The “Collective Rhythms Toolbox”: an audio-visual interface for coupled-oscillator rhythmic generation

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ABSTRACT

This paper presents a software package called the “Collective Rhythms Toolbox” (CRT), a flexible and responsive audio-visual interface that enables users to investigate the self-synchronizing behaviors of coupled systems. As a class of multi-agent systems, CRT works with networks of coupled-oscillators and a physical model of coupled-metronomes, allowing users to explore different sonification routines through real-time parameter modulation. Adjustable coefficient matrices allow for complex coupling topologies that can induce a diverse range of dynamic rhythmic states and audio-visual feedback facilitates user engagement and interactive flow. Similarly, several real-time analysis techniques provide the user with visual information pertaining to the state of the system in terms of group synchrony. Ultimately, this paper showcases how parameterizing coupled systems in specific ways allows different computer music and compositional techniques to be carried out through the lens of dynamical systems-based approaches.

1. INTRODUCTION

In the natural world, collective behavior arises in a number of biological systems that exhibit synchrony and self-organization. Simple repetitive behaviors from individuals responding to their environment can produce more complex forms of group behavior and research proposes a number of behavioural algorithms in order to account for the interplay between individual, interaction, and group adaptation [1]. The ecological validity of these sorts of interactions have inspired a range of experiments in sound and music computing, particularly those involved in simulating large-scale, behavioral dynamics as a medium through which to create sound.

Many such approaches to collective behavior in sound design make use of multi-agent based systems (MAS). MAS are a topic of research well-explored in computer music and many researchers and artists have explored several ways to generate sound using behavioral algorithms to control large groups of independent and interactive elements. This includes interactive evolutionary and genetic algorithms [2], ecosystems [3], boid/flocking based systems [4, 5], generative neural-networks [6], or even network-music [7, 8]. In these approaches, system or network behavior itself is an expressive medium which must be considered in the design of “metacreative processess”, a term used to denote autonomous behaviors that are generative in nature [9]. Similar coupled-oscillator models have been employed as a synchronizing algorithm to obtain more precise coordination between robots and humans performing in mixed cyborg orchestras [10]. Ultimately collective forms of synchronization, where self-organization arises spatially or temporally, can emerge from populations of agents interacting with one another using a limited, decentralized understanding of other agents’ states and has can be a flexible generative or analytical tool for use in controlling distributed networks.

The aim of the Collective Rhythms Toolbox (CRT) is to develop flexible and responsive tools for exploring collective behavior of ensembles of coupled oscillators and a physical system of coupled metronomes. Coupled oscillators are a broad class of systems often studied in applied physics and mathematics due to their unusual non-linear dynamics and self-synchronizing properties [11]. As such, they are typically comprised of limit-cycle oscillators who are connected together in various ways, often interacting in terms of phase using different coupling topologies to create network structure [12].

CRT allows a user to work with two well-studied coupled oscillator systems (pulse-coupling and Kuramoto Model) as well as a software interface for simulating a physical model of coupled metronomes. I briefly provide a mathematical definition of these two coupled-oscillator algorithms as well as the physics based model used to simulate systems of coupled metronomes. Next, I outline the unique design challenges for systems that make use of the collective behavior of multiple agents in producing sound. I also compare the advantages and disadvantages of designing sound using these models. Lastly, I present the ‘Collective Rhythms Toolkit’, an open-source platform for sound generation, analysis, and experimentation.

2. DYNAMICS OF SYNCHRONY

2.1 Kuramoto Model

Kuramoto oscillators are a type of limit-cycle oscillators with natural frequencies, ω_i, and a coupling coefficient, K_i, that continually adjusts their phases according to a sinusoidal phase response curve. The natural frequencies are typically drawn from different statistical distributions and since coupling is applied at all times, synchrony can result if coupling surpasses a critical coupling value. The governing equation for a group of N Kuramoto oscillators is shown in Equation (1) where $\dot{\phi_i}$ represents the time derivative of a single oscillator's phase variable within the group.

$$\dot{\phi_i} = \omega_i + \frac{K_i}{N} \sum_{j \neq i}^{N} \sin(\phi_j - \phi_i) \tag{1}$$

