

Periodic Mixed Convection of a Nanofluid in a Cavity with Top Lid Sinusoidal Motion

Arash Karimipour¹, M. Afrand², M. M. Bazofti³

Abstract—The periodic mixed convection of a water-copper nanofluid inside a rectangular cavity with aspect ratio of 3 is investigated numerically. The temperature of the bottom wall of the cavity is assumed greater than the temperature of the top lid which oscillates horizontally with the velocity defined as $u = u_0 \sin(\omega t)$. The effects of Richardson number, Ri , and volume fraction of nanoparticles on the flow and thermal behavior of the nanofluid are investigated. Velocity and temperature profiles, streamlines and isotherms are presented. It is observed that when $Ri < 1$, heat transfer rate is much greater than when $Ri > 1$. The higher value of Ri corresponds to a lower value of the amplitude of the oscillation of Nu_m in the steady periodic state. Moreover, increasing the volume fraction of the nanoparticles increases the heat transfer rate.

Keywords—Nanofluid, Top lid oscillation, Mixed convection, Volume fraction

I. INTRODUCTION

INVESTIGATIONS of mixed convection of a fluid confined in a cavity with moving wall have applications in problems such as manufacturing solar collectors, optimized thermal-designing of buildings, and cooling of electronic devices, and have attracted many researchers [1, 2].

Low thermal conductivity of conventional fluids such as water and oil in convection heat transfer is the main problem to increase the heat transfer rate in many engineering equipments. To overcome this problem, researchers have performed considerable efforts to increase conductivity of working fluid. An innovative way to increase conductivity coefficient of the fluid is to suspend solid nanoparticles in it and make a mixture called nanofluid, having larger thermal conductivity coefficient than that of the base fluid.

Free convection heat transfer and flow behavior of nanofluids for different volume fractions of nanoparticles in the cavities has been studied by some researchers [3-6]. These researchers used different models to calculate physical properties of nanofluid. They concluded that the increase of the volume fraction improves heat transfer rate. Moreover, Oztop and Abu-Nada [7] studied natural convection in a rectangular enclosure filled with a nanofluid containing Cu, Al_2O_3 and TiO_2 as nanoparticles. They concluded that the

highest value of heat transfer is obtained by using Cu nanoparticles.

In recent years, mixed convection heat transfer of a nanofluid has been studied. Tiwari and Das [8] studied heat transfer augmentation in a two-sided lid-driven square cavity filled with a Copper–Water nanofluid. They studied three cases depending on the direction of the moving walls and developed a model to analyze the behavior of nanofluids taking into account the solid volume fraction. Akbarinia and Behzadmehr [9] studied numerically fully developed laminar mixed convection of a nanofluid in horizontal curved tubes. Mirmasoumi and Behzadmehr [10] studied laminar mixed convection of a nanofluid in a horizontal tube using two-phase mixture model. In another work, Behzadmehr et al. [11] numerically studied turbulent forced convection heat transfer in a circular tube with a nanofluid consisting of water and 1% Cu. They showed that the particles can absorb the velocity fluctuation energy and reduce the turbulent kinetic energy.

Talebi et al. [12] numerically studied mixed convection flows in a square lid-driven cavity utilizing nanofluid. Their results showed that at a given Reynolds and Rayleigh numbers, solid concentration has a positive effect on the heat transfer enhancement. Moreover, their result indicated that for a given Reynolds number the increase of the solid concentration augments the stream function, particularly at the higher Rayleigh numbers. This result is also observed in the computation of the average Nusselt number. Muthtamilselvan et al. [13] studied heat transfer enhancement of copper-water nanofluids in a lid-driven enclosure. They studied the effect of aspect ratio and concluded that at higher values of aspect ratios, Nu is increased more with the increase of the volume fraction of nanoparticles. Khanafer et al. [14] numerically studied unsteady mixed convection of a simple fluid in a cavity using a lid sliding sinusoidally. They showed that the Reynolds and Grashof numbers have a profound effect on the structure of fluid flow and heat transfer fields. As a matter of fact, their effects are associated with the direction of the sliding lid. But, in the present paper, periodic mixed convection of a nanofluid in a cavity with top lid sinusoidal motion is studied numerically.

