

# Optimization of Doubly Fed Induction Generator Equivalent Circuit Parameters by Direct Search Method

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**Abstract**—Doubly-fed induction generator (DFIG) is currently the choice for many wind turbines. These generators, when connected to the grid through a converter, is subjected to varied power system conditions like voltage variation, frequency variation, short circuit fault conditions, etc. Further, many countries like Canada, Germany, UK, Scotland, etc. have distinct grid codes relating to wind turbines. Accordingly, following the network faults, wind turbines have to supply a definite reactive current. To satisfy the requirements including reactive current capability, an optimum electrical design becomes a mandate for DFIG to function. This paper intends to optimize the equivalent circuit parameters of an electrical design for satisfactory DFIG performance. Direct search method has been used for optimization of the parameters. The variables selected include electromagnetic core dimensions (diameters and stack length), slot dimensions, radial air gap between stator and rotor and winding copper cross section area. Optimization for 2 MW DFIG has been executed separately for three objective functions - maximum reactive power capability (Case I), maximum efficiency (Case II) and minimum weight (Case III). In the optimization analysis program, voltage variations (10%), power factor- leading and lagging (0.95), speeds for corresponding to slips (-0.3 to +0.3) have been considered. The optimum designs obtained for objective functions were compared. It can be concluded that direct search method of optimization helps in determining an optimum electrical design for each objective function like efficiency or reactive power capability or weight minimization.

**Keywords**—Direct search, DFIG, equivalent circuit parameters, optimization.

## I. INTRODUCTION

DFIGs are the preferred choice for many wind turbines. The stator of the generator is connected to grid directly while the wound rotor through slip-rings is connected through a power electronic converter. DFIGs have multiple advantages over synchronous and induction machines without slip-rings. The major advantage is of the power rating of the converter which is 30% of the power rating of turbine giving DFIGs an edge. DFIG is capable of power generation at sub-synchronous and super-synchronous speeds, which is typically done in the speed range of  $\pm 30\%$  of synchronous speed by controlling power in converter. The generators also provide the facility of active and reactive power control which is essential for satisfactory grid operation.

The DFIG performance depends on equivalent circuit

(shown in Fig. 1) parameters which in turn depend on the design of magnetic core and windings. Therefore, for better performance, a good design of its active materials is essential. Optimization tools help in designing for better performance. Depending on the DFIG's functionality, maximization of either reactive power capability or generator efficiency or minimization of weight is considered.

TABLE I  
 NOMENCLATURE

Symbol	Description
F	Supply frequency
$g_i$	Constraints
I	Phase current
L	Self inductance
$L_{sl}, L_{rl}$	Stator and rotor self leakage inductances
$L_m$	Mutual Inductance
p	Pole pairs
$P_g$	Power to grid
$P_m$	Mechanical power to DFIG
P, Q	Active and reactive power
R	Winding resistance referred to stator
$R_m$	Core loss resistance
$X_{sl}, X_{rl}$	Stator and rotor winding leakage reactance referred to stator
$X_m$	Magnetizing reactance
S	Rotor slip
s, r	Stator and rotor subscripts
$T_{em}$	Electromagnetic torque
U	Stator to rotor effective turns ratio
V	Volts per phase
$\omega$	Synchronous angular frequencies

## II. APPROACH

In this paper, an optimization mathematical tool "direct search method" has been adopted. Exploratory search, Pattern move and the method of "Golden section" have been used to find the optimum function [1].

In general terms, the problem statement is as follows: To determine  $X = (x_1, x_2, \dots, x_n)$  such that  $F(X)$  is minimum and  $g_i(X) \geq 0$  for  $i=1, 2, 3, \dots, m$ .  $F(X)$  is the objective function,  $g_i$  are constraints and  $x_1, x_2, \dots, x_n$  are variables. The objective function is defined as the sum total of deviations obtained from each constraint and its corresponding estimated value using the variables. For example, if the constraint value of torque is 90% and the estimated torque using the variables is 130%, then the deviation for torque constraint is 40%. Similarly, the deviation for each constraint has been calculated

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and the total deviation value is the objective function value and the same is minimized.

The details of variables and constraints used here are discussed in section IIC.

*A. Exploratory Search, Pattern Search and the Method of Golden Section*

Fig. 2 shows, the flow and logic followed for optimizing the function with exploratory, pattern search and golden section.

