

A Computational Model of Tonality Cognition Based on Prime Factor Representation of Frequency Ratios and Its Application

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ABSTRACT

We present a computational model of tonality cognition derived from physical and cognitive principles on the frequency ratios of consonant intervals. The proposed model, which we call the Prime Factor-based Generalized Tonnetz (PFG Tonnetz), is based on the Prime Factor Representation of frequency ratios and can be regarded as a generalization of the Tonnetz. Our assumed application of the PFG Tonnetz is a system for supporting spontaneous and improvisational participation of inexperienced citizens in music performance for regional promotion. For this application, the system needs to determine the pitch satisfying constraints on tonality against surrounding polyphonic music because inexperienced users frequently lack music skills related to tonality. We also explore a working hypothesis on the robustness of the PFG Tonnetz against recognition errors on harmonic overtones in polyphonic audio signals. On the basis of this hypothesis, the PFG Tonnetz has a good potential as a representation of the tonality constraints of surrounding polyphonic music.

1. INTRODUCTION

Musical tonality is an important cognitive element for listening to or playing tonal music. This cognitive phenomenon depends on the perception of the consonant/dissonant interval that can be physically explained with frequency ratios and the overlap of harmonic structures between multiple tones. There are three structural properties of melodic cognition [1]:

1. **Rhythm**: Ordinal duration ratios of adjacent notes.
2. **Pitch contour**: Pattern of ups and downs of pitch changes.
3. **Tonality**: Cognitive coherence of pitch combination related to consonance, harmony, key, scale, and chord.

Understanding the tonality comparatively requires more musical knowledge or experience than the rhythm and pitch contour. Although inexperienced users can intuitively input the

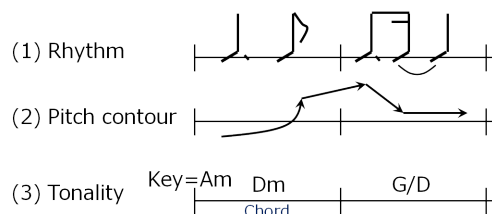


Figure 1. Three aspects for cognition of tonal melody.

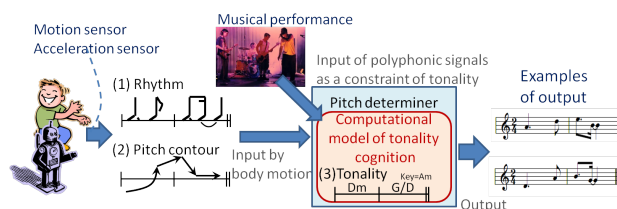


Figure 2. Application: Generating melody with tonality from only rhythm and pitch contour input by a user.

rhythm and pitch contour with their body motion, it is comparatively difficult for them to determine pitch with tonality. Hence, a computational model of tonality cognition can help support inexperienced users to play music with their intuitive body motion. We aim to formulate a computational model of tonality for enabling inexperienced users to participate in playing music by inputting the rhythm and pitch contour with their body motions. In this paper, we present a model of tonality derived from only frequency ratios against tonic without musical knowledge such as key and letter notation.

Recently, many participatory music events for regional promotion have been organized in Japan [2]. Since a broad range of participants are desired for the purpose of regional promotion, technology for supporting the participation of inexperienced citizens is important. Devices or techniques that enable non-experts to play music as emotion dictates could lead to the design of novel musical interaction between citizens for regional promotion.

Clapping to the beat, swaying to the rhythm, and “call-and-response” are basic ways for participating in musical performance without IT support. We aim to provide a novel method for supporting spontaneous and improvisational participation in music performance that does not require advanced music skills or experiences. We focus particularly on spontaneous participation with sustained har-

monic sound because such participation usually requires a certain level of musical skills related to tonality. To this end, we focus on a mechanism to determine pitch having a tonality coherent with the surrounding music performance from the spontaneous rhythm and pitch contour input by the users (Figure 2). This mechanism should be helpful for encouraging spontaneous and improvisational participation in playing tonal music.