II. MATHEMATICAL FORMULATION

2-1. Problem statement

Periodic mixed convection of a nanofluid consisted of Water and Cu nanoparticles and confined in a vertical two dimensional rectangular cavity with $AR=3$ is studied numerically. Bottom wall of the cavity is at higher temperature than its top lid. Vertical walls of the cavity are

¹ Islamic Azad University, Najafabad Branch, Iran, (corresponding author; e-mail: arash.karimipour@pmc.iaun.ac.ir and arashkarimipour@gmail.com).

² Islamic Azad University, Najafabad Branch, Iran; (e-mail: masoud.afrand@pmc.iaun.ac.ir).

³ Islamic Azad University, Najafabad Branch, Iran; (e-mail: milad_mis@yahoo.com).

assumed insulated. The top lid oscillates compulsorily and horizontally with the velocity $u = u_0 \sin(\omega t)$ in which t is time, u_0 is amplitude of the oscillation, and ω is angular frequency of the motion. Gravitational acceleration is in the downward direction as shown in Fig. 1. Thermo physical properties of the fluid, except density in the incompressible momentum equation, are constant. Boussinesq approximation is used to state density variation with temperature. Nanofluid inside the cavity is Newtonian and incompressible and the flow is laminar. Nanoparticles are spherical and at the same diameter. The base fluid and nanoparticles are at thermal equilibrium and move with the same velocity, meaning that nanofluid mixture is homogeneous. Radiation heat transfer compared to other modes of heat transfer is small and negligible.

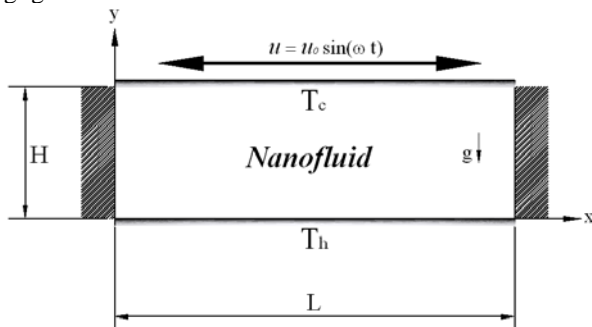


Fig. 1. Geometry and boundary conditions of the cavity.

2-2. Governing equations and boundary conditions

Nanofluid density and effective viscosity are defined as follows [15]:

$$\rho_{nf,0} = \chi \rho_{s,0} + (1 - \chi) \rho_{f,0} \quad (1)$$

$$\mu_{eff} = \mu_f / (1 - \chi)^{2.5} \quad (2)$$

Using formula presented by Xuan and Li [16], heat capacity of nanofluid is calculated as:

$$(\rho C_p)_{nf} = (1 - \chi)(\rho C_p)_f + \chi(\rho C_p)_s \quad (3)$$

in which subscript f represents base fluid, s solid particles and χ indicates volume fraction of nanoparticles.

The enhancement of effective thermal conductivity of nanofluid is mainly due to the localized Brownian movement of nanoparticles [17]. The following formula proposed by Chon et al. [18], which takes into account the Brownian motion and diameters of nanoparticles, is used to calculate effective conductivity of the nanofluid.

$$\frac{k_{eff}}{k_f} = 1 + 64.7 \times \chi^{0.7460} \left(\frac{d_f}{d_p} \right)^{0.3690} \times \left(\frac{k_s}{k_f} \right)^{0.7476} \left(\frac{\mu}{\rho_{f,0} \alpha_f} \right)^{0.9955} \left(\frac{\rho_{f,0} B_c T}{3\pi \mu^2 l_{BF}} \right)^{1.2321} \quad (4)$$

where l_{BF} is the mean free path of the base fluid, B_c is Boltzmann constant (1.3807×10^{-23} J/K) and μ is calculated as [10]:

$$\mu = A \times 10^{\frac{B}{T-C}}, \quad C = 140 \text{ (K)}, \quad B = 247 \text{ (K)}, \quad A = 2.414 \times 10^{-5} \text{ (Pa.s)} \quad (5)$$

The following dimensionless variables are introduced to convert the governing equations into non-dimensional ones.

$$Y = y/H, X = x/H, \quad V = v/u_0, U = u/u_0,$$

$$\theta = (T - T_c)/(T_h - T_c), \quad \tau = \frac{t u_0}{H},$$

$$S = \omega H / u_0, \quad P = \frac{P}{\rho u_0^2}, \quad Pr = \nu_f / \alpha_f, \quad (6)$$

$$Gr = g \beta_f H^3 (T_h - T_c) / \nu_f^2,$$

$$Re = u_0 H / \nu_f, \quad Ri = Gr / Re^2$$

Employing these dimensionless variables, the following non-dimensional equations are obtained [8].

