World Academy of Science, Engineering and Technology
International Journal of Industrial and Manufacturing Engineering
Vol:9, No:7, 2015

# A Survey of Discrete Facility Location Problems

Z. Ulukan, E. Demircioğlu

***Abstract*—**Facility location is a complex real-world problem which needs a strategic management decision. This paper provides a general review on studies, efforts and developments in Facility Location Problems which are classical optimization problems having a wide-spread applications in various areas such as transportation, distribution, production, supply chain decisions and telecommunication. Our goal is not to review all variants of different studies in FLPs or to describe very detailed computational techniques and solution approaches, but rather to provide a broad overview of major location problems that have been studied, indicating how they are formulated and what are proposed by researchers to tackle the problem. A brief, elucidative table based on a grouping according to "General Problem Type" and "Methods Proposed" used in the studies is also presented at the end of the work.

***Keywords*—**Discrete location problems, exact methods, heuristic algorithms, single source capacitated facility location problems.

## I. Introduction

FACILITY location models have been widely studied due to their application in many real situations. These models can differ in their objective function, the number of the facilities to locate, the solution space in which the problem is defined, and several other decision factors. The problem is called as a discrete facility location problem if there are a finite number of candidate facility locations. If the facilities can be placed anywhere in some continuous regions, then the problem is called as a continuous facility location problem. This work is concentrated on discrete facility location models. Discrete facility location problems (FLP) are concerned with choosing the best location for facilities from a given set of potential sites to minimize the total cost while satisfying customer demand.

In the uncapacitated facility location problem (UFLP), each facility is assumed to have no limit on its capacity where each customer receives all its demand from exactly one facility. The first models of UFLP date back to the 60's, when the Simple Plant Location Problem (SPLP), [1], [2] and the p-Median Problem (PM) [3], [4] were defined. It was later shown that these two models are particular cases of a more general formulation for deterministic, static, uncapacitated problems having a minisum objective function [5], [6].

The capacitated facility location problem (CFLP) is a well-known combinatorial optimization problem. It consists in deciding which facilities to open from a given set of potential facility locations and how to assign customers to those facilities. In the Single Source Capacitated Facility Location Problem (SSCFLP) each customer has to be assigned to one facility that supplies its whole demand. The total demand of customers assigned to each facility cannot exceed its capacity. An opening cost is associated with each facility. The objective is to minimize the total cost of opening the facilities and supply all the customers.

This research was financially supported by Galatasaray University Research Fund (15.402.001).

Ulukan, Z. and Demircioğlu, E. are with Industrial Engineering Department, Galatasaray University, Istanbul (e-mail: zulukan@gsu.edu.tr, edemircioglu@gsu.edu.tr).

## II. Literature Review

### A. Studies in UFLP and Its Variants

Kuehn and Hamburger [1] outlined a heuristic computer program for locating warehouses and compared it with recently published efforts at solving the problem either by means of simulation or as a variant of Linear Programming (LP). Erlenkotter, D. [7] proposed one of the earlier bounding procedures, called dual-ascent and dual-adjustment, based on the LP relaxation of the problem. Guignard [8] established a dual-ascent algorithm for solving Lagrangian dual problem obtained by relaxing constraints. Galvão and Raggi [6] developed a method for solving to optimality a general 0-1 formulation for UFLP. This is a three-stage method that solves large problems in reasonable computing times which is composed of a primal-dual algorithm, a subgradient optimization to solve a Lagrangean dual and a BB algorithm

Goldengorin et al. [9] presented a technique that enhances the performance of BB algorithms. The new algorithms thus obtained are called branch and peg algorithms, where pegging refers to assigning values to variables outside the branching process. Sun, M. [10] developed a Tabu Search (TS) heuristic procedure to solve UFLP. Hansen et al. [11] applied the Variable Neighbourhood Search metaheuristic to the primal simple plant-location problem (SPLP) and to a reduced dual that is obtained by exploiting the complementary slackness conditions to solve some very large-scale Euclidean instances.