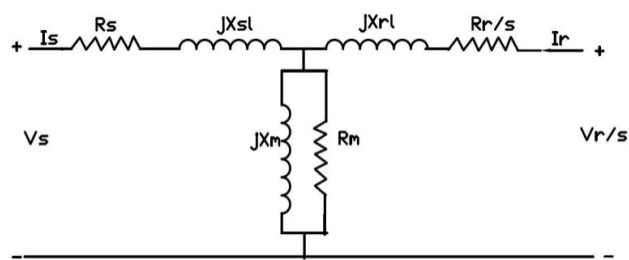


Fig. 1 DFIG Equivalent Circuit

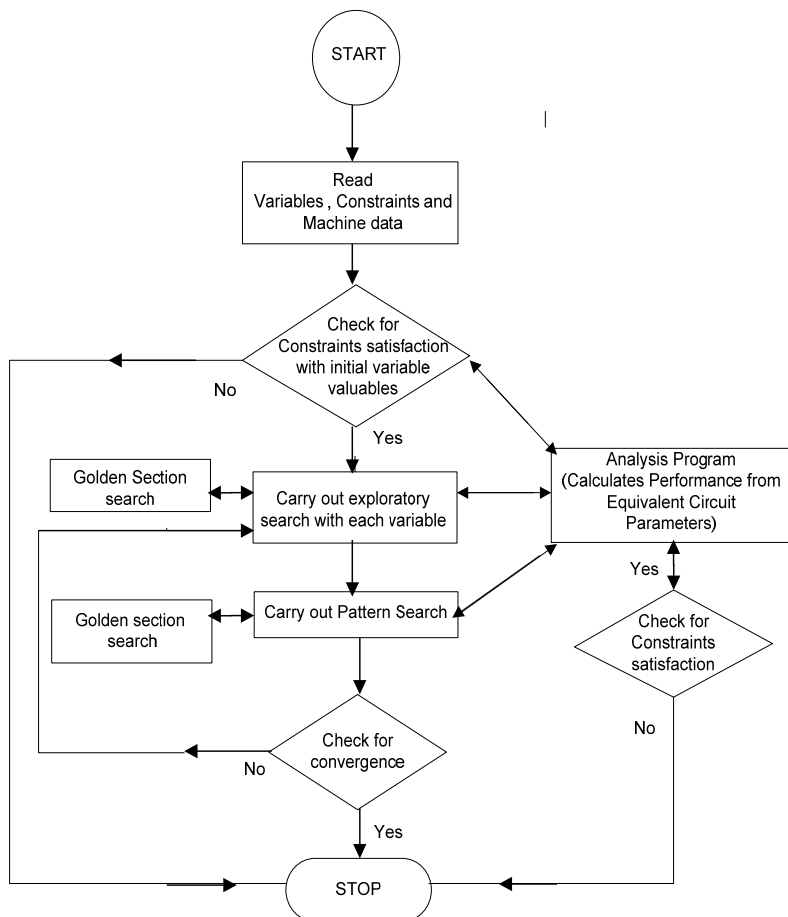


Fig. 2 Flow Chart for the computer program

In exploratory search, we determine the direction for each variable. The best way to improve the direction for an objective function is by perturbing the initial values of each variable [2]. Similar search is carried out for all the considered variables.

Once the direction has been obtained from exploratory search, pattern search is applied. In this search, a new point is established considering all the variables together [2].

In the above exploratory and pattern search methods, the “Golden section method” is used for finding out the minimum of the function [1].

With the help of the above search methods, the direction in which the minimum lies has been established. Let us assume two points, P and Q in the direction of minimum. After the segment PQ has been established, to reduce the interval of

uncertainty, R is to be placed in between. Golden search method helps locate R such that  $PR/QR = 1.618$ .



Fig. 3 “Golden Section”

*B. DFIG Rating Details*

For optimization and analysis, a 2000kW DFIG design has been selected (Table II).

During steady state and transient conditions, one of the essential requirements of DFIG is the capacity to supply reactive power to grid. To meet the requirements, DFIGs electrical loading has to be increased. This results in increase of current densities and temperature rise. Hence, it is

necessary to develop a design which meets the running and expected reactive power needs. Optimization technique helps in achieving such demands.

