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GTOC 9: Results from the Military College of South Carolina and Des Moines Area Community College (team Citadel-DMACC)

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Abstract. This report presents the results, methodology, and lessons learned for the Citadel-DMACC group concerning the 9th Global Trajectory Optimization Competition. As first-time entrants, the focus was on developing a fundamental understanding of the project scope and problem approach, while utilizing this competition format as an opportunity for undergraduate research, and in this instance, specifically for a first-year engineering student. The optimization strategy that was utilized focused on the debris selection process. Initial debris target selection was centered on possible orbital intersections between an initial debris orbit and the instantaneous orbital projection generated from another target debris at a given time step. This report will discuss two target selection processes and their applicability to the problem. Lessons learned from how to approach the problem are presented here along with an analysis on the reasons behind the team's failure to provide a successful submission.

1 Introduction

The Global Trajectory Optimization Competition (GTOC) is an international competition open to industry, governmental agencies, colleges, universities, and even savvy space mechanics enthusiasts. The goal is to find unique and creative solutions for solving complex orbital mechanics and trajectory optimization problems wherein the solutions can be applied to real-world missions [1]. The ninth iteration of the competition was developed and organized by the European Space Agency. The competition problem statement outlined the need to deorbit 123 pieces of orbital debris in order to halt the Kessler effect, brought on by the explosion of a Sun-synchronous satellite. The Kessler effect is the idea that a collision in space will generate impact debris, which will then cause more collisions with more debris, and so on until it becomes impractical to send spacecraft into or through a given orbital altitude [2]. In the given problem, the challenge presented is to find efficient procedures for navigating a spacecraft between debris orbits in order to remove this debris while minimizing cost.

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it was the team's first time competing in GTOC. As part of this first-time competition entry, a first-year undergraduate engineering student was brought onto the team to promote research, develop student interest in orbital mechanics, and reinforce concepts learned in first-year coursework. During the month-long window to generate a solution, the team developed two methods for target selection, which are described in detail herein. As the problem statement laid out, the spacecraft would start at a selected piece of debris (target) and then maneuver to successive targets in order to ultimately clear all of the debris, while minimizing fuel usage. As first-time entrants, this team's initial focus was to reduce the complexity of the problem and focus on finding a pair of orbits that would come close enough to an intersection to allow for a direct orbital transfer. From this point, a corresponding set of orbital maneuvers was calculated that would move the spacecraft from the initial orbit to the target orbit and rendezvous with the second piece of debris, producing a valid submission for this competition. While this target selection process was successful, issues ultimately arose in the precision of position and velocity calculations, which could not be overcome in the competition time frame. This report will therefore focus on the successful orbit selection process and transfer maneuvers utilized, followed by discussion of lessons learned from the competition.

2 Problem Approach

For the competition, the team utilized single-core processor operations on various Windows-based platforms, using MATLAB as the software of choice. The choice of hardware focus allowed code to be written in tandem across a variety of desktop and laptop computers owned by the various team members. Utilizing MATLAB with single-core processing allowed for reinforcement of concepts learned in the first year Engineering Computer Applications course at The Citadel, while directing focus towards learning concepts of orbital mechanics. Future entries will explore how to leverage the parallel processing capabilities of the software to expedite the trajectory optimization process, while expanding beyond the scope of existing course concepts.

The method utilized for optimizing target selection was to keep the process relatively simple and focus on identifying orbital intersections solely between two targets. Due to the wide variety of orbital parameters for the 123 pieces of space debris in this competition, not all of the orbits come close enough to each other to constitute a direct intersection, defined as having a minimum distance of less than 100 meters. Once an intersecting pair of orbits was found, the spacecraft transferred directly from the initial target orbit to the final target orbit at the intersection point. Because the starting orbit of the spacecraft was chosen, the spacecraft orbit was assumed to initially match that of the debris on the first orbit so that the deorbit package could be applied to this first target before the first orbital transfer maneuver was made. The starting orbit was determined based on the first two intersecting orbits that were found with a reasonable for ΔV for transferring orbits. This initial maneuver would enable the spacecraft and the second target debris to have matching eccentricity, inclination, right ascension of the ascending node (Ω), and argument of perigee (ω) to progress from. Upon transferring to the final target orbit, the spacecraft would then have to undergo a series of additional maneuvers, detailed later, in order to physically rendezvous at the final target location.

