

# Effect of Adaptation Gain on system Performance for Model Reference Adaptive Control Scheme using MIT Rule

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**Abstract**—Adaptive control involves modifying the control law used by the controller to cope with the fact that the parameters of the system being controlled change drastically due to change in environmental conditions or in system itself. This technique is based on the fundamental characteristic of adaptation of living organism. The adaptive control process is one that continuously and automatically measures the dynamic behavior of plant, compares it with the desired output and uses the difference to vary adjustable system parameters or to generate an actuating signal in such a way so that optimal performance can be maintained regardless of system changes. This paper deals with application of model reference adaptive control scheme in first order system. The rule which is used for this application is MIT rule. This paper also shows the effect of adaptation gain on the system performance. Simulation is done in MATLAB and results are discussed in detail.

**Keywords**—Adaptive control system, Adaptation gain, MIT rule, Model reference adaptive control.

## I. INTRODUCTION

A control system is an arrangement of different physical elements connected in such a manner so as to regulate, direct or command itself or some other system. In short, a control system is in the broadest sense, an interconnection of the physical components to provide a desired function, involving some type of control action with it. The requirement of high performance control system for industrial applications has produced great research efforts for the application of modern control theory and, in particular, adaptive control. Adaptive control is most recent class of control techniques though research in adaptive control has a long and vigorous history. In 1950s, it was motivated by the problem of designing autopilots for aircraft operating at wide range of speeds and altitudes. Consequently, gain scheduling based on some auxiliary measurements of air speed was adapted. Kalman developed the concept of a general self tuning regulator with explicit identification of the parameters of linear, single-input, single-output system and he used these

parameter estimates to update an optimal linear quadratic controller. Then Lyapunov's stability theory was established as tool for proving convergence in adaptive control scheme. Further, Parks found a way of redesigning the update laws for model reference adaptive control. Nowadays the adaptive control schemes are making their place where the conventional control system is not able to cope-up with the situation, like

- Loads, inertias and other forces acting on system change drastically.
- Possibility of unpredictable and sudden faults.
- Possibility of frequent or unanticipated disturbances.

The conventional PID controllers with fixed gain are unable to cop up with the problems discussed above. Though recently advanced fuzzy PID controllers [2-9] have been developed to deal with such problems for electrical and mechanical systems. Also the concept of neural network is applied to develop the PID controllers [10-12] to enhance the dynamic characteristics of controller. Still to obtain the complete adaptive nature specific adaptive control techniques are needed. Adaptive control changes the control algorithm coefficients in real time to compensate for variations in the environment or in the system itself. It also varies the system transfer function according to situation. In practical cases, a model reference adaptive control system is generally best implemented with a digital computer, owing to the complexity of the controller.

Out of many adaptive control schemes, this paper mainly deals with the model reference adaptive control approach based on MIT rule [13-16]. In MRAC [13-27], the output response is forced to track the response of a reference model irrespective of plant parameter variations. The controller parameters are adjusted to give a desired closed-loop performance. Apart from the MIT rule there are some other design techniques in Model reference Adaptive Control system, like Lyapunov theory [24-26] and the theory of augmented error [1]. In this paper the emphasis is given to MIT rule only for developing the MRAC scheme. Simulation is done in MATLAB and results are shown for the first order system. The results show that the nature of adaptation depends on a constant known as the adaptation gain. The parameter convergence rate is greatly affected by the correct choice of

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this adaptation gain. It is thus important to know the reasonable value of this parameter.

## II. MODEL REFERENCE ADAPTIVE CONTROL

This technique of adaptive control comes under the category of Non-dual adaptive control. A reference model describes system performance. The adaptive controller is then designed to force the system or plant to behave like the reference model. Model output is compared to the actual output, and the difference is used to adjust feedback controller parameters.

MRAS has two loops: an inner loop or regulator loop that is an ordinary control loop consisting of the plant and regulator, and an outer or adaptation loop that adjusts the parameters of the regulator in such a way as to drive the error between the model output and plant output to zero.

