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ON PSEUDO COMPATIBLE P-FUZZY SOFT RELATIONS

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Abstract:

We introduce the notion of pseudo compatible P-fuzzy soft relations of a sub group, cossets of a group, strongest fuzzy soft relations and how they are related with fuzzy soft normal subgroups.

Key Words: Soft Set, Null Soft Set, Injection Function, Fuzzy Set, P-Fuzzy Soft Middle Cosset, Pseudo Fuzzy Cosset**,** Strongest Fuzzy Relation & Compatible Fuzzy Soft Set.

Introduction:

The concept of fuzzy sets was first introduced by Zadeh [23]. Rosenfeld [16] used this concept to formulate the notion of fuzzy groups. Since then, many other fuzzy algebraic concepts based on the Rosenfeld's fuzzy groups were developed. Anthony and Sherwood [1] redefined fuzzy groups in terms of t- norm which is replaced the min operations of Rosenfeld's definition. Some properties of these redefined fuzzy groups, which we call t- fuzzy groups, have been developed by Sherwood [18], sessa [17], sidky and misherf (19). However the definition of t- fuzzy groups seems to be too general. Soft set theory was introduced in 1999 by Molodtsov [15] for dealing with uncertainties and it has gone through remarkably rapid strides in the mean of algebraic structures as in [1, 2, 11, 14, 15, 16, 18, 21, 23]. Moreover, Atagun and Sezgin [5] defined the concepts of soft sub rings and ideals of a ring, soft subfields of a field and soft sub modules of a module and studied their related properties with respect to soft set operations. Operations of soft sets have been studied by some authors, too. Ali et al. [4] introduced several operations of soft sets and Sezgin and Atagun [21] studied on soft set operations as well. Furthermore, soft set relations and functions [6] and soft mappings [14] with many related concepts were discussed. Here we introduce the notion of pseudo compatible P-fuzzy soft relations of a subgroup, cossets of a group, strongest fuzzy soft relations and how they are related with fuzzy soft normal subgroups.

Section-2 Preliminaries:

In this section, we recall basic definitions of soft set theory that are useful for subsequent sections. For more detail see the papers [[11], [15],] Throughout the paper, U refers to an initial universe, E is a set of parameters and P(U) is the power set of U. $\subset$ and $\supset$ stand for proper subset and super set, respectively.

Definition 2.1 [22]: A pair (F, A) is called a soft set over U, where F is a mapping given by F: A→P(U).

In other words, a soft set over U is a parameterized family of subsets of the universe U. Note that a soft set (F, A) can be denoted by F_A. In this case, when we define more than one soft set in some subsets A, B, C of parameters E, the soft sets will be denoted by F_A, F_B, F_C, respectively. On the other case, when we define more than one soft set in a subset A of the set of parameters E, the soft sets will be denoted by F_A,G_A, H_A, respectively. For more details, we refer to [11, 17, 18, 26, 29, 7]. Note that the set of all soft sets over U will be denoted by S(U).

Definition 2.2 [12]: Let $\lambda, \mu \in$S(U). Then

- (i) If λ(e) = $\emptyset$ for all e $\in$ E, λ is said to be a null soft set, denoted by $\emptyset$.
- (ii) If λ(e) = U for all e $\in$ E, λ is said to be an absolute soft set, denoted by U.
- (iii) λ is a soft subset of μ , denoted $\lambda \subseteq \mu$, if λ(e) $\subseteq$ μ(e) for all e$\in$ E.
- (iv) Soft union of λ and μ, denoted by $\lambda \cup \mu$, is a soft set over U and defined by $\lambda \cup \mu$: E $\rightarrow$ P(U) such that ($\lambda \cup \mu$)(e) = λ(e) $\cup$ μ(e) for all e $\in$ E.
- (v) $\lambda = \mu$, if $\lambda \subseteq \mu$ and $\lambda \supseteq \mu$.
- (vi) Soft intersection of λ and μ, denoted by $\lambda \cap \mu$, is a soft set over U and defined by $\lambda \cap \mu$: E $\rightarrow$ P(U) such that ($\lambda \cap \mu$)(e) = λ(e) $\cap$ μ(e) for all e $\in$ E.
- (vii) Soft complement of λ is denoted by λ^c and defined by λ^c : E $\rightarrow$ P(U) such that λ^c(e) = $U/\lambda(e)$ for all e $\in$ E.