Time-sensitive J2 perturbations on both the spacecraft and debris, due to the Earth's oblateness, made for a challenging approach to identifying possible initial and final targets. For each debris orbit, it was reasoned that at every instant in time, it is being perturbed by the Earth and therefore its Ω and ω are continually changing. This means that its orbital plane, and consequently its distance to any other orbit, are constantly varying. Therefore, the methodology began with selecting a starting target and calculating its trajectory for a given window of time. This would also constitute the starting position and trajectory for the spacecraft. For a given final target, a complete orbital path was calculated for a given time. When incorporating the J2 perturbations, the Ω and ω vary continually, requiring that the complete orbital path be recomputed each second. Possible intersections between this time-dependent, final-target, orbital path and the perturbed starting orbit trajectory were calculated and repeated until an orbital intersection was found.

Figure 1 is a highly conceptualized representation of the approach taken. In Figure 1.a, the black and blue curves represent the perturbed trajectories of the final target and the spacecraft/initial target, respectively. For each time step, the final target was considered to have an instantaneous orbital projection of possible intersection points for the spacecraft's trajectory, shown as a dashed red line. Figure 1.b, shows this again at a later time. It is not until the minimum cartesian distance between

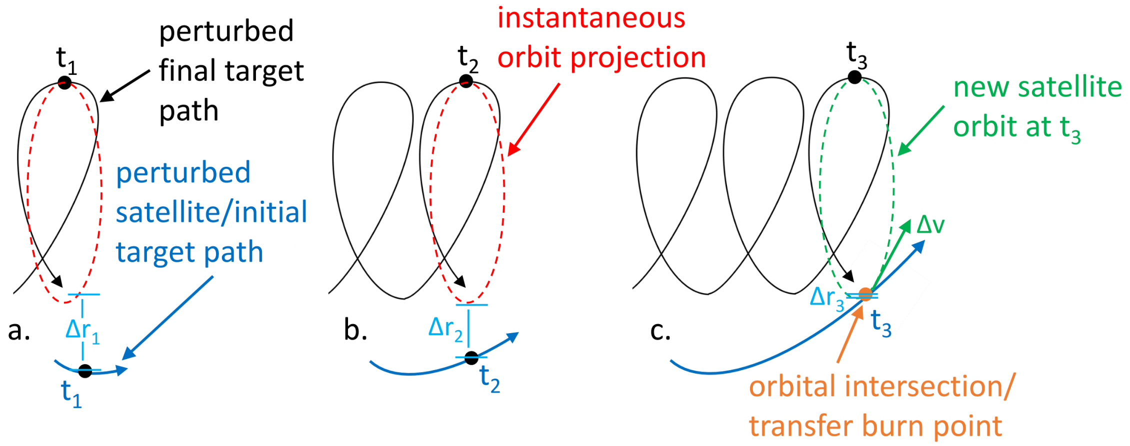

FIGURE 1. *Conceptualized interpretation of a perturbed orbital intersection between a spacecraft trajectory and the instantaneous orbit projection of the target debris*

the spacecraft and the final target's instantaneous orbit falls below 100 m, the threshold set in the competition's problem statement, that the orbits are considered to be intersecting (Figure 1.c).