Nezhad et al. [12] et al. focused on the UFLP with the presence of set-up cost. To efficiently tackle this decision problem, two Lagrangian-based heuristics are proposed one of which incorporates integer cuts to strengthen the formulation. Local Search (LS) operators are also embedded within these methods to improve the upper bounds as the search progresses. Kratica et al. [13] considered the multi-level UFLP. A new mixed integer linear programming (MILP) formulation is presented. Experimental results are performed on instances known from literature. Ardjmand et al. [14] proposed a discrete variant of Unconscious search (US) which mimics the process of psychoanalytic psychotherapy in which the psychoanalyst tries to access the unconscious of a mental patient to find the root cause his/her problem, which is encapsulated in unconsciousness. Letchford and Miller [15] presented an aggressive reduction scheme involving four different reduction rules, along with lower- and upper-bounding procedures in BB. Monabbati and Kakhki [16] presented a subadditive dual ascent procedure to find an optimal subadditive dual function based on Klabjan's generator subadditive function.

### B. Studies in CFLP (CMFLP) and Its Variants

Khumawala [17] introduced an efficient heuristic procedure for solving a special class of mixed integer programming problem called the capacitated warehouse (plant) location problem. Geoffrion and McBride [18], Nauss [19], Christofides and Beasley [20], Pirkul [21] and Shetty [22] considered Lagrangean Relaxation (LR) of the demand constraints with or without addition of an aggregate capacity constraint or another surrogate constraint for solving large scale CFLP. The algorithm consists of solving a SFLP and then solving a minimum cost network problem. The problem is solved by applying an iterative procedure in which lower bounds on the optimal objective value are generated through LR techniques. Upper bounds on the optimal objective value are obtained by modifying the Lagrangian solution to restore feasibility. However, Nauss [19] also presented a BB algorithm.

Magnanti and Wong [23] proposed methodology for improving the performance of Benders decomposition when applied to mixed integer programs. Jacobsen [24] generalized heuristics for CFLP that are ADD/DROP, SHIFT, ALA and VSM (Vertex Substitution Method). Domschke and Drexl [25] noted that ADD-heuristics normally lead to bad solutions. They presented some starting procedures in order to overcome this difficulty. Van Roy [26] presented an implementation of the Cross Decomposition method to solve CFLP. The method unifies Benders Decomposition and LR into a single framework that involves successive solutions to a transportation problem and a SPLP.

Leung and Magnanti [27] investigated the polyhedral structure of the CFLP. Their purpose was to identify facets and valid inequalities for a wide range of capacitated fixed charge problems that contain this prototype problem as a substructure. Mateus and Bornstein [28] examined CFLP where fixed costs, generally relating to the installation of warehouses and variable costs, consisting mainly of transportation costs, are minimized. Aardal et al. [29] examined the polyhedral structure of the convex hull of feasible solutions of CFLP. Rolland et al. [30] presented a new solution heuristic based on TS principles, and uses short term and long term memory, as well as strategic oscillation and random tabu list sizes for the PM problem.

Bornstein and Azlan [31] proposed simulated annealing (SA) for CFLP. Ghiani et al. [32] introduces CPLP with multiple facilities in the same site (CPLPM), a special case of the classical CPLP where several facilities can be opened in the same site. Lorena and Senna [33] presented a column generation approach to Capacitated P-Median Problem (CPMP).

Doong et al. [34] used a hybrid method of genetic algorithm and subgradient technique. However, Klose and Görtz [35] employed a (stabilized) column generation method and then the column generation procedure within a branch-and-price algorithm for computing optimal solutions to the CFLP.

Sambola et al. [36] formulated the problem as a two- stage stochastic program and two different recourse actions were considered. Küçükdeniz et al. [37] proposed a fuzzy c-means clustering algorithm based method which involves the integrated use of fuzzy c-means and convex programming. Rahmani, A. and MirHassani M.A. [38] proposed a new hybrid optimization method called Hybrid Evolutionary Firefly-Genetic Algorithm which is inspired by social behavior of fireflies and the phenomenon of bioluminescent communication. The method combines the discrete Firefly Algorithm (FA) with the standard Genetic Algorithm (GA).

Harris et al. [39] proposed an efficient evolutionary multi-objective optimization approach to the CFLP for solving large instances that considers flexibility at the allocation level, where financial costs and $CO_2$ emissions are considered simultaneously. Ozgen and Gulsun [40] combined a two-phase possibilistic LP approach and a fuzzy Analytical Hierarchical Process (AHP) approach to optimize two objective functions (''minimum cost'' and ''maximum qualitative factors benefit'') in a four-stage (suppliers, plants, distribution centers, customers) supply chain network in the presence of vagueness.

