Distributed Contaminant Detection and Isolation for Intelligent Buildings

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Abstract—The automatic preservation of the Indoor Air Quality is an important task of the intelligent building design in order to ensure the health and safety of the occupants. The indoor air quality, however, is often compromised by various airborne contaminants that penetrate the indoor environment as a result of accidents or planned attacks. In this work, we provide the detailed analysis, implementation and evaluation of a distributed methodology for detecting and isolating multiple contaminant events in large-scale buildings. Specifically, we consider the building as a collection of interconnected subsystems and we design a contaminant event monitoring software agent for each subsystem. Each monitoring agent aims to detect the contaminant and isolate the zone where the contaminant source is located, while it is allowed to exchange information with its neighbouring agents. For configuring the subsystems, we implement both exact and heuristic partitioning solutions. A main contribution of this work is the investigation of the impact of the partitioning solution on the performance of the distributed Contaminant Detection and Isolation scheme with respect to the detectability and isolability of the contaminant sources. The performance of the proposed distributed contaminant detection and isolation methodology is demonstrated using models of real building case studies created on CONTAM¹.

Index Terms—Indoor air quality monitoring, Intelligent buildings, Contaminant detection, Distributed algorithms, Fault Diagnosis

I. INTRODUCTION

INTELLIGENT buildings are all about making use of modern technology for ensuring the occupants' comfort, productivity and safety by autonomously governing and adapting the building environment [1]–[4]. Note that according to recent studies, European citizens spend approximately 75% of their time indoors [5]. Thus, Indoor Air Quality (IAQ) has been identified as one of the three most important factors (the other two are visual and thermal comfort) that influence the quality of occupants' life in a building environment [6].

The IAQ is often compromised by various airborne contaminants that are either generated indoors or penetrate into the

indoor environment with passive or active airflows [7], [8]. These contaminant events could be the result of an accident, such as carbon monoxide (CO) leakage from a malfunctioning furnace, or a terrorist attack. Under these safety-critical conditions, it becomes of paramount importance to promptly detect the presence of the contaminant event and identify the location of the source. Such information, is needed for taking the necessary measures (e.g., activation of the evacuation plan [9]) in order to ensure people's safety.

In the last decade, there have been major advancements in various sensing technologies that enabled the application of active protective measures. In this context, appropriate sensors that become alarmed in the presence of a particular contaminant are strategically placed throughout the intelligent building. The sensor measurements are collected and processed using Contaminant Detection and Isolation (CDI) algorithms for detecting the presence of the contaminant source and estimating its location. Related literature can be broadly categorized into (i) model-based [8], [10]–[12] and (ii) data-driven methods [13]–[19]. The main difference between them is that model-based methods assume the presence of a physical model for performing the estimation task, while data-driven methods utilize learning to either construct a model (from the data) or a scenario database before the event.

In our previous work [11], we have developed a model-based, state-space method based on multi-zone models. A key advantage of this solution compared to other related work, is that it does not require any prior information of the source characteristics (onset time, location and generation rate). Multi-zone models offer a computationally efficient solution for simulating contaminant transportation in the building interior by representing a building as a network of well-mixed zones. Temperature, humidity, air velocity and pollutant concentration are assumed uniform within a single zone. Different zones are connected by discrete flow paths such as doors, windows, wall cracks, ducts and hallways. In this framework, a zone maybe an entire room or part of a room. Note that multi-zone models also have their limitations, especially for cases where the well-mixed assumption does not hold [20]. For these cases, more sophisticated models based on Computational Fluid Dynamics (CFD) can be implemented instead at the expense of increased computational complexity and execution times. Indoor environment forecasting has also been used in recent years for utilizing sensor measurements to predict the contaminant transport and improve the accuracy of the contaminant dispersion models [21]. Under the state space

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¹CONTAM is a multi-zone simulation program developed by the US National Institute of Standards and Technology (NIST).

formulation, the contaminant event source is modeled as a fault in the process, which has to be detected and isolated. This enables the application of advanced fault diagnosis tools [22]–[26] to the problem of contaminant event monitoring in intelligent buildings [11]. Note that a number of other studies have utilized the state-space method to address indoor environment related problems such as security-oriented sensor placement [27], model predictive climate control [28], contaminant transportation [29], [30], and analysis of the dynamics and concentration of particulate matter (PM) [31] or CO₂ [32].

The CDI technique in [11], however, is centralized; it considers the whole building for applying the technique and consequently it faces the computational complexity problems that arise as the size of the building increases. In this paper, we develop a fully distributed CDI methodology for large-scale intelligent buildings. This includes developing partitioning algorithms for dividing the building into subsystems and designing CDI agents for each subsystem. In general, distributed approaches have significant benefits over the centralized ones due to (i) the lower complexity in designing the CDI method for smaller subsystems, (ii) the improved scalability derived from the easy adaptation to larger buildings by incorporating additional zones, (iii) the increased reliability due to the lack of a single point of failure, (iv) the reduction of communication requirements since there is no need of transmitting all the information globally to a single point, and (v) the improved ability in handling multiple contaminant sources.

In this work, a distributed CDI scheme is developed by considering the building as a collection of interconnected subsystems. Depending on the environmental conditions and the location of doors and windows, there are induced airflows between the various zones due to natural (wind) or forced (fan) ventilation conditions. In this framework, the entire building can be viewed as a system of interacting zones (through the airflows) which can potentially transport air contaminants from one zone of the building to another. Hence, the partitioning of the building into groups of zones can also be considered as the formation of subsystems for the application of the distributed CDI scheme. Note that, buildings offer a natural candidate for such structural partitioning, because they are distributed in space and a particular building zone is only interconnected with a limited number of neighboring zones, mainly through doors and windows. Each subsystem is assigned a monitoring agent, which is essentially a software program that processes the contaminant sensor measurements and aims to detect the presence of a contaminant and isolate the zone where its source is located. The decision logic implemented in a contaminant event monitoring agent is based on the development of distributed observer schemes for estimating the state (contaminant concentration) of the subsystem. The distributed observers are properly designed to handle the parameter varying nature of the system with respect to the airflows in the building interior, presented as modeling and interconnection uncertainty in the distributed system model. Moreover, each agent is allowed to exchange information about its state with its neighboring agents. The contaminant detection and isolation decisions are derived using observer based residuals and adaptive thresholds. The use of adaptive thresholds increases the sensitivity of the

detection and isolation algorithms, while at the same time avoids the presence of false alarms in the system.

Nonetheless, the success of the proposed distributed CDI approach relies on the proper partitioning of the building into subsystems. In fact, the partitioning solution plays a decisive role in the successful detection and isolation of the contaminant source, as it will be further demonstrated in the sequel. So a key focus of this work is to demonstrate the necessity to partition the system through proper partitioning algorithms in order to ensure high CDI performance. In particular, the subsystems should be chosen so as to (i) minimize the coupling between the various subsystems in terms of the interconnection airflows, (ii) ensure the connectivity of each subsystem so that all included zones are connected through at least one leakage path, and (iii) balance the size between the various subsystems with respect to the number of allocated zones.

Finally, in a real-building setting, the airflows may often change due to variable environmental conditions (temperature, wind direction and velocity), as well as the opening and closing of doors and windows. Under such conditions, we propose the on-line re-configuration of the building subsystems and the distributed CDI algorithms to retain high detection and isolation performance. For this purpose, we utilize two partitioning algorithms that we previously developed [33]: (i) an optimal Mixed Integer Linear Programming (MILP) solution, and (ii) a Building Partitioning Heuristic (BPH), that essentially provide a trade-off between complexity and performance. The former algorithm is applied *off-line* to provide an initially optimal partitioning solution, while the latter is then used *on-line* to re-partition the building in a timely manner.

At this point, we should mention that similar distributed schemes have been derived by the fault diagnosis community in different settings [34]–[37]; however, none of them considers explicitly the subsystem formation and its corresponding impact on the performance of the distributed schemes. The results of this work, although tailored for contaminant detection and isolation in buildings, are suitable for many other application domains with similar characteristics (i.e. with a structured environment).

In summary, the main contributions of this work are:

- 1) The development of a distributed CDI scheme for intelligent buildings based on the state-space, multi-zone model [11], taking into account modeling uncertainty and measurement noise. This involves designing the contaminant event monitoring agents, the observers, the adaptive thresholds and the decision logic. Some preliminary results of this work appear in [38].
- 2) The design of automatic building re-partitioning and CDI agent re-configuration processes that ensure high CDI performance under varying environmental conditions.
- 3) The analysis of the impact of partitioning solutions on the performance of CDI algorithms on real building case studies.

The rest of the paper is organized as follows. First, in Section II, we formulate the distributed framework for the problem of contaminant event monitoring. Next, in Section III, we present the distributed CDI scheme and in Section IV,

the partitioning algorithms for the effective automatic formation of the subsystems. Section V describes the performance evaluation of the proposed distributed CDI scheme through an extensive simulation campaign. Finally, Section VI provides some concluding remarks and our future research plans.