TABLE II  
DFIG RATING DETAILS

Particular	Value
Rating	2000 kW
Voltage	690 Volts $\pm$ 10%
Frequency	50 Hz $\pm$ 0.25%
Poles	4
Synchronous speed	1500 RPM
Maximum speed	1950 RPM
Minimum speed	1050 RPM
Torque	12486 Nm
Power factor	0.90 Cap / 0.9 Ind
Reactive Power	680.0 KVAR

### C. Variables and Constraints

The core physical dimensions, stator and rotor winding data decide the equivalent circuit parameters. They are  $R_s$ ,  $R_r$ ,  $X_{sl}$ ,  $X_{rl}$ ,  $X_m$ ,  $R_{fe}$  which in turn determine DFIGs performance.

Variables selected for optimization are:

- i) Stator outer and inner diameters,
- ii) Rotor outer and inner diameters,
- iii) Radial gap between stator and rotor,
- iv) Stator and Rotor slot dimensions,
- v) Lamination Core length,
- vi) Stator and Rotor winding copper section area.

The variables and the values have been detailed out in Table III.

Two types of constraints have been considered for optimization-Machine constraints and Performance constraints. Machine constraints are minimum radial air gaps between stator and rotor, stator and rotor teeth minimum widths and allowable maximum flux densities. The performance constraints are rotor currents, current densities in stator and rotor windings, maximum permissible electromagnetic material weight and minimum efficiency. At every stage of optimization, constraints are checked for the confirmation to the permissible values at rated operating conditions, at voltage variations of  $\pm$  10% and power factor 0.9 leading and lagging.

TABLE III  
COMPARISON OF VARIABLES, EQUIVALENT CIRCUIT PARAMETERS AND PERFORMANCE OF OPTIMIZED DESIGNS WITH INITIAL DESIGN

Parameters	Units	Initial Design (1)	"Q" Optimized Design (2)	"Efficiency" Optimized Design (3)	"Weight Reduction" Optimized Design (4)
VARIABLES					
Stator core outer diameter	mm	1060	1088.5	1080.9	955.6
Stator core inside diameter	mm	750	764.7	711.2	704.2
Rotor core inside diameter	mm	410	412.0	406.63	423.6
Radial air gap between stator and rotor	mm	3	2.8	3.194	5.9
Core length	mm	900	835.1	1041.4	899.4
Stator winding slot width	mm	18.8	19.28	18.69	12.03
Stator winding slot depth	mm	49	50.7	48.40	34.7
Rotor winding slot width	mm	10	10.66	13.99	10.00
Rotor winding slot depth	mm	53.7	61.48	63.48	43.87
Stator winding section area	mm <sup>2</sup>	123.06	131.4	120.6	50.16
Rotor winding section area	mm <sup>2</sup>	151.8	195.6	291.0	118.1
EQUIVALENT CIRCUIT PARAMETERS					
$R_s$ (Hot)	Ohms	0.00243	0.0022	0.002671	0.00597
$R_r$ (Hot)	Ohms	0.0046	0.00344	0.002589	0.00592
$X_{sl}$	Ohms	0.0419	0.0417	0.0442	0.031
$X_{rl}$	Ohms	0.0554	0.0558	0.054	0.0412
$X_m$	Ohms	3.91	3.97	3.98	1.802
$R_{fe}$	Ohms	75.86	70.84	119.1	77.4
PERFORMANCE AT 1950 RPM AND TORQUE 12.486kNm					
Q	KVAR	680	1000	680	680
Stator current per phase	Amps	1002	1065	1002	1002
Rotor current (referred to stator)	Amps	961	1008	962	965
Rotor Voltage (referred to stator)	Volts	216	207	210	220
Stator active power	kW	1969	1968	1969	1979
Rotor active power	kW	622.4	616	607	632
Power Delivered to Grid	kW	2590	2585	2577	2611
Stator copper loss	kW	7.36	7.49	8.08	18.02
Rotor copper loss	kW	12.75	10.5	7.32	16.46
Efficiency	%	97.82	97.87	98.24	97.34
Stator winding current density	amp/mm <sup>2</sup>	2.00	2.0	2.07	5.0
Rotor winding current density	amp/mm <sup>2</sup>	3.89	3.2	2.04	5.0
Electro Magnetic Material weight (approx.)	Kg	4400	4990	5497	3255

### III. CALCULATION PROCEDURE

From the stator and rotor lamination outer and inner diameters, and slot dimensions, an initial design winding data was worked out for a wound rotor induction motor. Equivalent circuit parameters  $R_s, R_r, X_{s1}, X_{r1}, X_m, R_m$  have been calculated using [3], [4]. For calculating the performance, the equations derived in [5] have been followed.

