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MULTIVARIABLE PID CONTROL VIA ILMIs: PERFORMANCES ASSESSMENT

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Abstract: This paper addresses the problem of performance assessment for MIMO PID controllers. An efficient comparative analysis is established using two Iterative Linear Matrix Inequality (ILMI) approaches. To evaluate the robustness and the performances of each ILMI approach, four tests are carried out on the closed-loop system including White Gaussian noise, set point changes, parametric perturbation, and multiple time-delays influence. Different performance criterion such as IAE, ISE, ITAE and ITSE are also presented to evaluate the closed-loop system performances. The quadruple-tank process as a benchmark of minimum/non-minimum phase systems is applied to show the effectiveness and utility of the proposed analysis. The comparative analysis between the two PID controllers leads to nearly similar performance results in parametric perturbation and noise attenuation but different results in terms of set point changes, multiple time-delays influence and performance indices.

Index terms: **PID control, Multivariable feedback control, ILMI, Non-minimum phase systems (NMPS), Performance assessment.**

I. INTRODUCTION

Proportional-integral-derivative (PID) controllers have found popularity in the industrial world since Ziegler and Nichols proposed their first PID tuning method. This is mainly due to their simplicity in controller structure, robustness to uncertainties and disturbances and availability of numerous tuning methods. Several approaches have been proposed to address tuning PID parameters [1]. However, tuning methods for MIMO PID controllers remain less understood than SISO ones. Numerous research works and books have been reported in this framework [2], [3], [4]. Recently, several emerging controller's design of industrial processes have been developed [5], [6], [7] and a large volume of published studies describing the implementation of PID controllers such as in water level regulation [8], temperature regulation [9], impressed current cathodic protection (ICCP) system [10], and conveyor belt system [11]. The PID controller tuning approaches are mainly used to ensure the stability of the closed-loop systems and meet objectives such as set-point changes, disturbance rejection, robustness against plant modelling uncertainty, noise attenuation ... Performances assessment of these approaches can be established by using different performance indices and robustness criteria [12]. Since major plants have time varying properties and changing operating regimes, it is difficult to find a suitable set of PID parameters that will provide optimal process performance under all conditions. Furthermore, for the case of non-minimum phase systems (NMPS), some performance limitations such as overshoot or undershoot are imposed by the characteristics of the system [13]. An overview of new results, particularly for NMPS , have been discussed through the literature [14], [15], [16], [17], [18].

On the other hand, Linear Matrix Inequalities (LMIs) are one of the most efficient tools to solve complex optimization problems in controller design [19]. A great deal of LMI-based design methods have been proposed for systems stabilization [20] and synchronization [21]. To tackle the problem of computing MIMO PID gain matrices, an Iterative Linear Matrix Inequality (ILMI) algorithm was proposed by [22] and later used to solve several MIMO PID controller design problems [23], [24], [25], [26], [27]. The basic idea is to transform a PID controller into an equivalent static output feedback (SOF) controller. Transformation of PID controllers to SOF controllers is a good alternative to solve such complex control problem. This can be realized by augmenting, using some new state variables, the dimension of the PID controller system. Established results in SOF field can be then used to design a multivariable PID controller for various specifications such as asymptotic stabilization, robustness, performances… Several ILMI algorithms were developed and lead to different approaches and methodologies. Very often, the different conditions derived are not readily implementable as numerical algorithms. Another major difficulty is due to the non-convexity of the static output feedback solution which gives an important computational task. These motivate the present work to study ILMI approaches, to detail resolution procedures and to deepen performance assessment studies for MIMO PID control of non-minimum phase systems using ILMI approaches. The main merit of our work is to propose a performance assessment for MIMO PID controllers using two ILMI approaches. The comparative analysis is based on four tests carried out on the system including Gaussian white noise perturbation, set-point changes, parametric perturbation, multiple delays influence. Four index performances (IAE, ISE, ITAE, ITSE) are also used to deal with closed-loop system performances. A quadruple-tank process, as a NMPS benchmark, is used to illustrate practicality and efficiency of the proposed analysis.

The paper is organized as follows. Section II presents the quadruple tank process and the minimum/non-minimum phase models. Section III details the MIMO PID tuning via two ILMI approaches. Finally, performance assessment of the two PID controllers using four tests and different performance indices is exposed in Section IV.

II. THE QUADRUPLE-TANK PROCESS

The quadruple-tank process [28], [29], is a multivariable process which consists of four interconnected water tanks and two pumps. The schematic diagram of the process is shown in Figure 1. The output of each pump is split into two using a three-way valve. The inlet flow of each tank is measured by an electro-magnetic flow-meter and regulated by a pneumatic valve. The level of each tank is measured by means of a pressure sensor. The regulation problem aims to control the water levels in the lower two tanks with two pumps. The two pumps convey water from a basin into the four tanks. The tanks at the top (tanks 3 and 4) discharge into the corresponding tank at the bottom (tanks 1 and 2, respectively). The positions of the valves determine the location of a zero for the linearized model. If γ_1 is the ratio of the valve for the first tank, then (1-γ_1) will be the valve ratio for the four tank. The voltage applied to pump is ϑ_i and the corresponding flow is $k_i\vartheta_i$. The parameters γ_1, $\gamma_2 \in [0,1]$ are determined from how the valves are set prior to an experiment. The flow to tank 1 is $\gamma_1 k_1 \vartheta_1$ and the flow to tank 4 is $(1-\gamma_1)k_1\vartheta_1$ and similarly for tank 2 and tank 3. The acceleration due to gravity is denoted by g. The measured level signals are $y_1 = k_c h_1$ and $y_2 = k_c h_2$.

For each tank i=1,...,4 , consideration of mass balances and Bernoulli's law yields :

$$q_{out_i} = a_i \sqrt{2gh_i} \tag{1}$$

$$A_i \frac{dh_i}{dt} = q_{in_i} - q_{out_i} \tag{2}$$

where q_{in_i} is the in-flow of the tank and q_{out_i} is the out-flow of the tank.

