EWASS Special Session 4 (2017): Star-planet interactions
Edited by Emeline Bolmont & Sergi Blanco-Cuaresma
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A double zone dynamical model for the tidal evolution of the obliquity

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Abstract

It is debated whether close-in giants planets can form in-situ and if not, which mechanisms are responsible for their migration. One of the observable tests for migration theories is the current value of the orbital obliquity, i.e. the angle between stellar equatorial plane and orbital plane. But after the main migration mechanism has ended, the combined effects of tidal dissipation and the magnetic braking of the star lead to the evolution of both the obliquity and the semi-major axis. The observed correlation between effective temperature and measured projected obliquity has been taken as evidence of such mechanisms being at play. Here I present an improved model for the tidal evolution of the obliquity. It includes all the components of the dynamical tide for circular misaligned systems. It uses an analytical formulation for the frequency-averaged dissipation for each mode, depending only on global stellar parameters, giving a measure of the dissipative properties of the convective zone of the host as it evolves in time. The model also includes the effect of magnetic braking in the framework of the double zone model. This results in the estimation of different tidal evolution timescales for the evolution of the planet's semi-major axis and obliquity depending on the properties of the stellar host. This model can be used to test migration theories, provided that a good determination of stellar radii, masses and ages can be obtained.

1 Introduction

Hot Jupiters, planets with masses comparable to that of Jupiter and orbital periods shorter than 10 days, challenge our understanding of planet formation and evolution. After the first detection of an exoplanet orbiting a solar-like star, which was a hot Jupiter, it was soon conjectured that those planets cannot form *in situ* in the protoplanetary disk. They would rather nucleate at several astronomical units from the star, beyond the snow line, where the formation of a solid core would allow the rapid gas accretion required to form a giant planet. This formation scenario compelled deeper studies of the processes involved in planetary migration, to bring the planet from its formation site to where it is observed, at a few tenths of an AU from the star.

Different scenarios have been put forward to explain planetary migration, they can be broadly divided in two categories. The first involves smooth interactions with a protoplanetary disk (Goldreich & Tremaine, 1980; Lin *et al.*, 1996). The second involves gravitational interactions with other massive bodies (Rasio & Ford, 1996; Nagasawa *et al.*, 2008) which would put the proto-hot Jupiter on a very eccentric orbit, eventually circularized by tidal interactions with the host. The former class of scenarios would in general result in short-period orbits that are well aligned with the spin axis of the host, while the latter naturally explains why a number of hot-Jupiters' orbits have non-null measured obliquities.

Observations seem to indicate that the value of the obliquity of hot-Jupiters is correlated with the effective temperature of the host. Most stars cooler than about 6250 K host mostly well-aligned planets, whereas there is a broad dispersion in the value of obliquity for hotter stars (Winn *et al.*, 2010; Albrecht *et al.*, 2012). This has been interpreted as evidence of tidal evolution, because tidal dissipation is expected to be more efficient in stars having important convective enveloppes, and cooler star's spin would realign with the orbit more rapidly. Because the mass of the outer convection zone on the main sequence decreases rapidly with increasing stellar mass after 1.2 $M_\odot$, which corresponds to an effective temperature of $T_{\rm eff} \approx 6250$ K, this would naturally explain the sharp increase in observed projected obliquities that occurs around this effective temperature. This interpretation would favor a single migration mechanism, capable of producing randomly distributed obliquities.

However there is a major flaw in this reasoning: if tides are efficient at damping the obliquity of hot-Jupiters, they would also lead to significant orbital decay, and the planet would be destroyed before re-alignment. To overcome the problem, Lai (2012) proposed that tidal dissipation efficiency can be different for different processes, e.g. spin-orbit alignment and orbital decay. This would be a natural outcome of the *dynamical tide* theory, where different components of the tidal potential, projected onto spherical harmonics, would excite different modes of oscillations of the star, and have different dissipation efficiencies.