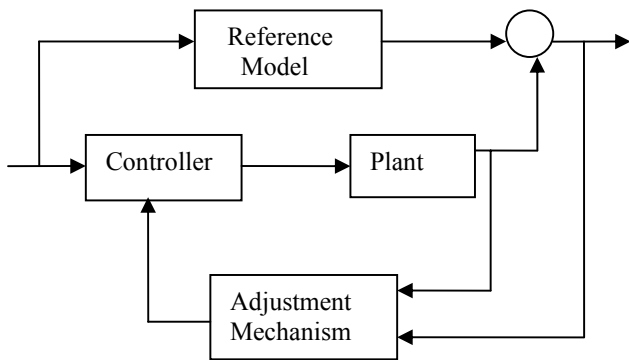


Fig. 1. Model Reference Adaptive Controller

### Components of Model Reference Adaptive Controller:

**Reference Model:** It is used to specify the ideal response of the adaptive control system to external command. It should reflect the performance specifications in control tasks. The ideal behavior specified by the reference model should be achievable for the adaptive control system.

**Controller:** It is usually parameterized by a number of adjustable parameters. In this paper two parameters  $\theta_1$  and  $\theta_2$  are used to define the controller law. The control law is linear in terms of the adjustable parameters (linear parameterization). Adaptive controller design normally requires linear parameterization in order to obtain adaptation mechanism with guaranteed stability and tracking convergence.

**Adaptation Mechanism:** It is used to adjust the parameters in the control law. Adaptation law searches for the parameters such that the response of the plant which should be same as the reference model. It is designed to guarantee the stability of the control system as well as conversance of tracking error to zero. Mathematical techniques like MIT rule, Lyapunov theory and augmented error theory can be used to develop the Adaptation mechanism. In this paper the MIT rule is used for this purpose.

**The MIT Rule:** This rule is developed in Massachusetts Institute of technology and is used to apply the MRAC approach to any practical system. In this rule the cost function or loss function is defined as

$$F(\theta) = e^2 / 2 \quad (1)$$

Where,  $e$  is the output error and is the difference of the output of the reference model and the actual model, while  $\theta$  is the adjustable parameter.

In this rule the parameter  $\theta$  is adjusted in such a way so that the loss function is minimized. For this it is reasonable to change the parameter in the direction of the negative gradient of  $F$ , that is,

$$d\theta/dt = -\gamma \partial F / \partial \theta \quad (2)$$

$$= -\gamma e \partial e / \partial \theta \quad (3)$$

The partial derivative term  $\partial e / \partial \theta$ , is called the sensitivity derivative of the system. This shows how the error is dependent on the adjustable parameter,  $\theta$ . There are many alternatives to choose the loss function  $F$ , like it can be taken as mode of error also. Similarly  $d\theta/dt$  can also have different relations for different applications.

Sign-sign algorithm:

$$d\theta/dt = -\gamma \text{sign}(\partial e / \partial \theta) \text{sign} e \quad (4)$$

Or it may be chosen as

$$d\theta/dt = -\gamma (\partial e / \partial \theta) \text{sign} e \quad (5)$$

Where  $\text{sign} e = 1$  for  $e > 0$

$$= 0 \text{ for } e = 0$$

$$= -1 \text{ for } e < 0$$

In some industrial applications it is found that the choice of adaptation gain is critical and its value depends on the signal levels. So MIT rule has to be modified as follows:

$$d\theta/dt = -\gamma \zeta e \quad (6)$$

Where  $\zeta = \partial e / \partial \theta$

$$\text{Also } d\theta/dt = -\gamma \zeta e / (\beta + \zeta^T \zeta) \quad (7)$$

Where  $\beta > 0$  is introduce to avoid the zero division when  $\zeta^T \zeta$  is small. This paper uses the MIT rule defined by equations (1), (2) and (3) for developing the control law.

## III. MATHEMATICAL MODELLING

In this paper Model Reference Adaptive Control Scheme is applied to first order system using MIT rule. The MIT rule which is used for simulation is described in equation (1).

Let the first order system is described by

$$dy/dt = -a y + b u \quad (8)$$

Let  $a=3$  and  $b=2$

where  $u$  is the controller output or manipulated variable. The transfer function can be written as

$$Y(s)/U(s) = 2/(s+3) \quad (9)$$

Similarly the reference model is described by

$$dy_m/dt = -a_m y_m + b_m r \quad (10)$$

Take  $a_m=4$  and  $b_m=4$

where  $r$  is the reference input.