II. PROBLEM FORMULATION

Given a n -zone building, the contaminant dispersion in the indoor environment of the building is considered as a monolithic system, described by the following multi-zone model:

$$\Sigma: \quad \dot{x}(t) = (A + \Delta A)x(t) + Q^{-1}Bu(t) + Q^{-1}Gg(t), \quad (1)$$

$$y(t) = Cx(t) + w(t), \quad (2)$$

where $x \in \mathbb{R}^n$ represents the concentrations of the contaminant in the building zones ($x^{(i)} \in \mathbb{R}$ is the element of x , denoting the concentration in the i -th zone), while $A \in \mathbb{R}^{n \times n}$ is the state transition matrix with its element $A(i, j)$ equal to zero, if there is no leakage path (door, window, etc.) between zone i and zone j , otherwise $A(i, j)$ depends on the airflow between zone i and zone j either through a physical connection (door, windows etc.) or through HVAC ducts. The term $\Delta A \in \mathbb{R}^{n \times n}$ collectively accounts for the presence of modeling uncertainty in the building envelope as a result of disturbances due to variable wind speed, variable wind direction and variable leakage openings. The controllable inputs in the form of doors, windows, fans and air handling units are represented by the variable $u \in \mathbb{R}^p$, while $B \in \mathbb{B}^{n \times p}$ is a zone index matrix concerning their locations, with $\mathbb{B} = \{0, 1\}$. The last term of (1) involves the location and evolution characteristics of the contaminant sources, represented by $G \in \mathbb{B}^{n \times s}$ and $g \in \mathbb{R}^s$ respectively. Note that $Q \in \mathbb{R}^{n \times n}$ is a diagonal matrix, i.e. $Q = \text{diag}(Q^{(1)}, Q^{(2)}, \dots, Q^{(n)})$ where $Q^{(i)}$ is the volume of the i -th zone. In (2), $y \in \mathbb{R}^m$ represents the output of the sensors monitoring Σ , $C \in \mathbb{B}^{m \times n}$ is a zone index matrix for the sensor locations and $w \in \mathbb{R}^m$ characterizes the additive measurement noise. More details on the state-space model formulation can be found in [11]. It is worth pointing out that in the context of fault diagnosis, the last term in (1), i.e. $Q^{-1}Gg(t)$, is equivalent to process faults that impact the normal system operation.

In this work, the monolithic system Σ is decomposed into K interconnected subsystems Σ_I , $I \in \{1, \dots, K\}$, where each subsystem corresponds to the dispersion of the contaminant in an area of n_I zones, described by:

$$\Sigma_I: \quad \dot{x}_I(t) = (A_I + \Delta A_I)x_I(t) + Q_I^{-1}B_Iu_I(t) + Q_I^{-1}G_Ig_I(t) \\ + (H_I + \Delta H_I)z_I(t), \quad (3)$$

$$y_I(t) = C_Ix_I(t) + w_I(t), \quad (4)$$

where $x_I \in \mathbb{R}^{n_I}$, $u_I \in \mathbb{R}^{p_I}$ and $y_I \in \mathbb{R}^{m_I}$ are the local state, local control input, and local measured output vectors, respectively, and $z_I \in \mathbb{R}^{l_I}$ is the vector of the interconnection variables. In particular, x_I represents a column vector made up of n_I elements of x (correspondingly for u_I and y_I), i.e. there is an index mapping function M_{x_I} such that $M_{x_I}: \{1, \dots, n\} \rightarrow \{1, \dots, n_I\}$ and $M_{x_I}^{-1}$ its inverse mapping function such that

$M_{x_I}^{-1}: \{1, \dots, n_I\} \rightarrow \{1, \dots, n\}$, satisfying

$$M_{x_I}(k) = \{j: x_I^{(j)} = x^{(k)}, k \in \{1, \dots, n\}, j \in \{1, \dots, n_I\}\}, \quad (5)$$

$$M_{x_I}^{-1}(j) = \{k: x^{(k)} = x_I^{(j)}, k \in \{1, \dots, n\}, j \in \{1, \dots, n_I\}\} \quad (6)$$

In a similar way, we determine the index mapping functions for u and y , i.e. M_{u_I} such that $M_{u_I}: \{1, \dots, p\} \rightarrow \{1, \dots, p_I\}$ and M_{y_I} such that $M_{y_I}: \{1, \dots, m\} \rightarrow \{1, \dots, m_I\}$. The interconnection variables z_I refer to the states of the neighboring subsystems in the building structure graph having a direct connection with the subsystem I , i.e. z_I is made up of states of x that belongs to subsystems different from Σ_I . The term $g_I \in \mathbb{R}$ represents the local contaminant source, $G_I \in \mathbb{B}^{n_I}$ is the local location index vector (i.e., a vector of zeros except from a single entry of one corresponding to the zone containing the source) and $w_I \in \mathbb{R}^{m_I}$ corresponds to the local noise vector. Note that, in this work, we assume a maximum of a single contaminant source per subsystem. Finally, A_I , ΔA_I , Q_I , G_I and C_I are sub-matrices of appropriate dimensions of the corresponding matrices of the monolithic system Σ , while H_I and ΔH_I are sub-matrices of A and ΔA respectively related to the interconnection variables z_I .

A. Holmes Building Case Study

In order to better illustrate the various concepts presented in this paper, we will be using a building case study corresponding to the Holmes house² simulated using CONTAM [40]. The specific building shown in Fig. 1, comprises of 14 zones: a garage (Z1), a storage room (Z2), a utility room (Z3), a living room (Z4), a kitchen (Z5), a corridor (Z8), three bedrooms (Z7, Z9, Z14), two bathrooms (Z6, Z13) and three closets (Z10, Z11, Z12); as well as 30 leakage path openings.

For the considered scenario, it is assumed that natural ventilation is the dominant cause of air movement in the building with a wind speed of 10 m/s blowing from the east (i.e., at 90°). The resulting airflows (direction and magnitude) are portrayed with green lines in Fig. 1 while the corresponding state transition matrix A is shown in Fig. 2. Note that the sum of incoming flows should be equal to the sum of outgoing flows for each zone. Hence, the entries in each row of matrix A should sum up to zero unless there are external incoming flows. Both the resulting airflows and the corresponding state transition matrix A are calculated using CONTAM.

In order to enable the distributed CDI, the building is partitioned in three subsystems, as shown in Fig.1, given by $\Sigma_1 = \{Z1, Z2, Z3, Z4, Z5\}$, $\Sigma_2 = \{Z6, Z8, Z9, Z12, Z13\}$ and $\Sigma_3 = \{Z7, Z10, Z11, Z14\}$. Note that the particular partitioning can be obtained using both the MILP and BPH algorithms, as it will be demonstrated in the sequel.

III. DISTRIBUTED CONTAMINANT DETECTION AND ISOLATION

The objective of this section is to design a methodology for detecting and isolating multiple contaminant sources in the

²Refers to a low-rise residential house model used by J.D. Holmes and then scaled up for Simulating Airflow and Contaminant Transport in and around Buildings [39].

Fig. 1: Holmes building case study consisting of 14 zones and 30 leakage paths with a wind speed of 10 m/s blowing from the east. The resulting airflows are portrayed with green lines (the length of the lines corresponds to the magnitude of the respective airflows). The building is partitioned into 3 subsystems (indicated with dashed lines of different colors).

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	0	-0.15	0	0.15	0	0	0	0	0	0	0	0	0	0
2	0	-0.42	0.42	0	0	0	0	0	0	0	0	0	0	0
3	0	0	-0.9	0	0.036	0	0	0	0	0	0	0	0	0
4	0	0	0	-0.292	0.277	0	0	0.015	0	0	0	0	0	0
5	0	0	0	0	-0.563	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	-0.1	0	0.1	0	0	0	0	0	0
7	0	0	0	0	0	0	-0.252	0	0	0	0	0	0	0
8	0	0	0	0	0	0.515	-0.695	0	0	0	0	0	0	0.18
9	0	0	0	0	0	0	0.232	-0.232	0	0	0	0	0	0
10	0	0	0	0	0	1.2E-05	0	0	-1.2E-05	0	0	0	0	0
11	0	0	0	0	0	0	0	0	1.12E-05	-1.12E-05	0	0	0	0
12	0	0	0	0	0	0	0	0.21	0	0	0	-0.31	0.1	0
13	0	0	0	0	0	0	0	0.24	0	0	0	0	-0.24	0
14	0	0	0	0	0	0	0	0	0	0	7.80E-06	0	0	-0.3

Fig. 2: State transition matrix A for the Holmes building case study shown in Fig. 1.

building subsystems, assuming a maximum of a single source per subsystem. For each of the interconnected subsystems Σ_I , a monitoring agent, denoted by $\mathcal{M}_I$, is designed to detect and isolate a contaminant source affecting the dynamics of Σ_I . The agent $\mathcal{M}_I$ is allowed to exchange information with its neighboring agents. The exchanged information is associated with the form of the physical interconnections, i.e., unidirectional or bidirectional interactions between the interconnected subsystems represented by the terms $H_I z_I$. Fig. 3 demonstrates the distributed agent architecture for the building case study described in Fig. 1, in terms of the various interconnections and information exchange between the three subsystems and the corresponding agents.

Each agent $\mathcal{M}_I$ consists of a contaminant detector, denoted by $\mathcal{D}_I$ and n_I isolators, denoted by $\mathcal{I}_{I,j}$, $j \in \{1, \dots, n_I\}$. Under normal conditions, the task of $\mathcal{D}_I$ is to detect the presence of a contaminant source in the subsystem Σ_I . If a contaminant is detected, then the corresponding isolators $\mathcal{I}_{I,j}$ are activated to localize the zone of subsystem Σ_I with the contaminant source. The decisions of the n_I isolators are combined for excluding the zones in the building subsystem Σ_I that do not contain the contaminant source.

The design of both detectors ($\mathcal{D}_I$) and isolators ($\mathcal{I}_{I,j}$) relies on comparing some generated residual signals to correspond-

Fig. 3: Distributed agent architecture for Holmes building case study shown in Fig. 1, demonstrating the various interconnections and information exchange between the three subsystems and the corresponding agents.

ing adaptive thresholds. The residuals express the difference of the system's actual behavior (i.e., expressed through the sensor measurements) with the nominal model behavior (i.e., non-faulty behavior as expressed through the estimation model of the observers). If there is no uncertainty, the residual signals should be normally zero or close to zero when no fault is present, but distinguishably different from zero when a fault occurs. On the other hand, the adaptive thresholds are designed to bound the residuals in the fault-free case while ensuring robustness of the decision in the presence of modeling uncertainty and sensor noise. Both the residuals and the adaptive thresholds are generated using an estimation model, which is produced by an observer based on the mathematical model given in (3)-(4). For more information on observers please see [26], [41]–[43].