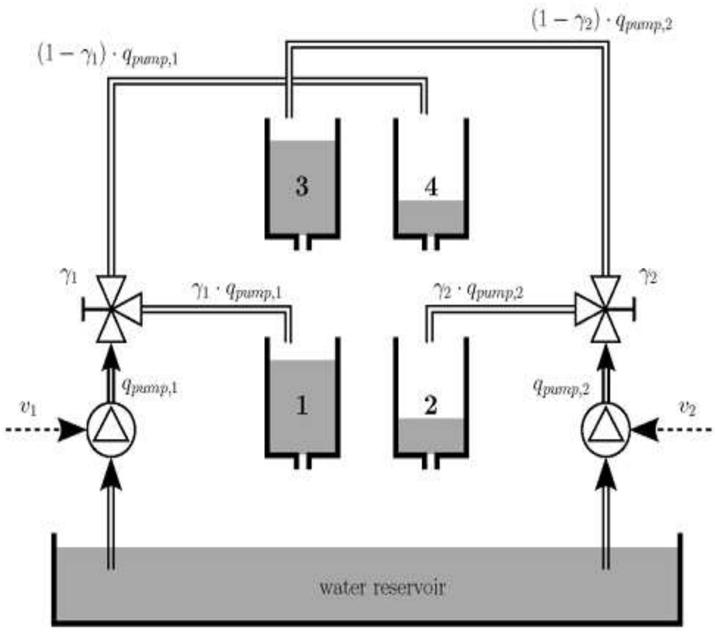


Figure 1. The quadruple tank process

Considering the flow in and out of all tanks simultaneously, the non-linear dynamics of the quadruple tank process is given by:

$$\begin{aligned}
\frac{dh_1}{dt} &= -\frac{a_1}{A_1}\sqrt{2gh_1} + \frac{a_3}{A_1}\sqrt{2gh_3} + \frac{\gamma_1 k_1 \vartheta_1}{A_1} \\
\frac{dh_2}{dt} &= -\frac{a_2}{A_2}\sqrt{2gh_2} + \frac{a_4}{A_2}\sqrt{2gh_4} + \frac{\gamma_2 k_2 \vartheta_2}{A_2} \\
\frac{dh_3}{dt} &= -\frac{a_3}{A_3}\sqrt{2gh_3} + \frac{(1-\gamma_2)k_2\vartheta_2}{A_3} \\
\frac{dh_4}{dt} &= -\frac{a_4}{A_4}\sqrt{2gh_4} + \frac{(1-\gamma_1)k_1\vartheta_1}{A_4}
\end{aligned} \tag{3}$$

Nomenclature

h_i	:	Level of water in tank i
a_i	:	Area of the pipe flowing out from tank i
A_i	:	Area of tank i
γ_1	:	Ratio of water diverting to tank 1 and tank 4
γ_2	:	Ratio of water diverting to tank 2 and tank 3
k_1	:	Gain of pump 1
k_2	:	Gain of pump 2
k_c	:	Level sensor
g	:	Gravitational constant
$q_{pump,1}$	:	Pump 1 flow
$q_{pump,2}$	:	Pump 2 flow
ϑ_1	:	Voltage input 1 (pump 1)
ϑ_2	:	Voltage input 2 (pump 2)
y_1	:	Voltage from level measurement devices of tank 1
y_2	:	Voltage from level measurement devices of tank 2
MPS	:	Minimum Phase System
NMPS	:	Non Minimum Phase System

Let note by $x = [h_i - h_{i0}]^T$, i=1,...,4, the state variable vector, $\vartheta = [\vartheta_1 - \vartheta_{10} \quad \vartheta_2 - \vartheta_{20}]^T$ the control vector and $y = [y_1 \quad y_2]^T$ is the output vector. The linearized model [28] around the equilibrium points ϑ_1^0, ϑ_2^0, h_1^0, h_2^0, h_3^0, h_4^0 , y_1^0, y_2^0 can be expressed by :

$$
\begin{aligned}
\dot{x} &= Ax + Bu \\
y &= Cx
\end{aligned}
\qquad (4)
$$

where

$$
A = \begin{bmatrix}
-\frac{a_1}{A_1}\sqrt{\frac{g}{2h_1^0}} & 0 & \frac{a_3}{A_1}\sqrt{\frac{g}{2h_3^0}} & 0 \\
0 & -\frac{a_2}{A_2}\sqrt{\frac{g}{2h_2^0}} & 0 & \frac{a_4}{A_2}\sqrt{\frac{g}{2h_4^0}} \\
0 & 0 & -\frac{a_3}{A_3}\sqrt{\frac{g}{2h_3^0}} & 0 \\
0 & 0 & 0 & -\frac{a_4}{A_4}\sqrt{\frac{g}{2h_4^0}}
\end{bmatrix}, \quad
B = \begin{bmatrix}
\frac{\gamma_1 k_1}{A_1} & 0 \\
0 & \frac{\gamma_2 k_2}{A_2} \\
0 & \frac{(1-\gamma_2)k_2}{A_3} \\
\frac{(1-\gamma_1)k_1}{A_4} & 0
\end{bmatrix},
$$

$$C = \begin{bmatrix} k_c & 0 & 0 & 0 \\ 0 & k_c & 0 & 0 \end{bmatrix}$$

The parameters of the quadruple–tank process are presented in [8]. The eigenvalues of the open-loop system are - 0.0159, - 0.0111, - 0.0419 and - 0.0333. The system admits two multivariable transmission zeros, which are determined by the zeros of its determinant as:

$$\det G(s) = \frac{T_1 T_2 k_1 k_2 \gamma_1 \gamma_2}{\prod_{i=1}^{4} (1 + sT_i)} \left[(1 + sT_3)(1 + sT_4) - \frac{(1-\gamma_1)(1-\gamma_2)}{\gamma_1 \gamma_2} \right] \tag{5}$$