Definition 2.3 [12]: Let E be a parameter set, S $\subset$ E and λ: S $\rightarrow$ E be an injection function. Then S$\cup$ λ(s) is called extended parameter set of S and denoted by ξ_S. If S=E, then extended parameter set of S will be denoted by ξ.

Definition 2.4 [6]: The relative complement of the soft set F_A over U is denoted by $F^r{}_A$, where $F^r{}_A : A \to P(U)$ is a mapping given as $F^r{}_A(a) = U \setminus F_A(a)$, for all $a \in A$.

Definition 2.5 [6]: Let F_A and G_B be two soft sets over U such that $A \cap B \neq \emptyset$,. The restricted intersection of F_A and G_B is denoted by $F_A \Cap G_B$, and is defined as $F_A \Cap G_B = (H,C)$, where $C = A \cap B$ and for all $c \in C$, $H(c) = F(c) \cap G(c)$.

Definition 2.6 [6]: Let F_A and G_B be two soft sets over U such that $A \cap B \neq \emptyset$,. The restricted union of F_A and G_B is denoted by $F_A \cup_R G_B$, and is defined as $F_A \cup_R G_B = (H, C)$, where $C = A \cap B$ and for all $c \in C$, $H(c) = F(c) \cup G(c)$.

Definition 2.7 [12]: Let F_A and G_B be soft sets over the common universe U and ψ be a function from A to B. Then we can define the soft set $\psi(F_A)$ over U, where $\psi(F_A) : B \to P(U)$ is a set valued function defined by $\psi(F_A)(b) = \bigcup\{F(a) \mid a \in A \text{ and } \psi(a) = b\}$, if $\psi^{-1}(b) \neq \emptyset$, $= 0$ otherwise for all $b \in B$. Here, $\psi(F_A)$ is called the soft image of F_A under ψ. Moreover we can define a soft set $\psi^{-1}(G_B)$ over U, where $\psi^{-1}(G_B) : A \to P(U)$ is a set-valued function defined by $\psi^{-1}(G_B)(a) = G(\psi(a))$ for all $a \in A$. Then, $\psi^{-1}(G_B)$ is called the soft pre image (or inverse image) of G_B under ψ.

Definition 2.8 [13]: Let F_A and G_B be soft sets over the common universe U and ψ be a function from A to B. Then we can define the soft set $\psi^\star(F_A)$ over U, where $\psi^\star(F_A) : B \to P(U)$ is a set-valued function defined by $\psi^\star(F_A)(b) = \bigcap\{F(a) \mid a \in A \text{ and } \psi(a) = b\}$, if $\psi^{-1}(b) \neq \emptyset$, $=0$ otherwise for all $b \in B$. Here, $\psi^\star(F_A)$ is called the soft anti image of F_A under ψ.

3. Structures of Fuzzy Soft Subgroup:

Definition 3.1: A mapping $\mu: X \to [0, 1]$, where X is an arbitrary non-empty set is called a fuzzy soft subset in X.

Definition 3.2: Let G be any group. A mapping $\mu: G \to [0, 1]$ is a fuzzy soft subgroup of G if (FSG1) $\mu(xy) \geq \min\{\mu(x), \mu(y)\}$ (FSG2) $\mu(x^{-1}) = \mu(x)$ for all $x, y \in G$.

Example:

Let Z be the additive group of all integers. For any integer n, nZ denote the set of all integers multiplies of n.