Continuity equation:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (7)$$

Momentum equation along the X coordinates:

$$\frac{\partial U}{\partial \tau} + U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = - \frac{\rho_{f,0}}{\rho_{nf,0}} \frac{\partial P}{\partial X} + \frac{1}{\nu_f \cdot Re} \frac{\mu_{eff}}{\rho_{nf,0}} \left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right) \quad (8)$$

Momentum equation along the Y coordinates:

$$\frac{\partial V}{\partial \tau} + U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = - \frac{\rho_{f,0}}{\rho_{nf,0}} \frac{\partial P}{\partial Y} + \frac{1}{\nu_f \cdot Re} \frac{\mu_{eff}}{\rho_{nf,0}} \times \left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) + \frac{\chi \rho_s \beta_s + (1 - \chi) \rho_f \beta_f}{\beta_f \rho_{nf,0}} \frac{Gr}{Re^2} \theta \quad (9)$$

Energy equation:

$$\frac{\partial \theta}{\partial \tau} + U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{\alpha_{nf}}{\alpha_f} \frac{1}{Re \cdot Pr} \left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right) \quad (10)$$

Uniform wall temperature and insulated wall are assumed as boundary conditions for horizontal and vertical walls of the cavity respectively. No slip conditions are assumed as velocity boundary conditions on the walls. Top wall oscillates horizontally as shown in Fig. 1. Thus, dimensionless boundary conditions are written as:

$$U = V = \frac{\partial \theta}{\partial X} = 0; \quad \text{for } X = 0 \text{ or } X = 3, \quad 0 \leq Y \leq 1. \\ U = V = 0, \quad \theta = 1; \quad \text{for } Y = 0, \quad 0 \leq X \leq 3. \\ V = \theta = 0; \quad \text{for } Y = 1, \quad 0 \leq X \leq 3. \quad (11)$$

$$U = \sin(S\tau), \quad S = 0.105, \quad \gamma = \frac{2\pi}{S} = 60.$$

in which S and γ are dimensionless frequency and period of the lid respectively. In this work it is assumed that $S = 0.105$, so that $\gamma = 60$.

Dimensional initial conditions for temperature and velocity fields are assumed as $T = T_c$ and $u = v = 0$ respectively, so that $\theta = U = V = 0$ are used as dimensionless initial conditions for these fields.

Spatial local and averaged Nusselt numbers on the horizontal walls are calculated using relations (12) and (13) respectively [2,5].

$$Nu_x)_h = -\frac{k_{eff}}{k_f} \left(\frac{\partial \theta}{\partial Y} \right)_{Y=0}, Nu_x)_c = -\frac{k_{eff}}{k_f} \left(\frac{\partial \theta}{\partial Y} \right)_{Y=1} \quad (12)$$

$$Nu_m = \frac{1}{L} \int_0^L Nu_x dx = \frac{1}{AR} \int_0^{AR} Nu_x dX \quad (13)$$

An implicit scheme is employed to deal with the time differential terms, therefore time step does not affect the convergence of the solution [19]. However, to see the effect of lid oscillation on the flow, it should be a proper fraction of the period, $\Delta\tau = \gamma/60 = 1$. Because of the initial conditions, the problem is unsteady and the solution consists of transient and steady periodic parts. As time passes, solution tends to the steady periodic state, which is studied in this work.

2-3. Grid independency study and validation

To study grid independency of the solution, we consider cavity shown in Fig. 1 and solve it for the case $Ri = 1$, $\chi = 0\%$ (pure water) and $Gr = 10^4$. Three non-uniform grids including 150×50 , 180×60 and 210×70 nodes are used to perform the numerical solution.

