Stellar Stream Track Reconstruction, with Errors

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Introduction

Stellar streams are sensitive probes of the Galactic potential. The likelihood of a model given stream data can only be assessed using simulations. However, comparison to simulation is challenging in a noisy 6D phase space in which even the stream paths are hard to quantify. Here we present a novel application of Self-Organizing Maps and first-order Kalman filters to reconstruct the stream path, propagating measurement errors and data sparsity into the stream path uncertainty. The technique is Galactic-model independent, non-parametric, and works on phase-wrapped streams. We can uniformly analyze and compare data with simulation.

Data Ordering

Using Self-Organizing Maps (SOMs) in conjunction with reference frame transforms, we discover the 1D structure of a stellar stream and a path-distance-minimizing ordering.

Self-Organizing Maps (SOMs)

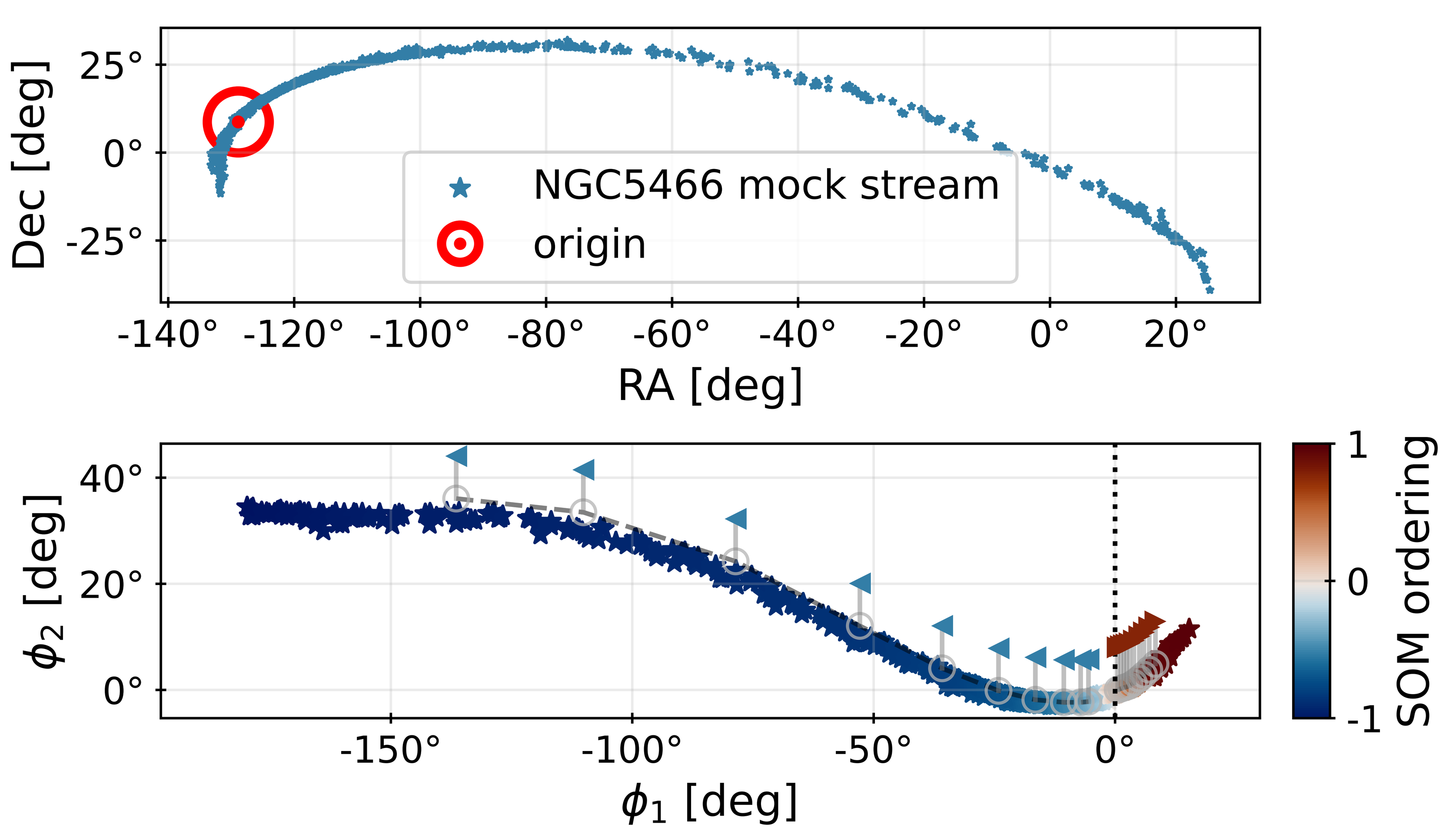
SOMs are a neural network for low-dimensional, discrete representation of data. Using linked prototype vectors, SOMs approximate the data by iteratively updating the topology of the prototypes to approximate the data distribution. In final form, each prototype maps nearby high-dimensional data to a lower-dimensional lattice [2] -- see lower figure.