Using the tidal prescription proposed by Lai (2012), Valsecchi & Rasio (2014) have computed the coupled evolution of the orbital elements and stellar spin of five representative systems, taking into account the combined effects of tides, stellar wind mass loss, magnetic braking and stellar evolution. Their results show that, accounting for all the relevant physical mechanisms, the current properties of the systems they consider can indeed be naturally explained. However, an important limitation of this study is the absence of a reliable estimation for the tidal quality factor of the obliquity tide. The principles of wave excitation and damping constitute an intricate problem, and there is no agreement to date on the estimation of the tidal quality factor associated with different components of the tides from first principles

(Ogilvie, 2013). Consequently, Valsecchi & Rasio (2014) consider the tidal dissipation quality factor as an ajustable parameter. For similar reasons, they consider that the rate of angular momentum loss due to magnetic braking can also be simply scaled differently for different host. Thus in Valsecchi & Rasio (2014), both parameters are allowed to vary within a broad range of values (instead of being imposed by stellar properties) to reproduce the observations. While this approach validates Lai's basic idea, it is not enough to accurately constrain the distribution of the obliquity of the population of known hot Jupiters before tidal evolution.

In this paper, we compute the tidal dissipation efficiency associated with different components of the tidal potential using the frequency-averaged formulation for dissipation in convective zones obtained by Ogilvie (2013). In a series of paper (Mathis, 2015; Lanza & Mathis, 2016; Bolmont & Mathis, 2016; Gallet *et al.*, 2017; Bolmont *et al.*, 2017), this approach has already shown the importance of the evolution of stellar structure and evolution, for the frequency-averaged dissipation efficiency. This compels the use of a double-zone model, one zone representing the radiative core and the other the convective envelope, and also account for the evolution of stellar structure in time. We also use a double-zone semi-empirical model for magnetic braking, (Spada *et al.*, 2011), to account for the evolution of stellar rotation.

The paper is organized in the following way : in Section 2, we compute the frequency averaged-dissipation for different modes involved in the evolution of the semi-major axis and inclinaison, and we discuss its dependance with mass and rotation. In Section 4, we present discuss the main results and perspectives of this work.

2 The double-zone weak friction model for tidal evolution

We consider a star of mass $M_\star$, which we model as a deformable core of homogenous density ρ_c and mean radius R_c in uniform rotation, surrounded by an enveloppe of homogenous lesser density ρ_e and mean radius $R_\star$ also rotating uniformly at the angular frequency Ω_e, but not necessarily at the same as that of the core Ω_c. We consider here the tides raised by a close-in planet orbiting the star, and we neglect the tides raised in the planet. We consider that the tide-generating potential of the planet is that of a mass-point. The planet generates a time-varying tidal potential Ψ which changes the shape and as a result, the exterior potential of the star. In polar coordinates, the time-varying tidal potential produced by the point-like planet can be expanded in terms of spherical harmonics Y_l^m of degree l and order m. For an eccentric orbit, Ψ must be expanded for an infinite number of terms. But for a circular orbit, not necessarily aligned in the equatorial plane of the reference frame, there is a finite number of tidal components for any given value of l. Here we are interested in the evolution of the inclination of the orbit resulting from the dissipation of the tides in the star, so we limit our study to the case of circular orbits.

When the amplitude of the tidal disturbance is small, as is generally the case for close-in exoplanetary systems, and for axisymmetric bodies, the tidal response of the perturbed body can be determined by treating each component of the tidal potential independently, and considering that the total response is simply the sum of each component. Each tidal component rotates with the angular velocity ω given by

$$\omega = (l - 2p)\Omega_0, \tag{1}$$

where p is an integer with $0 \leq p \leq l$ that arises from the inclination of the orbital plane. This defines the tidal frequency in the inertial frame. We associate to each component of the tidal potential a Love number $k_l^m(\omega)$, that quantify the frequency-dependent response of the star to tidal forcing.

2.1 Tidal dissipation

In the presence of dissipation, the perturbed external gravitational potential of the star involves complex Love numbers and the imaginary part of the Love numbers $\mathrm{Im}[k_l^m(\omega)]$ quantifies the part of the response that is out of phase with the tidal forcing, and is associated with transfers of energy and angular momentum.

