# Measurement Scheme Improving for State Estimation Using Stochastic Tabu Search

T. Kerdchuen

***Abstract*—**This paper proposes the stochastic tabu search (STS) for improving the measurement scheme for power system state estimation. If the original measured scheme is not observable, the additional measurements with minimum number of measurements are added into the system by STS so that there is no critical measurement pair. The random bit flipping and bit exchanging perturbations are used for generating the neighborhood solutions in STS. The $P\delta$ observable concept is used to determine the network observability. Test results of 10 bus, IEEE 14 and 30 bus systems are shown that STS can improve the original measured scheme to be observable without critical measurement pair. Moreover, the results of STS are superior to deterministic tabu search (DTS) in terms of the best solution hit.

***Keywords*—**Measurement Scheme, Power System State Estimation, Network Observability, Stochastic Tabu Search (STS).

## I. INTRODUCTION

THE actual system states are important for system operation. The inaccurate system states may lead to improper operation, leading to inadvertently insecure state. The power system state estimation is used to estimate the steady system states. The online measurement set provides the data into the state estimator in control center. Then, the system states are estimated in the state estimator. Thus, the measurement placement becomes an important problem to provide the sufficient system data for the state estimator.

The observability analysis is used to determine the system is observable or not. The observable system means it possible to estimate the system states. The numerical method uses the solvability from the flat start [1]. The triangular factorization is also arranged in numerical method [2, 3]. The topological method is also widely used for observability analysis. The rank of measurement topology matrix is judged the system observable or not [1-4]. Moreover, heuristic method is also used for measurement scheme design [5, 6]. However, the measurement graph is required. Meanwhile, the measurement scheme design is solved by many evolutionary algorithms [7-9].

In this paper, the minimum measurement pair number is considered for adding to the unobservable system. This additional measurement pair is considered as the optimization technique. The original measurement pair locations are fixed that unchanged in the neighborhood solution generating. Bit flipping and bit exchanging are used to generate the

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neighborhood solution of STS. Also, DTS [10] is applied to consider for measurement adding

## II. FUNDAMENTAL OF MEASUREMENT SCHEME FOR POWER SYSTEM STATE ESTIMATION

### A. Power System State Estimation

The conventional method for power system state estimation is the weight least squares (WLS) state estimation as in [1, 2]. The non-linear equations relating to the measurements and the state vector are

$$\mathbf{z} = \mathbf{h}(\mathbf{x}) + \mathbf{w} \qquad (1)$$

where $\mathbf{z}$ is the measurement vector, $\mathbf{h}(\bullet)$ is the $(m \times 1)$ non-linear function vector, $\mathbf{x}$ is the $(2N \times 1)$ system state vector, $\mathbf{w}$ is the $(m \times 1)$ measurement error vector, $m$ is the number of measurements, and $N$ is the number of buses. The WLS state estimation problem is solved by an iterative scheme:

$$\mathbf{G}(\mathbf{x}^k)[\Delta\mathbf{x}^k] = \mathbf{H}^T(\mathbf{x}^k)\mathbf{W}[\Delta\mathbf{z}^k], k = 0,1,2,... \qquad (2)$$

where

$$\Delta\mathbf{x}^k = \mathbf{x}^{k+1} - \mathbf{x}^k; \Delta\mathbf{z}^k = \mathbf{z} - \mathbf{h}(\mathbf{x}^k) \qquad (3)$$

$$\mathbf{G}(\mathbf{x}^k) = \mathbf{H}^T(\mathbf{x}^k)\mathbf{W}\mathbf{H}(\mathbf{x}^k). \qquad (4)$$

Matrix $\mathbf{W}$ is a diagonal matrix whose elements are the measurement weight factors, $\mathbf{H}(\mathbf{x}) = \partial\mathbf{h}(\mathbf{x})/\partial\mathbf{x}$ is the Jacobian matrix of $\mathbf{h}(\mathbf{x})$, and $\mathbf{G}$ is the gain matrix.

The state estimator needs a set of analog measurements and system topology to estimate the system state. In fact, the minimal measurements number is equal to $(2N-1)$ state variables. Therefore, the critical number of real and reactive measurement pairs is $(N-1)$ [5, 6] and additional one of bus voltage magnitude measurement at any bus.

### B. Network Observability

Network observability analysis is concerned with the power flows in the network and measurements made on the network.