Li et al. [41] studied multi-product facility location problem in a two-stage supply chain in which plants have production limitation, potential depots have limited storage capacity and customer demands must be satisfied by plants via depots. They developed a hybrid method. Aardal et al. [42] gave the first fully polynomial time approximation scheme (FPTAS) for the single-sink (single-client) CFLP. Then, they showed that the problem is solvable in polynomial time if the number of clients is fixed by reducing it to a collection of transportation problems.

### C. Studies in SSCFLP (SSCMFLP) and Its Variants

As a special case of CFLP, discrete facility location problems can be considered. There is a vast amount of research interest in literature devoted to the Single Source Capacitated (Multi) Facility Location Problem (SSCMFLP).

Nagelhaut and Thompson [43] gave two heuristic solution methods and a BB algorithm for solving single source transportation problems. Neebe and Rao [44] reformulate the SSCFLP as a set partitioning problem. They proposed a column generation and branch-and-price method to solve it.

Klincewicz and Luss [45] described a LR heuristic algorithm. By relaxing the capacity constraints, the uncapacitated facility location problem is obtained as a subproblem and solved by the well-known dual ascent algorithm. Darby-Dowman et al. [46] have considered SSCFLP by proposing the use of LR in which the capacity constraints were relaxed. Sridharan, R. [47] proposed a heuristic based on a LR. Tranganlaterngsak et al. [48] proposed a particular type of facility location problem in which there exist two echelons of facilities. Each facility in the second echelon has limited capacity and can be supplied by only one facility in the first echelon. Rönnqvist et al. [49]

proposed an approach which is based on a repeated matching algorithm which essentially solves a series of matching problems until certain convergence criteria are satisfied.

Delmaire et al. [50] proposed a Reactive GRASP heuristic; a TS heuristic; and two different hybrid approaches that combine elements of the GRASP and the TS methodologies. Holmberg et al. [51] developed a primal heuristic, based on a repeated matching algorithm which essentially solves a series of matching problems until certain convergence criteria are satisfied, and is incorporated into the Lagrangian heuristic.

Hindi and Pienkosz [52] combined LR with restricted neighbourhood search. However, Tragantalerngsak et al. [53] proposed a LR based BB algorithm for its solution. Cortinhal et al. [54] used a LR to obtain lower bounds for this problem. Upper bounds are given by Lagrangean heuristics followed by search methods and by one TS metaheuristic.

Ahuja et al. [55] presented a Very Large Scale Neighborhood (VLSN) search algorithm where the neighborhood structures are induced by customer multi-exchanges and by facility moves. Chen et al. [56] developed a Multiple Ant Colony System (MACS) and showed that the performance of their work is competitive with the Lagrangean heuristic. Chen et al. [57] proposed a hybrid algorithm, which combines Lagrangean heuristic and Ant Colony System (ACS), LH–ACS, is developed for the SSCFLP.

Correia and Captivo [58] proposed a discrete location problem, which they call the Single Source Modular Capacitated Location Problem (SS-MCLP) and they developed Lagrangean heuristic, enhanced by TS in order to obtain good feasible solutions. Lin [59] considered a stochastic version of SSCFLP. A set of capacitated facilities is to be selected to provide service to demand points with stochastic demand at the minimal total cost. Yang et al. [60] presented a cut-and-solve (CS) based exact algorithm for the SSCFLP. At each level of CS's branching tree, it has only two nodes, corresponding to the Sparse Problem (SP) and the Dense Problem (DP), respectively. Addis et al. [61] proposed a heuristic approach, based on VLSN search to tackle the problem, in which both ad hoc algorithms and general purpose solvers are applied to explore the search space.

Guastaroba, G. and Speranza, M.G. [62] extended the Kernel Search heuristic framework to general Binary Integer Linear Programming (BILP) problems, and apply it to the SSCFLP. Dantrakul et al. Reference [63] introduced three methods (greedy algorithm, PM algorithm and p-center algorithm) to solve the problem. Bieniek, M. [64] studied SSCFLP with general stochastic identically distributed. The unified a priori solution for the locations of facilities and for the allocation of customers to the operating facilities is found and a deterministic equivalent formulation of the model is presented.

