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Confidence Intervals for the Coefficients of Variation with Bounded Parameters

Jeerapa Sappakitkamjorn, Sa-aat Niwitpong

Abstract**—**In many practical applications in various areas, such as engineering, science and social science, it is known that there exist bounds on the values of unknown parameters. For example, values of some measurements for controlling machines in an industrial process, weight or height of subjects, blood pressures of patients and retirement ages of public servants. When interval estimation is considered in a situation where the parameter to be estimated is bounded, it has been argued that the classical Neyman procedure for setting confidence intervals is unsatisfactory. This is due to the fact that the information regarding the restriction is simply ignored. It is, therefore, of significant interest to construct confidence intervals for the parameters that include the additional information on parameter values being bounded to enhance the accuracy of the interval estimation. Therefore in this paper, we propose a new confidence interval for the coefficient of variance where the population mean and standard deviation are bounded. The proposed interval is evaluated in terms of coverage probability and expected length via Monte Carlo simulation.

Keywords**—**Bounded parameters, coefficient of variation, confidence interval, Monte Carlo simulation.

I. INTRODUCTION

THE coefficient of variance (CV) has been one of the most widely used statistical measures of the relative dispersion since it was introduced by Karl Pearson in 1896 [1]. This is due to its important property such that it is a dimensionless (unit-free) measure of variation and also its ability that can be used to compare several variables or populations with different units of measurement [2], [3]. As a result, it has been frequently used in numerous fields of knowledge. Here are some examples of the use of the CV. In science, the CV is often used as a measure of precision of measurement, and also used to compare the precision of laboratory experiments or techniques. In engineering, it is commonly used to evaluate the variability of strength of building materials. It is defined as a reference parameter for measurements in clinical diagnostics in medicine, and treated as a measure of risk to return in finance [3]–[5]. Recent applications of the CV in business, climatology and other fields are briefly reviewed in [6].

In most situations, the CV of a population is practically unknown. Therefore the sample CV is required to estimate the unknown value. However, for statistical inference purpose and to make best use of the sample CV, it is necessary to construct a confidence interval for the population CV. This is simply due to the fact that a confidence interval provides much more information about the parameter of interest than does a point estimate. As discussed in [7], the confidence interval is more informative than is the point estimates itself, because it contains all plausible values for the estimate of the unknown parameter with a specified level of confidence. In addition, the width of the confidence interval shows how accurate we believe our estimate is, i.e., the smaller width, the more precise our estimate of the parameter.

In general, confidence intervals of scale parameters, when parameter space is restricted, have received little attention. The development in this area is concentrated on location parameters (the population mean and the difference of two means) as presented in [8]–[12].

Although there have been a number of the confidence intervals for the CV, as appeared in recent literature [13], [14], the confidence intervals for the CV with restricted parameters have not much been done. It is, therefore, of significant interest to construct confidence intervals for the scale parameters. In this study we choose the CV as a parameter of our interest because of its widespread use in describing the variation within a data set. Moreover, among scale parameters, the CV is a more informative quantity than others. As noted in [15], the CV is preferred to the variance or standard deviation in various fields of interest, especially in biological and medical research.

The rest of the paper is organized as follows. Section II provides confidence intervals for the CV obtained by several existing methods when data are normally distributed. Then confidence intervals for the CV when the population mean and standard deviation are bounded are proposed in Section III. Section IV presents the results of simulation studies and reports on the performance of the proposed confidence intervals. Finally, Section V gives a conclusion with a few remarks.

II. CONFIDENCE INTERVAL FOR COEFFICIENT OF VARIATION FROM NORMAL DISTRIBUTION

In this section we review some of the existing methods for constructing confidence intervals of the CV when data are normally distributed. The population CV (denoted as τ) is defined as a ratio of the population standard deviation (σ) to the population mean (μ , $\mu \neq 0$), i.e.,

$$\tau = \frac{\sigma}{\mu} \qquad (1)$$

Jeerapa Sappakitkamjorn is with the Department of Applied Statistics, King Mongkut's University of Technology North Bangkok, Bangkok 10800 Thailand (phone: +662 555-2000 Ext. 4919; fax: +662 585-6105; e-mail: jsj@ kmutnb.ac.th).

Sa-aat Niwitpong is with the Department of Applied Statistics, King Mongkut's University of Technology North Bangkok, Bangkok 10800 Thailand (e-mail: snw@kmutnb.ac.th).

