

Simplified Models to Determine Nodal Voltages in Problems of Optimal Allocation of Capacitor Banks in Power Distribution Networks

L. A. Pereira, S. Haffner, and L. V. Gasperin

Abstract—This paper presents two simplified models to determine nodal voltages in power distribution networks. These models allow estimating the impact of the installation of reactive power compensations equipments like fixed or switched capacitor banks. The procedure used to develop the models is similar to the procedure used to develop linear power flow models of transmission lines, which have been widely used in optimization problems of operation planning and system expansion. The steady state non-linear load flow equations are approximated by linear equations relating the voltage amplitude and currents. The approximations of the linear equations are based on the high relationship between line resistance and line reactance (ratio R/X), which is valid for power distribution networks. The performance and accuracy of the models are evaluated through comparisons with the exact results obtained from the solution of the load flow using two test networks: a hypothetical network with 23 nodes and a real network with 217 nodes.

Keywords—Distribution network models, distribution systems, optimization, power system planning.

I. INTRODUCTION

PLANNING is a very important task for power system companies, as financial investment are based on the guidelines stated by the planning. In the specific case considered here the decision is about the location where new capacitor banks shall be installed. This action aims to adequate the distribution system to supply electric energy to new consumers and at the same time to keep node voltage levels within the required upper and lower limits. Costs and benefits coming from the choice of a specific location for the capacitors have to be judiciously evaluated once they have a strong influence on the final decision. Benefits from installing capacitors arise basically from: investment postponing, electric loss reduction, improvement of voltage profile,

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available capacity expansion, and load supply reliability [1]. These issues lead to development of software tools to aid decision makers in choosing the best alternative based on the technical and economic criteria. Reliable network models play in this context a crucial role, as they allow determining the network behavior under different conditions.

For power system analysis, loads are in general represented by constant current injections at the network nodes, while transmission lines and transformers are represented by fixed impedances. In this representation the power balance equations leads to a problem known as load flow, which is described by non-linear equations relating the constant power injections with the magnitude and phase angle of the nodal voltages [2]. Optimization problems that use this approach become very complicated, given that they have to handle non-linear constraints involving power flow and nodal voltage phasors. To overcome this difficulty in problems of planning and expansion of high and extra high voltage systems, network simplified models have been developed and successfully applied in [3]. Based on this approach new simplified models have been developed to represent distribution network for use in problems of feeder expansion. Excellent results were obtained with these models under several load conditions, network topology and conductor gauge, as reported in [4]. However, these models do not allow representing the effect of capacitor banks, as the nodal injection are represented only by the apparent power with no separation between reactive and active power. The present paper introduces an extension of the linearized models presented in [4], where the effect of capacitor banks is obtained by the superposition of the effects of active and reactive power. This kind of representation proved to be very adequate and allows representing the effect of installation of capacitor banks in the network with very good accuracy. The simplified models presented in this paper are derived from the models described in [5][6], where current injection has been successful applied to the problem of expansion planning of distribution network.

In the first part of this paper classical network models are briefly reviewed. Next, in the second part, the simplified distribution network models are presented. These models have been developed mainly to determine nodal voltage in optimization problems. In the third part, comparisons between

exact results – obtained with the solution of load flow – and the corresponding results obtained with the simplified models are presented and discussed in order to assess the model performance and accuracy. Finally, the main conclusions about the model application are presented and the main results are commented.

II. CLASSICAL NETWORK MODELS

In classical network models, the electric power system is represented by a set of nodes, where loads and generators are assumed to be located, and branches, representing transmission lines and transformers, both connecting two nodes. For the steady state system analysis two basic models are in general used to represent the relationship between node and branch variables: the exact model, called conventional AC load flow, and the approximate linearized model, called DC load flow.