Thus, the zeros can be computed analytically:

$$z_1(\eta) = \frac{-(T_3 + T_4) + \sqrt{(T_3 - T_4)^2 + 4T_3T_4\eta}}{2T_3T_4} \tag{6}$$

$$z_2(\eta) = \frac{-(T_3 + T_4) - \sqrt{(T_3 - T_4)^2 + 4T_3T_4\eta}}{2T_3T_4} \tag{7}$$

where

$$\eta = \frac{(1-\gamma_1)(1-\gamma_2)}{\gamma_1 \gamma_2} \in [0, \infty[$$

$$T_i = \frac{A_i}{a_i} \sqrt{\frac{2h_i^0}{g}}$$

The adjustable multivariable zero z_1 given by (6) can be set to a left or the right-half plane. The valves position adjustment determines if the system is minimum-phase or non-minimum phase. The results can be written in terms of the flow ratios γ_1 and γ_2 as shown in Table 1.

Table 1: Location of zeros on the linearized system as a function of the flow ratios γ_1 and γ_2

	z_1	z_2	System behavior
$1 < \gamma_1 + \gamma_2 \le 2$	negative	negative	Minimum phase
$\gamma_1 + \gamma_2 = 1$	zero	negative	Boundary
$0 < \gamma_1 + \gamma_2 \le 1$	positive	negative	Non minimum phase

The multivariable zeros being in the left or in right half-plane has a straightforward physical interpretation. Let q_i denote the flow through Pump i and assume that $q_1 = q_2 = q$. Then the sum of the flows to the upper tanks is $[2-(\gamma_1+\gamma_2)]q$ and the sum of the flows to the lower tanks is $(\gamma_1+\gamma_2)q$ which is greater than the flow to the upper tanks if $\gamma_1+\gamma_2>1$, i.e., if the system is in minimum phase, $\gamma_1+\gamma_2>1$. The flow in the lower tanks is smaller than the flow to the upper tanks if the system is non-minimum phase. In that case, $0<\gamma_1+\gamma_2\le 1$. It is easier to control y_1 with u_1 and y_2 with u_2, if most of the flows go directly to the lower tanks. The control problem is particularly hard if the total flow going to the left tanks (Tanks 1 and 3) is equal to the total flow going to the right tanks (Tanks 2 and 4). This corresponds to $\gamma_1+\gamma_2=1$ or a multivariable zero in the origin. Therefore, there is an immediate connection between the zero location of the model and physical intuition of controlling the quadruple-tank process [8]. The chosen operating points corresponds to the parameter values exposed in Table 2. The parameters values of the laboratory process are summarized in Table 3.

Table 2 : MPS and NMPS operating parameters of the quadruple-tank process

Parameters	Unit	MPS values	NMPS values
$h_1^0,\ h_2^0$	[cm]	12.4, 12.7	12.6 , 13
h_3^0, h_4^0	[cm]	1.8 , 1.4	4.8 , 4.9
$\vartheta_1^0,\ \vartheta_2^0$	[V]	3.00, 3.00	3.15 , 3.15
k_1, k_2	[cm^3/V.s]	3.33, 3.35	3.14 , 3.29
γ_1, γ_2	-	0.7 , 0.6	0.43 , 0.34

Table 3: Parameter values of the quadruple-tank process

Parameters	unit	Value
A_1, A_3	[cm^2]	28
A_2, A_4	[cm^2]	32
a_1, a_3	[cm^2]	0.071
a_2, a_4	[cm^2]	0.057
k_c	[V/cm]	0.5
g	[cm^2/s]	981

III. MIMO PID CONTROL VIA ILMI APPROACHES

Consider the linear time-invariant (LTI) MIMO system described by (4) where $x \in \Re^n$ is the state vector, $u \in \Re^m$ is the control vector, $y \in \Re^p$ is the output vector. The matrices A, B, C are with appropriate dimensions. The problem to be solved is to design the feedback gain matrices F_1, F_2, $F_3 \in \Re^{m\times p}$ such that system (4) is stabilized via a PID controller described by:

$$u = F_1 y(t) + F_2 \int_0^t y\,dt + F_3 \frac{dy}{dt} \tag{8}$$

where F_1, F_2 and F_3 are denoted by the proportional, time integral and time derivative gain matrices respectively. PID control synthesis can be easily reduced to a SOF stabilization problem. The main merit of this transformation is computing constant gains matrices hors line. The last problem is a difficult task but can be solved using Lyapunov theory and IILMI approaches. Consider then the augmented system:

$$\begin{aligned} \dot{z} &= \overline{A}z + \overline{B}u \\ \overline{y} &= \overline{C}z \\ u &= \overline{F}\overline{y} \end{aligned} \tag{9}$$

where:

$$\overline{A} = \begin{bmatrix} A & 0 \\ C & 0 \end{bmatrix}, \ \overline{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}, \ \overline{C} = \begin{bmatrix} C & 0 & CA \\ 0 & I & 0 \end{bmatrix} \ \overline{F} = \begin{bmatrix} \overline{F}_1 & \overline{F}_2 & \overline{F}_3 \end{bmatrix}$$

such that:

$$\overline{F} = \left[(I - F_3CB)^{-1}F_1 \quad (I - F_3CB)^{-1}F_2 \quad (I - F_3CB)^{-1}F_3 \right]$$

Let:

$$z = \left[z_1^T, z_2^T \right]^T, \ z_1 = x\,, \ z_2 = \int_0^t y\,dt$$

and:

$$\overline{y} = \left[\overline{y}_1 \quad \overline{y}_2 \quad \overline{y}_3 \right] = \overline{C}z$$

where:

$$\overline{y}_1 = y = Cz_1 = \left[C \quad 0 \right] z$$

$$\bar{y}_2 = \int_0^t y\,dt = z_2 = \begin{bmatrix} 0 & I \end{bmatrix} z$$

$$\bar{y}_3 = \frac{dy}{dt} = C\dot{x}(t) = \begin{bmatrix} CA & 0 \end{bmatrix} z$$