In case of measurement pair (PQ) is used in power system then the $\mathbf{H}$ matrix is block diagonal, the question of observability can be decoupled into $P\delta$ observability and $QV$ observability. A power system is defined to be $P\delta(QV)$ observable with respect to a measurement set if the $\mathbf{H}_{P\delta}$

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( $\mathbf{H}_{QV}$ ) matrix is of rank $N-1$ ( $N$ ) [1, 5]. Moreover, the triangular factorization is performed to the measurement gain matrix $\mathbf{G}_{P\delta}$ or $\mathbf{H}_{P\delta}^{T}\mathbf{H}_{P\delta}$. If the result of this factorization is only one zero pivot, the system is observable. This factorization is called numerical method..

*C. Single Measurement Pair Loss & Critical Measurement Free*

For single measurement pair loss contingency, any single injection or flow of real and reactive measurement pair can be lost from the power system. This loss is due to either communication failure or measurement failure. When a single measurement pair is lost, the entire row of measurement observability matrix is deleted. Then, the reduced measurement matrix is used to determine observability.

If a single measurement pair can be lost from the power system, the system is considered as critical measurement free. In the absence of critical measurement in the power system, the bad data in any single measurement pair can be detected. The residual vector **r** is defined as the difference between **z** and the corresponding filtered quantities $\hat{\mathbf{z}} = \mathbf{H}\hat{\mathbf{x}}$. The residuals in terms of the elements of matrix **E** as follows

$$\mathbf{r} = \mathbf{z} \cdot \hat{\mathbf{z}} = \mathbf{z} - \mathbf{H}\hat{\mathbf{x}} = \mathbf{z} \cdot \mathbf{H}(\mathbf{G}^{-1}\mathbf{H}^{T}\mathbf{z})$$
$$= (\mathbf{I} - \mathbf{H}\mathbf{G}^{-1}\mathbf{H}^{T})\mathbf{z} = \mathbf{E}\mathbf{z} \qquad (5)$$

where $\mathbf{E} = \mathbf{I} - \mathbf{H}\mathbf{G}^{-1}\mathbf{H}^{T}$, **z** is a unity vector [11] (This simplification is based on the fact that critical measurement is independently of the measurement values) and $\mathbf{G} = \mathbf{H}^{T}\mathbf{H}$ is a gain matrix. Therefore, the $i^{th}$ component of the residual vector is calculated by

$$r(i) = \sum_{k=1}^{m} E(i,k) \qquad (6)$$

For each $z(i)$ of the measurement set, if $r(i)$ and $E(i,i)$ are zero, $z(i)$ is defined as a critical measurement [9, 11]

## III. STS IMPLEMENTATION

The objective of this consideration is to improve the original unobservable system for making the state estimation possible. The additional measurement could at a low installation cost. Since, the installation is directly dependent to the measurement number. Thus, the objective is to minimize the total additional measurement pair numbers as follows.

$$\text{Min.} \quad \text{Number of } (M_O + M_A) = M_O + \sum_{j=1}^{M_A} M_{Aj} \qquad (7)$$

subjects to the observability constraints

$$zero\_pivot = 1 \qquad (8)$$

or

$$rank(\mathbf{G}_{P\delta}) = N-1 \qquad (9)$$

or

$$rank(\mathbf{H}_{P\delta}) = N-1 \qquad (10)$$

and subject to the measurement pair numbers limit constraints

$$(M_O + M_A) \geq (N-1) \qquad (11)$$

or

$$(M_O + M_A) \geq N \qquad (12)$$

where $M_O$ is the total number of measurement pairs in the original system, $M_A$ is the total number of power measurement pairs for adding into the system and making system observable. The matrix $\mathbf{G}_{P\delta}$ and $\mathbf{H}_{P\delta}$ in (9) and (10) are related to the locations of measurement pair installation. Constraint (8) is used when the triangular factorization or numerical method is used for observability analysis. When observable, only one zero pivot is encountered during the factorization. Constraints (9) and (10) are used based on $P\delta$ observability. Constraint (11) is used when only considers observability, while (12) is used when considers with single measurement pair loss contingency or critical measurement free.

STS in this paper is different from DTS [10] in the process of neighbor solution generating method. The neighbor solution is generated by random position bit flipping and random position bit exchanging. The STS process is shown as follows.