A. Conventional Load Flow Model

In the conventional load flow model there are four variables for each network node: voltage amplitude (V_k), voltage phase angle (θ_k), net active power injection (P_k) and net reactive power injection (Q_k). Network branches are associated with the current and power flow, which are obtained from the node voltages and from the parameters of the equivalent circuit. For each network branch having a transmission line – or a transformer – the current (\bar{I}_{km}), the active and reactive power flows (\bar{S}_{km}) are given by:

$$\bar{I}_{km} = -a_{km} \bar{y}_{km} \bar{V}_k + (\bar{y}_{km} + jb_{km}^{sh}) \bar{V}_m \quad (1)$$

$$\bar{S}_{km} = P_{km} + jQ_{km} = \bar{V}_k \bar{I}_{km}^* \quad (2)$$

Where $\bar{y}_{km} = g_{km} + jb_{km}$ is the series admittance, a_{km} is the transformer voltage ratio (for transmission lines $a_{km} = 1$ pu) and b_{km}^{sh} is the shunt admittance of the pi-model of the transmission line (for transformers $b_{km}^{sh} = 0$).

Given that the steady state currents and voltages are represented by phasors (complex numbers), in equations (1) and (2), current and power flows are described by non-linear equations in terms of amplitude and phase angle of their terminal voltages. For instance, the real part of the current and power flow (active power) from node k to m are defined as follows:

$$\text{Re}\{\bar{I}_{km}\} = a_{km}^2 V_k [g_{km} \cos \theta_k - (b_{km} + b_{km}^{sh}) \sin \theta_k] - a_{km} V_m (g_{km} \cos \theta_m - b_{km} \sin \theta_m) \quad (3)$$

$$\text{Re}\{\bar{S}_{km}\} = P_{km} = (a_{km} V_k)^2 g_{km} - a_{km} V_k V_m (g_{km} \cos \theta_{km} + b_{km} \sin \theta_{km}) \quad (4)$$

In the last expression the definition $\theta_{km} = \theta_k - \theta_m$ was used. Applying the Kirchhoff's Current Law (KCL) for each node, the following matrix relationship between current injections and nodal voltages can be obtained [2]:

$$\underline{I} = \underline{YV} \quad (5)$$

where \underline{I} is a vector of nodal current injections, \underline{V} is a vector of nodal voltages, and \underline{Y} is the nodal admittance matrix whose elements are given by:

$$\begin{aligned} Y_{kk} &= jb_k^{sh} + \sum_{m \in \Omega_k} (a_{km}^2 \bar{y}_{km} + jb_{km}^{sh}) \\ Y_{km} &= -a_{km} \bar{y}_{km} & m \in \Omega_k \\ Y_{km} &= 0 & m \notin \Omega_k \end{aligned} \quad (6)$$

where b_k^{sh} is the susceptance joining node k and the reference node (ground node), Ω_k is the set of all neighboring nodes of node k.

Applying equation (5) to a network with N nodes results in vectors with dimension N for voltages and currents, it also results in an admittance matrix of dimension $N \times N$. Finally, it must be pointed out that all voltages are referred to the ground node, whose order number is $N + 1$.

B. Linearized Load Flow Model

The linearized load flow model can be viewed as an approximation of the conventional load flow model. It was developed to represent high and extra high voltage networks, it is typically applied in cases such as: solution of a number of load flow problems, problems where the convergence is difficult to achieve, simplified representation of load flow equations in optimization problems [3]. The approximations carried out in this model aim primarily avoiding the use of the non-linear equations given by (1) and (2). Furthermore, this model has been widely used in the analysis of contingences and in optimization models applied to planning, expansion and operation of power system [7]. The linearized load flow model is obtained introducing the following approximations in the equations of the conventional load flow model [2]: the voltage amplitudes are assumed to be equal to their rated values ($V_k \approx V_m \approx 1$ pu); active power losses are disregarded; load angles are assumed small, implying $\sin \theta_{km} \approx \theta_{km}$; branch resistance are much lower than the branch reactance ($b_{km} \approx -(x_{km})^{-1}$). These approximations lead to the linearized model, in which each node has now only two variables: nodal voltage phase angle (θ_k) and net active power injection (P_k). Each branch has an associated active power flow (P_{km}) that is determined from the terminal voltage phase angle and from the branch reactance (x_{km}) applying the following equation:

$$P_{km} = (x_{km})^{-1} \theta_{km} \quad (7)$$

$$P_{mk} = -P_{km} = (x_{km})^{-1} \theta_{mk} \quad (8)$$

In this way, the relationship between active power flow and load angle becomes linear. In addition, this relationship is similar to that one existing between current flow and nodal voltages of a DC circuit. The name DC Load Flow comes from this similarity. As the losses are neglected the system of equations is singular. Therefore, it is necessary to eliminate one equation and assume one node as being the angular

reference. As a consequence a system of $N - 1$ non-singular equations and $N - 1$ unknown variables arises [2].