The variation of θ at the midpoint of the cavity with coordinates $x = L/2$ and $y = H/2$ versus time is plotted in Fig. 2. It is observed that difference between the results of the grids with 180×60 and 210×70 nodes are very small, thus the grid with 180×60 nodes is chosen for the subsequent calculations.

The mixed convection problem investigated in ref. [8] and consisted of Cu-Water nanofluid inside a square cavity is studied to validate the program. In this case the left vertical wall of this cavity has lower temperature than the right vertical wall and moves with constant velocity upward, while right wall moves with constant velocity downward. In Tables 1 and 2 the Nu_m on the vertical wall and maximum velocity on the vertical centerline obtained from the present work are compared with those of ref. [8] at $Gr = 10^4$ and different Ri 's and nanoparticle volume fractions. These Tables show good agreement between the results.

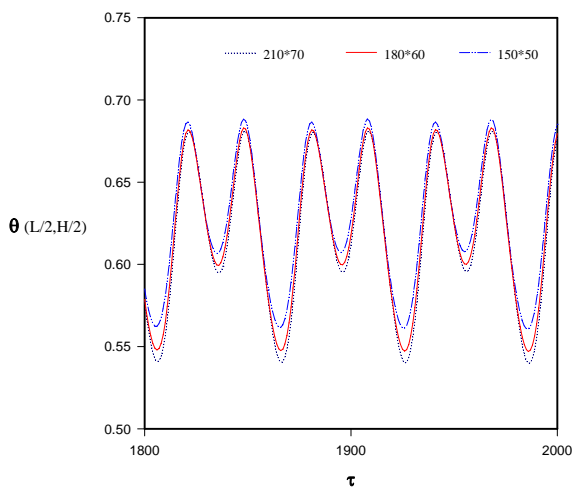


Fig. 2. Variation of θ versus time at the midpoint of cavity for $Ri = 1$, $\chi = 0\%$, $Gr = 10^4$ at three grids

TABLE I COMPARISON OF Nu_m ON THE VERTICAL WALL AND U_{MAX} ON THE VERTICAL CENTERLINE OBTAINED FROM THE PRESENT WORK WITH REF. [8] AT $Gr = 10^4$, $Ri = 0.1$

Parameter	$\chi = 0\%$		$\chi = 8\%$	
	Present work	Ref. [12]	Present work	Ref. [12]
U_{max}	0.47	0.48	0.45	0.46
Nu_m	30.68	31.64	43.16	43.37

TABLE II COMPARISON OF Nu_m ON THE VERTICAL WALL AND U_{MAX} ON THE VERTICAL CENTERLINE OBTAINED FROM THE PRESENT WORK WITH REF. [8] AT $Gr = 10^4$, $Ri = 10$

Parameter	$\chi = 0\%$		$\chi = 8\%$	
	Present work	Ref. [12]	Present work	Ref. [12]
U_{max}	0.19	0.18	0.15	0.15
Nu_m	1.38	1.55	1.85	2.03

III. RESULTS AND DISCUSSIONS

The results in this section obtained from the numerical solution of mixed convection of Cu-Water nanofluid inside a rectangular cavity as shown in Fig. 1 for the case $AR = 3$, $S = 0.105$, $Gr = 10^4$, $Pr = 6.2$ and thermophysical properties of water and nanoparticles stated as in Table 3. Using these results the effects of Ri and nanoparticle volume fraction are discussed in the following sections.

TABLE III THERMOPHYSICAL PROPERTIES OF THE BASE FLUID AND NANOPARTICLES

Property	base fluid (water)	nanoparticles (copper)
c_p (JKgK)	4179	383
ρ (Kg/m ³)	997.1	8954
K (W/mK)	0.6	400
β (K ⁻¹)	2.1×10^{-4}	1.67×10^{-5}