The original PID gain matrices can be recovered as:

$$F_3 = \bar{F}_3 (I + CB\bar{F}_3)^{-1} \tag{10}$$

$$F_2 = (I - F_3 CB)\bar{F}_2 \tag{11}$$

$$F_1 = (I - F_3 CB)\bar{F}_1 \tag{12}$$

a. Approach 1

Theorem 1 [24]:

The system (4) is stabilizable via SOF if and only if there exist P>0 and $\bar{F}$ satisfying the following matrix inequality:

$$\bar{A}^T P + P\bar{A} - P\bar{B}\bar{B}^T P + (\bar{B}^T P + \bar{F}\bar{C})(\bar{B}^T P + \bar{F}\bar{C}) < 0 \tag{13}$$

The negative sign of the term $-P\bar{B}\bar{B}^T P$ makes its solution very complicated. This approach introduced a new variable X to deal with the problem. Thus, we consider a matrix Ψ which depends on P affinely and satisfies:

$$\psi \leq P\bar{B}\bar{B}^T P \tag{14}$$

with $\psi = X^T \bar{B}\bar{B}^T P + P^T \bar{B}\bar{B}^T P - X^T \bar{B}\bar{B}^T X$ where X>0.

The system (13) can be stabilized if the following inequality has solution for (P, $\bar{F}$):

$$\bar{A}^T P + P\bar{A} - \psi + (\bar{B}^T P + \bar{F}\bar{C})^T (\bar{B}^T P + \bar{F}\bar{C}) < 0 \tag{15}$$

Using Schur complement, inequality (15) is equivalent to the following inequality:

$$\begin{bmatrix} \bar{A}^T P + P\bar{A} - \psi & (\bar{B}^T P + \bar{F}\bar{C})^T \\ (\bar{B}^T P + \bar{F}\bar{C}) & -I \end{bmatrix} < 0 \tag{16}$$

Once X is given, matrix inequality (16) can be solved very efficiently.

b. Approach 2

Theorem 2 [26]:

If

$$P(\overline{A}+\overline{B}\overline{F}\overline{C})+(\overline{A}+\overline{B}\overline{F}\overline{C})^{T}P-\alpha P<0 \tag{17}$$

holds, the closed-loop system matrix $\overline{A}+\overline{B}\overline{F}\overline{C}$ has its eigenvalues in the strict left-hand side of the line $\alpha/2$ in the complex s-plane. If a $\alpha \leq 0$ satisfying (17), the SOF stabilization problem is solved.

The key point of this approach is to divide the problem into two steps: the first one is to find an initial optimal P; the second step is to stabilize the system and thus compute the PID gains matrices. The ILMI algorithm corresponding to this approach is detailed in [26].

The two ILMI previous approaches are applied to design the feedback gain matrices of the MIMO PID controllers (10)-(12) of the quadruple-tank process. Sedumi and Yalmip toolbox [30] are used to solve the numerical problem. Simulation results are summarized for MPS and NMPS in Table 4 and Table 5, respectively.

Table 4: PID controllers for MPS

Approach	Feedback matrices	Poles
1	$F_1=\begin{bmatrix}3.1559 & 0.3655\\ 0.2936 & 3.1942\end{bmatrix}$ $F_2=\begin{bmatrix}0.2196 & 0.0838\\ 0.0708 & 0.1541\end{bmatrix}$ $F_3=\begin{bmatrix}24.3199 & 0.0646\\ 0.0208 & 32.2304\end{bmatrix}$	- 10.0175 - 6.1201 - 0.0166 - 0.0598 -0.0615±0.0149i
2	$F_1=\begin{bmatrix}0.1792 & 6.3362\\ 6.4028 & -2.1685\end{bmatrix}$ $F_2=\begin{bmatrix}1.3497 & 2.6548\\ 2.5015 & -2.3032\end{bmatrix}$ $F_3=\begin{bmatrix}30.4919 & 12.8140\\ 9.5034 & 18.8027\end{bmatrix}$	-0.2410±0.5686i -0.1671±0.3083i - 0.0585 - 0.0172