III. SIMPLIFIED MODELS FOR DISTRIBUTION NETWORKS

In general for power transmission lines the relationship between branch resistance and branch reactance (R/X) lies between 0.1 and 0.3. In addition, the power flow is strongly dependent on the phase angle of the nodal voltage. On the other hand, for distribution lines R/X is higher than 1.0 and the power flow is strongly dependent on the magnitude of the nodal voltage. Considering this fact, the approximations commonly used to derive the linearized model are not valid for distribution networks because the R/X ratio is different from the assumed value. This can be better understood considering Fig. 1 where the correct values of the phase angle are plotted together with the phase angles calculated with the linearized model and for a varying ratio R/X . Fig. 1 was based on a distribution network with 23 nodes, for which the ratio R/X of each branch was allowed to vary. For the case shown in Fig. 1(a) the mean value of R/X is 0.22 and for the case in Fig. 1(b) the mean value is 2.2. Moreover, the first case (Fig. 1(a)) represents the typical case of transmission lines. In this

case a good agreement between both results can be observed. In the second case (Fig. 1(b)) very different results are obtained, making the linearized model inadequate for such cases.

The models described in what follows are modified versions of the linearized network model. They have been specifically developed to represent distribution networks with high ratio R/X . The conventional linearized model uses constant power injections, phase angles of the node voltages and branch reactances. Instead of using these parameters, the two simplified models presented here use constant current injections – determined under assumption that the node voltages are at their rated values –, node voltage amplitude, and branch impedances. In both models, loads are represented by current injections. This way of representing loads can be considered as a new approach, given that, in general, power injection – as in the case of conventional load flow – or constant impedances is used to represent load [8]. This new kind of approach has as main advantage the fact that the current injections become independent from the nodal voltages, simplifying the set of equations.

A. Simplified Model 1

This model has been primarily developed to simplify the determination of the nodal voltages in distribution networks. In addition, the model should also allow determining the effect of the installation of capacitor banks along the network feeders. In this model the complex nodal voltages are replaced by real voltages representing their amplitudes. The currents, however, keep their complex form, having both real and imaginary parts. As detailed in what follows, the voltage drops (ΔV_{km}) are calculated using only the real part of the product from the complex branch impedance (z_{km}) and the complex branch current (f_{km}). Thus, the following expression can be defined for ΔV_{km} :

$$\Delta V_{km} = z_{km} \cdot f_{km} = (r_{km} + jx_{km}) \cdot (\text{Re}\{f_{km}\} + j\text{Im}\{f_{km}\}) \quad (9)$$

$$\Delta V_{km} = r_{km} \cdot \text{Re}\{f_{km}\} - x_{km} \cdot \text{Im}\{f_{km}\} + j(r_{km} \cdot \text{Im}\{f_{km}\} + x_{km} \text{Re}\{f_{km}\}) \quad (10)$$

In distribution networks the imaginary part of the voltage drop ΔV_{km} has practically no influence on the amplitude of ΔV_{km} . Therefore, only the real part of ΔV_{km} is accounted, resulting in an approximate expression given by:

$$\Delta V_{km} \cong \text{Re}\{z_{km} \cdot f_{km}\} = r_{km} \text{Re}\{f_{km}\} - x_{km} \text{Im}\{f_{km}\} \quad (11)$$

According to the preceding expression the voltage drop ΔV_{km} in each branch of the network can be determined by adding the effects of the real and imaginary part of the complex current. Each part of the current can be considered as circulating in two separate circuits, one formed by the branch resistance (r_{km}) and the other formed by the branch reactance (x_{km}):