## III. General Model Formulation for SSCMFLP

Let $I=\{1,\ldots,n\}$ be a set of potential locations and $J=\{1,\ldots,m\}$ be a set of customers. Each customer $j$ has an associated demand, $w_i$, that must be served by a single facility. Facilities can be located only at some prespecified sites in a plane and there is a limit, $s_i$, to the total demand that a facility located at a site $i$ can meet. The costs $c_{ij}$ of supplying the demand of a customer $j$ from a facility established at location $i$, as well as the fixed costs $f_i$ for opening a facility at $i$, are known. It should be noted that the distances values between customers and facilities situated at potential sites are embedded in supplying costs $c_{ij}$ Let

$$x_{ij} = \begin{cases} 1, & \text{if customer } j \text{ is assigned to a facility located at } i, \\ 0, & \text{otherwise} \end{cases}$$

$$y_i = \begin{cases} 1, & \text{if a facility is located at candidate site } i, \\ 0, & \text{otherwise} \end{cases}$$

Then, SSCMFLP can be mathematically stated as [51]:

$$\min z = \sum_{i\in I}\sum_{j\in J} c_{ij} x_{ij} + \sum_{i\in I} f_i y_i \tag{1}$$

subject to

$$\sum_{j\in J} w_j x_{ij} \le s_i y_i \qquad \forall i \in I, \tag{2}$$

$$\sum_{i\in I} x_{ij} = 1, \qquad \forall j \in J, \tag{3}$$

$$x_{ij} \in \{0;1\}, \qquad \forall i \in I,\ j \in J, \tag{4}$$

$$y_i \in \{0;1\}, \qquad \forall i \in I, \tag{5}$$

For a single-source assumption FLPs, the objective functions (1) calculate the total transportation. Constraints (2) ensure that the capacity of each facility is not exceeded while constraints (3) guarantee that every customer must be served from exactly one facility.

## IV. Conclusion

This paper presents a review of the fundamental models in location theory that find large applicability in many sectors, such as distribution planning systems, telecommunication networks design, supply chain decisions etc. We attempt to introduce the unfamiliar reader to the rich history of this problem that spans several decades. During this time period, the basic model and its variants have been the subject of considerable research. Numerous exact and approximate methods have been devised to solve the original uncapacitated problem and subsequent capacitated versions. These methodologies cover a broad range of techniques from Operations Research. Finally, a brief table consisting of the studies on FLPs is presented (See Appendix). The discussed problem type and proposed methods in every paper are clearly mentioned in this table. Thus, we believe that the table involving specified and brief information will be worthwhile for a reader interested in FLPs and this survey will hopefully encourage practitioners of OR to study this exciting area in more depth.