Since the (1) rhythm and (2) pitch contour depend less on musical knowledge or experience than (3) tonality, we assume that (1) and (2) can be input by an inexperienced user who has less experience with music performance. Body motion is suitable for inputting (1) and (2) because they are highly relevant to body motion. The affinity between pitch contour and body motion has been described in [3]. Here, we assume the use of motion sensors or acceleration sensors for the user's input. For example, the ups and downs of a hand motion can be used to input the pitch contour.

A computational model of tonality cognition is needed to determine the pitch satisfying constraint on tonality, as shown in Figure 2. The aim to develop a tonality model not for increasing the accuracy of estimating key or chord but for controlling harmony or consonance between the surrounding polyphonic music and the system-determined pitch. Considering the recognition error on harmonic overtone in polyphonic audio signals, the model of tonality cognition for our application should be directly based on physical and cognitive principles related to the pitch frequencies of consonant intervals.

2. PRIME FACTOR-BASED GENERALIZED TONNETZ

In this section, we describe our computational model of tonality based on prime factor representation of ratios between pitch frequencies. The proposed model is derived from only the essential principles on the integer frequency ratio of consonant interval and octave equivalence.

Table 1. Correspondence between frequency ratios of just intonation intervals and the exponents z_2 , z_3 , and z_5 .

Interval	I	#I	II	#II	III	IV
Frequency ratio	1	$\frac{16}{15}$	$\frac{9}{8}$	$\frac{6}{5}$	$\frac{5}{4}$	$\frac{4}{3}$
Cent	0.0	111.7	203.9	315.6	386.3	498.0
z_2	0	4	-3	1	-2	2
z_3	0	-1	2	1	0	-1
z_5	0	-1	0	-1	1	0

#IV	V	#V	VI	#VI	VII
64	3	8	5	9	$\frac{15}{8}$
45	2	5	3	5	$\frac{15}{8}$
609.8	702.0	813.7	884.4	1017.6	1088.3
6	-1	3	0	0	-3
-2	1	0	-1	2	1
-1	0	-1	1	-1	1

2.1 Deriving a Tonality Model from Cognitive Principles

Consonant intervals are usually formed by the frequency ratio of simple integers. When a frequency f_{tonic} of a tonic note and f form a consonant interval, the ratio of these frequencies consists of simple integers, e.g., $f = \frac{3}{2}f_{\text{tonic}}$ (perfect V) and $f = \frac{4}{3}f_{\text{tonic}}$ (perfect IV). Such a frequency ratio consisting of simple integers can be represented by the product of prime numbers, as

$$f = \left(\prod_{p \in \mathbb{P}_n} p^{(z_p)} \right) \cdot f_{\text{tonic}} \quad (z_p \in \mathbb{Z}), \quad (1)$$

where \mathbb{Z} is the set of integers and $\mathbb{P}_n = \{2, 3, 5, \dots, n\}$ is a set of prime numbers that are less than or equal to the upper limit n . For example, when the upper limit n of a prime number is set as 5, the perfect IV can be represented as

$$f = (2^2 \cdot 3^{-1} \cdot 5^0) \cdot f_{\text{tonic}}. \quad (2)$$

The consonant interval between f_{tonic} and f can be represented by a vector (z_2, z_3, \dots, z_n) consisting of the exponent z_p of a prime number p . This vector of the exponents is an expansion of the *prime factor representation* used in the field of number theory [4]. Although the original theory does not allow negative exponents (i.e., z_p should be a non-negative integer), we expand our representation of consonant interval to allow negative exponents so that we can represent the integer frequency ratios of consonant intervals. For example, when the upper limit n of a prime number is 5, representations of the following consonant intervals are represented as the following vectors:

- perfect IV up: $(2, -1, 0)$
- perfect V up: $(-1, 1, 0)$
- major III up: $(-2, 0, 1)$
- perfect unison (tonic itself): $(0, 0, 0)$ (the origin)

Table 1 shows the correspondence between the frequency ratios of the pure intervals of just intonation and exponents z_p of a prime number p .