Let $x_1, x_2, x_3, ..., x_n$ be an independently and identically distributed (iid) random sample of size n from a normal distribution, $N(\mu, \sigma^2)$. The sample mean ($\bar{x}$) and sample variance (s^2) are the unbiased estimates of μ and σ^2, respectively. Therefore the typical sample estimate of τ is given as

$$cv = \frac{s}{\bar{x}} \quad (2)$$

where $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ and $s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$.

To construct a confidence interval of the CV, there are several methods available. In this study we consider 6 methods namely Miller's, McKay's, Vangel's, two new methods proposed by Mahmoudvand, and Hassani [1] and the Method of Variance Estimates Recovery (MOVER) [16].

The lower and upper confidence limits of the $100(1-\alpha)$% confidence interval for τ from each method are obtained by the followings.

A. Miller's Confidence Interval

Miller [2] proposed a confidence interval based on the sample CV that approximates an asymptotic normal distribution. Miller's method, referred to as Mil, has confidence limits $[L_{Mil}, U_{Mil}]$ given by

$$\begin{cases} L_{Mil} = cv - z_{1-\alpha/2}\left[\dfrac{cv^2}{n-1}\left(\dfrac{1}{2} + cv^2\right)\right]^{1/2} \\ U_{Mil} = cv + z_{1-\alpha/2}\left[\dfrac{cv^2}{n-1}\left(\dfrac{1}{2} + cv^2\right)\right]^{1/2} \end{cases} \quad (3)$$

where $z_{1-\alpha/2}$ is the $100(1-\alpha/2)$% percentile of the standard normal distribution.

B. McKay's Confidence Interval

McKay [17] developed a confidence interval for normal CV by using the approximation method. McKay's method, referred to as McK, has confidence limits $[L_{Mck}, U_{Mck}]$ given by

$$\begin{cases} L_{Mck} = cv\left[\left(\dfrac{u_1}{n} - 1\right)cv^2 + \dfrac{u_1}{n-1}\right]^{-1/2} \\ U_{Mck} = cv\left[\left(\dfrac{u_2}{n} - 1\right)cv^2 + \dfrac{u_2}{n-1}\right]^{-1/2} \end{cases} \quad (4)$$

where $u_1 = \chi^2_{(n-1),1-\alpha/2}$ and $u_2 = \chi^2_{(n-1),\alpha/2}$ are respectively the $100(1-\alpha/2)$ and $100(\alpha/2)$ percentile of the chi-square distribution with $(n-1)$ degrees of freedom.

C. Vangel's Confidence Interval

Vangel [18] modified a confidence interval proposed byMcKay to obtain a nearly exact interval. Vangel's method, referred to as Van, provides confidence limits slightly different from those obtained by McKay's in (4); Vangel's confidence interval $[L_{Van}, U_{Van}]$ is given by

$$\begin{cases} L_{Van} = cv\left[\left(\dfrac{u_1+2}{n} - 1\right)cv^2 + \dfrac{u_1}{n-1}\right]^{-1/2} \\ U_{Van} = cv\left[\left(\dfrac{u_2+2}{n} - 1\right)cv^2 + \dfrac{u_2}{n-1}\right]^{-1/2} \end{cases} \quad (5)$$

D. Mahmoudvand, and Hassani's Confidence Intervals

Mahmoudvand, and Hassani [1] introduced two new confidence intervals for the CV when data are normally distributed. They are respectively referred to, in this study, as M&H(I)'s and M&H(II)'s.

M&H(I)'s confidence interval: by using the normal approximation, its confidence limits $[L_{H\&M(I)}, U_{H\&M(I)}]$ are given by

$$\begin{cases} L_{H\&M(I)} = cv\left(2 - c_n + z_{1-\alpha/2}\sqrt{1-c_n^2}\right)^{-1} \\ U_{H\&M(I)} = cv\left(2 - c_n - z_{1-\alpha/2}\sqrt{1-c_n^2}\right)^{-1} \end{cases} \quad (6)$$

where $c_n = (2/(n-1))^{1/2}[(\Gamma(n/2)/(\Gamma((n-1)/2)]$.