In what follows, we describe in more detail the detectors and isolators together with the corresponding decision logic. Note that the dependence of the signals on time (e.g. $x(t)$) will be dropped for notational brevity.

A. Contaminant Event Detection

1) *Residual Generation:* The estimation model of $\mathcal{D}_I$ is formulated by selecting the following observer

$$\dot{\hat{x}}_I = A_I \hat{x}_I + Q_I^{-1} B_I u_I + H_I y_{z_I} + L_I (y_I - C_I \hat{x}_I), \quad (7)$$

where $\hat{x}_I \in \mathbb{R}^{n_I}$ is the estimation of x_I (with initial conditions $\hat{x}_I(0) = 0$), $L_I \in \mathbb{R}^{n_I \times m_I}$ is the observer gain matrix (the design of L_I will be discussed in the sequel) and $y_{z_I} \in \mathbb{R}^{\ell_I}$ is the transmitted sensor information, defined based on (4) as

$$y_{z_I} = z_I + w_{z_I} \quad (8)$$

where $w_{z_I} \in \mathbb{R}^{\ell_I}$ is the corresponding noise vector.

The k -th residual of the detector $\mathcal{D}_I$, denoted by $\varepsilon_{y_I}^{(k)} \in \mathbb{R}$, is defined as

$$\varepsilon_{y_I}^{(k)} = y_I^{(k)} - C_I^{(k)} \hat{x}_I, \quad (9)$$

where $y_I^{(k)} \in \mathbb{R}$ is the k -th element of y_I and $C_I^{(k)}$ is the k -th row of C_I , $k \in \{1, \dots, m_I\}$.

2) *Computation of Adaptive Thresholds:* The k -th adaptive threshold of $\mathcal{D}_I$, denoted by $\bar{\varepsilon}_{y_I}^{(k)}(t)$, is designed in order to ensure robustness of the detector $\mathcal{D}_I$ with respect to modeling uncertainties ΔA_I and ΔH_I and noise w . The computation of adaptive thresholds is realized taking into account the following assumption.

Assumption 1:

The modeling uncertainty of ΔA_I , ΔH_I and the noise corrupting the measurements of each sensor, are unknown but uniformly bounded for all $I \in \{1, \dots, K\}$ and $k \in \{1, \dots, m_I\}$:

$$|\Delta A_I| \leq \overline{\Delta A_I}, \quad (10)$$

$$|\Delta H_I| \leq \overline{\Delta H_I}, \quad (11)$$

$$|w_I^{(k)}(t)| \leq \overline{w}_I^{(k)}. \quad (12)$$

It is noted that $|\cdot|$ denotes the euclidean norm in the case of matrices or vectors, while in the case of a scalar, $|\cdot|$ denotes the absolute value. The bounds in (10), (11) are commonly used for distinguishing disturbances and faults [44], while the noise bound in (12) corresponds to a practical representation of the available knowledge for the sensor noise that is typically stated in the sensor specifications provided by sensor manufacturers.

The k -th adaptive threshold of the detector $\mathcal{D}_I$ is designed to bound the residual under healthy conditions, i.e. taking into account that $g_I = 0$. Based on (4), (9), the residual can be expressed as

$$\varepsilon_{y_I}^{(k)} = C_I^{(k)} \varepsilon_{x_I} + w_I^{(k)}, \quad (13)$$

where ε_{x_I} is the state estimation error, defined as $\varepsilon_{x_I} \triangleq x_I - \hat{x}_I$. Under healthy conditions, the state estimation error is described by:

$$\dot{\varepsilon}_{x_I} = A_{L_I} \varepsilon_{x_I} - H_I w_{z_I} + \Delta H_I z_I + \Delta A_I x_I - L_I w_I, \quad (14)$$

$$A_{L_I} = A_I - L_I C_I. \quad (15)$$

satisfying the following Lemma.

Lemma 3.1: If for all $I \in \{1, \dots, K\}$, the observer gain L_I is selected such that: (i) the matrix $A_I - L_I C_I$ is stable, and (ii) there are positive constants ξ_I , ρ_I such that $|e^{(A_I - L_I C_I)t}| \leq \rho_I e^{-\xi_I t}$ and $\xi_I > \rho_I \overline{\Delta A_I}$, then the state estimation error described by (14), (15) is uniformly bounded.

Proof: The proof is included in Appendix A.

Based on (13), the adaptive thresholds are computed such that

$$|\varepsilon_{y_I}^{(k)}(t)| \leq \bar{\varepsilon}_{y_I}^{(k)}(t). \quad (16)$$

The details of the adaptive threshold derivation are provided in Appendix A.

3) *Detection Decision Logic:* The decision logic of the detector $\mathcal{D}_I$ is described by Lemma 3.2. The rationale behind this lemma is that the contaminant source is guaranteed to have occurred in the subsystem Σ_I , i.e. a contaminant event is concluded, when the behavior of Σ_I observed using the measurements y_I is not consistent with the expected behavior, represented by $\hat{x}_I$.

Lemma 3.2: If there is a time instant t at which

$$|\varepsilon_{y_I}^{(k)}(t)| > \bar{\varepsilon}_{y_I}^{(k)}(t), \quad (17)$$

for at least one $k \in \{1, \dots, m_I\}$, the detector $\mathcal{D}_I$ infers the presence of a contaminant source in subsystem Σ_I , $I \in \{1, \dots, K\}$.

Proof: The decision logic of the detector $\mathcal{D}_I$ relies on *reductio ad absurdum*; suppose that there is no contaminant source in the subsystem Σ_I (i.e., healthy conditions). Then, the k -th adaptive threshold $\bar{\varepsilon}_{y_I}^{(k)}(t)$ bounds by design at every time instant the magnitude of k -th residual $\varepsilon_{y_I}^{(k)}(t)$ for all $k \in \{1, \dots, m_I\}$. This contradicts the validity of (17). Thus, our assumption that there is no contaminant source in the subsystem Σ_I is incorrect.

The time instant of detection is defined as:

$$T_{D_I} = \min_t \bigcup_{k \in \{1, \dots, m_I\}} \left\{ \min_t \left\{ t : |\varepsilon_{y_I}^{(k)}(t)| > \bar{\varepsilon}_{y_I}^{(k)}(t) \right\} \right\} \quad (18)$$

After the time instant of detection, we determine the contaminant diagnosis set, denoted by $\mathcal{S}_{D_I}(t)$ that includes the zones in Σ_I that possibly contain the contaminant source. Hence, after the time of detection, the diagnosis set is defined as

$$\mathcal{S}_{D_I}(t) = \left\{ Z^{(j)}, \forall j \in \{1, \dots, n_I\} \right\}, \text{ for } t \geq T_{D_I} \quad (19)$$

where $Z^{(j)}$ denotes the zone j . Based on $\mathcal{S}_{D_I}(t)$, the contaminant event monitoring agent $\mathcal{M}_I$ initially assumes that every zone j in the building subsystem Σ_I possibly contains the contaminant source.

B. Contaminant Event Isolation

Suppose that the contaminant source lies in the r -th zone of building subsystem Σ_I , $r \in \{1, \dots, n_I\}$; i.e.,

$$\begin{aligned} \dot{x}_I &= (A_I + \Delta A_I)x_I + Q_I^{-1}B_I u_I + (H_I + \Delta H_I)z_I \\ &\quad + Q_I^{-1}G_{I,r}g_I, \end{aligned} \quad (20)$$

where $G_{I,r}$ is a vector with zeros except for the r -th row with ones. It is noted that the r -th zone of building subsystem Σ_I , $r \in \{1, \dots, n_I\}$ corresponds to the $M_{x_I}^{-1}(r)$ zone of the building system Σ . The difference between model (3) and (20) is that in (3) we model the unknown location of the contaminant by G_I and the unknown contaminant rate g_I , while in (20) we hypothesize that the contaminant is in the r -th zone so we know $G_{I,r}$ but we still do not know g_I . For this reason, we aim to estimate g_I .

At the time instant of detection T_{D_I} , the bank of n_I isolators $\mathcal{I}_{I,j}$, $j \in \{1, \dots, n_I\}$ is activated, aiming at isolating the contaminant source zone in the building subsystem Σ_I .

1) *Residual Generation:* The estimation model of $\mathcal{I}_{I,j}$ is formulated by selecting an adaptive observer, which is designed assuming the presence of the contaminant source in the zone j , $j \in \{1, \dots, n_I\}$; i.e.,

$$\begin{aligned} \dot{\hat{x}}_{I,j} &= A_I \hat{x}_{I,j} + Q_I^{-1}B_I u_I + H_I y_{z_I} + L_I(y_I - C_I \hat{x}_{I,j}) \\ &\quad + \Omega_{I,j} \hat{\phi}_{I,j} + Q_I^{-1}G_{I,j} \hat{\phi}_{I,j}, \end{aligned} \quad (21)$$

$$\dot{\Omega}_{I,j} = A_{L_I} \Omega_{I,j} + Q_I^{-1}G_{I,j} \quad (22)$$

$$\hat{\phi}_{I,j} = P_I \left\{ \Gamma_{I,j} (C_I \Omega_{I,j})^\top (y_I - C_I \hat{x}_{I,j}) \right\} \quad (23)$$

where $\hat{x}_{I,j} \in \mathbb{R}^{n_I}$ is the estimation of x_I , $L_I \in \mathbb{R}^{n_I \times m_I}$ is the observer gain matrix selected such that the matrix A_{L_I} (defined in (15)) is stable and where $G_{I,j}$ is a vector with zeros except for the j -th row with ones. Moreover, the adjustable parameter

of the adaptive approximator $\hat{\phi}_{I,j}(t) \in \mathbb{R}$ corresponds to the estimate of the contaminant source g_I . The initial parameter $\hat{\phi}(T_{D_I})$ and matrix $\Omega(T_{D_I})$ are chosen as $\hat{\phi}(T_{D_I}) = 0$ and $\Omega(T_{D_I}) = 0$, respectively. In the adaptive law (23), $\Gamma_{I,j} \in \mathbb{R}$ is a learning rate (positive), while the projection operator P_I restricts the adjustable parameter $\hat{\phi}_{I,j}(t)$ to the predefined interval Φ_I , within which g_I resides [45].