When plotting such vectors in the z_2 - z_3 - z_5 coordinate system, the following correspondences can be found, as shown in Figure 3. The origin point corresponds to the tonic f_{tonic} . The integer grid points close to the origin correspond to the frequency ratios of the consonant interval from f_{tonic} .

Here, *octave generalization* [5] can be applied with considering the octave equivalence between f_1 and f_2 , such as $f_1 = 2^z f_2$ (z is an integer). Concretely, each point can be octave-generalized by projecting the point onto the z_3 - z_5 plane (i.e., by letting $z_2 = 0$), as shown in Figure 3. In the case of the perfect IV, $(2, -1, 0)$ in the z_2 - z_3 - z_5 space can be projected onto $(-1, 0)$ on the z_3 - z_5 plane. The pitches octave-equivalent to $(2, -1, 0)$, i.e., $(2 \pm i, -1, 0)$ where i is an integer, are also projected onto $(-1, 0)$, at the same point. In the same way, integer grid points (z_2, z_3, z_5) within an octave from a tonic (i.e., such as $1 \leq 2^{z_2} \cdot 3^{z_3} \cdot 5^{z_5} < 2$) are octave-generalized as

- perfect IV up: $(2, -1, 0) \rightarrow (-1, 0)$
- perfect V up: $(-1, 1, 0) \rightarrow (1, 0)$
- major III up: $(-2, 0, 1) \rightarrow (0, 1)$

on the z_3 - z_5 plane, as shown in Figure 3.

The red triangles in Figure 4 consisting of $[(a, b), (a, b + 1), (a + 1, b)]$ can be regarded as representations of major triads on the root (a, b) , while the blue triangles consisting of $[(a, b), (a + 1, b - 1), (a + 1, b)]$ can be regarded as representations of minor triads on the root (a, b) .

Moreover, seventh and extended chords can be formed by alternately piling up the red and blue triangles to the positive direction of the z_3 axis, i.e., the right direction in Figure 4. Major seventh and extended chords are piled up on the right of a base red triangle corresponding to the major triad. Minor seventh and extended chords are piled up on the right of a base blue triangle corresponding to the minor triad. To formulate these structures shown in Figure

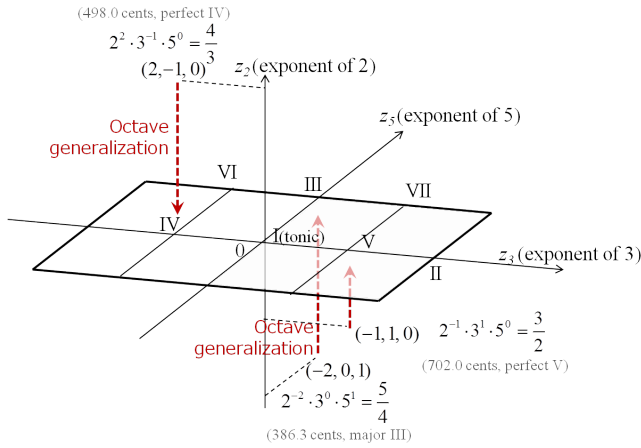


Figure 3. Octave generalization: projection onto z_3 - z_5 plane by omitting z_2 .

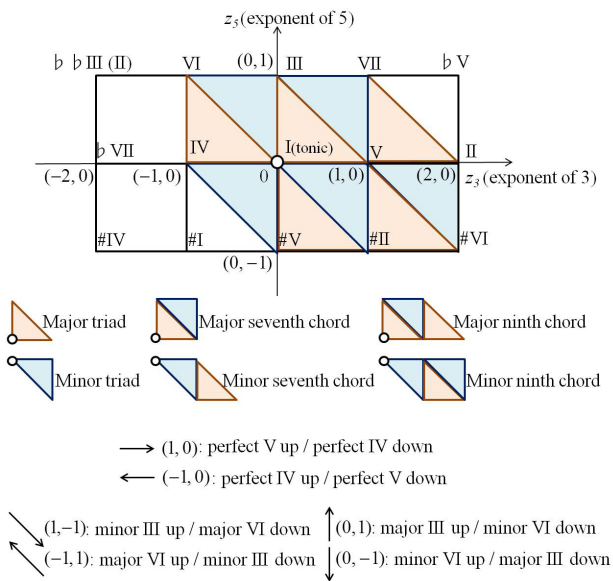


Figure 4. Proposed model: Prime Factor-based Generalized Tonnetz (5-limit).