The first challenge began in trying to determine if two given debris orbits could be considered intersecting. Two separate approaches were explored to determine the minimum distance between the initial and final target orbits before checking if that minimum distance fell below the intersection threshold. Figure 2 illustrates the first approach. In this method, the two orbital trajectories were discretized coarsely (approximately 1000 points each) and an initial guess was made as to which point on each orbit corresponded to the minimum distance. This guess on each of the two orbits is depicted in Figure 2.a as index m on the spacecraft path and index n on the target path. From this initial guess, the next point in each direction was considered (i.e. m-1 and m+1 on the spacecraft path) for each orbit. The cartesian distance between each of these 3 points on the spacecraft path and each of the three points on the target path was then calculated, forming a 3x3 local distance matrix, shown in Figure 2.b. The minimum of this 3x3 matrix was then found and the corresponding index was taken as the new initial guess. This process was repeated iteratively until the minimum of the matrix was found to occur at the center, resulting in a local minimum. Once the overall minimum distance between orbits was reached, the density of points on the orbits was refined by spreading 1000 points over what was previously 1 index on the course grid. This process was repeated until the minimum distance did not vary down to 16 significant figures (the highest default MATLAB precision). At that point, if the minimum distance was less than 100 m, the orbits were determined to have intersected and the analysis continued by calculating the necessary orbital maneuvers. If the minimum distance did not fall below the intersection threshold, two new target orbits were identified and the process was repeated. One drawback of this method is that it is possible to get a local minimum distance versus a global minimum and the particular local minimum is very dependent on the initial guess. Another drawback of this method is that even without incorporating J2 perturbations, the distance two orbits could theoretically have as many as four local minima. Once J2 perturbations are introduced, the minimum distance between orbits varies with each precession and it becomes necessary to analyze multiple orbital periods, each with their own set of local minima.

Incorporating J2 perturbations made it necessary to utilize a different approach for finding orbital intersections. As previously described, this approach involved generating a complete instantaneous orbit for the target orbit at each instant in time in order to find where the perturbed spacecraft trajectory intersected the target orbit as it moved along its own perturbed trajectory. Figure 3 illustrates the process to determine this intersection. For a given time, the Cartesian coordinates for the spacecraft and target debris were calculated based

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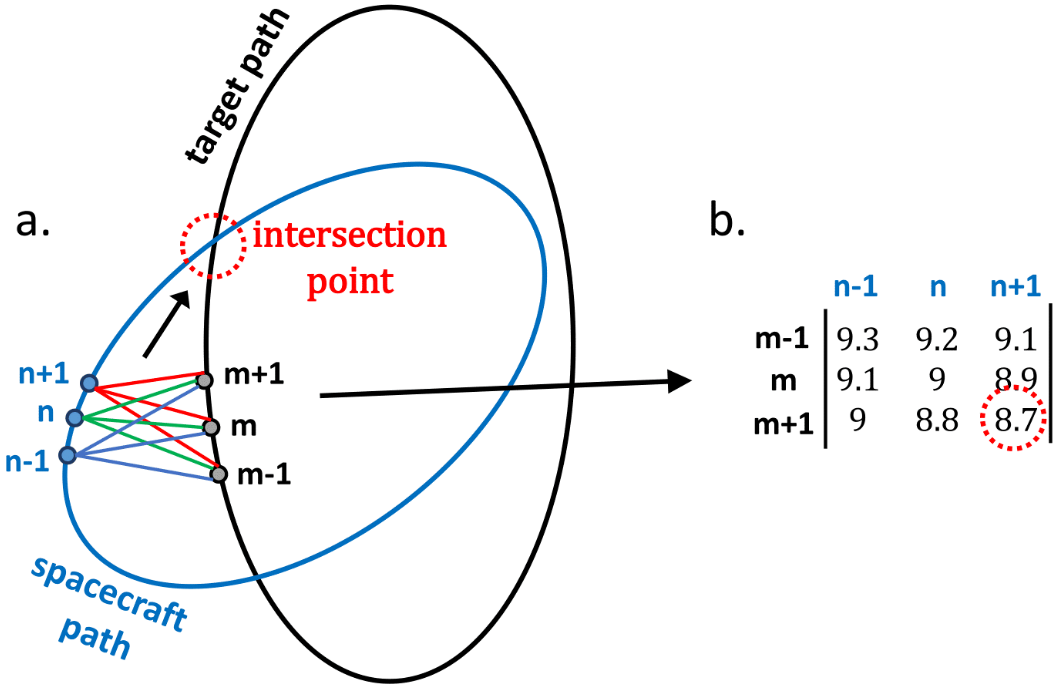