3-1. Effect of Richardson number

Fig. 3 shows the variation of horizontal component of dimensionless velocity, U , and temperature, θ , along the vertical centerline of the cavity for the case $Ri = 0.1$ and $\chi = 0\%$ at different λ . When $0 < \lambda < \gamma/2$, top lid velocity is positive, and thus applies shear stress on the fluid layers in the direction of positive X axis. On the other hand, when $\gamma/2 < \lambda < \gamma$, top lid moves in the direction of negative X axis, leading to the enhancements of fluid forced motions in the negative X axis. The direction of U is changing rapidly near the top lid, and its amplitudes decreases as we are approaching to the bottom wall of the cavity; as a result, the effect of the top lid velocity decreases with increasing distance from it, leading to the relative enhancement of buoyancy forces. Since these curves have been plotted in the steady periodic state, the profiles of $\lambda = 0$ and $\lambda = \gamma$ are similar and the profiles of $\lambda = \gamma/4$ with $\lambda = 3\gamma/4$ and also $\lambda = 0$ with $\lambda = \gamma/2$ are symmetric. It is also observed that the variations of temperature are lower in the core region of the cavity than the nearby- region of the top and bottom walls.

Fig. 4 shows the variation of U and θ along the vertical centerline of the cavity for the case $Ri = 10$ and $\chi = 0\%$ at different λ . It is observed that fluid flow near the top lid is strongly affected by its oscillatory motion. As we are

approaching the bottom wall, lower shear stress is applied on the fluid layers by the top lid, resulting in lower horizontal fluid velocity. As Ri increases, buoyancy motions dominate the forced motions in the core of the cavity and near the bottom wall, so unlike the case of Fig. 3, U at $\lambda = \gamma/4$ and $\lambda = \gamma/2$ are not symmetric with U at $\lambda = 3\gamma/4$ and $\lambda = \gamma$, respectively. In addition, temperature variations are almost small in these points.

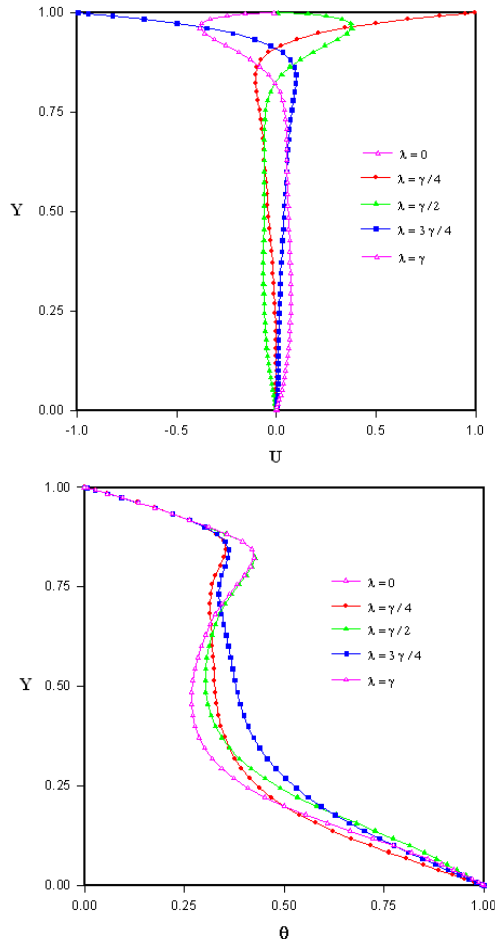


Fig. 3. Variation of U and θ along the vertical centerline for $Ri = 0.1$ and $\chi = 0\%$ at different λ 's

Fig. 5, 6 show the streamlines and isotherms for the cases $Ri = 0.1, 10$ respectively, each case at $\chi = 0\%$ and different λ 's, respectively. It is observed that at $\lambda = 0$ most of the cavity space is occupied by a circulation cell which its center is located adjacent to the top lid. At the same time a weak cell is being created in the bottom right side of the cavity which its intensity grows with time. At $\lambda = \gamma/4$ this cell covers almost all the cavity space and this process also occurs for other λ 's. Isotherms close to the top lid, especially in the middle parts, are almost horizontal, showing that heat transfer is one-dimensional in these regions. In this problem Gr remains constant, so a higher value of Ri corresponds to a lower value of Re . Fig. 6 shows that at $Ri = 10$, two cells occupy all of the cavity space without significant differences at different λ 's. It

means that steady periodic solution does not change significantly with time for this value of Ri .