4, we assume a list of integer grid points $\text{chord}(a, b, \delta, m)$ on the root note (a, b) .

$$\text{chord}(a, b, \delta, m) = [(a, b) + \sum_{i=0}^k \delta(i)]_{k=0,1,\dots,m} \quad (3)$$

$$\delta_{\text{maj}}(i) = \begin{cases} (0, 1) & (i = 2k + 1, k \in \mathbb{N}) \\ (1, -1) & (i = 2k, k \in \mathbb{N}) \end{cases} \quad (4)$$

$$\delta_{\text{min}}(i) = \begin{cases} (1, -1) & (i = 2k + 1, k \in \mathbb{N}) \\ (0, 1) & (i = 2k, k \in \mathbb{N}) \end{cases} \quad (5)$$

A list of integer grid points $\text{chord}(a, b, \delta_{\text{maj}}, m)$ represents the major triad where $m = 2$, the major seventh chord where $m = 3$, and the major ninth chord where $m = 4$. The member notes of these major chords are located in the area of $b \leq z_5 \leq b + 1 \wedge z_3 \geq a$ at the upper right side of the root note (a, b) . In contrast, $\text{chord}(a, b, \delta_{\text{min}}, m)$ represents the minor triad where $m = 2$, the minor seventh chord where $m = 3$, and the minor ninth chord where $m = 4$. The member notes of these minor chords are located in the area of $b - 1 \leq z_5 \leq b \wedge z_3 \geq a$ at the lower right side of the root note (a, b) .

The positional relationships of major/minor scales against the tonic note $(0, 0)$ are similar to those of major/minor chords against a root note (a, b) . Member notes of the major scale (I, II, III, IV, V, VI, VII) are distributed in the area of $0 \leq z_5 \leq 1$ at the upper side of the tonic $(0, 0)$, and those of the minor scale (I, II, #II, IV, V, #V, #VI) are distributed in the area of $-1 \leq z_5 \leq 0$ at the lower side of the tonic.

The above representation of tonality is derived only from the following two cognitive principles on frequency ratios.

1. Since the frequency ratios of a consonant interval are simple integer ratios, they can also be represented by the prime factor representation (z_2, z_3, z_5) , where the integer z_p is an exponent of a prime number p (expanded to allow $z_2, z_3, z_5 < 0$).
2. Since the interval with the frequency ratio 2^z (where z is integer) is octave equivalent, (z_2, z_3, z_5) can be projected onto the z_3 - z_5 plane by omitting z_2 for octave generalization.

In the other words, our tonality model is derived only from the cognitive principles on frequency ratios. Musical knowledge about the pitch notation, scale, and chord is used not for deriving our tonality model but rather for interpreting the representations appearing in the derived model. The interpretations of the representations are as follows:

- The origin $(0, 0)$: A tonic of scales
- Integer grid points (z_3, z_5) close to the origin: Candidates for scale notes
- A vector $(1, 0)$: Perfect V up
- A vector $(-1, 0)$: Perfect IV up
- A vector $(0, 1)$: Major III up

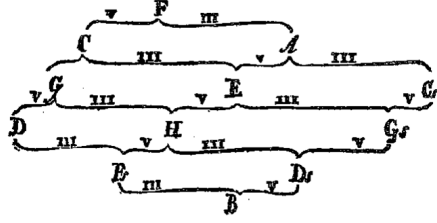


Figure 5. Euler's Tonnetz.

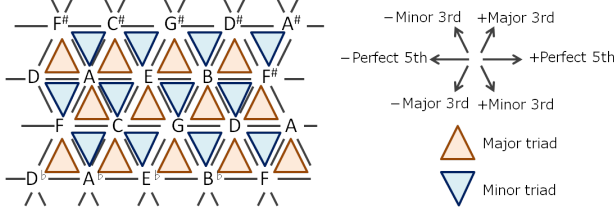


Figure 6. Riemann's Tonnetz.