The rates of transfer of energy and of the axial component of angular momentum from the orbit to the body, measured in an inertial frame, define the tidal power P, and the tidal torque T_z, respectively. So for each component of the tidal potential, averaged over azimuth in the case $m = 0$, it can be shown (Ogilvie, 2013) that $P = \omega\mathcal{T}$ and $T_\mathrm{z} = m\mathcal{T}$, where $\mathcal{T}$ depends on the amplitude of the tidal component and on $\mathrm{Im}[k_l^m(\omega)]$. For $m = 0$, there can be transfer of energy from the orbit to the star, but no associated change in the axial component of angular momentum of the star. In fact, even in the absence of a torque, the $m = 0$ mode is excited because the axisymmetric tidal deformation modulates the moment of inertia of the body.

Here, we use an approximation similar to the one made in the equilibrium theory tides. We consider that the tidal response resembles the hydrostatic one in the absence of dissipation, and that dissipation introduces a small frequency-dependent phase lag in the response of the body. However, contrary to the usual equilibrium theory of tides, we do not impose that all components have the same phase-lag. So for each component of the tidal potential, we assume that dissipation introduces separate frequency-dependent phase-lags $\Delta_l^m(\omega)$ which can be related to the imaginary part of the Love numbers (Efroimsky & Makarov, 2013; Ogilvie, 2013) with

$$\mathrm{Im}[k_l^m(\omega)] = |k_l^m(\omega)| \sin \Delta_l^m(\omega) \tag{2}$$

where $\mathrm{Im}[k_l^m(\omega)]$ has the same sign as $\hat{\omega}$. As detailed in Ogilvie (2014), typically $\mathrm{Re}[k_l^m(\omega)]$ is a quantity of order unity, only weakly dependent on m and ω, and can be well approximated by its hydrostatic value. The hydrostatic Love number k_l is a real quantity and does not depend on m. Its evaluation for a fluid body is a classical problem involving the Clairaut's equation. In the weak friction approximation, we thus consider that $|\Delta_l^m(\omega)| << 1$ so that

$$|k_l^m(\omega)| \approx k_l \tag{3}$$

and

$$\mathrm{Im}[k_l^m(\omega)] \approx k_l \Delta_l^m(\omega). \tag{4}$$

In general, the tidal response involves resonances with stellar oscillations when the tidal frequency matches that of an appropriate mode. Tidal dissipation has consequently a complex dependence on the tidal frequency, and thus on both the rotational and orbital period. Here, we use a simplified model for tidal dissipation, which does not involve the complicated details of the frequency-dependance of the response functions, and neglect the enhancement of dissipation due to resonances. Indeed, as shown by Ogilvie (2013), the typical level of dissipation can be approximated using a simple analytical formulation of the frequency-averaged dissipation $\int_{-\infty}^{\infty} \mathrm{Im}[k_l^m(\omega)]\mathrm{d}\omega/\omega$, obtained by means of an impulse calculation. Taking-up on their work, we use here $\int_{-\infty}^{\infty} \mathrm{Im}[k_l^m(\omega)]\mathrm{d}\omega/\omega$ as a measure of the tidal response in the low-frequency part of the spectrum, where inertial waves are found. Their solution depends on the internal structure of the body, either homogeneous or double-layered, but makes no assumption on the details of dissipation mechanism, so it is smooth and free from boundary-layers. Their derivations are done for arbitrary degree and order of the tidal components. In Ogilvie (2013), the computation of the frequency-averaged dissipation $\int_{-\infty}^{\infty} \mathrm{Im}[k_l^m(\omega)]\mathrm{d}\omega/\omega$ corresponding to inertial waves for a piece-wise homogeneous body is given in Appendix B for the $l = m = 2$ mode. Here we need to compute this value for arbitrary values of m. The determination of this quantity requires the formulation of the non-wavelike tide associated to the inhomogeneous Helmotz equation (Eq. 49, ibid.), expressed under the form of Eq. 117 (ibid.) for the two-layered fluid. The non-wavelike part is an instantaneous hydrostatic response to the tidal potential, parametrised in Ogilvie (2013) through coefficients B_1 and B_2, associated to a tidal potential of the form $A(r/R_\star)^l Y_l^m$, where $R_\star$ is the mean radius of the free surface the star. They are found by solving the matching conditions (Eq. 119, ibid.) and (Eq. 120, ibid.).