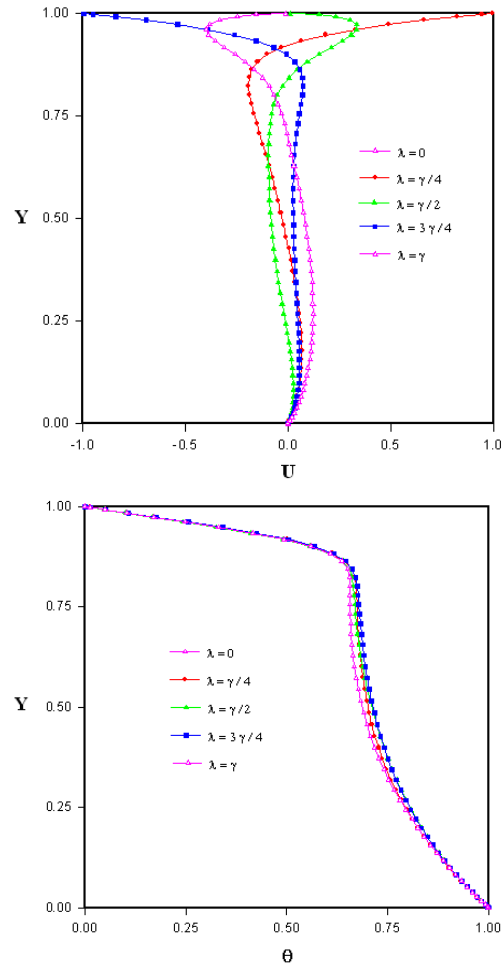


Fig. 4. Variation of U and θ along the vertical centerline for $Ri = 10$ and $\chi = 0\%$ at different λ 's

3-2. Effect of nanoparticle volume fraction

Figs. 7 and 8 show streamlines and isotherms for the cases $Ri = 0.1, Ri = 10$ and $\chi = 4\%$ at different λ 's. It is observed that for this case also at $Ri = 0.1$ a large rotational cell occupies and affects almost most of the cavity and its direction also depends strongly to the λ . At $Ri = 10$ two almost similar rotational cells at different λ 's fill the cavity. Comparison of these two figures with Fig's 5 and 6 show that at $Ri = 0.1$ the variations of streamlines and isotherms increases with the increase of nanoparticles volume fraction.

Fig. 9 and 10 show the variations of averaged Nusselt number of the cold wall, Nu_m , versus time at $\chi = 0, 2, 4\%$ for $Ri = 0.1, 10$, respectively. The increase of the nanoparticle concentration increases molecular thermal diffusion and then Nu_m . Previous studies of nanofluid mixed convection in an enclosure shows that Nu_m increases with the increase of volume fraction, but its growth rate is reduced with the increase of Ri [8]. However, the results of this work show that by adding wall oscillation, this weakness is compensated. For example, the results of Fig. 9 and 10 show that the increase of volume fraction increases Nu_m almost similarly at $Ri = 0.1$ 9

$Ri = 10$. Fig. 11 shows the variations of Nu_m for the cold wall versus time at $\chi = 0, 4\%$ for $Ri = 0.1, 10$, respectively. It is seen that at constant volume fraction, because of the less variation of the flow with time in this case, the amplitude of Nu_m decreases at higher value of Ri .

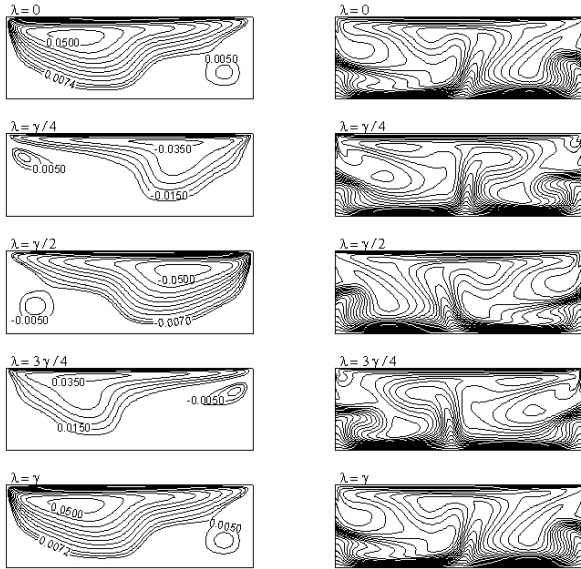


Fig. 5. Streamlines (left) and isotherms (right) for $Ri = 0.1$ and $\chi = 0\%$ at different λ 's