- A vector $(1, -1)$: Minor III up
- A triangle $[(a, b), (a, b + 1), (a + 1, b)]$:
Major triad on the root (a, b)
- A triangle $[(a, b), (a + 1, b - 1), (a + 1, b)]$:
Minor triad on the root (a, b)
- Integer grid points chord $(a, b, \delta_{\text{maj}}, m)$:
Major chords on the root (a, b)
- Integer grid points chord $(a, b, \delta_{\text{min}}, m)$:
Minor chords on the root (a, b)
- $b \leq z_5 \leq b + 1 \wedge z_3 \geq a$: Distribution area of major chords on the root (a, b)
- $b - 1 \leq z_5 \leq b \wedge z_3 \geq a$: Distribution area of minor chords on the root (a, b)
- $0 \leq z_5 \leq 1$: Distribution area of major scale notes on the tonic $(0, 0)$
- $-1 \leq z_5 \leq 0$: Distribution area of minor scale notes on the tonic $(0, 0)$

2.2 Comparison of Proposed Model and Tonnetz

Our derived model of tonality is topologically similar to the Tonnetz [6], which was originally proposed by Leonhard Euler in 1739 (Figure 5) and was expanded upon by Hugo Riemann in 1880 (Figure 6). In the Tonnetz, pitch notations are connected by three types of link: perfect V (opposite of perfect IV), major III (opposite of minor VI), and minor III (opposite of major VI). In our model, these links respectively correspond to the vectors $(1, 0)$ (opposite of $(-1, 0)$), $(0, 1)$ (opposite of $(0, -1)$), and $(1, -1)$ (opposite of $(-1, 1)$).

The Tonnetz was expanded as Neo-Riemannian theory and mathematically formulated in the 1980s [7, 8]. It was typically expanded to torus or spiral representations [9]

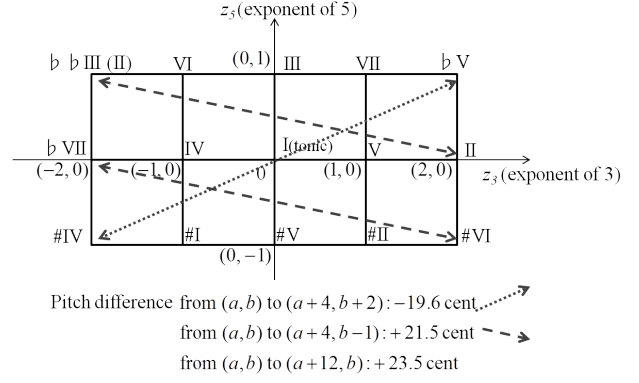


Figure 7. Pitch differences of enharmonic pairs of integer grid points.

considering circularity due to enharmonic equivalence. Enharmonic pairs of tones are also represented as vectors $(4, 2)$, $(4, -1)$, and $(12, 0)$ in our proposed model, as shown in Figure 7.

There are three key differences between our model and the conventional Tonnetz studies.

1. **Clear correspondence between the model and the physical and cognitive principles.** Although the conventional Tonnetz was originally formalized for representing the relationships among consonant intervals, the correspondence between the model and the principles on frequency ratios was not clear. Our proposed derivation process enables us to clearly understand the correspondence because it is directly derived from the principles on frequency ratios. Moreover, this feature should have a high affinity for processing polyphonic audio signals with the recognition error on harmonic overtone.
2. **Tonic representation.** In our proposed model (Figure 4), the origin $(0, 0)$ on the z_3 - z_5 plane has the role of the tonic of scale. Since the tonal characteristics of each point depend on the relative position from the tonic, equivalent scales on different tonics can be represented by a same pattern on the z_3 - z_5 coordination system. This feature also enables us to formulate computational representations of major/minor chords, such as Formulas (3), (4), and (5).
3. **Natural expandability to a higher dimensional space for the n -limit just intonation.** The above $n = 5$ setting for the upper limit of prime numbers can be varied to expand our tonality model. When $n = 7$, integer grid points in the z_3 - z_5 - z_7 space can represent the 7-limit just intonation [10], as shown in Figure 8. When $n = 11$, integer grid points in the z_3 - z_5 - z_7 - z_{11} space can represent the 11-limit just intonation.