$$\Delta V_{km} \cong \Delta V_{km}^A + \Delta V_{km}^B \quad (12)$$

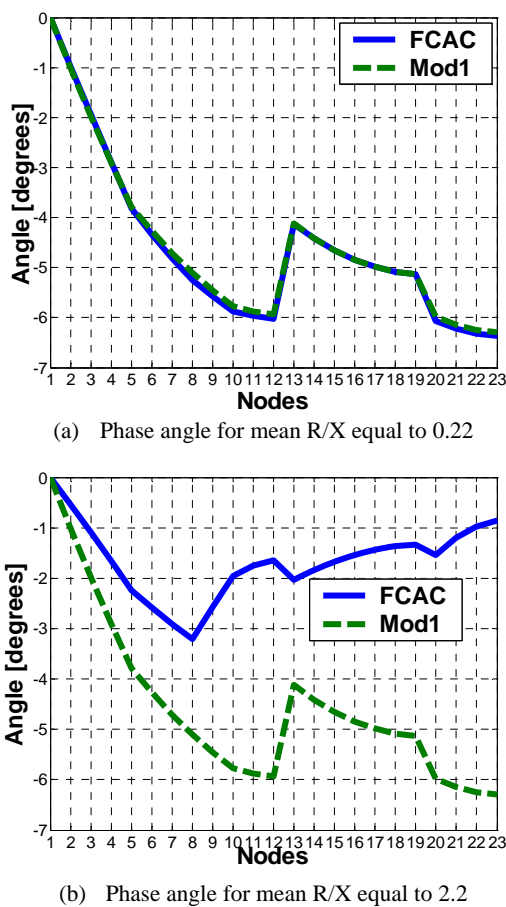


Fig. 1 Comparison of the results obtained with the linearized model (Mod1) and the exact model (FCAC) considering a varying ratio R/X and a distribution network with 23 nodes

The individual voltage drop is defined as:

$$\Delta V_{km}^A = r_{km} \cdot \text{Re}\{f_{km}\} = V_k^A - V_m^A \quad (13)$$

$$\Delta V_{km}^B = -x_{km} \cdot \text{Im}\{f_{km}\} = V_k^B - V_m^B \quad (14)$$

The amplitude of the nodal voltages can be obtained as follows:

$$\underline{V} = \underline{V}^A + \underline{V}^B \quad (15)$$

The terms \underline{V}^A and \underline{V}^B are obtained through the application of the Kirchhoff's Current Law to each network node, as stated by equation (5). Using the modified admittance matrix results in:

$$\underline{V}^A = [Y_R]^{-1} \cdot \text{Re}\{-d\} \quad (16)$$

$$\underline{V}^B = -[Y_X]^{-1} \cdot \text{Im}\{-d\} \quad (17)$$

$$d_k = \left(\frac{\bar{S}_k}{V_k} \right)^* \bar{v}_{k=1,0} \frac{0}{S_k} \quad (18)$$

where Y_R is the network modified admittance matrix that is obtained when only the branch resistances are considered; in a similar fashion Y_X is the modified admittance matrix which includes only branch reactances. With these definitions, equations (13) and (14) can now be rewritten in terms of nodal voltages:

$$\Delta V_{km}^A = V_k^A - V_m^A = \underline{e}_{km}^T \cdot \underline{V}^A \quad (19)$$

$$\Delta V_{km}^B = V_k^B - V_m^B = \underline{e}_{km}^T \cdot \underline{V}^B \quad (20)$$

The vector \underline{e}_{km}^T has zeros in all elements except in the positions k and m that have values +1 and -1, respectively.

B. Simplified Model 2

Through several tests, performed with a number of different networks and under different conditions, it was observed that the accuracy of model 1 can be improved introducing a correction factor K_{km}^R into the real part of the impedance. The factor K_{km}^R is determined from the voltage drops calculated with the simplified model 1 (ΔV_{km}^{MA1}) and also from the exact voltage drops given by the solution of the non-linear load flow (ΔV_{km}^{FCAC}):

$$\Delta V_{km} = K_{km}^R \cdot r_{km} \cdot \text{Re}\{f_{km}\} + x_{km} \cdot \text{Im}\{f_{km}\} = K_{km}^R \cdot \underline{e}_{km}^T \cdot \underline{V}^A + \underline{e}_{km}^T \cdot \underline{V}^B \quad (21)$$

$$\Delta V_{km}^{FCAC} = \underline{e}_{km}^T \cdot (K_{km}^R \cdot \underline{V}^A + \underline{V}^B) \quad (22)$$

$$K_{km}^R = \frac{\Delta V_{km}^{FCAC} - \underline{e}_{km}^T \cdot \underline{V}^B}{\underline{e}_{km}^T \cdot \underline{V}^A} \quad (23)$$

For a given network, the exact values of voltage amplitudes can be obtained, if the factor K_{km}^R is determined for each individual network branch. In optimization problems aiming to determine the best location for the capacitor banks, node voltages are sought for distinct network topologies and branch configurations. In such problems the node voltages have to be determined by an efficient algorithm, which allows the optimization problem to achieve a solution in reasonable time

and with affordable computational effort. Thus, the factors K_{km}^R are determined just once for the initial configuration of the network. They are then kept fixed for the whole optimization process.