Phase coherence is a summary statistics that gives an indication of the global synchrony of the oscillators in an ensemble. This is shown in Equation (2). r is the phase coherence magnitude and ψ is what is known as the average angle. Mapping each phase state of the oscillators above onto a circle (0-2π), we can derive an expression that relates the relative spread or dispersion of the swarm of phases of each oscillator to an r value between 0 and 1 and an average angle, ψ. This measure of phase coherence will become a useful metric to describe how 'in sync' the oscillators are as a group.

$$r\, e^{j\psi} = \frac{1}{N} \sum_{i=1}^{N} e^{j\phi_i} \tag{2}$$

2.2 Pulse Coupling

Kuramoto coupling is a specific instance of continuous coupling: oscillators are sharing phase information with each other continuously and making adjustments accordingly. In a pulse-coupling configuration, each oscillator triggers the other oscillators to make phase adjustments only at specific instances of time, for example every time it crosses some threshold. Consider a simple example of N pulse-coupled oscillators, where each oscillator contains a phase state, θ_i that evolves over time according to the differential equation in Equation 3 where ω_i is the natural frequency of oscillator i, K_i is the coupling coefficient, A_ij is the coupling matrix that determines the distributed coupling topology between oscillators, and $F(\theta_j - \theta_i)$ is a phase response curve (PRC) function that adjusts the phase response of the other oscillators from an input pulse from one oscillator in the group. The PRC function is assumed to be zero except for a small interval of time when the oscillator completes one cycle. Different PRC function affect the self-synchronizing dynamics of the system such as the critical coupling strength as well as the time and trajectory for synchronization to occur.

$$\dot{\theta_i} = \omega_i + \frac{K_i}{N} \sum_{j=1}^{N} A_{ij} F(\theta_j - \theta_i) \tag{3}$$

Pulse-coupling is often associated with 'integrate and fire' systems which model spiking neuron signaling in the brain. Due to this fundamental difference in coupling mechanisms, pulse-coupled oscillators exhibit distinct dynamics compared to Kuramoto oscillators; these dynamics also have greater real-world applicability, making this genre of interactive systems a useful paradigm for describing various systems, such as firefly synchronization [13], animal chorusing [14], cardiac pacemaker cells [15], and circadian rhythms [16] among others.

Figure 1. Coupled Metronomes

2.3 Coupled Metronomes

The Dutch mathematician, Christian Huygens, is often credited with initiating inquiry in coupled oscillators in 1665 when he observed how two pendulum clocks hung on the same wall would settle into anisochrony over time [17]. His observations regarding what he called the 'sympathy of two clocks' forms the basis of understanding the collective entrainment of coupled metronomes. In this physical system, mechanical pendulum metronomes are placed atop a movable surface and set into motion. Small moments of inertia from each metronome are applied to the table from which they sit; this aggregation of small forces compels the individual pendulums to phase align over time. Coupling in this sense is dependent on the mass and friction of the coupling platform as a lighter, more frictionless table allow this mean field force to exert more influence on the pendulums' phases. Research into coupled metronomes dynamics have been carried out in using both physical metronomes [18] and simulations involving computational modeling [17].

The following equation of motion shown in Equation (4) is taken from a model set forth from Panteleone et al. (2002) and shows a system of N metronomes of masses m_i with moments of inertia, I_i, and phase angle, θ_i [19].

$$\begin{aligned} \frac{d^2\theta_i}{dt^2} + \frac{m_i r_{cm,i} g}{I_i} \sin(\theta_i) + \epsilon_i * D(\theta_i) \frac{d\theta_i}{dt} \\ + \frac{m_i r_{cm,i} \cos(\theta_i)}{I_i} \frac{d^2 x}{dt^2} = 0. \end{aligned} \tag{4}$$

The first two terms represent the typical pendulum angular acceleration and the gravitational torque respectively. The third term models the mechanism for escapement (ϵ_i) and dampening (D) as a function of pendulum angle. Lastly, the fourth term accounts for the coupling of the table where x is the horizontal motion of the table in the direction of the pendulums' motion. Modifying these parameters changes the way in which the pendulums synchronize with one another or fail to do so entirely for example if the tempos of the pendulum (a determined by their length, r) are too far apart or if the mass of the table is too large. Synchronization time is proportional with natural tempo spread: metronomes take longer to phase align when their natural tempos are different. Similarly, if the dampening factor is too large, the escapement mechanism fails to induce the metronomes into periodic motion.