(i,e) $nZ = \{0, \pm n, \pm 2n, \pm 3n \ldots\ldots\}$. We have $Z > 2Z > 4Z > 8Z > 16Z$. Define $\mu : Z \to [0,1]$ by $\mu(x) = 1$, if x ἑ 16Z; $= 0.7$, if x ἑ 8Z -16Z; $= 0.5$ if x ἑ 4Z-8Z; $= 0.2$ if x ἑ 2Z- 4Z; $= 0$ if x ἑ Z-2Z . It can be easily verified that μ is fuzzy soft sub group of Z. If the Supplementary condition (FSG_3) $\mu(e_G) = 1$ are satisfied, then the fuzzy soft group is called a standardized fuzzy soft group where e_G is an identity of the group $(G, \cdot)$

Proposition 3.3:

A fuzzy soft subset μ of a group 'G' is a fuzzy soft subgroup of $\hat{G}$ if and only if $\mu(x y^{-1}) \geq \min\{\mu(x), \mu(y)$ for every x, y in G

Proof:

Let 'μ' be a fuzzy soft subgroup of $\hat{G}$. Form 'μ' is a fuzzy group (FSG_1) and (FSG_2) are satisfied.

$\mu(xy^{-1}) \geq \min\{\mu(x), \mu(y^{-1})\} = \min\{\mu(x)\ \mu(y)\}$ conversely let $\mu(x y^{-1}) \geq \min\{\mu(x), \mu(y)\}$ in equality be satisfied. Choosing $y = x$ we get that $\mu(xx^{-1}) = \mu(e) \geq \min\{\mu(x), \mu(x^{-1})\} = \mu(x)$. Hence for x=e. $\mu(y^{-1}) = \mu(ey^{-1}) \geq \min\{\mu(e)\ \mu(y)\} = \mu(x)$ consequently $\mu(xy^{-1}) \geq \min\{\mu(x)\ \mu(y^{-1})\} = \min\{\mu(x), \mu(y)\}$

Remarks 3.4: Let 'μ' be a fuzzy soft sub group of a group 'G' and $x \in G$. then $\mu(x\,y) = \mu(y)$ for every $y \in G$ if and only if $\mu(x) = \mu(e)$

Definition 3.5: Let 'μ' be a fuzzy soft sub group of a group 'G'. For any $a \in G$. are defined by $(a\,\mu)(x) = \mu(a^{-1} x)$ for every $x \in G$ is called the P-fuzzy soft cosset of the group G determined by 'a' and 'μ'

Definition 3.6: Let 'μ' be the fuzzy soft sub group of a group G. then for any $a, b \in G$ a P-fuzzy soft middle cosset $a\,\mu\,b$ of the group G is defined by $(a\,\mu\,b)(x) = \mu(a^{-1}\,x\,b_{-1})$ for every $x \in G$.

Definition 3.7: Let 'μ' be a fuzzy soft sub group of G and $a \in G$. Then the P-pseudo fuzzy cosset $(a\mu)^p$ is defined by $(a\,\mu)^p(x) = p(a)\,\mu(x)$ for every $x \in G$ and for some $p \in P$.

Example:

Let $G = \{1, w, w^2\}$ be a group with respect to multiplication where 'w' denotes the cube root of unity. Define a map $\mu : G \to [0,1]$ by

$$\mu(x) = \begin{cases} 0.7 & \text{if } x = 1 \\ 0.3 & \text{if } x = w, w^2 \end{cases}$$

The pseudo fuzzy soft cosset $(a\,\mu)^p$ for $p(x) = 0.4$ for every $x \in G$ to be equal to 0.28 if x =1 and 0.12 if $x = w, w^2$.

Definition 3.8: Let μ and λ be any two fuzzy soft subsets of a set 'X' and $p \in P$. the P-pseudo fuzzy soft double cosset to $(\mu \times \lambda)^p$ is defined as $((\mu \times \lambda)^p = (x\,\mu)^p \cap (x\,\mu)^p$ for $x \in X$.

Definition 3.9: Let λ and 'μ' be two fuzzy soft subgroups of a group 'G' then λ and μ are said to be P- fuzzy soft conjugate subgroups of G if for some $g \in G\lambda (x) = \mu (g^{-1} x g)$ for every $x \in G$.

4. Some Properties of Pseudo Fuzzy Softt Cosets:

Proposition 4.1:

Let 'μ' be a fuzzy soft subgroup of a group 'G'. Then P-pseudo fuzzy soft cosset $(a\ \mu)^{p}$ is a fuzzy soft sub group of 'G' for every $a \in G$.