## APPENDIX

TABLE I
DISCRETE LOCATION PROBLEMS

| Ref. No | Year | Author | General Problem Type | Methods Proposed |
|---|---|---|---|---|
| [1] | 1963 | Kuehn & Hamburger | UFLP | Linear Programming (LP) |
| [4] | 1965 | Hakimi | UFLP | P-Median (PM) |
| [7] | 1978 | Erlenkotter | UFLP | Linear Programming (LP) Relaxation, Branch-and-Bound (BB) |
| [8] | 1988 | Guignard | UFLP | Dual-Ascent algorithm, Lagrangean Relaxation (LR) |
| [6] | 1989 | Galvão & Raggi, | UFLP | Subgradient Optimization, Lagrangean Relaxation (LR), Branch-and-Bound (BB) |
| [9] | 2004 | Goldengorin et al. | UFLP | Branch and Peg algorithms |
| [10] | 2006 | Sun | UFLP | Tabu Search (TS) |
| [11] | 2007 | Hansen et al. | UFLP | Variable Neighbourhood Search (VNS) |
| [12] | 2013 | Nezhad et al. | UFLP | Lagrangean Relaxation (LR) |
| [15] | 2014 | Letchford & Miller | UFLP | Branch-and-Bound (BB) |
| [14] | 2014 | Ardjmand et al. | UFLP | Unconscious Search (US) |
| [13] | 2014 | Kratica et al. | UFLP | CPLEX solver |
| [16] | 2015 | Monabbati & Kakhki | UFLP | Subadditive Functions Method |
| [17] | 1974 | Khumawala | CFLP | Branch-and-Bound (BB) |
| [18] | 1978 | Geoffrion & McBride | CFLP | Lagrangean Relaxation (LR) |
| [19] | 1978 | Nauss | CFLP | Lagrangean Relaxation (LR), Branch-and-Bound (BB) |
| [23] | 1981 | Magnanti & Wong | CFLP | Benders' Decomposition |
| [20] | 1983 | Christofides & Beasley | CFLP | Lagrangean Relaxation (LR) |
| [24] | 1983 | Jacobsen | CFLP | Add/Drop Heuristic, Vertex Substitution Method (VSM) |
| [25] | 1985 | Domschke & Drexl | CFLP | ADD-heuristics with starting procedures, DROP-heuristics. |
| [26] | 1986 | Van Roy | CFLP | Cross Decomposition Algorithm |
| [21] | 1987 | Pirkul | CFLP | Lagrangean Relaxation (LR) |
| [27] | 1989 | Leung & Magnanti | CFLP | Polyhedral Technique |
| [22] | 1990 | Shetty | CFLP | Lagrangean Relaxation (LR) |
| [28] | 1991 | Mateus & Bornstein | CFLP | ADD/DROP |
| [29] | 1995 | Aardal et al. | CFLP | Branch-and-Cut Algorithm |
| [30] | 1996 | Rolland et al. | CFLP | Tabu Search (TS) |
| [31] | 1998 | Bornstein & Azlan | CFLP | Lagrangean Relaxation (LR), Simulated Annealing (SA) |
| [32] | 1999 | Ghiani et al. | CFLP | Lagrangean Relaxation (LR) |
| [33] | 2004 | Lorena & Senna | CFLP | Column Generation, Lagrangean/Surrogate Relaxation |
| [34] | 2007 | Doong et al. | CFLP | Genetic Algorithm (GA), Subgradient Technique |
| [35] | 2007 | Klose & Görtz | CFLP | Column Generation Method |
| [36] | 2011 | Sambola et al. | CFLP | Recourse Functions |
| [37] | 2012 | Küçükdeniz, T. | CFLP | Convex Programming |
| [38] | 2014 | Rahmani & MirHassani | CFLP | Hybrid Evolutionary Firefly-Genetic Algorithm |
| [39] | 2014 | Harris et al. | CFLP | Evolutionary Multi-Objective Optimization, Lagrangean Relaxation (LR) |
| [40] | 2014 | Ozgen & Gulsun | CFLP | Two-phase Possibilistic Linear Programming (LP), Fuzzy Analytical Hierarchical Process |
| [41] | 2014 | Li et al. | CFLP | Lagrangean Relaxation (LR), Dantzig–Wolfe Decomposition |
| [42] | 2015 | Aardal et al. | CFLP | N/P |
| [43] | 1980 | Nagelhout & Thompson | SSCFLP | Branch & Bound (BB) |
| [44] | 1983 | Neebe & Rao | SSCFLP | Column Generation, Branch-and-Price method |
| [45] | 1986 | Klincewicz & Luss | SSCFLP | Lagrangean Relaxation (LR) |
| [46] | 1988 | Darby-Dowman | SSCFLP | Lagrangean Relaxation (LR) |
| [47] | 1993 | Sridharan | SSCFLP | Lagrangean Relaxation (LR) |
| [48] | 1997 | Tragantalerngsak et al. | SSCFLP | Lagrangean Relaxation (LR) |
| [49] | 1999 | Rönnqvist et al. | SSCFLP | Repeated Matching Algorithm |
| [50] | 1999 | Delmaire et al. | SSCFLP | GRASP, Tabu Search (TS) |
| [51] | 1999 | Holmberg et al. | SSCFLP | Lagrangean Relaxation (LR), Branch & Bound (BB) |
| [52] | 1999 | Hindi & Pienkosz | SSCFLP | Lagrangean Relaxation (LR), Neighbourhood Search (NS) |
| [53] | 2000 | Tragantalerngsak et al. | SSCFLP | Lagrangean Relaxation (LR), Branch & Bound (BB) |
| [54] | 2003 | Cortinhal & Captivo | SSCFLP | Lagrangean Relaxation (LR), Tabu Search (TS) |
| [55] | 2004 | Ahuja et al. | SSCFLP | Very Large Scale Neighbourhood (VLSN) |
| [56] | 2006 | Chen & Ting | SSCFLP | Multiple Ant Colony System (MACS) |
| [58] | 2006 | Correia & Captivo | SSCFLP | Lagrangean Relaxation (LR), Tabu Search (TS) |
| [57] | 2008 | Chen & Ting | SSCFLP | Lagrangean Relaxation (LR), Multiple Ant Colony System (MACS) |
| [59] | 2009 | Lin | SSCFLP | Branch & Bound (BB), Lagrangean Relaxation (LR) |
| [60] | 2012 | Yang et al. | SSCFLP | Cutting Plane Algorithm |
| [61] | 2013 | Addis et al. | SSCFLP | Very Large Scale Neighbourhood (VLSN) |
| [62] | 2014 | Guastaroba & Speranza | SSCFLP | Kernel Search (KS) |
| [63] | 2014 | Dantrakul et al. | SSCFLP | Greedy, P-Median (PM), P-Center |
| [64] | 2015 | Bieniek | SSCFLP | N/P |

## REFERENCES


[1] A. A. Kuehn, and M. J. Hamburger, "A heuristic program for locating warehouses" *Management Science*, vol. 9, pp. 643-666, 1963.