FIGURE 2. *Determining the orbital intersection point of two non-perturbed orbits*

on the given orbital parameters for each piece of debris. Using this target debris position as well as the instantaneous Ω and ω for that orbit, an instantaneous orbit was calculated and discretized into 1000 points. The difference between the spacecraft position at that time and each point on the instantaneous orbit was determined and is shown as a row in Figure 3.b. Each row of this matrix corresponds to the distance from the instantaneous location of the spacecraft at a given time to each point on the instantaneous target debris orbit. This distance calculation was performed for 20 orbits of the spacecraft, each split into 1000 discrete cartesian locations. The resulting matrix formed using this approach had a size of 20000 x 1000 elements. While this is considerably larger in size than the localized distance matrix described in the first approach, the minimum of this matrix represents a global minimum over this 20 orbit sample. This global minimum is a necessity for determining the minimum distance between perturbed orbits given the cyclical nature of the distance between these orbits. If a minimum point for this specified time window was found to fall below 100 m, then an intersection was declared, the mesh was refined at that intersection point with another 1000 points, similar to the previous method, and the process was completed again until a final minimum distance was determined. Figure 4 shows the minimum distance between the path of target 16 and the instantaneous orbit of target 37 over a given 20 orbit sample. It is obvious that a search for a local minimum could result in any number of incorrect choices. The global minimum over this range is identified in the figure and can be seen to cross the 100 m threshold for intersection.

If no intersection was found between the spacecraft and target orbits over this 20 spacecraft period, the analysis would move to the next set of targets and the process was repeated. If an intersection was determined to occur, then a ΔV calculation was conducted to see how fuel intensive the maneuver would be to move from one orbit to the next. It was found that compensating for apse line rotations was the major factor in fuel consumption. Upon completing this analysis for the given targets, it was found that the orbits of targets 16 and 37 intersected at $t = 23472.33$ days and that the fuel consumption for this maneuver (Table 1) allowed for enough fuel for the spacecraft to reach target 37 after