M&H(II)'s confidence interval: Mahmoudv and, and Hassani introduced a new approximate point estimator $\hat{\tau} = cv/(2-c_n)$ for τ. They showed that the new estimator not only gives smaller variance than the typical estimator cv but it is also asymptotically unbiased. The M&H(II)'s confidence limits $[L_{H\&M(II)}, U_{H\&M(II)}]$ are given by

$$\begin{cases} L_{H\&M(II)} = \hat{\tau} - \dfrac{\hat{\tau}}{2-c_n} z_{1-\alpha/2}\sqrt{(1-c_n^2) + \dfrac{\hat{\tau}^2}{n}} \\ U_{H\&M(II)} = \hat{\tau} + \dfrac{\hat{\tau}}{2-c_n} z_{1-\alpha/2}\sqrt{(1-c_n^2) + \dfrac{\hat{\tau}^2}{n}} \end{cases} \quad (7)$$

E. MOVER's Confidence Interval

Donner and Zou [16] presented closed-form confidence intervals for functions of the normal mean and standard deviation including the coefficient of variation. By using the method of variance estimates recovery or MOVER, referred to as MOV, confidence limits $[L_{MOV}, U_{MOV}]$ for the CV based on separate confidence limits computed for the mean and standard deviation are given by

$$\begin{cases} L_{MOV} = (\bar{x} - \sqrt{\max\{0, \bar{x}^2 + ac(a-2)\}})\, s / c \\ U_{MOV} = (\bar{x} + \sqrt{\max\{0, \bar{x}^2 + bc(b-2)\}})\, s / c \end{cases} \tag{8}$$

where
$$\begin{cases} a = \sqrt{(n-1)/u_1} \\ b = \sqrt{(n-1)/u_2} \\ c = \bar{x}^2 - z^2_{\alpha/2} s^2 / n. \end{cases}$$

III. Confidence Interval for Coefficient of Variation with Bounded Parameters

Following the idea presented by Wang [19], we propose confidence intervals for the CV when the unknown parameters μ and σ are bounded. Although, a true value of a parameter of interest is practically unknown, the parameter space is often known to be restricted and the bounds of the parameter space are known.

When a parameter to be estimated is bounded, it is widely accepted that a confidence interval for a parameter θ when $a < \theta < b$ is the confidence interval of the intersection between $a < \theta < b$ and $[L_\theta, U_\theta]$, where L_θ and U_θ are lower and upper limits of the confidence interval for θ, thus in this situation the confidence interval for θ, denoted as CI_θ, is given by

$$CI_\theta = \left[\max\left(a, L_\theta\right), \min\left(b, U_\theta\right)\right] \tag{9}$$

There are four possible outcomes for the confidence interval in (9) as follows:

a) if $a > L_\theta$ and $b > U_\theta$ then CI_θ is reduced to
$$CI_{\theta a} = \left[a, U_\theta\right] \tag{10}$$

b) if $a > L_\theta$ and $b < U_\theta$ then CI_θ is reduced to
$$CI_{\theta b} = \left[a, b\right] \tag{11}$$

c) if $a < L_\theta$ and $b > U_\theta$ then CI_θ is reduced to
$$CI_{\theta c} = \left[L_\theta, U_\theta\right] \tag{12}$$

d) if $a < L_\theta$ and $b < U_\theta$ then CI_θ is reduced to
$$CI_{\theta d} = \left[L_\theta, b\right] \tag{13}$$

When the population mean is bounded, say $a < \mu < b$ where $0 < a < b$, it is straight forward to show that the population variance and the standard deviation are also bounded as follows:

$$\begin{aligned} a < \mu < b \quad &\Rightarrow \quad a^2 < \mu^2 < b^2 \\ &\Rightarrow \quad -b^2 < -\mu^2 < -a^2 \\ &\Rightarrow \quad \frac{\sum X_i^2}{N} - b^2 < \frac{\sum X_i^2}{N} - \mu^2 < \frac{\sum X_i^2}{N} - a^2 \\ &\Rightarrow \quad \sigma_b^2 < \sigma^2 < \sigma_a^2 \\ &\Rightarrow \quad \sigma_b < \sigma < \sigma_a \end{aligned}$$

where $\sigma_a = \sqrt{\frac{\sum X_i^2}{N} - a^2}$ and $\sigma_b = \sqrt{\frac{\sum X_i^2}{N} - b^2}$.