In this way, the computation of $\int_{-\infty}^{\infty} \mathrm{Im}[k_l^m(\omega)]\mathrm{d}\omega/\omega$ only requires the knowledge of $M_\star$, M_c, $R_\star$, R_c and Ω_e. We then define an average phase-shift $\bar{\Delta}_l^m$ for the tidal component of degree l and order m obtained using the frequency-averaged formulation of Ogilvie (2013)

$$\int_{-\infty}^{\infty} \mathrm{Im}[k_l^m(\omega)]\frac{d\omega}{\omega} \approx k_l \int_{-\infty}^{\infty} \Delta_l^m(\omega)\frac{d\omega}{\omega} \equiv k_l \bar{\Delta}_l^m. \quad (5)$$

Up to this point, all our derivations are valid for arbitrary value of l and m, and in principle, they allow the computation of the tidal response for any spherical harmonic of the multipole expansion of the tidal potential. The contribution of the spherical harmonic of degree l is proportional to r^l, so usually only the quadrupolar terms are retained in the formulation of the temporal evolution of orbital elements. While the tidal frequency can be positive or negative, the physical forcing frequency is positive so that the (m, p) component is physically identical to the $(-m, l-p)$ component. Moreover, since we use a frequency-averaged estimation of the tidal dissipation efficiency, we assume that dissipation is independent of the value of p, there are three components for which the dissipation can be computed corresponding to $m = 0, 1$ and 2.

2.2 Stellar angular momentum modelling

The angular momentum of the star $\mathbf{L}_\star = L_\star \hat{\mathbf{L}}_\star$ is modelled with core-envelope decoupling under the assumptions of the double-zone model (MacGregor & Brenner, 1991),

$$L_\star = L_\mathrm{c} + L_\mathrm{e} = I_\mathrm{c}\Omega_\mathrm{c} + I_\mathrm{e}\Omega_\mathrm{e} \quad (6)$$

where I_c and I_e are the moment of inertia for the core and the enveloppe respectively. The core and the enveloppe are assumed to rotate as solid bodies with different angular velocities. We simply parametrise the transfer of angular momentum between the zones by a quantity ΔL defined as

$$\Delta L = \frac{I_\mathrm{c} I_\mathrm{e}}{I_\mathrm{c} + I_\mathrm{e}} (\Omega_\mathrm{c} - \Omega_\mathrm{e}) \quad (7)$$

at a rate determined by a coupling time-scale τ_c. We follow the evolution of the star from its formation at until the end of its main sequence, i.e. before the core starts contracting. Initially the star is fully convective and is assumed to rotate rigidly. We consider a phase of disk-locking, where the net effect of interactions with the disk is that of keeping the surface angular velocity of the star constant for the disk lifetime, i.e.

$$\frac{\mathrm{d}\Omega_\mathrm{e}}{\mathrm{d}t} = 0, \quad \text{while} \quad t \le \tau_\mathrm{disk} \quad (8)$$

The disk lifetime τ_disk may vary, but it is in general shorter than the Pre-Main-Sequence (PMS) phase, as observations show that most primordial disks have disappeared by the first 10 Myr (Ribas *et al.*, 2014). As the radiative core develops, a quantity of material contained in a thin shell at the base of the convective zone, with a velocity of Ω_e, becomes radiative, producing an angular momentum transfer towards the core

$$\left.\frac{\mathrm{d}L}{\mathrm{d}t}\right|_\mathrm{growth} = \left(\frac{2}{3} R_\mathrm{c}^2 \frac{\mathrm{d}M_\mathrm{c}}{\mathrm{d}t}\right)\Omega_\mathrm{e} \quad (9)$$