[2] M. L. Balinski, "Integer programming: methods, uses, computation." *Management Science*, vol. 12, pp. 253-313, 1965.

[3] S. L. Hakimi, "Optimum locations of switching centers and the absolute centers and medians of a graph" *Operations Research*, vol. 12, pp. 450-459, 1964.

[4] S. L. Hakimi, "Optimum distribution of switching centers in a communication network and some related graph theoretic problems" *Operations Research*, vol. 13, pp. 462-475, 1965.

[5] G. Cornuejols, M. L. Fisher, and G. L. Nemhauser, "Location of bank accounts to optimize float: an analytic study of exact and approximate algorithms" *Management Science*, vol. 23, pp. 789-810, 1977.

[6] R. D. Galvão, and L. A. Raggi, "A method for solving to optimality uncapacitated location problems" Annals of Operations Research, vol. 18, pp. 225-244, 1989.

[7] D. Erlenkotter, "A dual-based procedure for uncapacitated facility location", *Operation Research*, vol. 26, no.6, pp. 992–1009, 1978.

[8] M. Guignard, "A Lagrangean dual ascent algorithm for plant location problems", *European Journal of Operational Research,* vol. 35, pp. 193–200, 1988.

[9] B. Goldengorin, D. Ghosh, and G. Sierksma, "Branch and peg algorithms for the simple plant location problem", *Computational Operations Research*, vol. 31, pp. 241–255, 2004.

[10] M. Sun, "Solving the uncapacitated facility location problem using tabu search", *Computers & Operations Research*, vol. 33, pp. 2563–2589, 2006.

[11] P. Hansen, J. Brimberg, D. Uroševic´, and N. Mladenovic´, "Primal-dual variable neighborhood search for the simple plant-location problem", *INFORMS Journal on Computing*, vol. 19, no. 4, pp. 552–564, 2007.

[12] A. M. Nezhad, H. Manzour, and S. Salhi, "Lagrangian relaxation heuristics for the uncapacitated single-source multi-product facility location problem", *Int. J. Prod. Economics*, vol. 145, pp. 713–723, 2013.

[13] J. Kratica, D. Dugošija, and A. Savic´, "A new mixed integer linear programming model for the multi level uncapacitated facility location problem", *Applied Mathematical Modelling*, vol. 38, pp. 2118–2129, 2014.

[14] E. Ardjmand, N. Park, G. Weckman, and M. R. Amin-Naseri, "The discrete unconscious search and its application to uncapacitated facility location problem", *Computers & Industrial Engineering*, vol. 73, pp. 32–40, 2014.

[15] A. N. Letchford, and S. J. Miller, "An aggressive reduction scheme for the simple plant location problem", *European Journal of Operational Research*, vol. 234, no. 3, pp. 674–682, 2014.

[16] E. Monabbati, and H. T. Kakhki, "On a class of subadditive duals for the uncapacitated facility location problem", *Applied Mathematics and Computation*, vol. 251, pp. 118–131, 2015.

[17] B. M. Khumawala, "An efficient heuristic procedure for the capacitated warehouse location problem", *Naval Research Logistics Quarterly*, vol. 21, pp. 609–623, 1974.

[18] A. M. Geoffrion, and R. McBride, "Lagrangean relaxation to capacitated facility location problems", *AIIE Transactions*, 10, pp. 40–47, 1978.

[19] Nauss, R.M., "An improved algorithm for the capacitated facility location problem", *The Journal of the Operational Research Society*, vol. 29, pp. 1195–1201, 1978.

[20] N. Christofides, and J. E. Beasley, "Extensions to a lagrangean relaxation approach for the capacitated warehouse location problem", *European Journal of Operational Research*, vol. 12, pp. 19–28, 1983.

[21] H. Pirkul, "Efficient algorithms for the capacitated concentrator location problem", *Computers & Operations Research*, vol. 14, pp. 197–208, 1987.