Proof: Let 'μ' be a fuzzy soft sub group of G, for every x, y in G we have $(a\ \mu)^{p} (xy^{-1}) = p(a)\ \mu (xy^{-1}) \geq p (a) \min \{ \mu(x), \mu (y)\} = \min \{p(a)\ \mu (x), p (a), \mu (y)\} \geq \min \{ a\ \mu\}^{p} (x), (a, \mu)^{p} (y)\}$ for every $x \in G$. This proves that $(a\ \mu)^{p}$ is a fuzzy soft subgroup of G.

Remark 4.2: A fuzzy soft subgroup 'μ' of a group G is said to be positive fuzzy soft subgroup of 'G' if 'μ' is positive fuzzy soft subset of the group 'G'.

Proposition 4.3:

Every P- pseudo fuzzy soft double cosset is a fuzzy soft subgroup of a group 'G'

Proof:

(i) $(\mu \times \lambda)^{p} (x y) = \{ (x\ \mu)^{p} \cap (x \lambda)^{p} \} (xy) = (x\ \mu)^{p} (x y)$ and $(x \lambda)^{p} (xy) \}$
$= p (x)\ \mu (x y)$ and $p (x)\ \lambda (xy)\}$
$\geq p (x) \min \{ \mu (x), \mu (y) \}$ and $p (x) \min \{ \lambda (x), \lambda (y)\}$
$\geq \min \{p (x)\ \mu (x), p (x)\ \mu (y)\}$ and $\min \{ p (x)\ \lambda (x), p (x)\ \lambda (y)\}$
$\geq \min \{p (x)\ \mu (x), p (x)\ \mu (x)\}, \min\{p (x)\ \mu (y)$ and $p (x)\ \lambda (y)\}$
$= \min \{(x\ \mu)^{p} \cap (x \lambda)^{p}\} (x), (x\ \mu)^{p}\ n\ (x \lambda)^{p}) (y)\}$
$\geq \min \{ (\mu \times \lambda)^{p} (x), (\mu \times \lambda)^{p} (y) \}$

(ii) $(\mu \times \lambda)^{p} (x) = \{(x\ \mu)^{p} \cap(x \lambda)^{p}\} (x) = (x\ \mu)^{p} \cap (x \lambda)^{p} (x)$
$= p (x)\ \mu (x)$ and $p (x)\ \lambda (x) = p (x)\ \mu (x)^{-1}$ and $p (x)\ \lambda (x)^{-1}$ (since λ and μ are fuzzy subsets) $= (x\ \mu)^{p} (x)^{-1}$ and $(x \lambda)^{p} (x)^{-1} = \{ (x\ \mu)^{p}\ n (x \lambda)^{p} \} (x)^{-1} = (\mu \times \lambda)^{p} (x)^{-1}$

Theorem is proved.

Proposition 4.4:

Every P-fuzzy soft middle cosset of a group 'G' is a fuzzy soft subgroup of G.

Proof:

Let $a\ \mu\ b$ be a P-fuzzy soft middle cosset of the group 'G' and 'λ' and 'μ' be two P-conjugate fuzzy soft subgroups of G.

(i) $(a\ \mu\ b) (x y) = \mu (a^{-1} x y b^{-1}) = \lambda (x y)$ [$\because$ λ and μ conjugate fuzzy soft subgroups]
$\geq \min \{ \lambda (x), \lambda (y) \} \geq \min \{ \mu (a^{-1} x b^{-1}), \mu ((a^{-1} y b^{-1})\}$
$\geq \min \{ (a\ \mu\ b) (x), (a\ \mu\ b) (y)\}$

(ii) $(a\ \mu\ b) (x) = \mu (a^{-1} x b^{-1})= \mu (a^{-1} x^{-1} b^{-1})$($\because$ 'μ' fuzzy sub group)$=(a\ \mu\ b) (x^{--1})$ Theorem is proved.