**Step 1.** Stator current,  $\bar{I}_s$  has been calculated from a given torque (T), initial reactive KVAR of 680 and at slip of -3% at rated voltage.

$$\bar{I}_s = \frac{Q_s}{3V_s} + j \frac{\omega_s T_{em}}{3pV_s} \quad (1)$$

**Step 2.** The stator flux linkage  $\bar{\lambda}_s$  is calculated using (2)

$$\bar{\lambda}_s = \frac{\bar{V}_s - \bar{I}_s R_s}{j\omega_s} \quad (2)$$

**Step 3.** Using (3)-(6), rotor voltage  $V_r$  has been calculated

$$L_s \bar{I}_s + L_m \bar{I}_r = L_{sl} \bar{I}_r \quad (3)$$

$$L_r \bar{I}_r + L_m \bar{I}_s = L_{rl} \bar{I}_s \quad (4)$$

$$s = \frac{L_m^2}{L_s L_r} \quad (5)$$

$$\bar{V}_r = \omega_r \left[ \frac{\omega_s T_{em} L_s L_r \sigma}{3pV_s L_m} \right] + j\omega_r \left[ \frac{V_s L_r}{\omega_s L_m} - \frac{Q_s \sigma L_s L_r}{3V_s L_m} \right] \quad (6)$$

**Step 4.** Rotor current  $I_r$  has been calculated using (7):

$$\bar{I}_r = \left[ \frac{V_s}{\omega_s L_m} - 2 \frac{Q_s}{3V_s} \left( \frac{L_s}{L_m} \right) \right] + j \left[ \frac{\omega_s T_{em} L_s}{3pV_s L_m} \right] \quad (7)$$

**Step 5.** The rotor active power  $P_r$ , reactive power  $Q_r$  have been calculated as per (8) and (9):

$$P_r = 3 \operatorname{Re} \left\{ \bar{V}_r \bar{I}_r' \right\} \quad (8)$$

$$Q_r = 3 \operatorname{Im} \left\{ \bar{V}_r \bar{I}_r' \right\} \quad (9)$$

**Step 6.** The stator active power  $P_s$  and power to grid  $P_g$  is calculate as per (10) and (11):

$$P_s = \frac{\omega_s T_{em}}{p} \quad (10)$$

$$P_g = P_s + P_r \quad (11)$$

**Step 7.** From stator and rotor currents and resistances, copper losses have been calculated. Stray loss is taken as 0.5% of input. Windage loss has been captured from previously tested machines having same frame size and same speed. The input to DFIG is calculated by adding losses to power supplied to grid ( $P_g$ ). The efficiency has been computed from (12):

$$\eta = \frac{P_g}{P_m} \quad (12)$$

### IV. ANALYSIS

#### A. "Q" Optimization

Wind turbine manufacturer decides the operating speed range, and required 'torque vs. speed' characteristics. In this paper, slip range selected is  $\pm 3\%$  of synchronous speed. i.e. speeds from 1050 rpm to 1950 rpm and torque at maximum speed is 12,480 kNm. Torque reduction at lower speeds is directly proportional to square of speed reduction. Optimization has been conducted at maximum speed and its corresponding torque.

Constraints were checked at all stages of optimization. Stator and rotor currents and currents densities in windings were checked at operating voltage  $\pm 10\%$  and power factors of  $\pm 0.9$ .

The maximum reactive power requirement depends on grid strength and country standards of power grid requirement. In the present optimization, a maximum requirement of 1000 KVAR has been considered. Therefore, an initial design of 680 KVAR was optimized to 1000 KVAR.

The variables obtained before and after optimization are shown in Table III, columns 1 and 2.

Performance parameters stator and rotor currents, voltage, power delivered to grid, stator and rotor copper loss have been computed and compared with initial design. Table III highlights the comparison.

#### B. "Efficiency Maximization" Optimization

In situations where better efficiency is required, it can be maximized by keeping reactive power and torque constant. In the present case, reactive power (680 KVAR) and torque of 12, 480 Nm have been kept constant at 1950 RPM and efficiency has been maximized. Optimized variables, equivalent circuit parameters and the performance values are shown in Table III, column 3. Further, efficiency values at lower operating speeds are also computed and the comparison is shown in Fig. 4.