The k -th residual of the isolator $\mathcal{I}_{I,j}$, denoted by $\varepsilon_{y_{I,j}}^{(k)} \in \mathbb{R}$, is defined as

$$\varepsilon_{y_{I,j}}^{(k)} = y_I^{(k)} - C_I^{(k)} \hat{x}_{I,j}, \quad (24)$$

where $y_I^{(k)} \in \mathbb{R}$ is the k -th element of y_I and $C_I^{(k)}$ is the k -th row of C_I .

2) *Computation of adaptive thresholds:* The adaptive threshold of the isolator $\mathcal{I}_{I,j}$ is designed to bound the residual $\varepsilon_{y_{I,j}}^{(k)}$, i.e.

$$\left| \varepsilon_{y_{I,j}}^{(k)}(t) \right| \leq \bar{\varepsilon}_{y_{I,j}}^{(k)}(t), \quad (25)$$

assuming that the contaminant source lies in the j -th zone, $j \in \{1, \dots, n_I\}$, i.e. $G_{I,r} = G_{I,j}$ in (20) and taking into account the following assumption.

Assumption 2: The rate of evolution of the j -th contaminant $g_I \in \mathbb{R}$, for all $I \in \{1, \dots, K\}$ is uniformly bounded, $|\dot{g}_I(t)| \leq \bar{g}_I$, where $\bar{g}_I \in \mathbb{R}$ is a known constant bound.

Based on (4), (24), the residual can be expressed as $\varepsilon_{y_I}^{(k)} = C_I^{(k)} \varepsilon_{x_{I,j}} + w_I$, where $\varepsilon_{x_{I,j}} \triangleq x_I - \hat{x}_{I,j}$; taking into account (20) and (21), the state estimation error is described by:

$$\begin{aligned} \dot{\varepsilon}_{x_{I,j}} &= A_{L_I} \varepsilon_{x_{I,j}} - H_I w_{z_I} + \Delta A_I x_I + \Delta H_I z_I - L_I w_I \\ &\quad + Q_I^{-1} G_{I,r} g_I - Q_I^{-1} G_{I,j} \hat{\phi}_{I,j} - \Omega_{I,j} \dot{\hat{\phi}}_{I,j}. \end{aligned} \quad (26)$$

By adding and subtracting $\Omega_{I,j} \dot{g}_I$ and $Q_I^{-1} G_{I,j} g_I$ and using (22), we obtain

$$\begin{aligned} \dot{\tilde{\varepsilon}}_{x_{I,j}} &= A_{L_I} \tilde{\varepsilon}_{x_{I,j}} - H_I w_{z_I} + \Delta A_I x_I + \Delta H_I z_I - L_I w_I \\ &\quad + Q_I^{-1} (G_{I,r} - G_{I,j}) g_I - \Omega_{I,j} \dot{g}_I \end{aligned} \quad (27)$$

$$\varepsilon_{x_{I,j}} = \tilde{\varepsilon}_{x_{I,j}} + \Omega_{I,j} \tilde{\phi}_{I,j} \quad (28)$$

where $\tilde{\phi}_{I,j} = g_I - \hat{\phi}_{I,j}$.

By introducing the solution of (27) in (28), the adaptive threshold of the isolator $\mathcal{I}_{I,j}$ is calculated following the procedure outlined in Appendix B.

3) *Isolation Decision Logic:* The decision logic of the isolator $\mathcal{I}_{I,j}$, $j \in \{1, \dots, n_I\}$ is expressed in Lemma 3.3. The rationale behind this lemma is that it is guaranteed that the zone j is safe, i.e. the contaminant source is excluded from zone j , when the behavior of Σ_I observed through y_I is not consistent with the expected behavior represented by $\hat{x}_{I,j}$.

Lemma 3.3: If there is a time instant at which

$$\left| \varepsilon_{y_{I,j}}^{(k)}(t) \right| > \bar{\varepsilon}_{y_{I,j}}^{(k)}(t), \quad (29)$$

for at least one $k \in \{1, \dots, m_I\}$, the isolator $\mathcal{I}_{I,j}$ infers that zone j , $j \in \{1, \dots, n_I\}$, of the building subsystem Σ_I , or equivalently, zone $M_{x_I}^{-1}(j)$ of the building system Σ , does not contain the contaminant source.

Proof: The decision logic of the isolator $\mathcal{I}_{I,j}$ relies on *reductio ad absurdum*; suppose that there is a contaminant source

in zone j , $j \in \{1, \dots, n_I\}$. Then, the k -th adaptive threshold $\bar{\varepsilon}_{y_{I,j}}^{(k)}(t)$ bounds by design at every time instant the magnitude of k -th residual $\varepsilon_{y_{I,j}}^{(k)}(t)$ for all $k \in \{1, \dots, m_I\}$, as described in Section III-B2. This contradicts the validity of (29). Thus, our assumption that there is a contaminant source in zone j is incorrect. The decision of $\mathcal{I}_{I,j}$ is represented by a boolean function, $D_{I,j}$ defined as

$$D_{I,j}(t) = \begin{cases} 1, & \text{for } t < T_{I,j} \\ 0, & \text{otherwise} \end{cases}, \quad (30)$$

$$T_{I,j} = \min_t \bigcup_{k \in \{1, \dots, m_I\}} \left\{ \min_t \left\{ t : \left| \varepsilon_{y_{I,j}}^{(k)}(t) \right| > \bar{\varepsilon}_{y_{I,j}}^{(k)}(t) \right\} \right\}. \quad (31)$$

Hence, when $D_{I,j}(t) = 0$, the isolator $\mathcal{I}_{I,j}$ guarantees that zone j is safe (i.e. no contaminant event has occurred), or equivalently, zone $M_{x_I}^{-1}(j)$ in the building system Σ is safe.

The decision of the contaminant event monitoring agent $\mathcal{M}_I$, $I \in \{1, \dots, K\}$ on which zone in the building subsystem Σ_I contains the contaminant source, is obtained by combining the decisions of the n_I isolators that updating the diagnosis set $\mathcal{S}_{D_I}(t)$ defined by (19). In particular, the diagnosis set is determined as follows

$$\mathcal{S}_{D_I}(t) = \mathcal{S}_{D_I}(T_{D_I}) \setminus \left\{ Z^{(j)}, \forall j \in W_I \right\}, \quad (32)$$

$$W_I = \{j : D_{I,j} = 0\} \quad (33)$$

Proposition 3.4: If there is a contaminant source in Σ_I and every isolator $\mathcal{I}_{I,j}$ except for one, isolator $\mathcal{I}_{I,r}$, $j \neq r$, $j, r \in \{1, \dots, n_I\}$, excludes the contaminant source from zone $Z^{(j)}$, then it is guaranteed that a contaminant source is located in zone r of the building subsystem Σ_I , or equivalently, in zone $M_{x_I}^{-1}(r)$ of the building system Σ .

Proof: Suppose that there is no contaminant source in zone r . Then, two situations can be valid: (i) there is no contaminant source in the building subsystem Σ_I , or (ii) a contaminant source is located in one of remainder $n_I - 1$ zones. If the first situation is valid, then the detector $\mathcal{D}_I$ should not detect any contaminant source. This contradicts the fact that the isolators have been activated due to the contaminant source detection. If the second situation is valid, then for at least one of the isolators $\mathcal{I}_{I,j}$, $j \neq r$, (25) is valid. This contradicts the fact that for all isolators $\mathcal{I}_{I,j}$ except for $\mathcal{I}_{I,r}$, (29) is activated. Thus, our assumption that there is no contaminant source in zone r is incorrect.

C. Distributed CDI scheme

Each agent $\mathcal{M}_I$ is responsible for configuring a monitoring scheme based on the partitioning solution provided by the partitioning algorithms (i.e., MILP and BPH described in Section IV) and utilize observer based residuals for detecting and isolating a contaminant source. At first, each agent constructs the observers and the adaptive thresholds based on the A_I, H_I matrices configuration provided by the partitioning algorithms. Then, it calculates the residuals and the adaptive thresholds for detection and isolation using the measurements (y_I) from its own subsystem (Σ_I) as well as the transmitted measurements (y_{z_I}) from its neighboring subsystems. The adaptive thresholds and the observer based residuals, allow each agent to detect

Algorithm 1 : CDI Procedure for agent $\mathcal{M}_I$

```

1: Receive  $A_I, H_I$ 
2: Configure detector  $\mathcal{D}_I$  and isolators  $\mathcal{I}_{I,j}, j \in \{1, \dots, n_I\}$  in
   terms of gains and parameters needed for the calculation
   of the adaptive thresholds
3: Set  $T_{D_I} = 0$  and  $D_{I,j} = 1, j \in \{1, \dots, n_I\}$ 
4: for each time step  $t$  do
5:   Read sensor measurements  $y_I$ 
6:   Exchange measurements with neighboring agents
7:   DETECTION PROCEDURE
8:   for all  $k \in \{1, \dots, m_I\}$  do
9:     if  $T_{D_I} == 0$  then
10:      Compute detection residual  $\varepsilon_{y_I}^{(k)}$  (9)
11:      Compute detection threshold  $\bar{\varepsilon}_{y_I}^{(k)}$  (39)
12:      if  $|\varepsilon_{y_I}^{(k)}| > \bar{\varepsilon}_{y_I}^{(k)}$  then  $\triangleright$  Contaminant detected!
13:        Set  $T_{D_I} = t$  (18)
14:        Construct diagnosis set  $\mathcal{S}_{D_I}$  (19)
15:   ISOLATION PROCEDURE
16:   if  $T_{D_I} > 0$  then
17:     for all  $j \in \{1, \dots, n_I\} : Z^{(j)} \in \mathcal{S}_{D_I}$  do
18:       Estimate the contaminant source  $\hat{\phi}_{I,j}$  (23)
19:       for all  $k \in \{1, \dots, m_I\}$  do
20:         Compute isolation residual  $\varepsilon_{y_{I,j}}^{(k)}$  (24)
21:         Compute isolation threshold  $\bar{\varepsilon}_{y_{I,j}}^{(k)}$  (42)
22:         if  $|\varepsilon_{y_{I,j}}^{(k)}| > \bar{\varepsilon}_{y_{I,j}}^{(k)}$  then
23:           Set  $D_{I,j} = 0$  (30)
24:           Update diagnosis set  $\mathcal{S}_{D_I}$  (32)
25:   if  $||\mathcal{S}_{D_I}|| == 1$  then  $\triangleright$  Contaminant Isolated!