On the basis of the above, our proposed model can be regarded as a generalization of the Tonnetz. We call it Prime Factor-based Generalized Tonnetz (PFG Tonnetz).

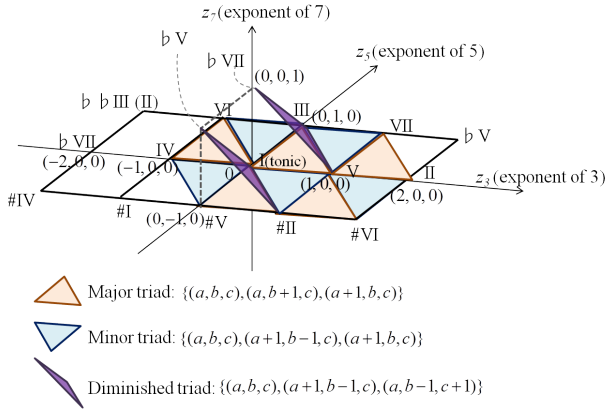


Figure 8. 7-limit PFG Tonnetz.

The model based on the z_3 - z_5 - \dots - z_n space is called n -limit PFG Tonnetz because it represents the n -limit just intonation [10].

Hereafter, we regard PFG Tonnetz without specifying n as the 5-limit PFG Tonnetz (Figure 4) because the 5-limit PFG Tonnetz represents the usual just intonation, i.e., 5-limit just intonation. The 5-limit PFG Tonnetz can easily be visualized on the z_3 - z_5 plane. The topological similarity of the 5-limit PFG Tonnetz to the conventional Tonnetz is easier to understand than that of other n -limit PFG Tonnetz.

3. APPLYING PFG TONNETZ TO DETERMINING PITCH WITH TONALITY CONSTRAINT

As discussed in Section 1, we aim to apply our model, the PFG Tonnetz, to a module to determine the pitch satisfying constraint on coherence of tonality against surrounding polyphonic music. 3D motion sensors, such as the Microsoft Kinect¹ or the Intel RealSense 3D Camera², can be used for recognizing users' hand motions, e.g., the heights of hands. Our system needs to convert a recognized hand height $x(t)$ at given time t into a tonal pitch frequency $f(t)$ that satisfies a constraint on the tonality of the surrounding polyphonic music, as

$$f(t) = \arg \min_{f(t) \text{ satisfies } c(t)} (|f(t) - f'(t)|), \quad (6)$$

$$f'(t) = f_{\text{tonic}} \cdot \exp(\alpha(x(t) - x_{\text{tonic}})), \quad (7)$$

where $f'(t)$ is an atonal frequency that simply corresponds to $x(t)$, $c(t)$ is a tonality constraint at the time t , $\exp(\cdot)$ is the exponential function, x_{tonic} is a basis location corresponding to f_{tonic} , and α is a parameter to adjust the ratio between the change of $x(t)$ and that of $f'(t)$.

A module for the online F0 estimation of the surrounding polyphonic music is needed to deal with the tonality constraint $c(t)$. Although F0 estimation of the polyphonic audio signals generally cannot avoid recognition errors on

harmonic overtones, a representation of the tonality constraint based on the PFG Tonnetz should be robust against such errors because the frequencies of harmonic overtones, i.e., integral multiples of a true frequency, are located at grid points close to the true pitch in the PFG Tonnetz space.

In future work, we intend to empirically verify this working hypothesis on the robustness against the recognition error on harmonic overtones. We primarily need to formulate a representation of tonality constraint $c(t)$ by integrating the PFG Tonnetz and the F0 estimation of polyphonic audio signals. This representation should be learnable from training data of polyphonic music and should be empirically compared with a representation by integrating the conventional chroma vector and the F0 estimation through an experiment. For example, if the tonality constraint is represented as probabilistic prediction models over the PFG Tonnetz space or over the chroma vector, the two representations can be compared by prediction ability such as the perplexity metric.