From the moment the disk disappears, the star experiences angular momentum loss through its magnetized wind. We assume that because of wind braking, the enveloppe loses angular momentum at a rate given by the following parametric formula

$$\left.\frac{\mathrm{d}L_\mathrm{e}}{\mathrm{d}t}\right|_\mathrm{wind} = K_\mathrm{w} \left(\frac{R_\star}{\mathrm{R}_\odot}\right)^{\frac{1}{2}} \left(\frac{M_\star}{\mathrm{M}_\odot}\right)^{-\frac{1}{2}} \min\left(\Omega_\mathrm{e}^3, \Omega_\mathrm{sat}^2\Omega_\mathrm{e}\right), \quad (10)$$

where K_w determine the braking intensity and Ω_sat is the angular frequency threshold defining a saturated (when $\Omega_\mathrm{e} \ge \Omega_\mathrm{sat}$) regime of angular momentum loss.

In fine, the evolution of the stellar angular momentum is modelled with the following differential equations

$$\frac{\mathrm{d}L_\mathrm{c}}{\mathrm{d}t} = -\frac{\Delta L}{\tau_c} + \left.\frac{\mathrm{d}L}{\mathrm{d}t}\right|_\mathrm{growth} \quad (11)$$

$$\frac{\mathrm{d}L_\mathrm{e}}{\mathrm{d}t} = +\frac{\Delta L}{\tau_c} - \left.\frac{\mathrm{d}L}{t}\right|_\mathrm{growth} - \left.\frac{\mathrm{d}L_\mathrm{e}}{\mathrm{d}t}\right|_\mathrm{wind} \quad (12)$$

This neglects the effect of the tidal torque on the stellar rotation, but we are interested here in quantifying the different dissipation efficiencies of distinct tidal component, a study of the joined evolution of semi-major and obliquity will be presented in a forthcoming paper. We use the prescription given in Spada *et al.* (2011) for the values of K_w, Ω_sat and τ_disk for a solar-mass star and simply divide K_w by ten (Barker & Ogilvie, 2009) to reflect the less efficient loss of hotter stars.

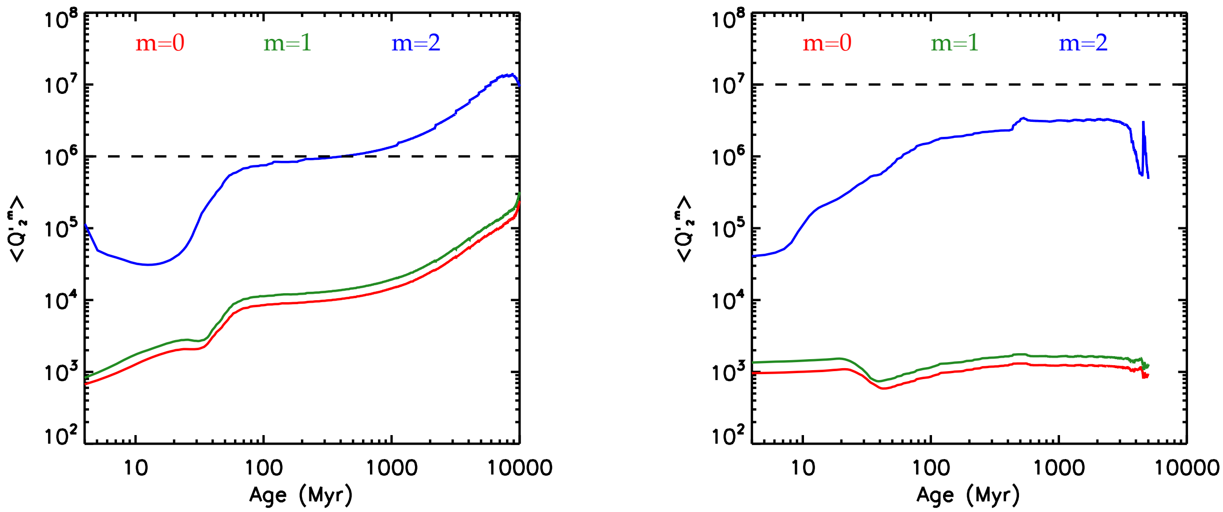

Figure 1: Left: Frequency-averaged modified tidal quality factor corresponding to the sectoral harmonics component of the tidal potential $<Q_2^{'m}>$ for a solar-like star (left) and for an F-type star (right). The colors correspond to different values of m as indicated on the figure. The constant Q' value of the equilibrium tide calibrated on hot-Jupiters is also shown with dotted-lines