```

and isolate a contaminant source. The procedure followed by each agent $\mathcal{M}_I$ is summarized in Algorithm 1.

IV. BUILDING PARTITIONING ALGORITHMS

In this section, we present the two proposed partitioning algorithms (MILP and BPH), for structurally decomposing the intelligent building into different subsystems, for the effective application of the distributed Contaminant Detection and Isolation (CDI) algorithms described in Section III. Before presenting the details of the two partitioning solutions, we start by motivating the design choices made in creating these algorithms.

A. Design criteria for partitioning algorithms

Given a state transition matrix A of a building system Σ , the general objective is to define the building subsystems Σ_I such as to: (i) minimize the airflows between them, (ii) control their size and (iii) ensure their connectivity. In addition, (iv) in the case of a change in the environment affecting the airflows, the partitioning algorithm should be able to reconfigure the solution in a timely manner.

According to these design criteria, the primary objective for designing effective partitioning algorithms is to form subsystems such that the airflows between them are minimized. Note that by minimizing the flows that traverse between the

subsystems, we are effectively minimizing the interconnection effects $(H_I + \Delta H_I)z_I$. Hence, a contaminant release in one subsystem will have minimal impact on the detection and isolation sensitivity of its neighboring subsystems (see (14),(39),(27),(42)). This is actually equivalent to minimizing the sum of the entries of the interconnection matrices H_I , defined by

$$\text{Partitioning Cost (PC)} = \sum_{I=1}^K \sum_{(i,j) \in \mathcal{H}_I} H_I(i,j) \quad (34)$$

where $\mathcal{H}_I$ denotes the set of elements $A(i,j)$ which belong to H_I (i.e., they are not included in any of the subsystems). So the primary objective of any partitioning solution is to reduce the PC as much as possible.

For effectively partitioning the building into subsystems according to the aforementioned design criteria, we use two partitioning solutions developed in our previous work, BPH and MILP. On the one hand, MILP provides optimal results with respect to minimizing the PC , while on the other hand, the heuristic BPH provides a trade-off between performance and execution time which makes it appropriate for real-time implementations. Both algorithms use the state transition matrix A as a starting point for automatically dividing the building into subsystems.

B. MILP

The MILP formulation uses the state transition matrix in order to generate a graph representation of the building. Buildings are a natural candidate for graph transformation since they are distributed in space and the zones are connected with a limited number of flows. On this note, the formation of subsystems is realized as a graph partitioning problem where the produced sub-graphs represent the building's subsystems.

Consider an undirected, connected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ with the vertices representing the buildings' zones and the edges indicating the existence of airflow connections between the zones. Note that the edge weights $d_{i,j}, (i,j) \in \mathcal{E}$, correspond to the elements of matrix $D = A + A^T \in \mathbb{R}^{N \times N}$, $N = |\mathcal{V}|$ defined in terms of the state transition matrix A so that the edge weights are symmetric. The objective is to partition the graph into $K \in \mathbb{Z}^+$ subgraphs, while minimizing the edge weights that are not included in any subgraph which represent the PC defined in (34). Let variables $z_{i,j}, (i,j) \in \mathcal{E}$ indicate whether edge (i,j) belongs to any subgraph ($z_{i,j} = 1$) or not ($z_{i,j} = 0$). The objective of the problem can then be expressed as:

$$\min \sum_{(i,j) \in \mathcal{E}} \frac{1}{2} (1 - z_{i,j}) d_{i,j}. \quad (35)$$

The above objective is subject to three sets of constraints: (i) Basic Partitioning Constraints, to partition the graph, (ii) Connectivity Constraints, to ensure continuity of zones in each partition, and (iii) Size constraints, to control the size of the partitions. For more details on the MILP partitioning algorithm see [46].

For example, Fig. 4 shows the building graph transformation for the considered case study. The vertices in Fig. 4 represent the zones while the edges represent the existence of airflow

Fig. 4: Graph transformation and MILP solution using $K = 3$ for the building case study introduced in Section II-A. The partitioning cost for this solution is $PC = 0.015 + 0.515 + 0.18 = 0.71$ which corresponds to the sum of weights of the edges connecting the three subsystems.

between two zones. The weights are defined according to the airflow values of the state transition matrix portrayed in Fig. 2. The optimal partitioning solution after running the MILP algorithm is also shown in Fig. 4 with partitioning cost $PC = 0.015 + 0.515 + 0.18 = 0.71$ and the following subsystems: $\Sigma_1 = \{Z1, Z2, Z3, Z4, Z5\}$, $\Sigma_2 = \{Z6, Z8, Z9, Z12, Z13\}$ and $\Sigma_3 = \{Z7, Z10, Z11, Z14\}$.

C. BPH

The Buildings Partitioning Heuristic (BPH) is based on matrix reordering techniques. Although the particular heuristic does not always provide optimal solutions, the advantage of using BPH over MILP solvers is that it can obtain fast close-to-optimal solutions in polynomial complexity time, whereas the MILP is a NP-hard combinatorial optimization problem with exponential computational complexity in the worst case.

The BPH, is a matrix clustering algorithm that rearranges the rows and columns of the state transition matrix A in such a way, so that blocks of values are formed indicating the

	1	3	2	5	4	10	11	7	14	8	9	6	13	12
1	0	0.15	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0.036	0	0	0	0	0	0	0	0	0	0
2	0	0.42	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0.277	0	0	0	0	0	0.015	0	0	0	0
10	0	0	0	0	0	0	0	1.2E-05	0	0	0	0	0	0
11	0	0	0	0	0	1.12E-05	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	7.8E-06	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0.515	0.18	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0.232	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0.1	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0.24	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0.21	0	0.1

Fig. 5: Resulting state transition matrix after executing the BPH algorithm for the building case study in Section II-A. The three resulting subsystems are demonstrated with blocks of different colors. The partitioning cost for this solution is $PC = 0.015 + 0.515 + 0.18 = 0.71$ which corresponds to the sum of nonzero entries that are not included in any of the formed blocks (colored in red).

resulting subsystems. Its objective is to minimize the values that are not included in any block (partitioning cost) and retain their individual connectivity, while also enforcing size and relative size difference constraints between the formed blocks. Furthermore, possibly its biggest advantage, is the ability to utilize an existing subsystem configuration as a starting point and optimize it on-line; hence, compensating for the dynamic nature of the buildings' airflows.

The heuristic algorithm is composed of two phases: (a) the “Dividing” and (b) the “Regrouping” phase. In the first phase, “Dividing”, the matrix is separated into blocks that satisfy the constraint regarding the maximum number of zones inside a subsystem while also minimizing the PC . In the second phase, “Regrouping”, the smaller blocks are merged to form bigger ones in order to satisfy the constraint regarding the minimum number of zones inside a subsystem. For more details on the BPH algorithm see [47].

For example, Fig. 5 depicts the resulting partitioning of the considered case study after running the BPH algorithm, with three resulting blocks corresponding to the three formed subsystems: $\Sigma_1 = \{Z1, Z2, Z3, Z4, Z5\}$, $\Sigma_2 = \{Z7, Z10, Z11, Z14\}$ and $\Sigma_3 = \{Z6, Z8, Z9, Z12, Z13\}$. The partitioning cost of this solution is $PC = 0.015 + 0.515 + 0.18 = 0.71$. Note that for the considered case study, the BPH solution is exactly the same as the optimal solution provided by the MILP in the previous section.

D. On-line re-partitioning

The unpredictability of the airflow variations in the building interior due to changes in ambient (e.g. temperature, wind speed or direction) or interior (e.g. opening and closing of doors/windows) conditions, may result in a performance degradation of the distributed CDI algorithms. To prevent this, when the airflow conditions change significantly, the proposed BPH algorithm can be used to reconfigure the subsystems on-line (since it is able to provide a close to the optimal partitioning solution in a timely manner). This way, as the airflow conditions change, the fast reconfiguration of the subsystems is ensured and the performance of the CDI

Fig. 6: Three different case studies using the Holmes house demonstrating the importance of on-line re-partitioning in case of a major airflow change. In all three cases, a contaminant source is introduced in zone Z9 at time $t = 2$ hr with a release rate of 225 g/hr. The source is, (i) correctly detected using an optimal partitioning solution given by the MILP (left figure), (ii) is not detected at all after a major change in wind direction (middle figure) and (iii) is detected correctly again after the reconfiguration of the distributed CDI algorithms using BPH (right figure). Note that correct detection implies that the residual in Z9 exceeds the adaptive threshold.

approach is maintained. This is further demonstrated by the following case study.