Definition 4.5: Let 'G' be a group. A fuzzy soft subgroup 'μ' of 'G' is called normal if $\mu (x) = \mu (y^{-1} x y)$ for all x, y in G. (or) A fuzzy soft subgroup μ_{H} of G is called a fuzzy soft normal subgroup of 'G' if $\mu_{H} (x y) = \mu_{H} (y x)$ for all x, y in G.

Proposition 4.6:

Every P-pseudo fuzzy soft cosset is a fuzzy soft normal subgroup of a group 'G'

Proof:

Let $(a\ \mu)^{p}$ be any P-pseudo fuzzy cosset. $a \in G$ and for some $p \in P$. Now $(a\ \mu)^{p} (x) = p (a)\ \mu (x)= p (a) \min \{ \mu (e) , \mu (x) \}= p (a) \min \{ \mu (y^{-1} y), \mu(x) \}$
$\geq p (a) \min \{ \min \{\mu (y)^{-1,}\ \mu (y)\},\mu (x) \}\geq p (a) \min \{\mu (y)^{-1} ,\min \{ \mu (y), \mu (x) \}$
$= p (a)\ \mu (y^{-1} x y)$ for all $y \in G$.

Aliter:

Let $(a\ \mu)^{p}$ be any P-pseudo fuzzy soft cosset and $a \in G$ for some $p \in P$, Let μ_{H} is a fuzzy soft normal subgroup of G. Now $(a\ \mu\ H)^{p} (x y) = p (a)\ \mu_{H} (xy)= p (a)\ \mu_{H} (y x)$ (μ_{H} is fuzzy soft normal)$= (a\ \mu_{H})^{p} (y x)$

Proposition 4.7:

The intersection of two P-pseudo fuzzy soft cosset normal subgroup is also fuzzy soft normal subgroup of a group.

Proof:

Let $(a\ \mu)^{p}$ and $(b\ \mu)^{p}$ be any two P-pseudo fuzzy soft cosset normal subgroup of G.

$(a\ \mu)^{p} (x) = (a\ \mu^{p} (y^{-1} x y), y \in G$--- (1)

$(b\ \mu)^{p} (x) = (a\ \mu)^{p} (y^{-1} x y), y \in G$—(2)

Now, $\{(a\mu)^{p} \cap (b\mu)^{p} (x) = ((a \cap b)\mu)^{p} (x) = p (a \cap b)\ \mu (x) = p (a). p (b)\ \mu (x) = p (a). \mu (x)$ and $p (b)\ \mu (x)= (a\ \mu)^{p} (x)$ and $(b\ \mu)^{p} (x) = (a\ \mu)^{p} (y^{-1} x y)$ and $(b\ \mu)^{p} (y^{-1} x y)$ by ((i) & (ii))$= p (a). p (b)\ \mu (y^{-1} x y)= ((a \cap b)\ \mu)^{p} (y^{-1} xy) = \{ (a\ \mu)^{p} \cap (b\ \mu)^{p}\} (y^{-1} x y)$.

Theorem is proved

Aliter:

Let $(a\,\mu_H)^p \cap (b\,\mu_H)^p\}(x\,y) = ((a \cap b)\,\mu_H)^p\}(x\,y) = p\,(a \cap b)\,\mu\,(x\,y) = p\,(a \cap b)\,\mu_H\,(y\,x)$ (μ_H is fuzzy soft normal)$= (a \cap b)\,\mu_H)^p\}(y\,x) = \{a \cap b)^p \cap (b\,\mu_H)^p\}\,(y\,x)$

Proposition 4.8:

P-Pseudo fuzzy soft double cosset is a fuzzy soft normal subgroup of a group 'G'

Proof:

Let $(\mu \times \lambda)^p$ be any P- pseudo fuzzy soft double cosset for $x \in X$.