Similarly, the bounded population mean can lead to the bounded population CV.

$$\begin{aligned} a < \mu < b \quad &\Rightarrow \quad \frac{1}{a} > \frac{1}{\mu} > \frac{1}{b} \\ &\Rightarrow \quad \frac{1}{b} < \frac{1}{\mu} < \frac{1}{a} \\ &\Rightarrow \quad \frac{\sigma}{b} < \frac{\sigma}{\mu} < \frac{\sigma}{a} \end{aligned}$$

Since $\sigma_b < \sigma < \sigma_a$, we have

$$\begin{aligned} &\Rightarrow \quad \frac{\sigma_b}{b} < \frac{\sigma}{\mu} < \frac{\sigma_a}{a} \\ &\Rightarrow \quad \frac{\sigma_b}{b} < \tau < \frac{\sigma_a}{a} \end{aligned}$$

Thus the CV is also bounded when the mean and standard deviation are bounded.

According to Wang [19] and Niwitpong [20], the proposed confidence interval for τ with bounded mean and standard deviation based on (9) is given by

$$CI_\tau = \left[\max\left(\frac{\sigma_b}{b}, L_\tau\right), \min\left(\frac{\sigma_a}{a}, U_\tau\right)\right] \tag{14}$$

Equation (14) and confidence limits from existing methods presented in Section II are then used to obtain confidence intervals for τ when the mean and standard deviation are bounded.

IV. Simulation Studies

In this study, we examine the performance of the proposed confidence intervals for the CV of a normal distribution under the additional information that the population means lies in some bounded interval. In addition, we compare the proposed confidence intervals to those obtained from the existing methods in terms of coverage probability and average length of the confidence intervals. Simulation studies using different values of sample size (n = 5, 10, 15, 25, 50, and 100) and coefficients of variation (CV = 0.05, 0.10, 0.20, 0.33, and 0.50) are considered. Without loss of generality, the population variance is set to 1, i.e., we consider a sample taken from a population that has $N(\mu, 1)$, where μ is adjusted to get the required CV. Thus μ = 2, 3, 5, 10, and 20. Each value of μ is set to lie in a bounded interval that has two standard deviation wide. Then 95% confidence intervals are constructed based on the existing methods with unbounded and bounded parameters.

The results via Monte Carlo simulation with 10,000 runs for each combination of n and CV, using functions written in R, are summarized in Tables I and II. By detailing the estimated coverage probabilities and the average lengths (in parentheses) for the 95% confidence intervals based on six methods including sample sizes and the corresponding CV, Tables I and II present the simulation results for the cases of unbounded and bounded parameters, respectively.

In terms of coverage probabilities, the results in Tables I and II have identical coverage probabilities for sample size larger than 5. All methods perform very well as the coverage probabilities exceed the nominal level of 95% and reached 100%. However, the average lengths of the proposed intervals from all methods are similar or shorter in length.

When $n = 5$ and CV=0.05, Miller's method and H&M(II)'s method are inferior to the other methods in terms of coverage probabilities. As seen in Tables I and II, these two methods have coverage probabilities lower than the nominal level. This is because; their confidence intervals are much shorter than those obtained by other methods. For CV = 0.50, MOVER's intervals become negative and have notably wider interval lengths than the other intervals.

In most cases, the coverage probabilities of the proposed intervals that include the bounds of parameters in Table II and those of the existing intervals that exclude bounds of parameters in Table I are not only higher than the nominal level but also identical. Thus, in order to access the performance of confidence intervals, the average lengths are compared. For this purpose, the ratios of average lengths from unbounded intervals to those from bounded intervals are calculated and presented in Table III. If the ratio is greater than one, the average lengths from unbounded intervals are wider than those from bounded intervals. It can be easily seen that the ratios are equal or greater than one in most cases, i.e., the proposed confidence intervals have narrower widths. We can conclude that the proposed confidence intervals are superior to the intervals obtained from the existing methods that do not take the bounds of parameters into account.