[22] B. Shetty, "Approximate solutions to large scale capacitated facility location problems", *Applied Mathematics and Computation*, vol. 39, pp. 159–175, 1990.

[23] T. L. Magnanti, and R. T. Wong, "Accelerating benders decomposition: Algorithmic enhancement and model selection criteria", Operations Research, vol. 29, pp. 464–484, 1981.

[24] S. K. Jacobsen, "Heuristics for the capacitated plant location model" *European Journal of Operational Research*, vol. 12, pp. 253–261, 1983.

[25] W. Domschke, and A. Drexl, "ADD-heuristics' starting procedures for capacitated plant location models", *European Journal of Operational Research*, vol. 21, pp. 47–53, 1985.

[26] T. J. Van Roy, "A cross decomposition algorithm for capacitated facility location", *Operations Research*, vol. 34, pp. 145–163, 1986.

[27] J. M. Y. Leung, and T. L. Magnanti, "Valid inequalities and facets of the capacitated plant location problem", *Mathematical Programming*, vol. 44, pp. 271–291, 1989.

[28] G. R. Mateus, and C. T. Bornstein, "Dominance criteria for the capacitated warehouse location problem", *The Journal of the Operational Research Society*, vol. 42, 145–149, 1991.

[29] K. Aardal, Y. Pochet, and L. A. Wolsey, "Capacitated facility location: Valid inequalities and facets", *Mathematics of Operations Research*, vol. 20, pp. 552–582, 1995.

[30] E. Rolland, D. A. Schilling, and J. R. Current, "An efficient tabu search procedure for the p-median problem", *European Journal of Operational Research*, 96, pp. 329–342, 1996.

[31] C. T. Bornstein, and H. B. Azlan, "The use of reduction tests and simulated annealing for the capacitated location problem", *Location Science*, vol. 6, pp. 67–81, 1998.

[32] G. Ghiani, F. Guerriero, and R. Musmanno, "The capacitated plant location problem with multiple facilities in the same time", *Computers and Operations Research*, vol. 50, pp. 268–274, 1999.

[33] L. A. N. Lorena, and E. L. F. Senne, "A column generation approach to capacitated p-median", *Computers & Operations Research*, vol. 31, pp. 863–876, 2004.

[34] S. H. Doong, C. C. Lai, and C. H. Wu, "Genetic subgradient method for solving location-allocation problems", *Applied Soft Computing*, 7 (1), pp. 373-386, 2007.

[35] A. Klose, and S. Görtz, "A branch-and-price algorithm for the capacitated facility location problem", *European Journal of Operational Research*, vol. 179, pp. 1109–1125, 2007.

[36] M. A. Sambola, E. Fernandez, and F. S. Gama, "The facility location problem with Bernoulli demands", *Omega*, vol. 39, pp. 335–345, 2011.

[37] T. Küçükdeniz, A. Baray, K. Ecerkale, and Ş. Esnaf, "Integrated use of fuzzy c-means and convex programming for capacitated multi-facility location problem", *Expert Systems with Applications*, vol. 39, pp. 4306–4314, 2012.

[38] A. Rahmani, and M. A. MirHassani, "A hybrid firefly-genetic algorithm for the capacitated facility location problem", *Information Sciences*, vol. 283, 70–78, 2014.

[39] I. Harris, C. L. Mumford, and M. M. Naim, "A hybrid multi-objective approach to capacitated facility location with flexible store allocation for green logistics modeling", *Transportation Research*, E 66, pp. 1–22, 2014.

[40] D. Ozgen, and B. Gulsun, "Combining possibilistic linear programming and fuzzy AHP for solving the multi-objective capacitated multi-facility location problem", *Information Sciences*, vol. 268, pp. 185–201 2014.

[41] J. Li, F. Chu, C. Prins, and Z. Zhu, "Lower and upper bounds for a two-stage capacitated facility location problem with handling costs", *European Journal of Operational Research*, vol. 236, pp. 957–967, 2014.

[42] K. Aardal, P. L. V. D. Berf, D. Gijswijt, and S. Li, "Approximation algorithms for hard capacitated k-facility location problems", *European Journal of Operational Research*, vol. 242, pp. 358–368, 2015.

[43] R. V. Nagelhout, and G. L. Thompson, "A single source transportation algorithm", *Computers & Operations Research*, vol. 7, no. 3, pp. 185-198, 1980.