Consider the Holmes house case study with the distributed CDI algorithms configured under natural flow conditions for a wind speed of 20 m/s and a wind direction of 150° relative to the north. The uncertainty bounds are calculated by considering a $\pm 10^\circ$ deviation in the wind direction. The optimal partitioning solution for this scenario is given by the MILP algorithm as $\Sigma_1 = \{Z1, Z2, Z3, Z5\}$, $\Sigma_2 = \{Z7, Z10, Z11, Z14\}$ and $\Sigma_3 = \{Z4, Z6, Z8, Z9, Z12, Z13\}$. Now, assume a contaminant source is introduced in zone Z9 at time $t = 2$ hours with a release rate of 225 g/hr. Since the CDI is properly configured, it is able to quickly detect the presence of the source as illustrated in Fig. 6 (left figure). Note that detection is decided as soon as the residual exceeds the adaptive threshold which is calculated using the observer schemes detailed in Section III.

Next, consider a major change in airflows caused by a sudden shift in the wind direction to 20° (a 130° relative change from before) and consider the same contaminant source scenario as before. The results, illustrated in Fig. 6 (center figure), show that the CDI algorithms are not able to detect the contaminant source after this major airflow change. Note from the plot, that the residual never exceeds the adaptive threshold in this case. This inability to detect the contaminant is a direct result of using the wrong partitioning solution in the distributed CDI configuration. Note that the CDI configuration is exactly the same (in terms of the observer gains for the detectors and isolators and the parameters for calculating the adaptive thresholds) as the one used before the airflow change.

To resolve this issue, a re-partitioning solution is derived for the new wind conditions using the BPH algorithm as $\Sigma_1 = \{Z1, Z2, Z3, Z4, Z5\}$, $\Sigma_2 = \{Z7, Z10, Z11, Z14\}$ and $\Sigma_3 = \{Z6, Z8, Z9, Z12, Z13\}$. Note that the only change in the new partitioning solution compared to the previous one, is concerning Z4 which has been moved from Σ_3 to Σ_1 . The distributed CDI scheme is also reconfigured in this case, according to the new partitioning solution, in terms of the observer gains for the detectors and isolators and the parameters for calculating the adaptive thresholds. Using the new partitioning solution, the

correct detection of the contaminant source is now possible, as illustrated in Fig. 6 (right figure).

Thus, the combination of the MILP, for ensuring optimality of an initial partitioning solution, and the BPH, for re-partitioning the initial solution in a timely manner after a major airflow change, can maintain the high performance of the distributed CDI scheme under different airflow conditions. This also presents an interesting design trade-off for distributed CDI schemes between: (i) high uncertainty bounds that cover multiple scenarios so frequent re-configuration is not required, and (ii) low uncertainty bounds that increase the detection performance but for limited uncertainty variations so frequent re-configuration becomes necessary. We will further investigate this trade-off in our future research.

E. Reconfiguration Process

The subsystem reconfiguration process is described in Algorithm 2. The initial airflow conditions are calculated using CONTAM and the initial subsystem configuration (i.e., matrices A_I, H_I) is given by the MILP algorithm. The matrices A_I, H_I are used by the CDI agents for configuring and running the detection and isolation procedures described in Algorithm 1. In the event that the environmental conditions change by more than the acceptable deviation, as provided by the uncertainty bounds (e.g., ambient wind direction changes by more than $\pm 10^\circ$), the new airflows are calculated on-line using the CONTAM software. This change in airflow conditions initiates the reconfiguration of the subsystems using the BPH algorithm which provides the new matrices A_I and H_I for configuring the CDI agents.

Algorithm 2 : Reconfiguration Process

- 1: Use MILP to find matrices A_I, H_I for initial subsystem configuration
 - 2: **while** airflow conditions do not change **do**
 - 3: Run Algorithm 1 for each CDI agent
 - 4: Use BPH to reconfigure matrices A_I, H_I
 - 5: Go to Step 2
-

V. SIMULATION RESULTS

In this section, an extensive simulation campaign is carried out using real building models, which assesses and validates the distributed CDI approach. First, the validity of the proposed approach is shown in a three contaminant source scenario for the Holmes case study with missing sensors. Next, the effect of the partitioning solution to the CDI performance is demonstrated by comparing both centralized and distributed approaches with both random and optimal partitioning configurations. Finally, the effective applicability of the distributed CDI approach is presented with results for multiple contaminant release rate scenarios and multiple subsystem numbers for a large-scale complex building.

A. Detection and Isolation of Multiple Contaminant Sources

In this section, we demonstrate how the methodology developed in Section III can be applied towards the distributed detection and isolation of multiple contaminant sources in an indoor building test case environment.

For the investigated scenario, we use Holmes house case study (see Fig. 1) with three contaminant sources released at the same time (at $t = 2$ hours) in zones Z3, Z7 and Z9, with a steady generation rate of 140 g/hr. During the release, the random variations of the wind speed (in the range of 10 ± 0.5 m/s) and direction (in the range of $90^\circ \pm 10^\circ$) constitute the only sources of modeling uncertainty for the specific scenario. Based on these values, the uncertainties were calculated as $\overline{\Delta A}_1 = 0.29$, $\overline{\Delta A}_2 = 0.3$, $\overline{\Delta A}_3 = 0.13$, $\overline{\Delta H}_1 = 0.021$, $\overline{\Delta H}_2 = 0.20$ and $\overline{\Delta H}_3 = 0$. Sensors are placed in 11 out of the 14 zones (Z1, Z2, Z4, Z6, Z7, Z8, Z9, Z10, Z11, Z12, Z14). Note that the resulting partitioning solution after removing the sensors in Z3, Z5 and Z13 is manually checked to ensure that the formed subsystems remain observable in the considered scenario. The objective of the scenario is to promptly detect and accurately isolate all three contaminant sources in the presence of noise and modeling uncertainty.

The detection and isolation results for the three agents (as previously portrayed in Fig. 3) are presented in Fig. 7 and Fig. 8 respectively. From the detection results it becomes evident that, the three sources are correctly detected by the detectors $\mathcal{D}_1$, $\mathcal{D}_2$ and $\mathcal{D}_3$, since the magnitude of the residual surpasses the threshold. Specifically, the contaminants are detected at Z3 in $t = 0.171$ hr, at Z7 in $t = 0.0207$ hr, at Z9 in $t = 0.0207$ hr and isolated in $t = 1.4068$ hr, $t = 0.1543$ hr and $t = 0.1209$ hr, respectively. Note that as a result of the missing sensors in the affected zone (Z3) and one of its incident zones (Z5) in subsystem Σ_1 , the detection and isolation are delayed in comparison with the other subsystems.

Following detection, the isolators for each subsystem are activated by the respective agents. As observed from the isolation results (Fig. 8), following the isolation logic presented in Section III-B3, the sources are correctly isolated in each subsystem when all isolator logic signals *except one* go to zero. The remaining isolator (for which the logic signal remains one) is the only one out of the isolators in the particular subsystem for which the residual remains below the threshold, therefore constituting the only possible hypothesis

of containing the contaminant source. It is worth pointing out, that in the considered scenario all three sources are accurately detected and isolated, even though three zones have no sensors including one of the source zones.

The proposed distributed approach assumes one contaminant source per subsystem. For multiple contaminants in the same subsystem, the CDI algorithm would detect the existence of the contaminants but it will not be able to isolate the source due to the hypothesis used in designing the bank of isolators where only one contaminant per subsystem is assumed. However, the proposed approach could be generalized to handle multiple contaminants in each subsystem through additional banks of isolators that consider multiple contaminant sources in each subsystem. We plan to further investigate this issue in our future research.

B. Effect of partitioning on CDI performance

In order to optimize the performance of the CDI, the right partitioning solution has to be chosen. Hence, automatic algorithms for finding the proper partitioning solutions are necessary in order to ensure the effectiveness of the CDI approach.

In order to better illustrate the importance of choosing the correct partition when running distributed CDI algorithms, consider the following building simulation example using Holmes's house, created in CONTAM [40]. Specifically, we consider two different partitions for running the distributed CDI algorithms assuming sensors for measuring the contaminant concentration in *all* the building zones: (i) the Optimal Partition (OP) shown in Fig. 9a, which is the result of running the exact MILP formulation (with the constraint of a single zone difference with respect to the size of subsystems) and (ii) another randomly created partition which we refer to as Random Partition (RP) which is demonstrated in Fig. 10a. Note that both partitions result in the creation of three subsystems but with different zone configurations. For testing our hypothesis, we assume ambient temperature of 25° C, external wind of 10 m/s speed blowing from the east and all the controllable inputs (e.g doors and windows) in the fully-open position. Furthermore, we assume a steady contaminant release of 5 g/hr in Z7 starting at $t = 1$ hours. The detection results for the OP and RP are depicted in Fig. 9b and Fig. 10b respectively. Note, that according to the distributed CDI observer schemes described in Section III, detection occurs when the computed residual exceeds an adaptive threshold which depends on the underlying uncertainties. From Fig. 9b, it becomes evident that using the OP the contaminant is correctly detected at $t = 3$ hours, whereas using the RP, shown in Fig. 10b, the contaminant is not detected at all.

To better understand the benefit of OP over RP, we need to take a closer look at the differences between them; note that OP is more balanced with respect to the number of zones allocated in each subsystem. Since the uncertainty ΔA_l is a result of the changing airflows in the buildings' interior then it can be deduced that the addition of a zone in a subsystem (i.e., one more state in the state vector of (3)) increases its uncertainty. Add to this, the equally divided computational

Fig. 7: Contaminant Detectors for the 3 Subsystems. The residual $\varepsilon_{y_I}^{(k)}(t)$, is displayed using solid lines while the corresponding adaptive thresholds $\bar{\varepsilon}_{y_I}^{(k)}(t)$ are displayed using dashed lines.