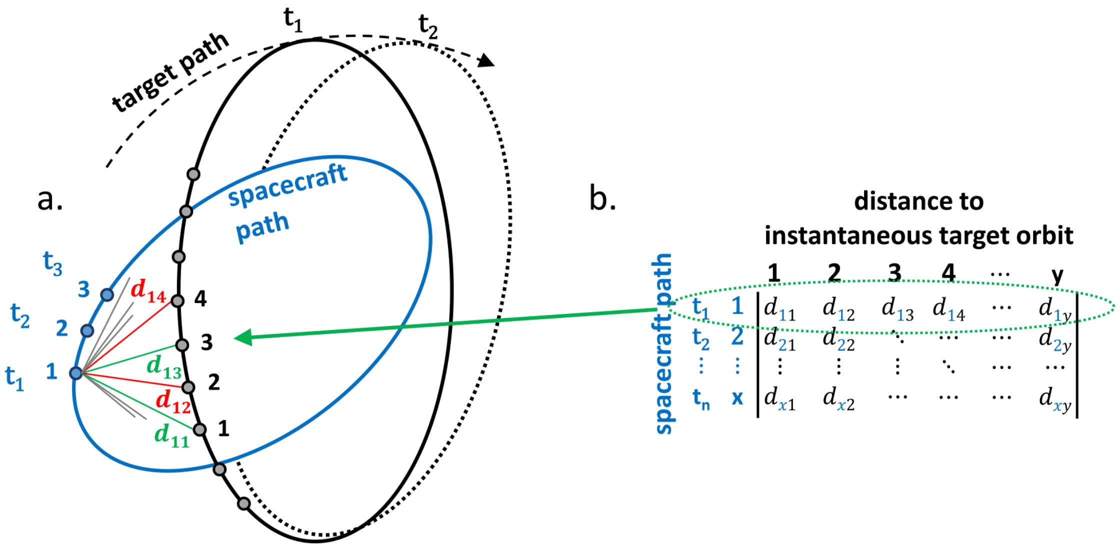

FIGURE 3. *Determining distance between the time-sensitive, perturbed spacecraft path and the time-sensitive instantaneous-orbit projection from the perturbed target path*

TABLE 1. *Mission Submission for Transiting from Target 16 to 37*

t (mjd2000)	x (km)	y (km)	z (km)	v_x (m/s)	v_y (m/s)	v_z (m/s)	m (kg)	ΔV_x (m/s)	ΔV_y (m/s)	ΔV_z (m/s)	event id
23467	-4149.3	-5770.5	-606.9	-535.5	1098.9	-7351.1	4560	0	0	0	16
23472.33	2680.5	5652.9	-3270.3	-452.8	-3592	-6580.1	2236	2104.3	-1053.4	31.84	16
23472.36	-2671.6	-5640.8	3267.3	445.93	3543.3	6481.7	2153	-7.533	-59.95	-109.7	-1
23473.33	-2597.9	-5857.6	2927.5	245.84	3275.1	6771.2	2074	4.078	54.488	112.7	37
23478.33	8516.9	3161.1	6306.3	2253.3	6411.9	-3208.8	2044	0	0	0	37

two additional phasing burns.

Given the spirit of the competition, optimizing the orbital maneuvers that the spacecraft would undergo when moving from target to target is of clear importance. However, as will be discussed in further detail below, errors in calculating spacecraft position and velocity propagated with respect to time and these inaccuracies could not be overcome in time for a successful rendezvous submission. This prevented full vetting and optimization for the intended maneuvering procedure, when transiting from one target to another, so these maneuvers will only be mentioned briefly in this report. Upon selecting targets, as detailed above, and making the commensurate burn to move the spacecraft onto the target's orbit, two more burns were reasoned to be needed for moving the spacecraft into a rendezvous position with the target debris. A phasing maneuver was initiated with the final phasing maneuver occurring 15 orbits after the initial burn was conducted [3]. The initial and final burns were performed at the target's perigee point. This procedure consisted of 3 total burns for moving between two targets, consisting of: 1) RAAN/inclination/AP burn from starting target to final target orbit burn, 2/3) initial and final phasing burns. For the targets selected (transiting from target 16 to 37), these maneuvers resulted in a total ΔV of 2.6145 km/s and a consumption of 2522.0 kg of fuel.

3 Lessons Learned

Despite the inability to overcome errors in position and velocity calculations in order to submit a successful solution, a variety of lessons were learned throughout this process and these lessons will inform the team's approach to future competitions. First, during the solu-

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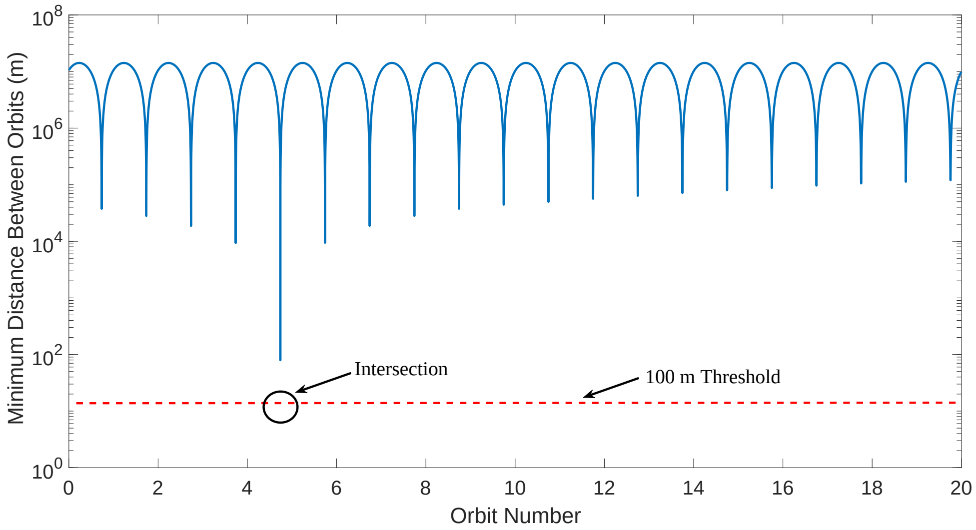

