingly, we will reconsider the mathematical formulation to better reproduce the nonlinearity of the PPG signal in the future.

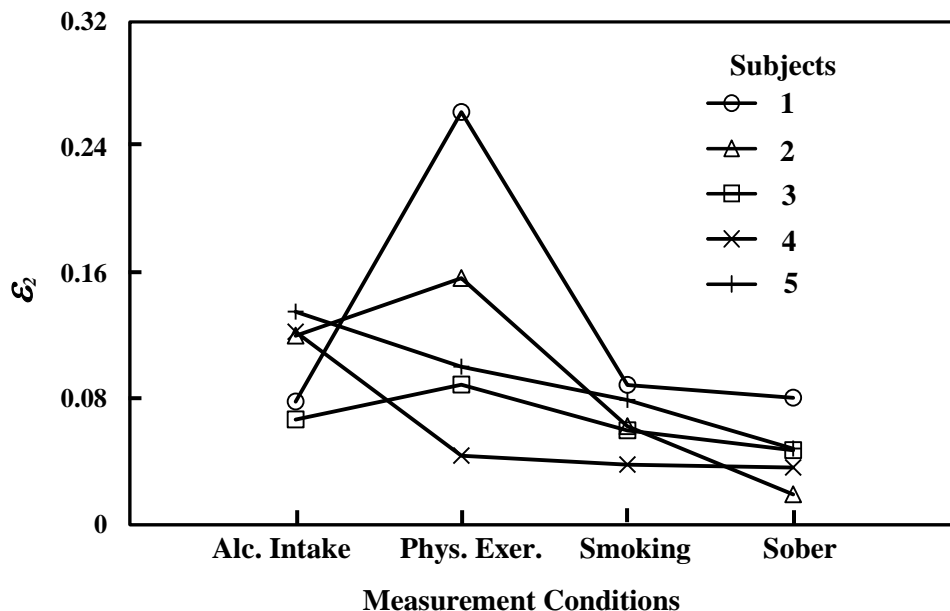


Figure 11. Average values of nonlinearity factor ε_2 for five subjects under all conditions.

VI. CONCLUSION

This paper proposed an advanced nonlinear signal model to analyze pulsation-derived photoplethysmogram (PPG) signals because the pulsation is intrinsically nonlinear. This model consists of three-different sinusoidal source signals and basically reproduces the primary harmonic, the second harmonic, the respiratory and the other spectra stemming from the interaction of the three source signals. The model was developed by solving the nonlinear differential equations. It was demonstrated that the model well reproduces important aspects of the nonlinear PPG signal for various conditions except a sub-Hertz range. The results strongly suggest that alcohol intake yields a large impact on the systolic process of cardiovascular motion, and that physical exercise strongly impacts the diastolic process of cardiovascular motion.

We also examined whether the model reproduces the specific spectral intensity ratio defined for the purpose of alcohol-intake detection. It has been demonstrated that the model basically

reproduces the time evolution of the specific spectral intensity ratio assuming a large nonlinearity factor. As a result, the results shown in this study will be very useful for alcohol-intake detection by categorizing the difference between alcohol intake and physical exercise.

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Appendix: Mathematical Model for PPG Signal Based on Nonlinear Signal Analysis

a. Solution of nonlinear equation

The purpose of modeling is to obtain the intrinsic mode function that can reproduce the spectra of the signal. However, generally speaking, there is no best way to do that. With regard to this point, the following Hilbert spectra representation gives us a better idea [10].

$$X(t) = \sum_{j=1}^N A_j(t) \exp\left(i \int \omega_j(t) dt\right), \quad (\text{A-1})$$

where $X(t)$ is an arbitrary time series, $A_j(t)$ is non-stationary amplitude, and $\omega_j(t)$ is instantaneous angular frequency. If we successfully propose appropriate functions for $A_j(t)$ and $\omega_j(t)$, Eq. (A-1) will be a trial function for signal analysis.

When the nonlinear oscillation term is small, we can use a well-known approximate solution of the differential equation describing oscillation for signal analysis [10, 24, 25]. We start from the following differential equation for small ε .

$$\ddot{x} + \omega_0^2 x + \varepsilon x^n = 0, \quad (\text{A-2})$$

where n is an integer [$n = (2, 3, \dots)$] and ω_0 is the angular frequency of the primary linear wave. We solve Eq. (A-2) for the following conditions.

(i) n is an even number ($n = 2m$ ($m \geq 1$)):

The Lagrangean equation yielding Eq. (A-2) is expressed as

$$L = \frac{1}{2} (\dot{x}^2 - \omega_0^2 x^2) - \frac{\varepsilon}{(n+1)} x^{n+1}. \quad (\text{A-3})$$

It is well known that Eq. (A-3) doesn't contain any important information on the nonlinearity of the oscillation other than the level of oscillation distortion. Accordingly, we don't assume this condition.