Table 5: PID controllers for NMPS

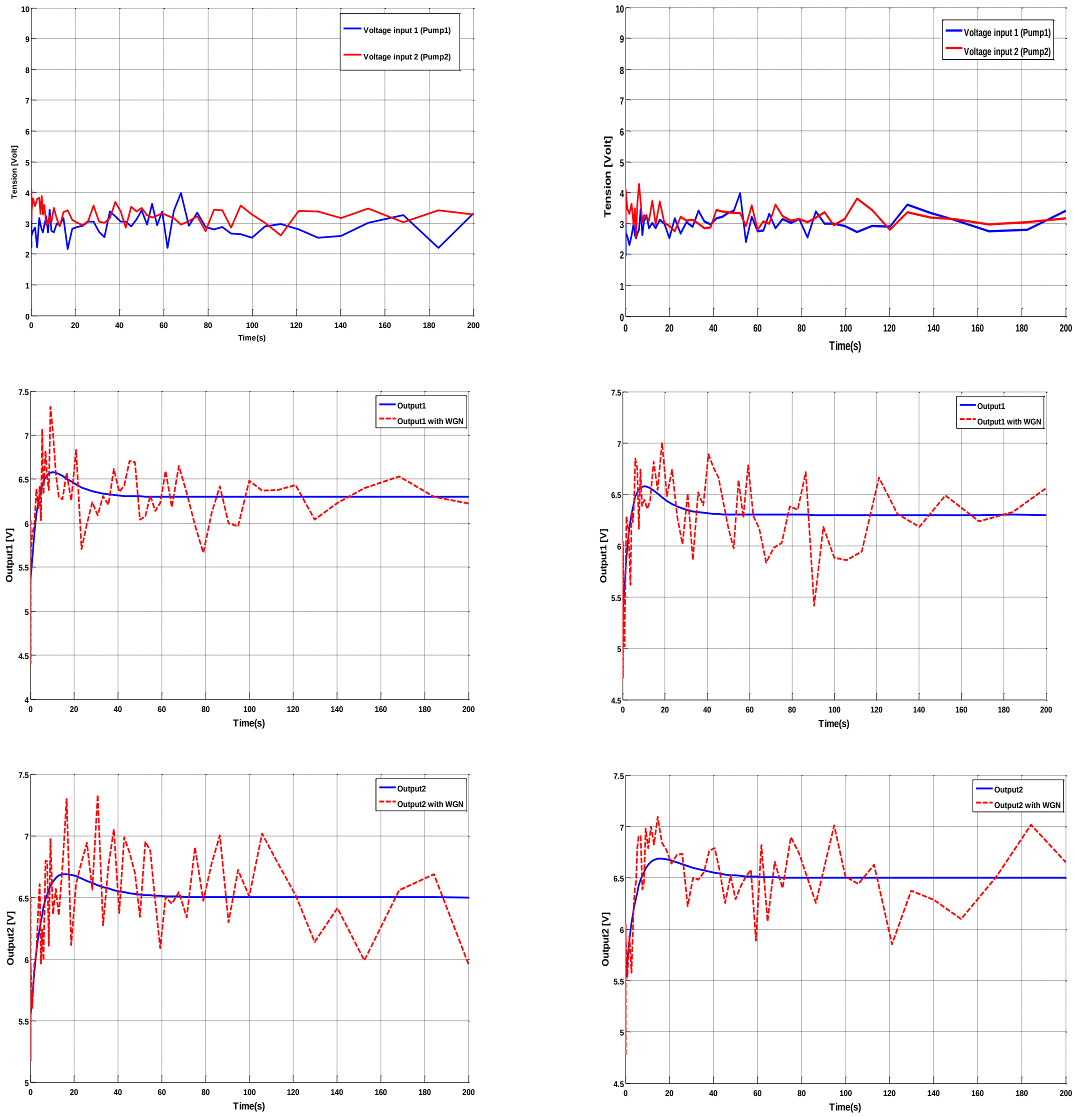
Approach	Feedback matrices	Poles
1	$F_1 = \begin{bmatrix} 0.1792 & 6.3362 \\ 6.4028 & -2.1685 \end{bmatrix}$ $F_2 = \begin{bmatrix} 1.3497 & 2.6548 \\ 2.5015 & -2.3032 \end{bmatrix}$ $F_3 = \begin{bmatrix} 30.4919 & 12.8140 \\ 9.5034 & 18.8027 \end{bmatrix}$	- 1.4903 - 0.3065 -0.0228 ± 0.0083i - 0.0130 - 0.0173
2	$F_1 = \begin{bmatrix} 2.4164 & 0.9490 \\ 1.8290 & 1.6698 \end{bmatrix}$ $F_2 = \begin{bmatrix} 0.2995 & 0.2334 \\ 0.2678 & 0.2447 \end{bmatrix}$ $F_3 = \begin{bmatrix} 41.2985 & 0.2831 \\ -0.0432 & 57.6159 \end{bmatrix}$	-1.5404±11.6372i -0.0003±0.0124i -0.0771±0.0158i

IV. PERFORMANCES ASSESSMENT

To elaborate an efficient performance assessment for the NMPS controlled via the PID designed by the ILMI approaches, four tests are carried out including white noise disturbance, set points changes, parametric uncertainties and time-delays influence. The four performance indices, ISE, IAE, ITAE and ITSE are also used to complete the performance analysis.

a. White Gaussian noise disturbance

To evaluate the effect of a noise disturbance, we have performed simulation results with White Gaussian noise with a variation of 0.5 acting on the outputs of the system. The influence of this noise is observed in Figure 2. It is shown that the closed loop system is very sensitive to noise.



(a) (b)