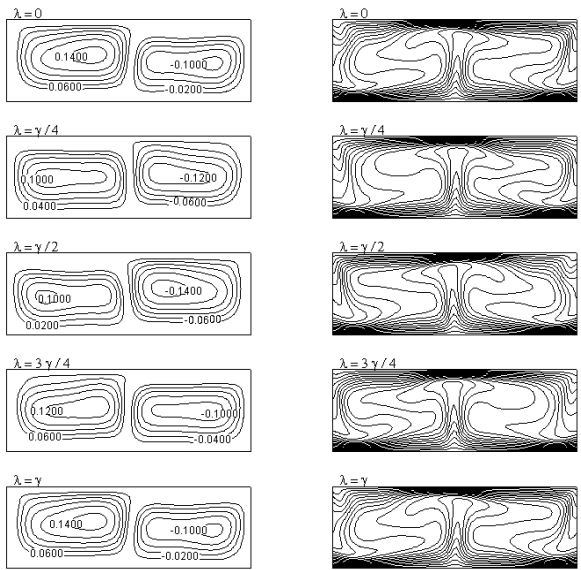


Fig. 6. Streamlines (left) and isotherms (right) for $Ri = 10$ and $\chi = 0\%$ at different λ 's

IV. CONCLUSION

A computer program was developed to study periodic mixed convection of a Water-Cu nanofluid inside a two dimensional rectangular cavity. Horizontal bottom wall of this cavity was at higher temperature than its top lid, and its vertical walls were insulated. In addition, top lid had sinusoidal motion horizontally. The effects of the variation of Ri and volume fraction on fluid flow and heat transfer in the

transient and steady periodic parts were studied and the following results were obtained.

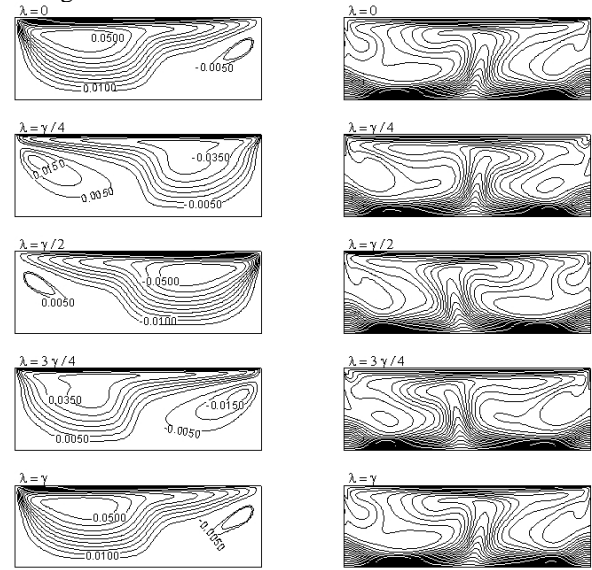


Fig. 7. Streamlines (left) and isotherms (right) for $Ri = 0.1$ and $\chi = 4\%$ at different λ 's

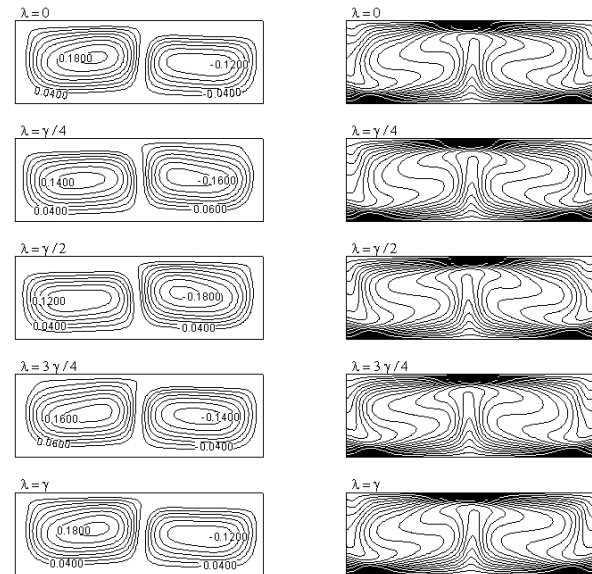


Fig. 8. Streamlines (left) and isotherms (right) for $Ri = 10$ and $\chi = 4\%$ at different λ 's