3. COLLECTIVE RHYTHMS TOOLBOX SOFTWARE OVERVIEW

The ‘Collective Rhythms Toolbox’ (CRT) [1] is a package of software interfaces that allow for sound generation and analysis that allows a user to interact with the aforementioned dynamical coupled oscillator systems in sound. As such, users can control an audio-visual environment built in Processing to generate a wide range of output behaviors and dynamic states. Processing is a widely-used scripting language based on Java for creative production and its relative ease of use encourages exploration and extension of these models by code adaptation and modification. CRT relies on a controller-model paradigm where the system state is visually rendered onto a model window.

To encourage cross-platform interactivity, CRT sends out OSC messages to an audio client so that audio synthesis can take place on other platforms more suitable for sound design (e.g. Supercollider, Ableton Live). Therefore, CRT requires the OSCP5 and controlP5 libraries as dependencies.

A number of predefined keystrokes, mapped to system parameters, allow users to make quick adjustments to parameter states using just their keyboards to interact with the interface (add or remove oscillators, (de)select all oscillators, change coupling type, change view, parameter setting). This are detailed on the help page of the software repository and can be modified in the open-source code.

Figure 2 shows the visual interface of CRT. Individual oscillators can be added to the ensemble and once activated, each oscillator can be addressed by selecting the corresponding oscillator on the selector matrix. Individual oscillator’s (or groups of oscillators) parameter values are adjusted with sliders and set using a trigger key.

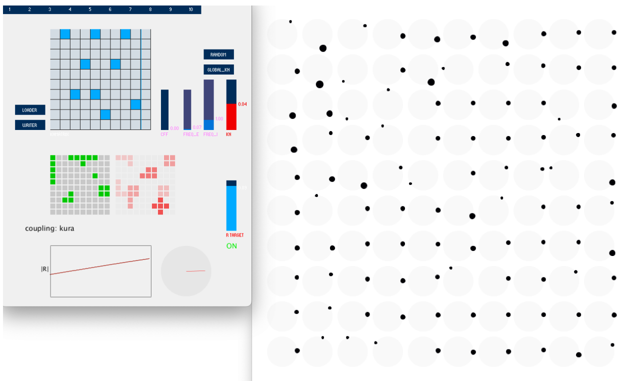

Figure 2. Visual interface to Collective Rhythms Toolkit

CRT allows a user to modify the following model parameters associated with coupled-oscillator systems: add/remove oscillators, coupling type (pulse/continuous), coupling matrix, coupling coefficients, intrinsic frequency, external forcing function, phase coherence target value, and coupling delay. In terms of visual feedback, it provides two visualizations of the oscillators phases (grid view and swarm of points circle map), two indicators of phase coherence over time, coupling strength indicator, oscillator selection, and a sequencer grid. Lastly, it allows input/output in the form of the saving and loading of system states by writing the current parameter states to a text file. It also allows the user the option to save the individual oscillator’s zero crossings to a text file.

CRT also comes with a separate program for simulating the coupled metronomes described in the previous section. Figure 3 shows the interface for the coupled metronome physical model. In this model, the user can interact in real time with the coupling, tempo, and dampening as referenced by the respective terms in Equation (4). Additionally, an external driving force allows the user to apply oscillatory motion onto the shared platform itself which facilitates the exploration of more unusual collective rhythms from the metronome ensemble. This physical model uses simple forward Euler integration with a step size of 17 ms which is constrained by the frame rate.