Fig. 8: Contaminant Isolation results for the three subsystems ($\Sigma_1, \Sigma_2, \Sigma_3$). The last figure of each column displays the isolation diagnosis decision logic $\mathcal{S}_{D_I}(t), I \in \{1, \dots, K\}$ while the other figures display the isolation logic signal for each individual isolator $D_{I,j}, j \in \{1, \dots, n_I\}$. Note that the sources are correctly isolated in each subsystem when all isolator logic signals *except one* go to zero.

complexity in each subsystem, especially for the isolation procedure where equal number of candidate zones are included in each subsystems and the result is an overall increase in the performance of the CDI approach for OP. Thus, choosing the correct partition is extremely important when designing distributed CDI schemes.

Next, we evaluate the performance of the centralized CDI approach with the two distributed solutions (OP and RP), for different contaminant release rates in Z7, starting from a low rate (i.e 3 g/hr) that is not detectable and moving upwards in equal increments until all approaches have almost the same performance (i.e 55 g/hr). Fig. 11 shows the detection and isolation results for the considered scenario. Note that the performance of the CDI approach is measured in terms of the ability to quickly and correctly detect and isolate the contaminant source. Also, the lack of results on the plot for a specific contaminant release rate indicates the inability of a particular method to detect or isolate it. On that note, OP clearly achieves the best performance both in terms of detection and isolation, with RP achieving slightly better

results than the centralized CDI algorithms. From Fig. 11, it becomes evident that OP is able to detect contaminants starting from a small generation rate of 3 g/hr, while for RP and the Centralized approach a generation rate of at least 7 – 8 g/hr is required. The advantage of OP is even more evident when it comes to isolation; the OP approach is able to isolate contaminants with a generation rate as low as 13 g/hr, whereas RP and the centralized approach require at least 31 – 33 g/hr.

One of the main reasons for the lower performance of the centralized approach is the increased uncertainty since the system is affected by all the airflows. On the other hand, in the distributed case, each subsystem is only affected by its own included flows which are significantly smaller than in the centralized case and the interconnection flows which are minimized as part of the partitioning process. Furthermore, the centralized approach has higher complexity due to the increased number of zones and variables involved; hence, sometimes the calculation of the adaptive thresholds required both in the detection and isolation procedure become a formidable task.

Fig. 9: a) Holmes house Optimal Partitioning (OP) solution using $K = 3$ b) Corresponding contaminant detection result for Z7

Fig. 10: a) Holmes house Random Partitioning (RP) solution using $K = 3$ b) Corresponding contaminant detection result for Z7

In order to confirm the superiority of the OP partitioning, all possible configurations for $K = 3$ subsystems that follow the connectivity constraints have been compared in terms of detection and isolation performance (detailed results are not shown due to space limitations). The partitioning solutions were tested separately for contaminant sources in zones Z5, Z7 and Z14 and multiple contaminant release rates, starting from 3 g/hr and moving upwards in equal increments until a rate of 17 g/hr. The tests showed that, only a small percentage of the random partitioning solutions were successful in detecting the contaminant for release rates below 5 g/h while even for a release rate of 17 g/hr more than 50% of the partitioning solutions were not successful in isolating the contaminant source for zones Z7 and Z14. Whereas multiple partitioning solutions were able to detect and isolate a contaminant source, the OP partitioning exhibited the best overall CDI performance by combining both equal partitions and minimization of the interconnections between the subsystems.

C. Large-scale building case study

Distributed approaches have shown significant benefits over centralized approaches. Simplicity, scalability, increased detectability and isolability are among the major improvements that distributed CDI approaches bring along. Having this in mind, this case study presents the capabilities of the proposed distributed approach over the centralized with a focus on the

scalability advantages using a large-scale complex building (hospital [48]) consisting of 96 zones. The hospital has 5 floors and 4 HVAC systems that span to multiple floors with different ventilation requirements. Two of the HVAC systems are responsible for heating and ventilating the “critical” zones (i.e., operating rooms, exam rooms, patient rooms) and the remaining two are responsible for the “administrative” zones (e.g., dining rooms, lobbies, offices, corridors). The HVAC systems are modeled as simple air handling units in CONTAM with different supply flow rates. The existence of HVAC systems in a building facilitates the contaminant transfer between different zones since multiple zones are connected together even if they are physically located far from each other. Note that, HVAC systems are already addressed in the proposed approach and they are reflected in the state transition matrix A . The existence of an HVAC systems has as a result the increase of non-zero values in the state transition matrix A , which indicates an increase in the number of airflow connections between the various zones. In spite of the larger number of connections, the complexity of the CDI approach is not affected. On the other hand, the complexity of optimally partitioning the building into the individual subsystems is significantly increased. This highlights the necessity of utilizing the partitioning algorithms in order to minimize the interconnection values between the different subsystems, especially in large-scale complex buildings.

Fig. 11: Detection and isolation performance comparison between Centralized and Distributed CDI schemes for Holmes house case study (OP and RP shown in Fig. 9 and Fig. 10 respectively), by varying the source generation rate in Z7.

The contaminant source is placed in Z7 (NE Stairs) and is released at $t = 2$ hours. We further assume a wind speed of 10m/s, a wind direction in the range of $270 \pm 10^\circ$ and sensors in all zones. Results comparing the centralized and distributed CDI performance with respect to detection and isolation capabilities are illustrated in Fig. 12 and Fig. 13 respectively. In the figures, the contaminant release rates are varied in the range of 20g/h–180g/h, while for the distributed CDI approaches we compare the results for different numbers of optimally partitioned subsystems using the MILP; namely 2, 4, 6, and 8. From the two figures, it becomes evident that all the distributed approaches outperform the centralized CDI approach in both detection and isolation. Specifically, the performance of the detection procedure increases with the number of subsystems from 2 to 4 to 6 where it saturates. On the contrary, the isolation performance appears to improve as we increase the number of subsystems. Increasing the number of subsystems will increase the partitioning cost and therefore the dependencies between the subsystems. The communication cost will also increase due to the increased information exchange between the subsystems. In addition, considering the case where fewer sensors than zones are placed in the system, smaller subsystems (larger selection of K) could impose problems with the observability of the subsystems which is a necessary condition for designing the observers. In practice, there is a trade-off between the chosen number of subsystems and the performance of the approach. Therefore, the selection of K should be made empirically for each specific system by taking into account the different parameters including the size of the system, the number of sensors, the magnitude of the expected faults and the resulting communication cost. Most importantly, this example highlights the scalability of the proposed distributed CDI approach and its ability to handle large-scale complex buildings.

Fig. 12: Detection performance comparison between centralized and distributed CDI approaches (using different numbers of optimal partitions $K = 2, 4, 6$ and 8) for a Hospital building case study with 97 zones. For the considered scenario, the contaminant source is placed in Z7 (NE Stairs).

Fig. 13: Isolation performance comparison between centralized and distributed CDI approaches (using different numbers of optimal partitions $K = 2, 4, 6$ and 8) for a Hospital building case study with 97 zones. For the considered scenario, the contaminant source is placed in Z7 (NE Stairs).

VI. CONCLUSION

Distributed contaminant detection and isolation schemes are needed in intelligent buildings for automatically preserving the IAQ and ensuring the occupants' health and safety. In this paper, we provide the details for the design of such a distributed methodology for detecting and isolating multiple contaminant events in a large-scale building. Specifically, we consider the building as a collection of interconnected subsystems and we design a contaminant event monitoring agent for each subsystem for detecting and isolating the contaminant source introduced in the underlying subsystem. Successful deployment of such distributed CDI approaches, requires a

procedure for defining the subsystems while minimizing their interdependencies, controlling their size and ensuring their connectivity. Therefore, in this paper we have used both an exact MILP formulation and a heuristic algorithm (BPH) for partitioning the building based on the airflows between the zones. Both approaches can effectively partition buildings following size and connectivity constraints aiming at ensuring the successful deployment of distributed CDI approaches. In addition, the BPH can provide on-line reconfiguration in case of a major change in the airflows. The performance of the proposed distributed CDI scheme was evaluated in real building models using extensive simulations.

Most of the proposed system, including the detection and isolation agents and the partitioning algorithms can be easily implemented in software. In fact, the adaptive thresholds and residuals utilized in the proposed approach can be easily implemented using simple linear filters. Therefore, the proposed approach is ideal for large-scale complex buildings where prompt detection and isolation of a possible contaminant source could be the deciding factor for the occupants safety. In this context, the proposed approach is mainly targeted to the buildings' operators who can effectively use this system in order to obtain the proper information and proceed with the necessary actions for a given contaminant event (e.g., activation and execution of the evacuation plan). Regarding hardware, the only need for realizing the proposed solution is the incorporation of sensors in the different building zones that can measure the concentration of the contaminant of interest. Although this system can be easily retrofitted to existing structures (i.e., using a wireless sensor network), designers should also be aware of its capabilities in order to include it in the building designing phase (i.e., as part of the HVAC plan).

In the future, we plan to investigate the optimal sensor placement problem for situations with a limited number of sensors. In this case, we need to incorporate additional constraints into our partitioning algorithms to ensure that the resulting solutions remain observable in order to maintain the performance of the CDI algorithms. Finally, an interesting research direction involves controlling the building (by automatically opening and closing the doors and windows) as well as the Air Handling Units, in order to improve the CDI capabilities on one hand (i.e. faster detection, more accurate isolation), and contain the contaminant or drive it away from the occupants on the other hand.