3 Results

In Figure 1, we show the resulting frequency-averaged modified tidal quality factor

$$< Q_l^{'m} >= \frac{3}{2k_l \bar{\Delta}_l^m} \tag{13}$$

for the $l = m = 2$ sectoral harmonics, the $l = 2, m = 0$ zonal harmonic, and the $l = 2, m = 1$ tesseral harmonic. The effect of stellar structure and evolution on the frequency-averaged dissipation of the sectoral harmonic have already been computed in other studies (Mathis, 2015; Lanza & Mathis, 2016; Bolmont & Mathis, 2016; Gallet *et al.*, 2017; Bolmont *et al.*, 2017). Here we present it also for the zonal and tesseral harmonic. In the impulsive forcing problem, the value of m affects the energy dissipated by inertial waves through the coupling of the spheroidal and toroidal wavelike velocity components due to the Coriolis force. The only difference between the energy transfer from the sectoral harmonic and the zonal and tesseral ones stems from the contribution of the toroidal part, related to the coupling coefficient $\tilde{q}_l$,

$$\tilde{q}_l = \frac{1}{l}\left(\frac{l^2 - m^2}{4l^2 - 1}\right)^{1/2} \tag{14}$$

Thus the $m = 1$ and $m = 0$ components are almost identical due to similar coupling coefficient values.

When the obliquity is large, all the components of the tidal potential participate to tidal dissipation. The dominant term is then the $<Q_2^{'1}>$ component, which correspond to a tidal dissipation that is more efficient by three orders of magnitude than in the classical equilibrium tide. This results in characteristic timescales of evolution much shorter than the main-sequence of the star, but could still allow the re-alignement of the orbit before the engulfment of the planet, depending on the ratio of orbital to spin angular momentum. Even though the value of $<Q_2^{'1}>$ for an F-type star is most of the time smaller than that of a solar-like star, it is possible that the resulting temporal evolution of the obliquity may be comparatively smaller, since in depends on the moment of inertia in the convective enveloppe, which is significantly smaller for hotter stars. When the obliquity is small, the $<Q_2^{'2}>$ component becomes dominant, and the survival of the planet is actually favoured around G-type stars, especially after the first Gyr, when the spin-down of the star brings $<Q_2^{'2}>$ to values higher than 10^6. Qualitatively, our computations show that tidal evolution could indeed explain the observed correlation between obliquity and effective temperature.

4 Conclusions

We have computed the frequency-averaged tidal dissipation associated to inertial waves in the frame of the dynamical tide, for all the components of the tide. We use an analytical formulation which only depends on global stellar parameters, and gives a measure of the dissipative properties of the convective zone of the host as it evolves in time, also accounting for the effect of magnetic braking. A preliminary study show that different tidal evolution timescales for the evolution of the planet's semi-major axis and obliquity are expected depending on the properties of the host. However, simple timescales estimates such as discussed here can be different by orders of the magnitude to the actual values obtained by computing the coupled evolution of the orbital parameters (Barker & Ogilvie, 2009). Moreover, dissipation efficiency may still be less important if the tidal frequency does not fall within the range of inertial waves. Thus, the phase of enhanced dissipation through the dynamical tide is likely to occur during the PMS if it is ever to occur. Further development will be given in a forthcoming paper. Eventually, the complete dynamical model will be used to test migration theories. This requires also a good determination of stellar radii, masses and ages. Major advances are thus expected with the results of the PLATO 2.0 mission, selected as the next M-class mission of ESA's Cosmic Vision plan, that will allow the complete characterisation of host stars using asteroseismology.

Acknowledgments : This work was supported by the German Space Agency under PLATO grant 50OO1501.

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