The transfer function can be written as

$$Y_m(s)/U(s) = 4/(s+4) \quad (11)$$

Here the object is to compare the actual output ( $y$ ) and the reference output ( $y_m$ ) and by applying Model Reference Adaptive Control Scheme the overall output will be improved. Let the controller is described by the law

$$u(t) = \theta_1 r(t) - \theta_2 y(t) \quad (12)$$

The controller parameters are chosen as

$$\theta_1 = b_m / b \text{ and } \theta_2 = (a_m - a) / b \quad (13)$$

The update rule for the controller parameters using MIT rule is described by  

$$d\theta_1/dt = -\gamma e \partial e / \partial \theta_1$$

$$= -\gamma e [b r / (p + a_m)]$$

$$= -\alpha e [a_m r / (p + a_m)] \quad (14)$$

$$\text{and } d\theta_2/dt = -\alpha e [a_m y / (p + a_m)] \quad (15)$$

where  $\alpha = \gamma b/a_m$  is the adaptation gain.

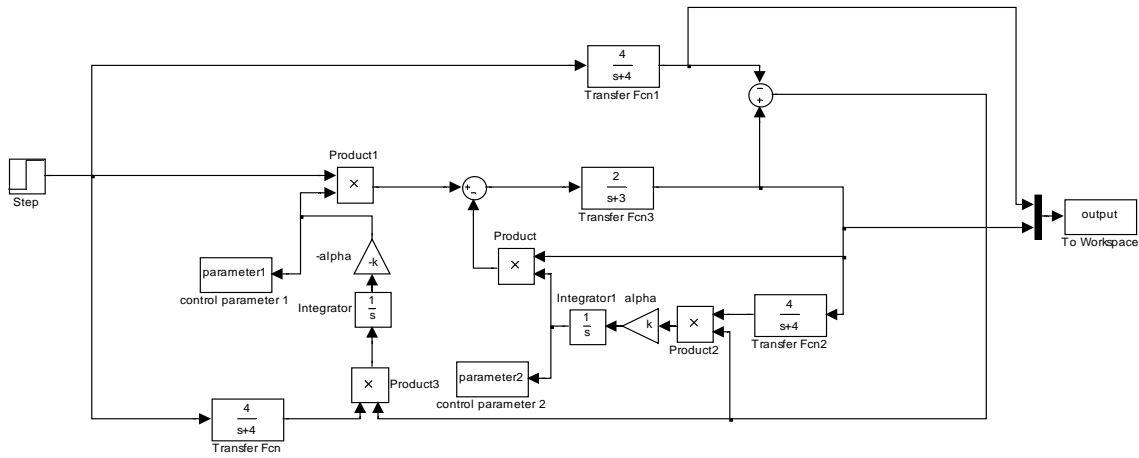


Fig. 2 Simulated Model for MIT rule

### III. SIMULATION AND RESULT

In this paper the model reference adaptive control model is simulated in MATLAB and simulink. The simulation model for the MIT rule is applied to the first order system is shown in fig 2.

The time response characteristic for adaptation gain  $\alpha=1$  is shown in fig 3. It can be observed that the characteristic is oscillatory and the settling time is about 3.74 seconds.

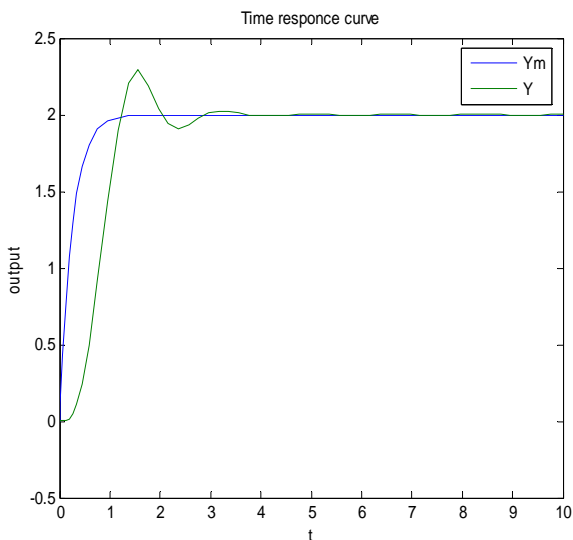


Fig. 3 Time response curve for  $\alpha = 1$

Variation of control parameters  $\theta_1$  and  $\theta_2$  is shown in fig 4 and 5. From fig 4 and fig 5 it can be observed that controller parameter 1 ( $\theta_1$ ) converges to 1.46 and controller