4. CONTEXT OF THIS STUDY AND RELATED WORKS

4.1 Tonality Models

Pitch representations based on Prime Factor Representation have been proposed in other studies [11, 12]. However, these works did not consider the relationship between their model and the Tonnetz. Direct derivation of the Tonnetz on the basis of the Prime Factor Representation of frequency ratio is a viewpoint unique to the present study.

There have been many models and theories related to tonality cognition. For example, a key estimation method based on the Cycle of Fifth [13] has been proposed. The Tonnetz has also been studied by Neo-Riemannian theorists [7] and applied to instrument interfaces such as the isomorphic keyboard [14]. The PFG Tonnetz we propose in the present work has three advantages as aforementioned: (1) it clearly corresponds to physical and cognitive principles on the integer frequency ratios, (2) it has tonic representation, and (3) it is naturally expandable to higher dimensions for n -limit just intonation. If the hypothesis on the robustness against recognition errors on harmonic overtones is empirically verified in future work, it can also be our contribution.

4.2 Related Applications

Figure 9 shows a smartphone application, TonalityTouch³, developed in our past study. TonalityTouch can convert the user's multi-touch location into consonant pitch frequencies with tonality. The scale for converting the location to the pitch frequency can be automatically generated on the basis of the PFG Tonnetz. However, TonalityTouch does not consider the constraint on tonality against the surrounding music.

KAGURA [15] is a digital instrument with visual effects based on body motion sensing. SWARMED [16] and mass-Mobile [17] are systems for supporting participatory mu-

¹ <https://www.microsoft.com/en-us/kinectforwindows/>

² <http://www.intel.com/content/www/us/en/architecture-and-technology/realsense-3d-camera.html>

³ <https://play.google.com/store/apps/details?id=org.toralab.music.beta>



Figure 9. TonicityTouch: Smartphone application based on PFG Tonnnetz.

sical performance using smartphones. Although these systems are related to our application, they do not focus on any computational model for converting the spontaneous input of pitch contour to pitch frequency satisfying the constraint of the tonality against surrounding polyphonic music, which is our focus in the current study.

4.3 Application to Music Event for Regional Promotion

We have been dealing with technologies for supporting public participation and collaboration [18, 19]. To facilitate public collaboration in local communities, building a conciliatory community through “ice-breaker activities” is important. Since music has a social functionality for enhancing positive emotions through sharing body motion [20], we aim to apply our model to such ice-breaker activities through participatory local music events. We need to investigate and verify whether the PFG Tonnnetz can contribute to spontaneous and improvisational participation in music performance and whether such support can contribute to ice-breaking in local communities.

5. CONCLUSION AND FUTURE WORK

We formulated the PFG Tonnnetz, a model of tonality cognition based on simple principles on frequency ratios of consonant intervals, i.e., consonant intervals can be represented by the frequency ratios of simple integers. The derivation of the proposed model is based on the Prime Factor Representation of the frequency ratios. The PFG Tonnnetz can be regarded as a generalization of the conventional Tonnnetz and can be applied to a representation of the tonality constraint of surrounding polyphonic music. The representation should be robust against recognition errors on harmonic overtone in polyphonic audio signals because the frequencies of harmonic overtones are located at grid points close to the true pitch in the PFG Tonnnetz space. This working hypothesis should be empirically verified through experiments in future.

We are also planning to apply the PFG Tonnnetz to support the spontaneous and improvisational participation of experts in local music events for regional promotion. We will utilize such functionality for ice-breaker activities in local

communities. To do this, we will develop a system based on the PFG Tonnnetz by integration with motion sensors.