IV. MODEL PERFORMANCE AND PRACTICAL RESULTS

The accuracy of the proposed simplified models has been evaluated based on two distribution networks: a theoretical network with 23 nodes and a real network with 217 nodes. The problem considered here is the optimal location of capacitor banks along the feeders of the distribution network. In fact, node voltage are an essential information for the optimization process, because the main reason to install capacitors in the network is namely to improve the voltage level at each node (voltage profile). The test consists in placing capacitor banks of 600 and 1,200 kVar in several distinct network nodes and evaluating the model performance. After placing a capacitor bank, the errors in the node voltages are obtained through a comparison with the exact values from the AC flow solution. In what follows, the results for each example networks are presented and discussed.

A. Test Network with 23 Nodes

This network is analyzed in [9] and has 13.8 kV as rated voltage; its nodes are numbered from 2 to 23, as shown schematically in Fig. 2. For the sake of test each node is loaded with 189 kW and 124 kVar summing up 4.158 MW e 2.728 MVar in the whole network. The network branches are all 2 km long and are wired with three different aluminum cable types: 336.4 AAC, 2/0 AAC e 1 AAC. The impedance of each cable type is $0.348+j0.854 \Omega$, $0.946+j0.844 \Omega$ and $1.528+j0.916 \Omega$, respectively.

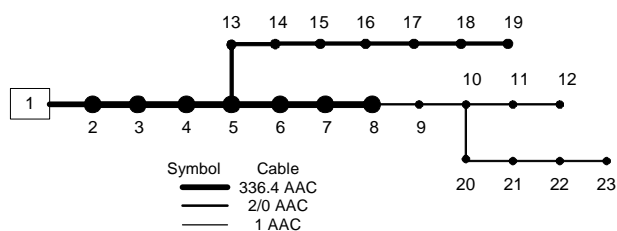


Fig. 2 Example network with 23 nodes

The first test consisted in placing a single capacitor bank of 600 kVar in one of the 22 network nodes. For this network the correction factors calculated with equation (23) are within the range going from 1.2210 to 1.5089. The capacitor bank was located initially at node 2 and moved successively to the next, until all 22 nodes have been evaluated with the capacitor placed once at each node. Each time the capacitor bank was moved all node voltages were calculated and compared with their exact values obtained with the exact load flow. The plot in Fig. 3 shows the mean percent error – obtained considering all 23 nodes - for each of the 22 capacitor bank locations along the network. The biggest percent errors found in all the 22 situations analyzed were 3.19% and 0.19% for the

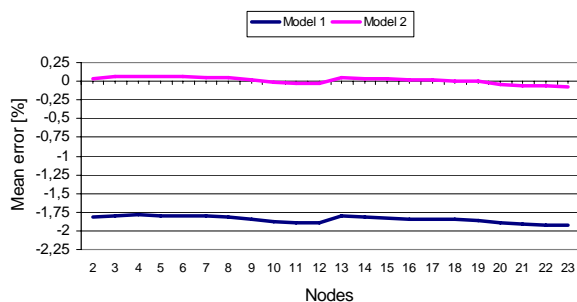


Fig. 3 Mean error in nodal voltages for the example network with 23 nodes and a capacitor bank of 600 kVAr placed one time at each node