[44] A. W. Neebe, and M. R. Rao, "An algorithm for the fixed-charge assigning users to sources problem", *The Journal of the Operational Research Society*, vol. 34, pp. 1107–1113, 1983.

[45] J. G. Klincewicz, and H. Luss, "A lagrangian relaxation heuristic for capacitated facility location with single-source constraints", *Journal of the Operational Research Society,* vol. 37, no. 5, pp. 495–500, 1986.

[46] K. Darby-Dowman, and H. S. Lewis, "Lagrangian relaxation and the single source capacitated facility location problem", *Journal of the Operational Research Society,* vol. 39, pp. 1035–1040, 1988.

[47] R. Sridharan, "A Lagrangian heuristic for the capacitated plant location problem with single source constraints", *European Journal of Operational Research*, vol. 66, pp. 305–312, 1993.

[48] S. Tragantalerngsak, J. Holt, and M. Rönnqvist, "Lagrangian heuristics for the two-echelon, single-source, capacitated facility location problem", *European Journal of Operational Research,* vol. 102, pp. 611–625, 1997.

[49] M. Rönnqvist, S. Tragantalerngsak, and J. Holt, "A repeated matching heuristic for the single-source capacitated facility location problem", *European Journal of Operational Research*, vol. 116, pp. 51-68, 1999.

[50] H. Delmaire, J. A. Dı´az, and E. Ferna´ndez, "Reactive GRASP and tabu search based heuristics for the single source capacitated plant location problem" INFOR, *Canadian Journal of Operational Research and Information Processing*, vol. 37, pp. 194–225, 1999.
[51] K. Holmberg, M. Ronnqvist, and D. Yuan, "An exact algorithm for the capacitated facility location problems with single sourcing", *European Journal of Operational Research*, vol. 113, pp. 544-559, 1999.
[52] H. Hindi, and K. Pienkosz, "Efficient solution of large scale, single-source, capacitated plant location problem", *Journal of the Operational Research Society*, vol. 50, 268–274, 1999.
[53] S. Tragantalerngsak, J. Holt, and M. Rönnqvist, "An exact method for the two-echelon, single source, capacitated facility location problem", *European Journal of Operational Research*, vol. 123, no. 3, pp. 473–489, 2000.
[54] M. J. Cortinhal, and M. E. Captivo, "Upper and lower bounds for the single source capacitated location problem", *European Journal of Operational Research,* vol. 151, no. 2, pp. 333–351, 2003.
[55] R. K. Ahuja, J. B. Orlin, S. Pallottino, M. P. Scaparra, and M. G. Scutellá, "A multi-exchange heuristic for the single-source capacitated facility location problem", *Management Science*, vol. 50, no. 6, pp. 749-760, 2004.
[56] C. H. Chen, and C. J. Ting, "Applying multiple ant colony system to solve the single source capacitated facility location problem", *Lecture Notes in Computer Science*, vol. 4150, pp. 508–509, 2006.
[57] C. H. Chen, and C. J. Ting, "Combining lagrangian heuristic and ant colony system to solve the single source capacitated facility location problem", *Transportation Research Part E*, vol. 44, no. 1, pp. 1099–1122, 2008.
[58] I. Correia, and M. E. Captivo, "Bounds for the single source modular capacitated plant location problem", *Computers & Operations Research*, vol. 33, pp. 2991–3003, 2006.
[59] C. K. Y. Lin, "Stochastic single-source capacitated facility location model with service level requirements", *International Journal of Production Economics*, 117, pp. 439–451, 2009.
[60] Z. Yang, F. Chu, and H. Chen, "A cut-and-solve based algorithm for the single-source capacitated facility", *European Journal of Operational Research*, vol. 221, pp. 521–532, 2012.
[61] B. Addis, G. Carello, and A. Ceselli, "Combining very large scale and ILP based neighborhoods for a two-level location problem", *European Journal of Operational Research*, vol. 231, pp. 535–546, 2013.
[62] G. Guastaroba, and M. G. Speranza, "A heuristic for BILP problems: The single source capacitated facility location problem", *European Journal of Operational Research,* vol. 238, pp. 438–450 2014.
[63] S. Dantrakul, C. Likasiri, and R. Pongvuthithum, "Applied p-median and p-center algorithms for facility location problems", *Expert Systems with Applications*, vol. 41, pp. 3596–3604, 2014.
[64] M. Bieniek, "A note on the facility location problem with stochastic demands", Omega, vol. 55, pp. 53–60, 2015.