#### C. "Weight Minimization" Optimization

The operation height of DFIG depends on WTGs rotor diameter and the desired power generation. For power generation in Megawatts, the tower heights which are dependent on wind turbine rotors are usually above 60 meters [6]. Hence operation and maintenance becomes critical and

calls for high reliability. Minimum space and weight are an advantage. Present optimization tool aids in achieving this objective. Optimized variables, equivalent circuit parameters and the performance values for the “optimized for minimum weight” design are as shown in the comparison Table III, column 4.

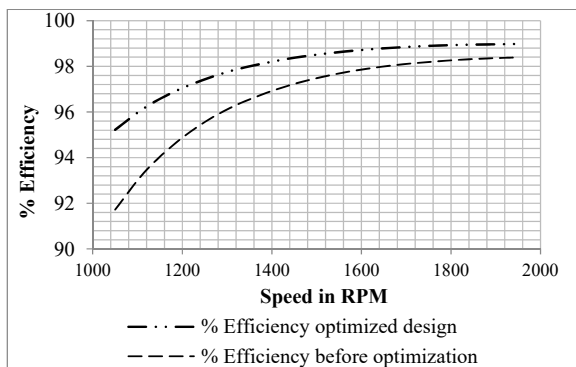


Fig. 4 Speed vs. Efficiency (Before and after “Efficiency” optimization)

#### V. RESULTS OF OPTIMIZED DESIGNS

From Table III, it can be observed that, there is considerable variation in optimized variables and equivalent circuit parameters. They are discussed below.

##### A. “Q” Optimization

In “Q” optimized design, for an increased capacity from 680 KVAR to 1000 KVAR, the active material has been increased from 4400 kg to 4974 kg an increase of 13%. The efficiency has marginally decreased from 97.82% to 97.63%. The radial air gap has reduced from 3.0 mm to 2.74 mm. The salient feature in “Q” optimization is, the optimized design has less stator and copper loss than original design value for all Q values. The variation is shown in Fig. 5.

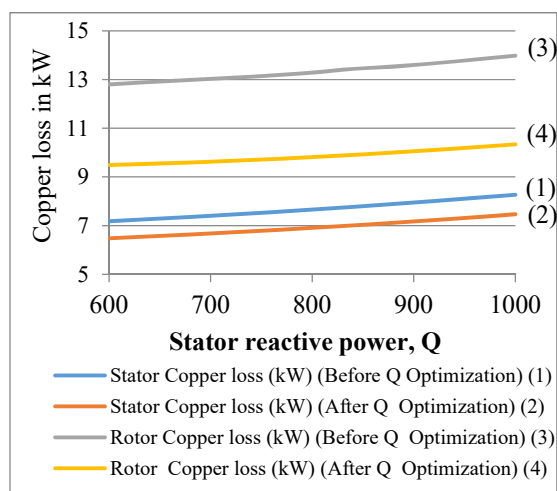


Fig. 5 Reactive Power vs. Stator and rotor copper loss (Before and after “Reactive power optimization”)

##### B. Efficiency Optimization

In case of efficiency optimization, improvement in

efficiency is from 97.82% to 98.24%. The weight of active material has gone up from 4400 kg to 5497 kg, an increase of 25%, which is high. The dimensional change of the variables including the stator core outer diameter, core length, stator winding copper and rotor winding copper, increased substantially; there was no change in radial air gap. With the optimized design, the efficiency was calculated at all operating speeds and the variation of efficiency is shown in Fig. 4.

##### C. Weight Optimization

In case of weight reduction optimization, significant reduction of 47% was achieved in material. The current densities in windings have increased to maximum stipulated value of 5 amp/mm<sup>2</sup>. The air-gap between stator and rotor also increased. The efficiency has reduced from 97.82% to 97.24%.

#### VI. DISCUSSION OF RESULTS

DFIGs are the preferred choice in wind turbines due to its speed control capability, size and cost of controllers as compared to synchronous machines. However, with varying power grid requirements by country, coupled with operating conditions, specific designs of DFIG need to be developed. In the present case, three specific requirements like reactive power requirement, efficiency and weight reduction have been considered. Parameters have been optimized and compared in steady state conditions. The optimized values obtained depend on the variables and constraints imposed. One has to decide the constraints considering the practical aspects prevailing in the manufacturing and operating conditions. It may be noted that, this analysis and optimization can be extended for transient conditions also.

#### ACKNOWLEDGMENT

M. Ramakrishna Rao thanks Ms. Mamidi Sri Rekha for her sincere help in preparing the paper and presentation.

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