LIST OF VARIABLES

A, A_I	State transition matrix for Σ and Σ_I
B, B_I	Controllable input zone-index matrix for Σ and Σ_I
C, C_I	Sensor location zone-index matrix for Σ and Σ_I
$\Gamma_{I,j}$	Learning rate for the contaminant estimator
$\Delta A, \Delta A_I$	Modeling uncertainty for Σ and Σ_I
$\Delta A_I, \Delta H_I, \bar{w}_I$	Modeling uncertainty, interconnection uncertainty and sensor noise bounds
$D_{I,j}$	Boolean decision function for isolator $\mathcal{I}_{I,j}$
$\mathcal{D}_I$	Contaminant Detector of Σ_I

ε_{x_I}	State estimation error vector of Σ_I
$\varepsilon_{y_I}^{(k)}$	k -th residual of the detector $\mathcal{D}_I$
$\varepsilon_{y_{I,j}}^{(k)}$	k -th residual of the isolator $\mathcal{I}_{I,j}$, $j \in \{1, \dots, n_I\}$
$\bar{\varepsilon}_{y_I}^{(k)}$	k -th adaptive threshold of Σ_I
G, G_I	Contaminant generation sources zone-index matrix for Σ and Σ_I
g, g_I	Contaminant generation rate for Σ and Σ_I
$\bar{g}_I$	Contaminant rate of evolution bound
$H_I, \Delta H_I$	Interconnection matrix and uncertainty for Σ_I
$\mathcal{I}_{I,j}$	j -th isolator of Σ_I , $j \in \{1, \dots, n_I\}$
K	Number of subsystems
L_I	Observer gain matrix
ℓ_I	Number of interconnections in Σ and Σ_I
M_{x_I}	Mapping function
$\mathcal{M}_I$	Monitoring agent for Σ_I
m, m_I	Number of sensors in Σ and Σ_I
n, n_I	Number of zones in Σ and Σ_I
P_I	Projection operator
p, p_I	Number of controllable inputs in Σ and Σ_I
Q, Q_I	Diagonal matrix with zones's volumes for Σ and Σ_I
$\mathcal{S}_{D_I}$	Diagnosis set for detector $\mathcal{D}_I$
s	Number of contaminant sources
T_{D_I}	Time of detection
u, u_I	Controllable inputs vector for Σ and Σ_I
w, w_I	Additive measurement noise vector for Σ and Σ_I
x, x_I	State vector of contaminant concentrations for Σ and Σ_I
$\hat{x}_I, \hat{x}_{I,j}$	State estimator vector for Σ_I and the j -th $\in \{1, \dots, n_I\}$ zone of Σ_I
y, y_I	Output vector of sensor measurements for Σ and Σ_I
y_{z_I}	Transmitted sensor information
z_I	Vector of interconnection variables of Σ_I
$\rho_I, \xi_I, \alpha_I^k, \zeta_I^k$	Positive constants used in the adaptive thresholds of Σ_I
$\hat{\phi}_{I,j}$	Estimate of the contaminant source g_I
Ω	State vector for the estimation model of isolator $\mathcal{I}_{I,j}$
$\tilde{\phi}_{I,j}$	Contaminant rate estimation error

APPENDIX A

ADAPTIVE THRESHOLDS FOR DETECTION

The adaptive thresholds of the detectors D_I , $I \in \{1, \dots, K\}$ can be calculated using the following equations:

$$|\varepsilon_{x_I}(t)| \leq E_1(t) + \int_0^t \rho_I e^{-\xi_I(t-\tau)} \bar{\Delta A}_I |\varepsilon_{x_I}(\tau)| d\tau, \quad (36)$$

$$E_1(t) = \rho_I e^{-\xi_I t} \bar{x}_I + \frac{\rho_I}{\xi_I} ((|H_I| + \bar{\Delta H}_I) \bar{w}_{z_I} + |L_I| \bar{w}_I) (1 - e^{-\xi_I t}) + \int_0^t \rho_I e^{-\xi_I(t-\tau)} (\bar{\Delta H}_I |y_{z_I}(\tau)| + \bar{\Delta A}_I |\hat{x}_I(\tau)|) d\tau \quad (37)$$

where $\bar{x}_I \in \mathbb{R}$ is a bound such that $|x_I(t)| \leq \bar{x}_I$ and ρ_I, ξ_I are positive constants selected such that $|e^{A_{L_I} t}| \leq \rho_I e^{-\xi_I t}$, for all t .

Applying the Bellman-Gronwall Lemma [49] results in

$$|\varepsilon_{x_I}(t)| \leq E_1(t) + \int_0^t \rho_I e^{-(\xi_I - \rho_I \bar{\Delta} A_I)(t-\tau)} E_1(\tau) d\tau \triangleq \bar{\varepsilon}_{x_I}(t) \quad (38)$$

In order for $\bar{\varepsilon}_{x_I}(t)$ to be finite, ρ_I, ξ_I are selected such that $\xi_I > \rho_I \bar{\Delta} A_I$.

Using the solution of (14), the adaptive threshold bounding the residual under healthy conditions $\varepsilon_{y_I}^{(k)}$ is

$$\begin{aligned} \bar{\varepsilon}_{y_I}^{(k)}(t) = & \frac{\alpha_I^{(k)}}{\zeta_I^{(k)}} ((|H_I| + \bar{\Delta} H_I) \bar{w}_{z_I} + |L_I| \bar{w}_I) \left(1 - e^{-\zeta_I^{(k)} t}\right) \\ & + \alpha_I^{(k)} e^{-\zeta_I^{(k)} t} \bar{x}_I + \int_0^t \alpha_I^{(k)} e^{-\zeta_I^{(k)}(t-\tau)} (\bar{\Delta} H_I |y_{z_I}(\tau)| \\ & + \bar{\Delta} A_I |\hat{x}_I(\tau)| + \bar{\Delta} A_I \bar{\varepsilon}_{x_I}(\tau)) d\tau + \bar{w}_I^{(k)} \end{aligned} \quad (39)$$

where $\bar{\varepsilon}_{x_I}$ is the right hand side of (38) and $\alpha_I^{(k)}, \zeta_I^{(k)}$ are positive constants chosen such that $|C_I^{(k)} e^{A_{L_I} t}| \leq \alpha_I^{(k)} e^{-\zeta_I^{(k)} t}$, for all t . It is noted that the adaptive threshold can be implemented using linear filtering techniques.

APPENDIX B

ADAPTIVE THRESHOLDS FOR ISOLATION

The adaptive threshold of the isolator $\mathcal{I}_{I,j}$, $I \in \{1, \dots, K\}$, $j \in \{1, \dots, n_I\}$ can be calculated using the following equations:

$$\begin{aligned} |\varepsilon_{x_{I,j}}(t)| \leq & E_2(t) + \int_0^t \rho_I e^{-(\xi_I - \rho_I \bar{\Delta} A_I)(t-\tau)} E_2(\tau) d\tau = \bar{\varepsilon}_{x_{I,j}}(t), \quad (40) \\ E_2(t) = & \bar{\phi}_I |\Omega_{I,j}(t)| + \rho_I e^{-\xi_I(t-T_{D_I})} \bar{x}_I \\ & + \frac{\rho_I ((|H_I| + \bar{\Delta} H_I) \bar{w}_{z_I} + |L_I| \bar{w}_I)}{\xi_I} \left(1 - e^{-\xi_I(t-T_{D_I})}\right) \\ & + \int_{T_{D_I}}^t \rho_I e^{-\xi_I(t-\tau)} (\bar{\Delta} H_I |y_{z_I}(\tau)| + \bar{\Delta} A_I |\hat{x}_I(\tau)| \\ & + \bar{g}_I |\Omega_{I,j}(\tau)|) d\tau \\ & + \int_{T_{D_I}}^t \rho_I e^{-\xi_I(t-\tau)} |Q_I^{-1} (G_{I,r} - G_{I,j}) g_I| d\tau \end{aligned} \quad (41)$$

where $\bar{\phi}_I$ is such that $|g_I - \hat{\phi}_{I,j}| \leq \bar{\phi}_I$ and is related with the interval Φ , ρ_I, ξ_I are selected as in the design of detector $\mathcal{D}_I$, i.e. $\xi_I > \rho_I \bar{\Delta} A_I$, leading to a finite bound $\bar{\varepsilon}_{x_{I,j}}(t)$ and $\bar{g}_I$ is a bound such that $|\hat{g}_I| \leq \bar{g}_I$, computed based on the Assumption 2.

The adaptive threshold bounding the residual under healthy conditions $\varepsilon_{y_I}^{(k)}$ is

$$\begin{aligned} \bar{\varepsilon}_{y_I}^{(k)}(t) = & \alpha_I^{(k)} e^{-\zeta_I^{(k)}(t-T_{D_I})} \bar{x}_I + |C_I^{(k)} \Omega_{I,j}(t)| \bar{\phi}_I \\ & + \frac{\alpha_I^{(k)}}{\zeta_I^{(k)}} ((|H_I| + \bar{\Delta} H_I) \bar{w}_{z_I} + |L_I| \bar{w}_I) \left(1 - e^{-\zeta_I^{(k)} t}\right) \\ & + \int_0^t \alpha_I^{(k)} e^{-\zeta_I^{(k)}(t-\tau)} (\bar{\Delta} H_I |y_{z_I}(\tau)| + \bar{\Delta} A_I |\hat{x}_I(\tau)| \\ & + \bar{\Delta} A_I \bar{\varepsilon}_{x_{I,j}}(\tau) + \bar{g}_I |\Omega_{I,j}(\tau)|) d\tau + \bar{w}_I \end{aligned} \quad (42)$$

where $\bar{\varepsilon}_{x_{I,j}}$ is the right hand side of (40) with $G_{I,r} = G_{I,j}$ in (41) and $\alpha_I^{(k)}, \zeta_I^{(k)}$ are selected as in the design of detector $\mathcal{D}_I$.

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