TABLE I
COVERAGE PROBABILITY AND AVERAGE LENGTH OF 95% CONFIDENCE INTERVALS FOR CV WITH UNBOUNDED PARAMETERS

n	Method	CV				
		0.05	0.10	0.20	0.33	0.50
5	Miller	0.848 (0.065)	0.977 (0.132)	0.997 (0.274)	1.000 (0.508)	1.000 (0.936)
	McKay	0.952 (0.109)	0.998 (0.228)	1.000 (0.598)	1.000 (1.184)	1.000 1.350)
	Vangel	0.952 (0.108)	0.998 (0.220)	1.000 (0.484)	1.000 (0.917)	1.000 (1.173)
	H&M(I)	0.944 (0.093)	0.997 (0.186)	1.000 (0.372)	1.000 (0.634)	1.000 0.987)
	H&M(II)	0.797 (0.056)	0.961 (0.113)	0.992 (0.232)	0.999 (0.424)	1.000 0.762)
	MOVER	0.954 (0.108)	0.998 (0.224)	1.000 (0.518)	1.000 (1.512)	1.000 (0.074)
10	Miller	1.000 0.045)	1.000 0.092)	1.000 0.189)	1.000 0.340)	1.000 (0.588)
	McKay	1.000 0.056)	1.000 (0.114)	1.000 0.245)	1.000 0.518)	1.000 (1.109)
	Vangel	1.000 (0.055)	1.000 (0.113)	1.000 (0.238)	1.000 (0.461)	1.000 0.930)
	H&M(I)	1.000 (0.052)	1.000 (0.105)	1.000 (0.210)	1.000 (0.352)	1.000 (0.536)
	H&M(II)	1.000 (0.042)	1.000 (0.086)	1.000 (0.182)	1.000 0.340)	1.000 (0.621)
	MOVER	1.000 (0.057)	1.000 (0.122)	1.000 (0.312)	1.000 (1.083)	1.000 (-0.971)
15	Miller	1.000 (0.036)	1.000 0.073)	1.000 (0.152)	1.000 (0.273)	1.000 (0.465)
	McKay	1.000 0.042)	1.000 0.084)	1.000 (0.179)	1.000 (0.343)	1.000 (0.740)
	Vangel	1.000 0.042)	1.000 (0.084)	1.000 (0.176)	1.000 (0.328)	1.000 (0.645)
	H&M(I)	1.000 0.040)	1.000 0.080)	1.000 0.161)	1.000 0.269)	1.000 (0.407)
	H&M(II)	1.000 0.035)	1.000 (0.072)	1.000 (0.155)	1.000 (0.302)	1.000 (0.568)
	MOVER	1.000 0.043)	1.000 (0.095)	1.000 (0.263)	1.000 0.915)	1.000 5.952)
25	Miller	1.000 0.028)	1.000 (0.057)	1.000 (0.117)	1.000 (0.209)	1.000 (0.351)
	McKay	1.000 (0.030)	1.000 0.061)	1.000 (0.128)	1.000 (0.236)	1.000 (0.441)
	Vangel	1.000 (0.030)	1.000 (0.061)	1.000 0.127)	1.000 (0.232)	1.000 (0.418)
	H&M(I)	1.000 0.030)	1.000 0.059)	1.000 0.119)	1.000 (0.198)	1.000 (0.298)
	H&M(II)	1.000 (0.028)	1.000 (0.057)	1.000 (0.129)	1.000 (0.266)	1.000 (0.518)
	MOVER	1.000 0.032)	1.000 0.074)	1.000 0.229)	1.000 (0.841)	1.000 (11.867)
50	Miller	1.000 (0.020)	1.000 (0.040)	1.000 (0.082)	1.000 (0.146)	1.000 0.244)
	McKay	1.000 0.021)	1.000 0.041)	1.000 0.086)	1.000 (0.155)	1.000 0.272)
	Vangel	1.000 0.021)	1.000 0.041)	1.000 (0.085)	1.000 (0.154)	1.000 (0.267)
	H&M(I)	1.000 0.020)	1.000 (0.040)	1.000 (0.081)	1.000 (0.135)	1.000 0.203)
	H&M(II)	1.000 (0.020)	1.000 0.043)	1.000 0.104)	1.000 0.233)	1.000 0.480)
	MOVER	1.000 0.023)	1.000 0.059)	1.000 (0.206)	1.000 (0.794)	1.000 (82.440)
100	Miller	1.000 (0.014)	1.000 0.028)	1.000 (0.058)	1.000 0.103)	1.000 (0.171)
	McKay	1.000 (0.014)	1.000 (0.029)	1.000 (0.059)	1.000 0.106)	1.000 (0.182)
	Vangel	1.000 0.014)	1.000 0.029)	1.000 0.059)	1.000 0.106)	1.000 (0.180)
	H&M(I)	1.000 (0.014)	1.000 0.028)	1.000 (0.056)	1.000 (0.094)	1.000 (0.141)
	H&M(II)	1.000 (0.014)	1.000 (0.033)	1.000 (0.089)	1.000 (0.215)	1.000 (0.459)
	MOVER	1.000 0.017)	1.000 (0.050)	1.000 (0.195)	1.000 (0.778)	1.000 8.729)

V. CONCLUSIONS

This study looks at the performance of the proposed confidence interval for the normal population CV by taking into account the bounds of the population mean and standard deviation. We consider six existing methods for constructing confidence intervals for the normal population CV and apply them into two main situations, unbounded and bounded parameter spaces.