When $0 < \lambda < \gamma/2$, velocity of the top lid is in the positive direction, and thus applies positive shear stress to the fluid layers. On the other hand, When $\gamma/2 < \lambda < \gamma$, velocity of the top lid is in the negative direction, so it applies negative shear stress to the fluid layers. At $Ri = 0.1$, velocity profiles at $\lambda = \gamma/4$ and $\lambda = 3\gamma/4$ are symmetric with opposite directions, and similarly, this behavior happens at $\lambda = \gamma/2$ and $\lambda = \gamma$. However, at $Ri = 10$, the symmetry of velocity profile decreases.

It is observed that at constant volume fraction the higher value of Ri corresponds to a lower value of the amplitude of the oscillation of Nu_m in the steady periodic state. The increase

of volume fraction increases Nu_m almost similarly at $Ri = 0.1$, $Ri = 10$. So, by using wall with oscillatory-motion, nanofluid effects to increase heat transfer through cold or hot wall are maintained even at higher Ri numbers.

$\chi = 0, 4\%$ and different Ri 's.

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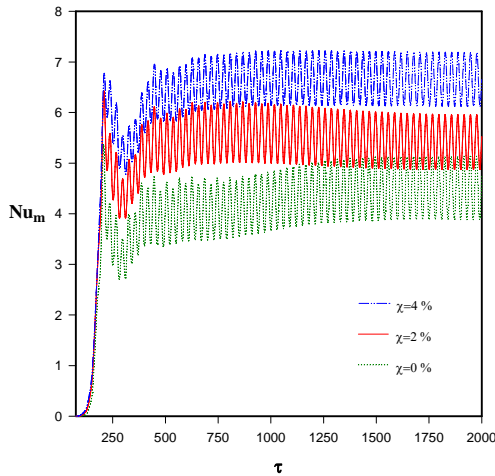


Fig. 9. Nu_m of the top lid versus time for $Ri = 0.1$ and different χ 's

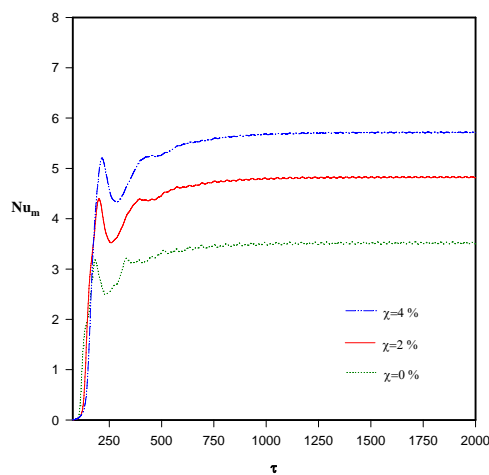


Fig. 10. Nu_m of the top lid versus time for $Ri = 10$ and different χ 's

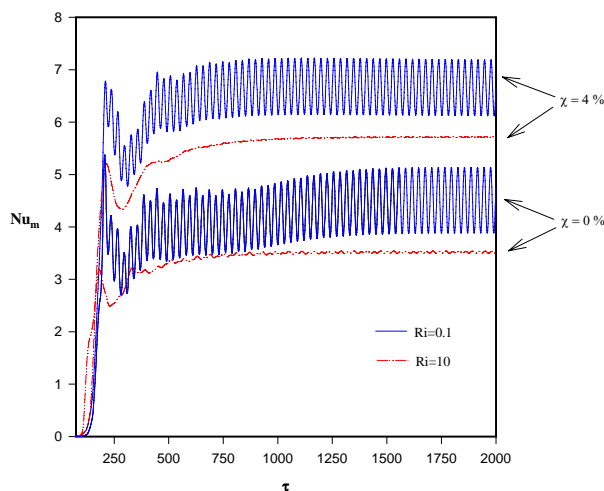


Fig. 11. Nu_m of the top lid versus time for