FIGURE 4. *Minimum orbital distances between debris 16 and 37, as a function of debris 16's orbit number, with an indicated intersection point between orbits 4 and 5.*

tion process, the team was able to work through most of the issues for making simple maneuvers between targets, but in the end, propagating errors with the position/velocity vectors were found. The code used was repeatedly refined and re-compared to the charts provided in the problem statement, but the differences could not be overcome in the allotted timeframe and therefore, no solution was submitted successfully. At best, the team could match 6-7 significant digits with the numbers provided in the problem statement which had 17 significant figures. When submitting possible solutions, it was found that the propagating error resulted in position differences of 300 m between the calculated and actual positions. After reviewing procedures graciously posted by one of the other competitors, it was found that the issue with the calculations was with the level of precision that was being rendered through MATLAB when solving the set of ordinary differential equations that describe the motion of the spacecraft [4]. For future competitions and research, understanding how to effectively use the precision options for MATLAB's ordinary differential equation solvers is paramount to obtaining accurate solutions. The second lesson learned stems from an understanding of the equations provided in the problem statement for determining the Cartesian vectors of the spacecraft/target position and velocity. It was initially incorrectly assumed that these relationships provided the same results as if positions were calculated in the geocentric equatorial frame. This error was discovered at roughly the half-way point of the competition and meant a near total rework of the code produced. The last lesson to be discussed stemmed from a passage in the problem statement, where it is mentioned that the spacecraft feels the full J2 perturbations once it leaves a given target. From this statement, it was incorrectly assumed that the transit time of the spacecraft was the only time that perturbations were necessary to consider. It was found that the intent of the statement was quite the opposite and meant that perturbations had to be considered at all times, with the effects needing to be calculated separately when the spacecraft was away from any target. In future competitions, these lessons will be taken into consideration and will form a basis from which to implement more complex concepts into the approach to a solution.

4 Conclusions

In all, this was a challenging competition that allowed for creativity and flexibility in solving a complex, open-ended problem. The competition prompted many interesting discussions and brain-storing sessions, as well as a number of teachable moments, for both the student and faculty members. Had the team been able to overcome the precision errors that were encountered, the target selection processes detailed herein would have been expanded to transiting between multiple targets versus the bulk of our endeavors being solely focused on moving between two. As an opportunity to promote undergraduate research, this competition has been a great success, by reinforcing and giving engineering context to concepts delivered in first-year coursework. This experience was a great opportunity for fostering the student's academic interests and he is excited about contributing to future GTOC iterations.

References

[1] D. Izzo and M. Märtens. The Kessler Run: On the Design of the GTOC9 Challenge. *Acta Futura*, 11:11–24, 2018.

[2] D. Kessler and B. Cour-Palais. Collision frequency of artificial satellites: The creation of a debris belt. *Journal of Geophysical Research: Space Physics*, 83:2637–2646, 1978.

[3] H. Curtis. *Orbital Mechanics for Engineering Students*. Elsevier Butterworth Heinemann, 2013.

[4] M. Hallmann. Sharing code - position and velocity vectors. `https://kelvins.esa.int/gtoc9-kessler-run/discussion/368/`, 2017. Online; accessed 4-May-2017.

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