The important result from this research is that when we take into account the bounds of the population mean and standard deviation, the confidence interval obtained from each method provides a smaller width in most cases. Moreover, the results of this study not only provide useful insights into the interval estimation when parameters being estimated are bounded but also support the argument by Koopmans et al. [21] that with some prior information about the range of the parameter μ , it is possible to obtain confidence intervals for τ that have finite length with probability one for all values of μ and σ .

A key advantage of adding the bounds of parameters into account is that we are certain that the confidence limits never go beyond their parameter bounds. As a result, it should be noted that if there is a presence of outliers in a data set, the width of a confidence interval obtained by including the bounds of the parameter space will be less effect from observations with extreme values.

TABLE II
COVERAGE PROBABILITY AND AVERAGE LENGTH OF 95% CONFIDENCE INTERVALS FOR CV WITH BOUNDED PARAMETERS

n	Method	CV				
		0.05	0.10	0.20	0.33	0.50
5	Miller	0.838 (0.050)	0.975 (0.102)	0.996 (0.214)	0.999 (0.382)	1.000 (0.645)
	McKay	0.950 (0.100)	0.997 (0.207)	1.000 (0.455)	1.000 (0.777)	1.000 (1.036)
	Vangel	0.950 (0.099)	0.997 (0.203)	1.000 (0.427)	1.000 (0.753)	1.000 (1.037)
	H&M(I)	0.944 (0.087)	0.997 (0.174)	1.000 (0.351)	1.000 (0.587)	1.000 (0.886)
	H&M(II)	0.781 (0.043)	0.956 (0.087)	0.990 (0.177)	0.997 (0.303)	1.000 (0.478)
	MOVER	0.951 (0.100)	0.998 (0.207)	1.000 (0.459)	1.000 (-0.138)	1.000 (-5.054)
10	Miller	1.000 (0.041)	1.000 (0.083)	1.000 (0.172)	1.000 (0.306)	1.000 (0.506)
	McKay	1.000 (0.055)	1.000 (0.111)	1.000 (0.240)	1.000 (0.491)	1.000 (0.932)
	Vangel	1.000 (0.055)	1.000 (0.111)	1.000 (0.233)	1.000 (0.449)	1.000 (0.842)
	H&M(I)	1.000 (0.051)	1.000 (0.102)	1.000 (0.205)	1.000 (0.342)	1.000 (0.519)
	H&M(II)	1.000 (0.038)	1.000 (0.077)	1.000 (0.154)	1.000 (0.260)	1.000 (0.400)
	MOVER	1.000 (0.056)	1.000 (0.119)	1.000 (0.303)	1.000 (0.776)	1.000 (-17.477)
15	Miller	1.000 (0.035)	1.000 (0.071)	1.000 (0.146)	1.000 (0.259)	1.000 (0.439)
	McKay	1.000 (0.041)	1.000 (0.084)	1.000 (0.176)	1.000 (0.338)	1.000 (0.700)
	Vangel	1.000 (0.041)	1.000 (0.083)	1.000 (0.174)	1.000 (0.323)	1.000 (0.627)
	H&M(I)	1.000 (0.039)	1.000 (0.079)	1.000 (0.158)	1.000 (0.264)	1.000 (0.402)
	H&M(II)	1.000 (0.033)	1.000 (0.067)	1.000 (0.135)	1.000 (0.227)	1.000 (0.353)
	MOVER	1.000 (0.042)	1.000 (0.094)	1.000 (0.258)	1.000 (0.815)	1.000 (-18.644)
25	Miller	1.000 (0.028)	1.000 (0.056)	1.000 (0.116)	1.000 (0.207)	1.000 (0.344)
	McKay	1.000 (0.030)	1.000 (0.061)	1.000 (0.128)	1.000 (0.236)	1.000 (0.436)
	Vangel	1.000 (0.030)	1.000 (0.061)	1.000 (0.127)	1.000 (0.232)	1.000 (0.413)
	H&M(I)	1.000 (0.029)	1.000 (0.059)	1.000 (0.118)	1.000 (0.198)	1.000 (0.296)
	H&M(II)	1.000 (0.027)	1.000 (0.054)	1.000 (0.110)	1.000 (0.186)	1.000 (0.288)
	MOVER	1.000 (0.032)	1.000 (0.074)	1.000 (0.227)	1.000 (0.802)	1.000 (-11.817)
50	Miller	1.000 (0.020)	1.000 (0.040)	1.000 (0.082)	1.000 (0.146)	1.000 (0.244)
	McKay	1.000 (0.021)	1.000 (0.041)	1.000 (0.086)	1.000 (0.155)	1.000 (0.272)
	Vangel	1.000 (0.021)	1.000 (0.041)	1.000 (0.085)	1.000 (0.154)	1.000 (0.267)
	H&M(I)	1.000 (0.020)	1.000 (0.040)	1.000 (0.081)	1.000 (0.135)	1.000 (0.203)
	H&M(II)	1.000 (0.019)	1.000 (0.039)	1.000 (0.079)	1.000 (0.137)	1.000 (0.219)
	MOVER	1.000 (0.023)	1.000 (0.058)	1.000 (0.206)	1.000 (0.786)	1.000 (-18.303)
100	Miller	1.000 (0.014)	1.000 (0.028)	1.000 (0.058)	1.000 (0.103)	1.000 (0.172)
	McKay	1.000 (0.014)	1.000 (0.029)	1.000 (0.059)	1.000 (0.106)	1.000 (0.183)
	Vangel	1.000 (0.014)	1.000 (0.029)	1.000 (0.059)	1.000 (0.106)	1.000 (0.181)
	H&M(I)	1.000 (0.014)	1.000 (0.028)	1.000 (0.056)	1.000 (0.094)	1.000 (0.141)
	H&M(II)	1.000 (0.014)	1.000 (0.028)	1.000 (0.057)	1.000 (0.102)	1.000 (0.170)
	MOVER	1.000 (0.017)	1.000 (0.050)	1.000 (0.195)	1.000 (0.775)	1.000 (-33.970)