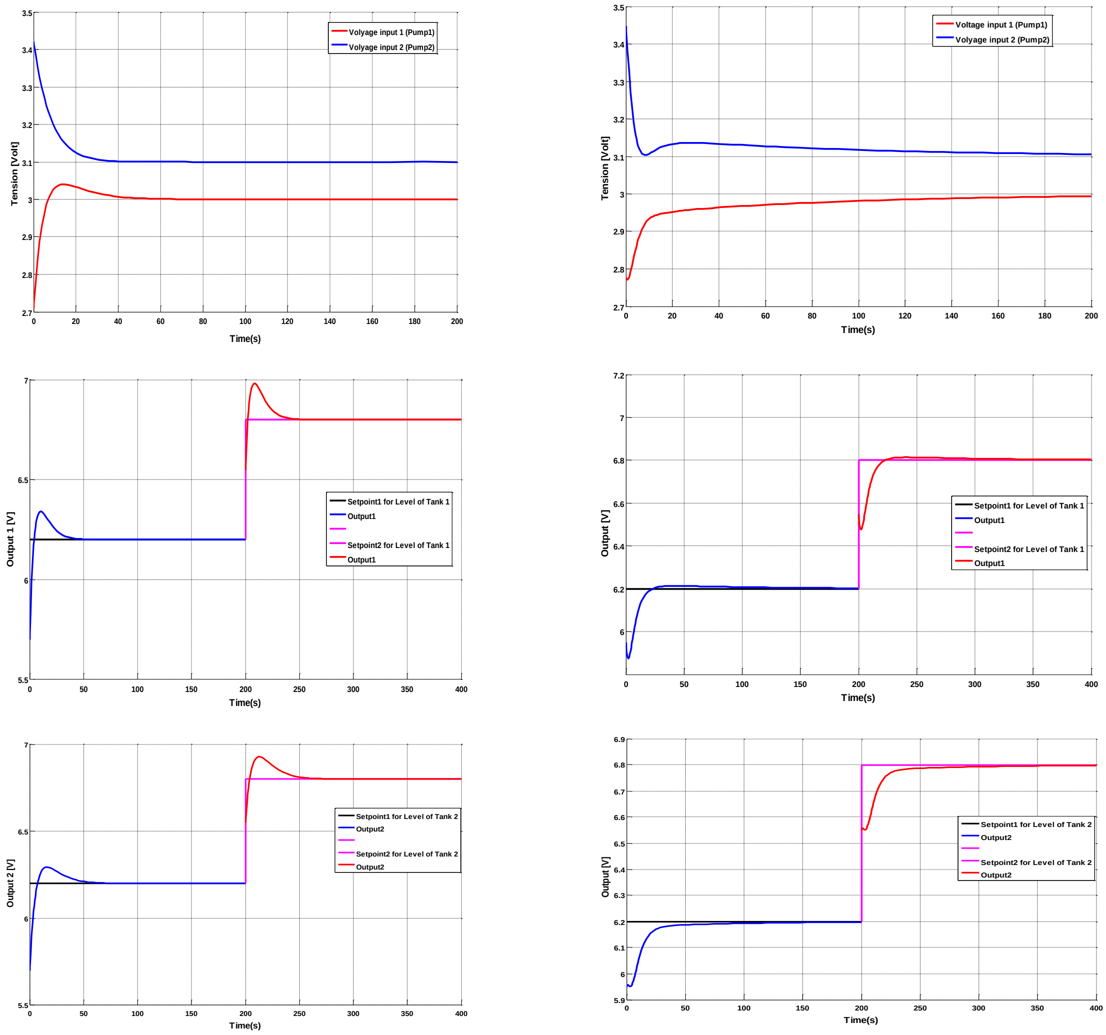
Figure 2. Test 1: (a) Approach 1- (b) Approach 2

b. Test 2: Set point change

In this section, the 4-tank process is subject to set point changes. The water level is regulated at 12.4 cm and 13.6cm, respectively. For the approach 2, better performances in terms of rise time and settling time are obtained, as shown by Table 6, compared to approach 1. Note that the system outputs converge to the set points with a small undershoot for the two approaches. Asymptotic stabilization with zero steady state error is achieved for the two approaches as shown in Figure 3.

Table 6: Quantitative comparison between approach 1 and approach 2

PID Controller via ILMI approach	**Parameters**	**Minimum phase System**		**Non-Minimum phase System**	
		Tank Level 1	Tank Level 2	Tank Level 1	Tank Level 2
Approach 1	Rise Time (s)	3	5	2.3	3.73
	Peak Over-shoot (%)	3.66	2.41	4.51	3.22
	Settling time (s)	27.1	32.76	22.31	27.72
Approach 2	Rise Time (s)	18.7	19.12	8.3	9.75
	Peak Over-shoot (%)	0	0	0.43	0
	Settling time (s)	24.82	25.6	13.29	17.7

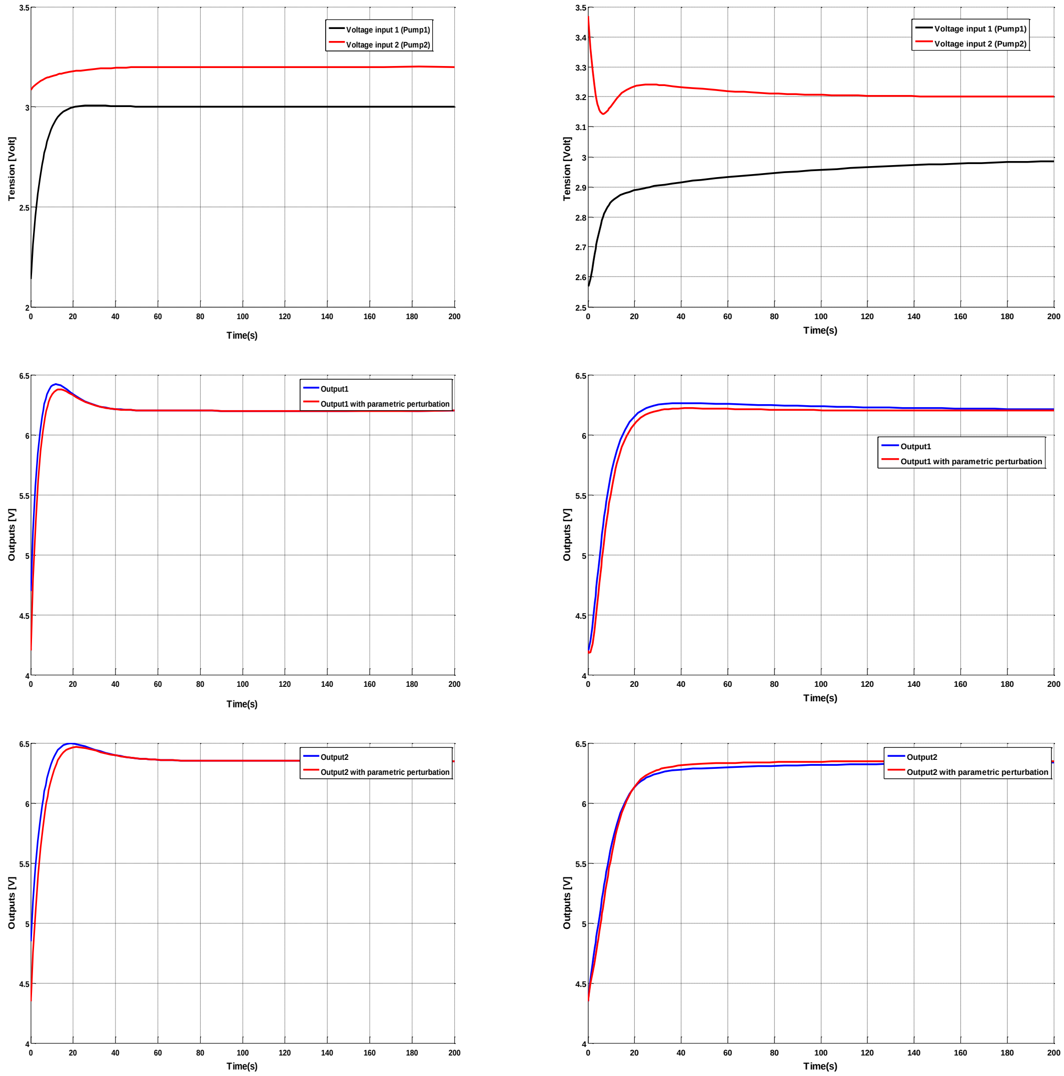


(a) (b)