parameter 2 ( $\theta_2$ ) converges to -0.04. Now by using equation (13) the estimated plant parameters come out to be  

$$b' = b_m / \theta_1 = 2.739$$

$$\text{and } a' = a_m - b' \theta_2 = 4.109$$

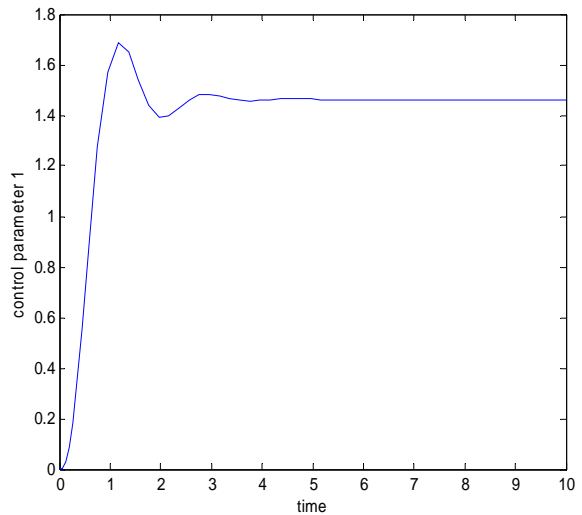


Fig. 4 Variation of control parameter  $\theta_1$  with time

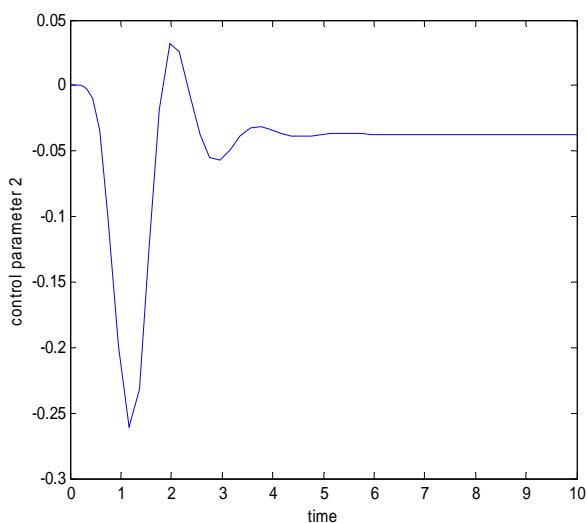


Fig. 5 Variation of control parameter  $\theta_2$  with time

The nature of time response curve greatly depends on the value of adaptation gain,  $\alpha$ . Higher the value of  $\alpha$  better the response. The effect of  $\alpha$  on the time response curve is shown in fig 6 (for  $0.1 > \alpha > 5$ ).

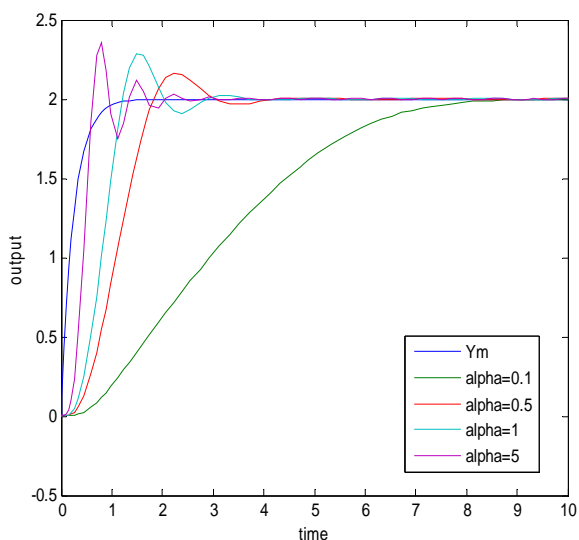


Fig. 6 Effect of  $\alpha$  on time response curve

It can be observed that if the value of adaptation gain is increased then the settling time is reduced but at the same time the maximum overshoot is increased. Also with the increment in  $\alpha$ , number of oscillation cycles are increased. But beyond the certain value of  $\alpha$  (say for  $\alpha > 5$ ), the maximum overshoot is decreased (Fig. 8).