$$
\begin{aligned}
\text{Now } (\mu \times \lambda)^p\,(x) &= \{(x\,\mu)^p \cap (x\,\lambda)^p\}(x) = (x\,\mu)^p(x) \cap (x\,\lambda)^p\}(x) \\
&= p\,(x) \cap \mu\,(x) \cap p\,(x)\,\lambda\,(x) = p\,(x) \min\{\mu\,(x), \mu\,(e)\} \cap p\,(x) \min\{\lambda\,(x), \lambda\,(e)\} \\
&= p\,(x) \min\{\mu\,(x), \mu\,(y^{-1}y)\} \cap p\,(x) \min\{\lambda\,(x), \lambda\,(y^{-1}y^{1})\} \\
&\geq p(x) \min\{\mu(x), \min \mu\,(y^{-1}), \mu(y)\} \cap \\
&\quad p\,(x) \min\{\lambda\,(x), \min\{\lambda\,(y^{-1}), \lambda\,(y)\} \\
&= p\,(x) \min\{\mu\,(y^{-1}), \mu\,(x\,y)\} \cap p\,(x) \min\{\lambda\,(y^{-1}), \lambda\,(x\,y)\} \\
&= p\,(x)\,\mu\,(y^{-1}x\,y) \cap p\,(x)\,\lambda\,(y^{-1}x\,y) = \{x\,\mu\}^p \cap (x\,\lambda)^p\}\,(y^{-1}x\,y) \\
&= (\mu \times \lambda)^p\,(y^{-1}x\,y)
\end{aligned}
$$

Theorem is proved.

Proposition 4.9:

P-Fuzzy soft middle cossets forms a fuzzy soft normal subgroup of G.

Proof:

$$
\begin{aligned}
(a\,\mu\,b)\,(x) &= \mu\,(a^{-1}x\,b^{-1}) = \lambda\,(x) = \min\{\lambda\,(x), \lambda\,(e)\} \\
&= \min\{\lambda\,(x), \lambda\,(y^{-1}y)\} \geq \min\{\lambda\,(x), \min(\lambda\,(y^{-1}), \lambda\,(y))\} \\
&= \min\{\lambda\,(y^{-1}) \min(\lambda\,(x), \lambda\,(y)\} = \min(\lambda\,(y^{-1}), \lambda\,(x\,y)\} = \lambda\,(y^{-1}x) \\
&= \mu\,(a^{-1}(y^{-1}x\,y)\,b^{-1}) = (a\,\mu\,b)\,(y^{-1}x\,y)
\end{aligned}
$$

Definition 4.10: The strong fuzzy soft α-cut is defined as $A^{+}_{\alpha} = \{x/A(x) > \alpha\}$ where A is any fuzzy soft set .

Definition 4.11: Let 'A' be a fuzzy soft set in a set S. Then the strongest fuzzy soft relation on 'S' (ie) fuzzy soft relation on 'A' is $\mu_A(x,y) = \min\{(A(x), A(y)\}$.

Definition 4.12: Cartesian Product: Let λ and μ be any two fuzzy soft sets in X. Then the cartesian Product of λ and μ is $\lambda x \mu$: $x \times x \rightarrow [0, 1]$ defined by $(\lambda \times \mu)\,(x,y) = \min\{\lambda(x), \mu(y)\}$ for all $x,y\ \varepsilon$ X.

Proposition 4.13:

Let μ_A be a strongest Fuzzy soft relation on 'S' and 'A^{+}_{α}' be the strong α-cut .Then μ_A forms a strong α- cut fuzzy soft group on S.

Proof:

Let $A:S \rightarrow [0,1]$ be any function and μ_A be the strongest fuzzy soft relation on S.

(i) Let $x,y\ \varepsilon$ S

$\mu_A(x,y) = \min\{A(x), A(y)\} \geq \min\{\alpha,\alpha\} \geq \alpha$

(ii) $\mu_A(x^{-1},y^{-1}) = \min\{A(x^{-1}), A(y^{-1})\} = \min\{A(x), A(y)\} = \mu_A(x,y)$

(ii) $\mu_A(e, e) = \min\{A(e), A(e)\} = \min\{1,1\} = 1$

μ_A forms a strong fuzzy group α- cut on S.

Proposition 4.14:

Let λ and μ be strong fuzzy soft α- cuts on S. Then $\lambda \times \mu$ is a strong fuzzy soft group α- cut.

Proof: Let $x,y\varepsilon S$ and λ: $x \times x \rightarrow [0,1]$ be any function.