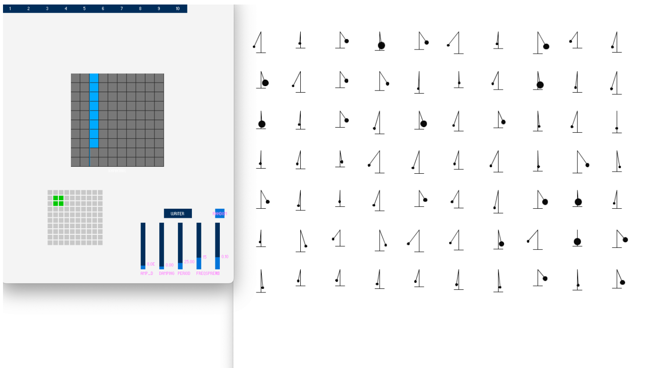

Figure 3. Simulation of coupled metronome physical model in CRT

3.1 Advanced Features

3.1.1 Phase Coherence Graphs and Feedback

The phase coherence in Equation (2) is visually depicted as a phasor rotating about a circle over time. A real-time graph also shows the magnitude of the phase coherence on a 2-D plot. This provides an indication of the system’s global synchrony via oscillator phase alignment. In order to ‘tune’ the system into a desired synchronous state, I implemented a moving average filter that averages the phase coherence magnitude every 10 frames. This value is compared to the target value and the oscillators’ coupling coefficients are incremented or decremented proportionally until the target r value is achieved. This setting requires that all of the oscillators’ individual coupling strengths be the same or else target phase coherence values are not not assured.

3.1.2 Selector Matrices and Coupling Strength Indicators

A selector matrix allows a user to set coupling coefficients and intrinsic frequencies for individual oscillators or groups of oscillators. As coupling strength is set by a slider and “burned in”, its corresponding cell in the coupling strength matrix is colored and tinted accordingly.

[1] https://github.com/nolanlem/CollectiveRhythmsToolbox

Design Consideration	Features	Interface
Flexibility	Real-time vs Compositional Tool	Instrument for real-time performance settings vs. aid in compositional process
Accessibility	"Ease of Use" and open-source	Facilitate interactive behaviors and learnability
Reusability	Data Flow	Provide functionality that allows for saving and loading from memory
Modularity	Interface with other software	Communicate with other platforms often used in sound and music production
Feedback	Audio-Visual cues	States can be interacted with sonically or visually
Controlability	Responsive and open-ended	Parameter space has perceptual outcomes

Table 1. Design Criteria for Collective Rhythms Toolbox

This has the benefit of providing the user with a visual indication of the parameter states of individual oscillators relative to others in the group and is particularly useful when implementing more complex coupling topologies.

3.1.3 Sound Event Triggers with Sonification Sequencer

The default sonification mode for triggering a sound event is when an oscillator completes one cycle which implies that full synchrony results in the entire oscillator ensemble outputting a simple, isochronous rhythm. Users can opt to trigger sound events at other points along the phase trajectory of the circle using the sequencer matrix to allow the system to converge into polyrhythyms at quantized intervals.

3.2 Challenges in System Control and Design

Once parameters become time-varying, coupled oscillator dynamics become very complex, which is the basis for real-time interaction. Consequently, it is challenging to anticipate how a swarm will react when parameters are adjusted on-the-fly. For instance, the synchronization time relies on the number of oscillators in the group, their connectivity topology, and their PRC function. The visual feedback implemented in CRT aims to enhance user control by providing visual and numerical information about the swarm's dynamics, particularly when such information is not readily apparent through sound. This increases the accessibility of composing for and interacting with the swarm and may facilitate different interactive behaviors. Furthermore, the system states can be saved, allowing users to retrieve interesting system states at will and is particularly advantageous for live performances and model reusability.

Ultimately, this toolbox focuses on optimizing and controlling collective behaviors that generate interesting audio-visual and perceptual outcomes, rather than serving as a platform for precise numerical simulations that require advanced integration methods. Table 1 lists several design categories related to these considerations.

4. CONCLUSION

This toolbox is an exploratory interface for experimenting with coupled oscillator and metronome systems for sound generation. Dynamical systems approaches can be a powerful tool in the production technologies intended for performers, composers, or creative technologists. These generative models propose novel methods for synthesizing sound, controlling collective behavior, and the user interaction of dynamic parameter spaces as evidenced by the rich behaviors that arise when phase-coupling simple oscillator systems. CRT provides one useful research direction for users looking to experiment with novel generative methods with multi-agent systems in which synchrony and rhythmic self-organization are paramount.

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