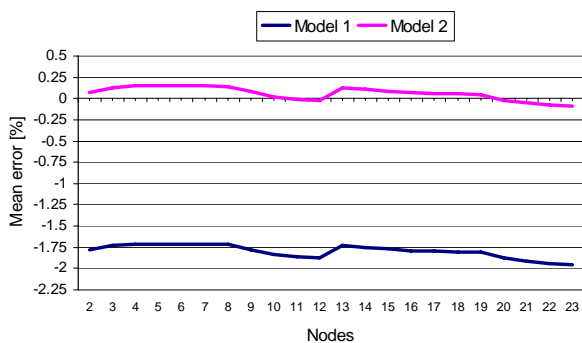


Fig. 4 Mean error in nodal voltages for the example network with 23 nodes and a capacitor bank of 1,200 kVAr placed one time at each node

simplified model 1 and 2 respectively and with the capacitor bank located at node 23. The errors resulting from model 1 are significantly bigger when compared to the errors obtained with the use of model 2. The latter performed very well, leading to errors below 0.25% in comparison to the exact node voltages in all situations studied. Furthermore, the mean error is about the same irrespective the capacitor location in the network.

Similar to the first test, for the second test a single capacitor bank of 1,200 kVAr was successively placed at each node. The same correction factors as in the preceding test have been used for this test. In this case, the biggest mean errors are 3.29% e 0.30% for the simplified model 1 and 2 respectively and with the capacitor bank located at node 23. Fig. 4 illustrates the mean percent errors which were obtained considering the voltage of all nodes for each capacitor bank location.

Also in this test the simplified model 2 performed better in terms of accuracy, as the errors are much smaller in comparison to the model 1. In all situations evaluated the error was less than 0.5% in relation to the exact value. Finally, it is worthwhile to note that Fig. 3 and 4 have practically the same shape but at different scales.

B. Test Network with 217 Nodes

This network is part of a real distribution network made up of a three phase distribution feeder which is 8.519 km long and operates at 13.8 kV as rated the voltage [9]. The feeder

total load is 7.46 MW and 2.64 MVar. The topology of this network is represented in Fig. 5, while a three dimensional view of the load distribution can be seen in Fig. 6.

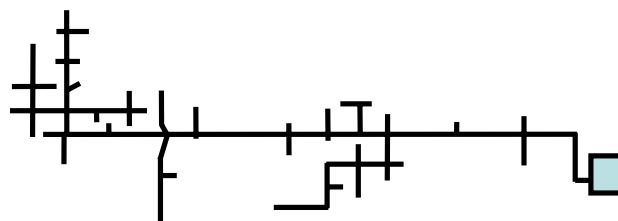


Fig. 5 Diagram of the example network with 217 nodes

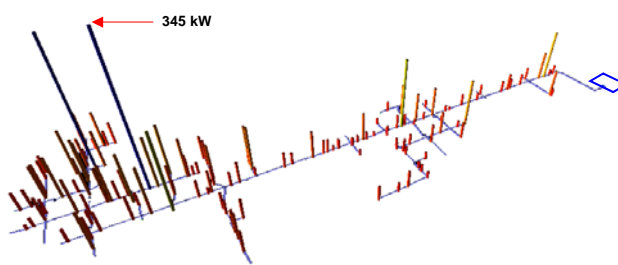


Fig. 6 Three dimensional view of the active load distribution along the network

Similar to the procedure adopted for the example network with 23 nodes, first the correction factors K_{km}^R for this network have been determined based on equation (23). This procedure resulted in values for the correction factors ranging from 1.0249 to 1.3554. In Fig. 7 the mean percent errors are plotted for the case of a single capacitor bank of 600 kVAr. The capacitor bank was successively placed at each of the network nodes, as proceeded before. The biggest errors in this case have been found as 1.14% for the simplified model 1 and 0.045% for the simplified model 2.

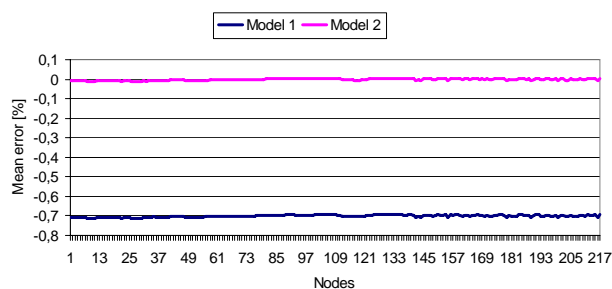


Fig. 7 Mean error in nodal voltages for the example network with 217 nodes and a capacitor bank of 600 kVAr placed one time at each node

It can be recognized from Fig. 7 that the model 2 performed better than model 1, as the mean errors are much smaller. Even for networks with a large number of nodes the mean percent errors are within acceptable limits.