(ii) n is an odd number ($n = 2m + 1$ ($m \geq 1$)):

The Lagrangean equation yielding Eq. (A-2) is also expressed by Eq. (A-3). In this case, however, we have the following result of the line integral.

$$\begin{aligned} I_\omega &= \int_0^{2\pi/\omega} dt \left(\frac{1}{2} (\dot{x}^2 - \omega_0^2 x^2) - \frac{\varepsilon}{n} x^n \right) \\ &= \frac{1}{2} A^2 \frac{\pi}{\omega} (\omega^2 - \omega_0^2) - \frac{\varepsilon A^n}{n} 4 \frac{\pi}{2\omega} \frac{n-1}{n} \frac{n-3}{n-2} \dots, \end{aligned} \quad (\text{A-4})$$

where A is the signal amplitude and ω is the angular frequency of the signal. The derivative of I_ω yields

$$\begin{aligned} \frac{\partial I_{\omega}}{\partial A^2} &= \frac{1}{2} \frac{\pi}{\omega} (\omega^2 - \omega_0^2) - \frac{n}{2} \frac{\varepsilon A^{n-2}}{n} 4 \frac{\pi}{2\omega} \frac{n-1}{n} \frac{n-3}{n-2} \dots \\ &= \frac{1}{2} \frac{\pi}{\omega} \left\{ \omega^2 - \omega_0^2 - \frac{\varepsilon A^{n-2}}{n} (n-1) \frac{n-3}{n-2} \dots \right\}. \end{aligned} \quad (\text{A-5})$$

Thus, the solutions are specific frequencies that characterize the oscillation and the intrinsic mode function.

$$\omega_1 = \sqrt{\omega_0^2 + \frac{\varepsilon A^{n-2}}{n} (n-1) \frac{n-3}{n-2} \dots}, \quad (\text{A-6a})$$

$$\omega_2 = -\sqrt{\omega_0^2 + \frac{\varepsilon A^{n-2}}{n} (n-1) \frac{n-3}{n-2} \dots}, \quad (\text{A-6b})$$

$$x_n = A \sin \left(\pm \sqrt{\omega_0^2 + \frac{\varepsilon A^{n-2}}{n} (n-1) \frac{n-3}{n-2} \dots} t \right). \quad (\text{A-7})$$

Eq. (A-6) yields side bands of the primary linear wave. As is well known, signal amplitude (A) is a function of the phase of the wave and, as a result, the frequency alters instantaneously. As these solutions have nonlinear properties, we apply them to analysis of the nonlinear PPG signals.

b. Nonlinear signal model

Based on the analysis given in the previous section, we introduce the following two side-band components as nonlinear signal functions.

$$f_1(t) = A_1 \sin(\omega_1 t) = A_1 \sin \left(\sqrt{\omega_0^2 + \frac{\varepsilon A^{n-2}}{n} (n-1) \frac{n-3}{n-2} \dots} t \right), \quad (\text{A-8})$$

$$\begin{aligned}
 f_2(t) &= A_2 \sin(\omega_2 t) = A_2 \sin\left(-\sqrt{\omega_0^2 + \frac{\varepsilon A^{m-2}}{m} (m-1) \frac{m-3}{m-2} \dots} t\right) \\
 &= -A_2 \sin\left(\sqrt{\omega_0^2 + \frac{\varepsilon A^{m-2}}{m} (m-1) \frac{m-3}{m-2} \dots} t\right)
 \end{aligned} \tag{A-9}$$

In the above solutions, we assume that the parameter used in the two functions should be determined independently. Given a trial function with a certain frequency, we take the following set of approximate solutions for Eq. (A-2).

$$f_0(t) = A_0 \sin(\omega_0 t), \tag{A-10a}$$

$$f_1(t) = A_1 \sin\left(\sqrt{\omega_0^2 + \frac{\varepsilon_1 A_1^{n-2}}{n} (n-1) \frac{n-3}{n-2} \dots} t\right), \tag{A-10b}$$

$$f_2(t) = A_2 \sin\left(-\sqrt{\omega_0^2 + \frac{\varepsilon_2 A_2^{m-2}}{m} (m-1) \frac{m-3}{m-2} \dots} t\right), \tag{A-10c}$$

where the amplitudes (A_0 , A_1 , and A_2) and exponents (n and m) of Eq. (A-10) are different from those of the others as suggested previously; nonlinearity factors (ε_1 and ε_2) are excepted. They are self-consistently determined so that they reproduce the entire signal (here, the PPG signal).

In this paper, based on Eq. (A-1), we assume the following signal form, $f(t)$, in analyzing the PPG signal.

$$f(t) = f_0(t) + f_1(t) + f_2(t) + f_1(t)f_2(t) + f_0(t)f_1(t) + f_0(t)f_2(t) + f_0(t)f_1(t)f_2(t). \tag{A-11}$$

This expression consists of the primary signal and others with non-stationary amplitude and instantaneous angular frequency.

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