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6. REFERENCES

- [1] J. B. Prince, “Contributions of pitch contour, tonality, rhythm, and meter to melodic similarity,” *Journal of Experimental Psychology: Human Perception and Performance*, vol. 40, no. 6, pp. 2319–2337, 2014.
- [2] Sakai Urban Policy Institute, “Report on investigating regional promotion by citizens’ initiative through organizing music events,” <http://www.sakaiupi.or.jp/30.products/31.resarch/H22/H22.music.pdf>, 2011, (in Japanese).
- [3] M. Kan, “An audience-participatory concert emphasizing physical expression : A case study promoting an understanding of polyphonic music,” *Bulletin of the Center for Educational Research and Training, Faculty of Education, Wakayama University*, vol. 18, pp. 121–129, 2008, (in Japanese).
- [4] R. L. Graham, D. E. Knuth, and O. Patashnik, *Concrete Mathematics: A Foundation for Computer Science*. Addison-Wesley, 1989.
- [5] E. M. Burns and W. D. Ward, “Intervals, scales, and tuning,” *The psychology of music*, vol. 2, pp. 215–264, 1999.
- [6] R. Behringer and J. Elliot, *Linking Physical Space with the Riemann Tonnnetz for Exploration of Western Tonality*. Nova Science Publishers, 2010, ch. 6, pp. 131–143.
- [7] W. Hewlett, E. Selfridge-Field, and E. Correia, *Tonal Theory for the Digital Age*, ser. Computing in Musicology. Center for Computer Assisted Research in the Humanities, Stanford University, 2007, vol. 15.
- [8] D. Tymoczko, “The generalized tonnetz,” *Journal of Music Theory*, vol. 56, no. 1, pp. 1–52, 2012.
- [9] E. Chew, *Mathematical and Computational Modeling of Tonality: Theory and Applications*, ser. International Series in Operations Research & Management Science. Springer, 2013, vol. 204.
- [10] H. Partch, *Genesis of a music: an account of a creative work, its roots and its fulfilments*. Da Capo Press, 1974.
- [11] J. Monzo, *JustMusic: A New Harmony Representing Pitch as Prime Series*, 4th ed. J. Monzo, 1999.
- [12] D. Keislar, “History and principles of microtonal keyboards,” *Computer Music Journal*, pp. 18–28, 1987.
- [13] T. Inoshita and J. Katto, “Key estimation using circle of fifths,” in *Advances in Multimedia Modeling*. Springer, 2009, pp. 287–297.
- [14] A. Milne, W. Sethares, and J. Plamondon, “Isomorphic controllers and dynamic tuning: Invariant fingering over a tuning continuum,” *Computer Music Journal*, vol. 31, no. 4, pp. 15–32, 2007.
- [15] SHIKUMI DESIGN, “Kagura - the motion perform instrument,” <https://www.youtube.com/watch?v=SvOfu9NifyY>, 2015.

- [16] A. Hindle, “Swarmed: Captive portals, mobile devices, and audience participation in multi-user music performance,” in *Proceedings of the 13th International Conference on New Interfaces for Musical Expression*, 2013, pp. 174–179.
- [17] N. Weitzner, J. Freeman, Y.-L. Chen, and S. Garrett, “mass-mobile: towards a flexible framework for large-scale participatory collaborations in live performances,” *Organised Sound*, vol. 18, no. 01, pp. 30–42, 2013.
- [18] S. Shiramatsu, T. Ozono, and T. Shintani, “Approaches to assessing public concerns: Building linked data for public goals and criteria extracted from textual content,” in *Electronic Participation. 5th IFIP WG 8.5 International Conference, ePart 2013*, ser. Lecture Notes in Computer Science, vol. 8075. Springer, 2013, pp. 109–121.
- [19] S. Shiramatsu, T. Tossavainen, T. Ozono, and T. Shintani, “A goal matching service for facilitating public collaboration using linked open data,” in *Electronic Participation. 6th IFIP WG 8.5 International Conference, ePart 2014*, ser. Lecture Notes in Computer Science, vol. 8654. Springer, 2014, pp. 114–127.
- [20] H. Terasawa, R. Hoshi-Shiba, T. Shibayama, H. Ohmura, K. Furukawa, S. Makino, and . Okanoya, “A network model for the embodied communication of musical emotions,” *Japanese Cognitive Science Society*, vol. 20, no. 1, pp. 112–129, 2013, (in Japanese).