Fig. 7 shows the effect of adaptation gain on output error ( $y-y_m$ ). The amount of error is continuously decreasing with the increment in adaptation gain.

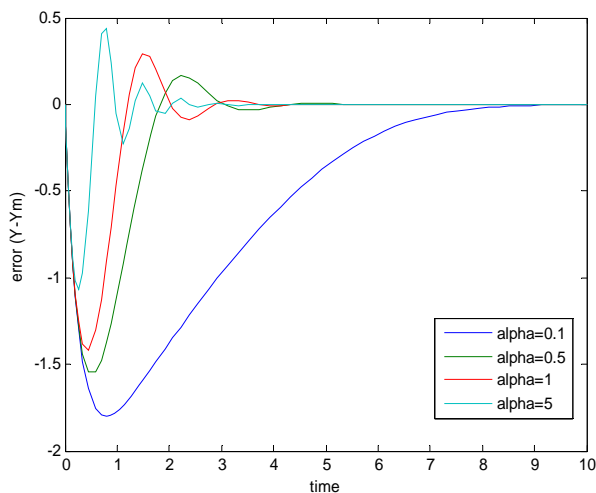


Fig. 7 Effect of  $\alpha$  on output error

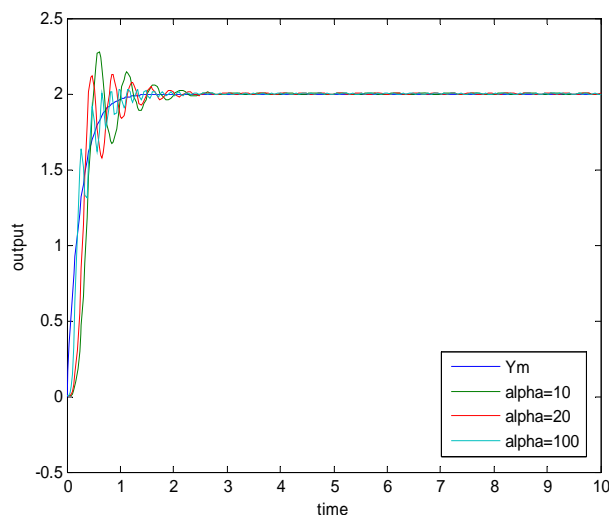


Fig. 8 Effect of  $\alpha$  on time response curve ( $\alpha \geq 10$ )

## V. CONCLUSION

As compared to conventional fixed gain controllers (PID Controllers) Adaptive Controllers are very effective to handle the situations where the parameter variations and environmental changes are frequent have been demonstrated clearly in results. The controller parameters are adjusted to give a desired closed-loop performance. The adaptive controller maintains constant dynamic performance in the presence of unpredictable and immeasurable variations. This paper describes the behavior of a system controlled by model reference adaptive control scheme using MIT rule. Also the effect of adaptation gain,  $\alpha$  is checked on the time response characteristic of the first order system. It has been seen that response is very slow with the smaller value of adaptation gain but there are no oscillations in the response. With the increment in  $\alpha$ , the maximum overshoot is increased but at the same time speed becomes faster (the settling time is reduced). Again the maximum overshoot starts reducing for  $\alpha$

greater than 5. Though undershoot is somewhat increased for the higher range of  $\alpha$ . If  $\alpha$  is increased further the time response specifications are almost become constant and there is very less effect of variation of  $\alpha$  on time response curve. Table I shows the effect of adaptation gain  $\alpha$  on different time response specification. So for suitable value of adaptation gain in MIT rule can make the plant output as close as possible to the reference model in Model Reference Adaptive Control scheme.

TABLE I  
EFFECT OF ADAPTATION GAIN ON TIME RESPONSE CURVE

S.No	Adaptation Gain ( $\alpha$ )	Maximum Overshoot (%)	Peak Time (Second)	Settling Time (Second)
1	0.1	-	-	8.3
2	0.5	8%	2.23	5.15
3	1	15%	1.5	3.74
4	5	18.5%	0.756	3
5	10	12.5%	0.6	3.2
6	20	6%	0.44	3
7	100	5%	0.44	3

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