*Step 1*: Initial a feasible solution from search space ( $x_0$ ) as a best solution $x''$
     and a global best $Gb = x''$
*Step 2*: Initial empty tabu list and aspiration level (*AL*)
*Step 3*: Iteration starting with condition of the cost of *Gb* is not equal 1
     *Step 3.1*: Generate neighborhood solutions using $x''$ to be initialized and
         find the current best solution ( $x'$ )
     *Step 3.2*: If $x'$ is not in tabu list,
         Update best solution by $x'$
         Update tabu list and *AL* by $x'$ and cost of $x'$
     *Step 3.3*: If cost of $x' < AL$
         Update best solution by $x'$
         Update tabu list and *AL* by $x'$ and cost of $x'$
         *Step 3.3.1*: If cost of $x' <$ cost of $Gb$
             Update $Gb$ by $x'$
     *Step 3.4*: Incremental iteration
     *Step 3.5*: If the *Gb* is not updated for long time,
         some of tubu list is used to generate the current solution (back
         tracking)
     *Step 3.6*: Return to step 3
*Step 4*. The best answer is *Gb*.

The initial feasible solution of STS is generated from the original measurement pair in a power system. The original measurement pair is fixed while the additional measurement pair is random search from the search space. For example, the 5-bus system with 6 branches, the original power injection measurement pairs have been installed at buses number 2 and 3. Also, the power flow measurement pair has been installed on the branch number 2. Thus, those positions of installed

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measurement pairs in the solution are fixed. The random search for new adding measurement pair is introduced to other positions that out of the original measurement positions. A typical solution is shown as follows.

Power Injection       Power Flow

| x | 1 | 1 | x | x | x | 1 | x | x | x | x |

Fig. 1: Typical Solution

In Fig. 1, the x positions are obtained by random search between 0 and 1. 0 means no measurement pair installs at that bus or that branch and 1 otherwise.

The cost evaluation of each solution includes the measurement pair number and observability part as follows.

$$cost = abs((N-1)-(M_O + M_A)) + (N - rank(\mathbf{H}_{P\delta})) \tag{13}$$

$$cost = abs(N-(M_O + M_A)) + (N - rank(\mathbf{H}_{P\delta})) + (Cm)(N) \tag{14}$$

In Fig. 1, (13) and (14), the original measurement pair $M_O$ is 3. The measurement matrix $\mathbf{H}_{P\delta}$ is formed from all measurement pairs $(M_O + M_A)$. These $cost$ are used for updating of aspiration level. The lower cost is a high aspiration level. The (13) is considered when without contingency consideration, while (14) is used when the critical measurement pair is considered. Note the minimum of (13) and (14) are 1. The $Cm$ in (14) is the number of critical measurement pairs that is identified by residual analysis.

The STS perturbation is introduced for generating the neighbor solutions. The bit flipping and bit exchanging are used for perturbation. Bit flipping is introduced to only one bit with random position in the solution, except that bit position belongs to the original measurement pairs installed. The bit exchanging is introduced into both power injection measurement pair and power flow measurement pair. The 50 probability is used for operation between of bit flipping and bit exchanging. The number of neighborhood solutions can be adjusted for suitability to a problem size. The typical of neighborhood solution generating is shown in Fig. 2.

The perturbation of DTS is only introduced to the x parts. The number of neighborhood solutions is equal to $(N + N_L - M_O)$ since the reason of one at a time flipping used. One at a time flipping for generating the neighborhood solution is applied only at the out of positions of $M_O$ in Fig. 3.
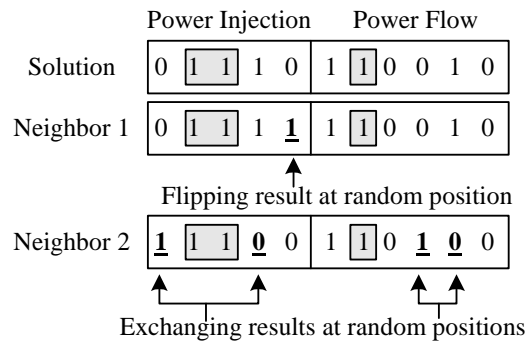


Fig. 2: Typical neighborhood solution generating of STS



Fig. 3: One at a time flipping of DTS

## IV. NUMERICAL RESULTS

The STS and DTS use the tabu list length is 5. The number of neighborhood solutions of DTS is depending on the position remaining of original measurement $(N + N_L - M_O)$ while STS depends to system buses. However, the neighborhood solution numbers of STS is considered as DTS for equality of comparison. The original measurement pairs are shown as Table 1. The typical of additional measurement pairs following the running of STS and DTS are also shown in Table 1. In Table 1, the additional measurement pairs are added by consideration without critical measurement pair. Also, the STS performance is shown as Tables 2-7. The total number of runs is 20.