(i) $(\lambda \times \mu)\,(x,y) = \min\{\lambda\,(x), \mu\,(y)\} \geq \min\{\alpha,\alpha\} \geq \alpha$

(ii) $(\lambda \times \mu)\,(x^{-1},y^{-1}) = \min\{\lambda\,(x^{-1}), \mu\,(y^{-1})\} = \min\{A(x), A(y)\} = (\lambda \times \mu)\,(x, y)$

(ii) $(\lambda \times \mu)\,(e,e) = \min\{\lambda\,(e), \mu\,(e)\} = \min\{1,1\} = 1$

$(\lambda \times \mu)$ forms a strong fuzzy soft group α-cut on S.

Remark 4.15: i) $\min(a,b)^i = \min\{a^i,b^i\}$ for all Positive integer 'i'}

ii) $\mu_A i(x,y) = (\mu_A(x,y))^i = \min\{A(x), A(y)\}^i = \min\{A^i(x), A^i(y)\}$

Proposition 4.16:

Let $\mu_A{}^i$ and $\mu_A{}^j$ be two strong fuzzy soft relations and $A_\alpha{}^+$ be strong fuzzy soft α- cut .Then $\mu_{A^i{}_{UA}{}^j}$ forms a strong fuzzy soft α-cut on S.

Proof:

Since i<j

$$
\begin{aligned}
\mu_{A^i{}_{UA}{}^j}\,(x,y) &= \{(A^iUA^j,)\,(x),(A^i \cup A^j)\,(y)\} = \min\{\max\{A^i(x), A^j(x)\}, \max\{A^i(y), A^j(y)\} \\
&= \max\{\min\{A^i(x), A^i(y)\}, \min\{A^j(x), A^j(y)\} \\
&= \max\{\min\{A(x), A(y)\}^I, \min\{A(x), A(y)\}^j\} \\
&\geq \max\{\min\{\alpha,\alpha\}^i, \min\{\alpha,\alpha\}^j\} \\
&\geq \max\{\min\{\alpha^i,\alpha^i\}, \min\{\alpha^j,\alpha^j\}\} \geq \max\{\alpha^i,\alpha^j\} \geq \alpha^i
\end{aligned}
$$

$\mu_A{}^i{}_{UA}{}^j$ is a strong fuzzy soft α-cut on S.

Remark 4.17: Let $\mu_A{}^i$ and $\mu_A{}^j$ be two strong fuzzy soft relations and $A_\alpha{}^+$ be strong fuzzy soft α-cut. Then $\mu_A{}^i{}_{nA}{}^j$ is a strong fuzzy soft α-cut on S.

Proof: It is obvious

Definition 4.18 : A fuzzy soft binary relation μ on a semi group 'S' is called P-fuzzy soft compatible iff μ (ac, bd) $\geq$ min {μ (a,b) , μ(c,d)} for all a,b,c,d ε S.

Preposition 4.19:

Let μ_A be the strongest fuzzy soft relation on S. Then $A_\alpha{}^+$ is a strong α-cut then μ_A forms P-fuzzy soft compatible.

Proof:

Now μ_A(ac, bd) = min {A(ac), (A(bd)} $\geq$ min {min {A(a), A(c)}, min (A(b) A(d)}

$\geq$ min {μ_A(a,b), μ_A(b,d)}

3.18 Proposition:

Let μ_A be a P-fuzzy soft compatible. Then μ_A is a strong fuzzy soft α-cut.

Proof:

Now μ_A (ac, bd) $\geq$ {μ_A (a, b), μ_A(b, d)} =min {min (A(a), A(b)}, min {A(b), A(d)}}>min {min {α,α}, min {α,α}}>min {α,α}> α. Hence μ_A is P-fuzzy soft compatible forms a strong fuzzy soft α-cut.

Conclusion:

Here we introduce the notion of pseudo compatible P-fuzzy soft relations of a subgroup, cossets of a group, strongest fuzzy soft relations and how they are related with fuzzy soft normal subgroups**.** One can obtain the similar ideal into Soft G-modular and L- fuzzy structures.

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