TABLE III
RATIOS OF AVERAGE LENGTHS FROM UNBOUNDED INTERVALS TO AVERAGE LENGTHS FROM BOUNDED INTERVALS

n	Method	CV				
		0.05	0.10	0.20	0.33	0.50
5	Miller	1.300	1.294	1.280	1.330	1.451
	McKay	1.090	1.101	1.314	1.524	1.303
	Vangel	1.091	1.084	1.133	1.218	1.131
	H&M(I)	1.069	1.069	1.060	1.080	1.114
	H&M(II)	1.302	1.299	1.311	1.399	1.594
	MOVER	1.080	1.082	1.129	-10.957	-0.015
10	Miller	1.098	1.108	1.099	1.111	1.162
	McKay	1.018	1.027	1.021	1.055	1.190
	Vangel	1	1.018	1.021	1.027	1.105
	H&M(I)	1.020	1.029	1.024	1.029	1.033
	H&M(II)	1.105	1.117	1.182	1.308	1.553
	MOVER	1.018	1.025	1.030	1.396	-0.056
15	Miller	1.029	1.028	1.041	1.054	1.059
	McKay	1.024	1	1.017	1.015	1.057
	Vangel	1.024	1.012	1.011	1.015	1.029
	H&M(I)	1.026	1.013	1.019	1.019	1.012
	H&M(II)	1.061	1.075	1.148	1.330	1.609
	MOVER	1.024	1.011	1.019	1.123	-0.319
25	Miller	1	1.018	1.009	1.010	1.020
	McKay	1	1	1	1	1.011
	Vangel	1	1	1	1	1.012
	H&M(I)	1.034	1	1.008	1	1.007
	H&M(II)	1.037	1.056	1.173	1.430	1.799
	MOVER	1	1	1.009	1.049	1.004
50	Miller	1	1	1	1	1
	McKay	1	1	1	1	1
	Vangel	1	1	1	1	1
	H&M(I)	1	1	1	1	1
	H&M(II)	1.053	1.103	1.316	1.701	2.192
	MOVER	1	1.017	1	1.010	-4.504
100	Miller	1	1	1	1	0.994
	McKay	1	1	1	1	0.995
	Vangel	1	1	1	1	0.994
	H&M(I)	1	1	1	1	1
	H&M(II)	1	1.179	1.561	2.108	2.700
	MOVER	1	1	1	1.004	-0.257

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