TABLE I
RESULTS OF MEASUREMENT PAIRS ADDITION

| Systems | Original Meas. Pairs | Additional Meas. Pairs |
|---|---|---|
| 10-bus | Inj. 2, 3, 9 | Inj. 6, 8<br>Fl. 1-7, 2-5, 3-4, 5-7, 5-9 |
| IEEE 14-bus | Inj. 4, 5, 6, 13<br>Fl. 6-11 | Inj. 7<br>Fl. 1-2, 3-4, 7-8, 7-9, 9-10, 10-11, 12-13, 13-14 |
| IEEE 30-bus | Inj. 2, 6, 10, 12, 25<br>Fl. 5-7, 9-11, 18-19 | Inj. 3, 4, 9, 15, 20, 27, 28, 29<br>Fl. 1-3, 2-4, 9-10, 12-13, 12-16, 14-15, 10-17, 10-21, 21-22, 15-23, 22-24, 25-26, 27-30, 8-28 |

In Table 1 for 10-bus system, the original measurement scheme includes 3 power injection measurement pairs at buses

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2, 3 and 9. After running, the 7 additional power measurement pairs are added that includes 2 power injection measurement pairs at buses 6 and 8 and 5 power flow measurement pairs at lines 1-7, 2-5, 3-4, 5-7 and 5-9. Total number of power measurement is 10 that is no critical measurement pair in the finally measurement scheme.

TABLE II
PERFORMANCE OF STS FOR 10-BUS

| Detail | STS | DTS |
|---|---|---|
| Percent of the Best Hit | 100 | 100 |
| Average of Evaluation Time (sec.) | 0.09 | 0.21 |
| Average of the Best Hit Generation | 2 | 6.2 |

TABLE III
PERFORMANCE OF STS FOR IEEE 14-BUS

| Detail | STS | DTS |
|---|---|---|
| Percent of the Best Hit | 100 | 95 |
| Average of Evaluation Time (sec.) | 0.14 | 0.39 |
| Average of the Best Hit Generation | 2.8 | 11.26 |

TABLE IV
PERFORMANCE OF STS FOR IEEE 30-BUS

| Detail | STS | DTS |
|---|---|---|
| Percent of the Best Hit | 100 | 80 |
| Average of Evaluation Time (sec.) | 0.53 | 2.65 |
| Average of the Best Hit Generation | 5.55 | 41 |

From Tables 2-4, the results are considered for network observability only. STS yields the best answer with 100 percent of the best hit. Also, the computing time is lower than DTS. The generation of solution hit is also lower than DTS.

TABLE V
PERFORMANCE OF STS FOR 10-BUS WITHOUT CRITICAL MEASUREMENT PAIR

| Detail | STS | DTS |
|---|---|---|
| Percent of the Best Hit | 100 | 100 |
| Average of Evaluation Time (sec.) | 0.15 | 0.12 |
| Average of the Best Hit Generation | 2.75 | 2.35 |

TABLE VI
PERFORMANCE OF STS FOR IEEE 14-BUS WITHOUT CRITICAL MEASUREMENT PAIR

| Detail | STS | DTS |
|---|---|---|
| Percent of the Best Hit | 100 | 50 |
| Average of Evaluation Time (sec.) | 0.76 | 0.4 |
| Average of the Best Hit Generation | 14.5 | 7.5 |

TABLE VII
PERFORMANCE OF STS FOR IEEE 30-BUS WITHOUT CRITICAL MEASUREMENT PAIR

| Detail | STS | DTS |
|---|---|---|
| Percent of the Best Hit | 80 | 20 |
| Average of Evaluation Time (sec.) | 12.19 | 2.09 |
| Average of the Best Hit Generation | 53.75 | 9.75 |

From Tables 5-7, the results are considered for without critical measurement in the measurement scheme. STS still yields the best answer with high percent of the best hit. Computing time of STS is higher than DTS that DTS includes many inferior solutions. The inferior solution is not required to identify the critical measurement pair thus the computing time is low.

## V. CONCLUSION

The STS is effectively and efficiently added of the additional measurement pairs into the original unobservable power system. The random bit position and bit exchanging are used for generating the neighborhood solutions in STS. This STS is superior to DTS in terms of percent of the hit.

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