Figure 3. Test 2. a) Approach 1 b) Approach 2

c. Test 3: Parametric uncertainties

In this section, we take into account uncertainties in the process parameters. Consider the case in which the flow ratios γ_1 and γ_2 are subject to an uncertainty of $\pm 20\%$. The influence of such uncertainties is shown by Figure 4. The values of the valve parameters γ_1 and γ_2 are very important as they determine if the system is MPS or NMPS. The two approaches are less sensitive to the parametric perturbations. Therefore, PID controllers designed via Approach 1 and Approach 2 are then robust.



(a) (b)

Figure 4. Test 3: (a) Approach 1- (b) Approach 2

d. Test 4: Multiple time-delays

In this section, we consider that the NMPS is subject to multiple delays. To have a more realistic description of the quadruple-tank process, we take into account transport delays between valves and tanks. Consider the following linear system with time-delayed state and control [31]:

$$\dot{x}(t) = A_0 x(t) + A_1 x(t-\tau_1) + B_0 u(t-\tau_2) + B_1 u(t-\tau_3)$$

$$y(t) = Cx(t) \tag{18}$$

where $x(t) \in \Re^n$, $u(t) \in \Re^m$, $y(t) \in \Re^p$ are the state vector, the control vector and the output vector respectively , $A_0 \in \Re^{n\times n}$, $A_1 \in \Re^{n\times n}$, $B_0 \in \Re^{n\times m}$ and $B_1 \in \Re^{n\times m}$ are known constant matrices with appropriate dimensions and τ_1, τ_2 and τ_3 are known and constant time-delays.

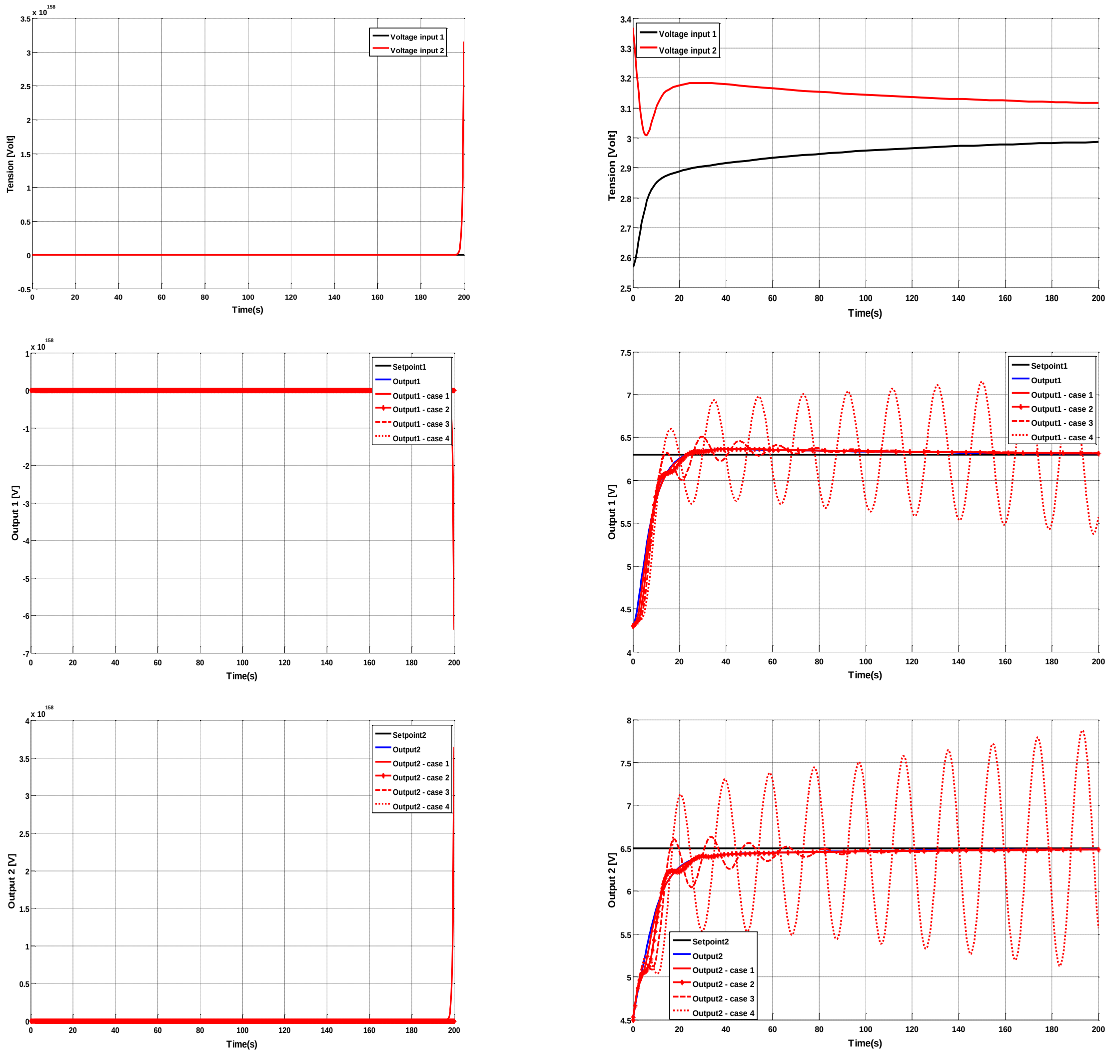
To evaluate the time-delays effect , 4 case studies corresponding to different transport delays between tanks and valves are considered : $\tau_1 = 1s$, $\tau_2 = 1s$ and $\tau_3 = 1s$ for case 1, $\tau_1 = 2s$, $\tau_2 = 2s$ and $\tau_3 = 4s$ for case 2, $\tau_1 = 3s$, $\tau_2 = 3s$ and $\tau_3 = 3s$ for case 3 and $\tau_1 = 4s$, $\tau_2 = 4s$ and $\tau_3 = 4s$ for case 4.