For the next test a capacitor bank of 1,200 kVAr was used and the same procedure already outlined was repeated. The

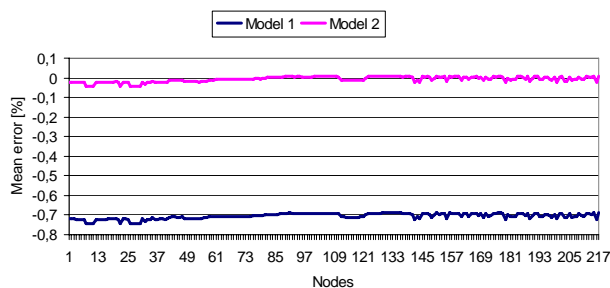


Fig. 8 Mean error in nodal voltages for the example network with 217 nodes and a capacitor bank of 1,200 kVAr placed one time at each node

mean errors for this case are reproduced in Fig. 8. The biggest mean error obtained with the simplified model 1 is 1.21%; using the model 2 the error becomes 0.11%. As in the preceding cases, the errors obtained with the model 2 are very low, they are, however, bigger for the capacitor bank of 1,200 kVAr than for 600 kVAr.

A further test has been carried out using this network, this time using two capacitor banks of 1,200 kVAr each one. Their locations have been chosen in such a way to improve the node voltages or to reduce the active power losses in the feeder, as described in [9]. The mean percent errors under this condition are reproduced in Table I. For both models the mean errors increased in comparison to the case of only one capacitor bank. However, the errors can still be considered acceptable considering the main purpose of the model. The first three rows in Table I refer to the choice of the capacitor location aiming at voltage level improvement. This strategy lead to bigger errors compared to the choice of the location aiming to reduce active power loss, given by the four last rows in Table I. The results obtained with model 2 are far better than those obtained with model 1, being the errors significantly smaller. The biggest mean error for the case of two capacitor banks was 1.26% for the model 1 and 0.16% for the model 2. These errors correspond to the location of the capacitor at nodes 7 and 27 respectively.

V. CONCLUSION

The simplified linear models of distribution network presented in this paper showed excellent results when applied to determine the node voltage amplitudes, as the practical test with example networks demonstrated. The differences in the voltage values obtained with the simplified models and the exact values obtained with the non-linear load flow are well acceptable, keeping in mind the context of optimization problems and approximate network representation. Thus, using the proposed models in optimization problems it is possible to handle voltage limit constraints and at the same time to keep the optimization problem linear. Non-linear optimization problems are far more complex than linear problems, being the solution more difficult to obtain, in some cases it is even impossible to obtain a solution.

The practical tests showed that the simplified model 2 give

better results than the model 1 due the difference in the correction factor used. The correction factors are determined only once and using the initial network topology with no capacitor banks. These factors are then kept the same during the whole optimization process despite the installation of capacitor banks at some nodes. The application of the model to example networks showed that the influence of capacitor banks in the correction factor is in practice very small, which validates the procedure adopted. These issues make the models very appropriated for using in optimization problems aiming to determine the best location for the capacitor banks to improve the network voltage profile. The use of the models allows solving a linear optimization problem instead of a non-linear problem, which is a very important advantage.

TABLE I

MEAN ERRORS IN THE NODE VOLTAGES CALCULATED WITH SIMPLIFIED MODELS. ERRORS ARE EXPRESSED IN PERCENT IN RELATION TO EXACT VALUES DETERMINED FROM THE SOLUTION OF THE LOAD FLOW

Nodes	Node voltage error (%)			
	Model 1		Model 2	
	Mean	Maximum	Mean	Maximum
23, 30	0.7834	1.2426	0.0845	0.1456
7, 27	0.7886	1.2588	0.0898	0.1607
7, 22	0.7883	1.2571	0.0895	0.1591
31, 77	0.7299	1.1154	0.0311	0.0397
20, 77	0.7289	1.1130	0.0301	0.0388
17, 77	0.7307	1.1172	0.0319	0.0404
52, 77	0.7282	1.1113	0.0294	0.0382

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