Figure 8 shows the input and the output variables for the two PID approaches. Approach 1 seems to be very sensitive to multiple time delays since unstable dynamics appears. However, approach 2 appears to be more robust for small delays.

e. Performance criteria

The performance of the control system is usually evaluated based on its transient response behavior. This response is the reaction, when subjecting a control system, to inputs or disturbances. The characteristics of the desired performance are usually specified in terms of time domain quantities. Commonly, unit step responses are used in the evaluation of the control system performance due to their ease of generation. In the design of an efficient PID controller, the objective is to improve the unit step response by minimizing the domain parameters such as the maximum over-shoot, the rise time, the settling time and the steady state error [32]. The most commonly used functions are the time domain integral error performance criteria which are based on calculating the error signal between the system output and the input reference signal [33]. Generally, the error signal is expressed as:

$$e(t) = y_c(t) - y(t) \tag{19}$$



(a) (b)

Figure 5. Test 4- (a) Approach 1-(b) Approach 2

For this signal error, the common integral performance function types are integral of absolute error (IAE), integral of time multiplied by absolute error (ITAE), integral of squared error (ISE), integral of time multiplied by squared error (ITSE), and integral of squared time multiplied by squared error (ISTE) [34]. The indices that involve time (ITAE and ITSE) evaluate the error occurring late in the response because t is small in the early stages. Both indices, IAE and ISE, intend to evaluate the errors at the early stages of the response (during the transient) regardless of the error sign, and finally the ISE index evaluates higher emphasis on large errors. Taking into account the methodology using performance evaluation criteria and the 4-tank process specifications, we introduce the following errors as:

$$e_1(t) = y_{c1}(t) - y_1(t) \tag{20}$$

$$e_2(t) = y_{c2}(t) - y_2(t) \tag{21}$$

The criteria IAE_1, ISE_1, $ITAE_1$, $ITSE_1$, IAE_2, ISE_2, $ITAE_2$, $ITSE_2$, and their formulas are as follows :

$$IAE_1 = \int_0^{t_{ss}} |e_1(t)|dt \tag{22}$$

$$ISE_1 = \int_0^{t_{ss}} e_1^2(t)dt \tag{23}$$

$$ITAE_1 = \int_0^{t_{ss}} t|e_1(t)|dt \tag{24}$$

$$ITSE_1 = \int_0^{t_{ss}} te_1^2(t)dt \tag{25}$$

$$IAE_2 = \int_0^{t_{ss}} |e_2(t)|dt \tag{26}$$

$$ISE_2 = \int_0^{t_{ss}} e_2^2(t)dt \tag{27}$$

$$ITAE_2 = \int_0^{t_{ss}} t|e_2(t)|dt \tag{28}$$

$$ITSE_2 = \int_0^{t_{ss}} te_2^2(t)dt \tag{29}$$

Assume that the two references of the quadruple tank process are subject to a unitary step $y_{c1}(t) = 1V$ and $y_{c2}(t) = 1V$ over $0 \le t \le 100s$. Note that t_{ss} is taken as 100s in all simulation results. The computed performance indexes (22)-(29) using the two ILMI approaches are shown by Table 7. The comparison over $0 \le t \le 100s$ shows that the ISE index gives the best value more heavily the ISE index for the Approach 1 whereas only the ISE index is the best for approach 2. We have also noted the important value of the ITAE index for the two approaches.

Table 7: Closed-loop performance Indexes

PID Controller via ILMI approach	Error type	**Performances indexes**			
		IAE	**ISE**	**ITAE**	**ITSE**
Approach 1	e_1	4.0478	2.6713	30.0264	1.0055
	e_2	4.7743	2.9162	32.6258	2.3794
Approach 2	e_1	10.684	4.4182	170.1213	34.1761
	e_2	8.6124	3.4360	129.8044	19.8623

f. Comparison analysis

After carrying out this comparative analysis, one can deduce the following remarks: On the one hand, PID controller using Approach 1 provide good results in terms of rise time and appears to be robust to parametric uncertainties. However, it has a long setting time and fails to deal with multiple time-delays changes. On the other hand, PID controller using Approach 2 provide good results in terms of settling time and minimized overshoot and tends to be robust to parametric uncertainties assigned to the valves and multiple time-delays changes. A comparison established between different performances indices proves that Approach 1 is better than Approach 2 in terms of error dynamics precision. Thus, a suitable choice of PID parameters auto-tuning using ILMI approach can lead to a PID controller with higher performances in terms of stabilization, set-point tracking, disturbances rejection, robustness to parametric uncertainties... This study shows that these objectives are mainly reached by Approach 2 and partially by Approach 1.

V. CONCLUSION

In this paper, two ILMI approaches for MIMO PID controllers are revised and compared using the quadruple tank process as a benchmark of non-minimum phase system. Simulation results show clearly that Approach 2, described in [26], achieves globally the best compromise between robustness and performance tests compared to Approach 1, described in [24], especially with respect to the multiple time-delays influence test, set-point changes test and performance indices.

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