Optimal Reference Frames

Streams orbit the Galactic center of mass. The orbit path for the majority of streams, distant and less sensitive to Galactic structure, approximately describe a great circle.

Therefore, for a fixed origin point (α^*, δ) and rotation θ the transformation to a great circle frame defines a locally-linearizing set of sky coordinates (ϕ_1, ϕ_2) [1]. By ordering data along ϕ_1 we discover the approximate data ordering and decrease the SOM's network burn-in phase.

Figure: Mock Stellar Stream of NGC 5466



Kalman Filter Math

Kalman filters estimate a joint probability distribution over the variables in a timeseries. For a hidden-velocity filter, the state $\mathbf{x}$ encodes the position and Kalman-velocity, and $\mathbf{P}$ the error therein. The Newtonian-dynamics transition matrix $\mathbf{F}$ gives the (prior) dynamics between states, and $\mathbf{Q}$ the modeling uncertainty. $\mathbf{H}$ is the observation function, bringing states into measurement space, where $\mathbf{z}$, $\mathbf{R}$ are the mean and noise covariance [3].

$$\mathbf{x} = \begin{bmatrix} x & v_x & y & v_y & z & v_z \end{bmatrix}^T \qquad \mathbf{F} = \mathrm{diag}_3\left(\begin{bmatrix} 1 & \Delta t \\ & 1 \end{bmatrix}\right) \qquad \mathbf{H} = \begin{bmatrix} 1 & 0 & & & & \\ & & 1 & 0 & & \\ & & & & 1 & 0 \end{bmatrix} \tag{1}$$

The Kalman filter operates by updating at each time step (Δt); however, the times are not known. As a proxy, before each predict-update iteration we tune the time-step in the state transition matrix $\mathbf{F}$ by the smoothed point-to-point distance.

For each data point the Kalman filter process is two steps [2, 3]:

Predict

$$\mathbf{x}_{k|k-1} = \mathbf{F}_k \mathbf{x}_{k-1|k-1} + \mathbf{B}_k \mathbf{u}_k \qquad \textit{a priori state estimate} \tag{2}$$

$$\mathbf{P}_{k|k-1} = \mathbf{F}_k \mathbf{P}_{k-1|k-1} \mathbf{F}_k^{\intercal} + \mathbf{Q}_k \qquad \textit{a priori estimate covariance} \tag{3}$$

Update

$$\tilde{\mathbf{y}}_k = \mathbf{z}_k - \mathbf{H}_k \mathbf{x}_{k|k-1} \qquad \text{pre-fit residual} \tag{4}$$

$$\mathbf{S}_k = \mathbf{H}_k \mathbf{P}_{k|k-1} \mathbf{H}_k^{\intercal} + \mathbf{R}_k \qquad \text{pre-fit residual covariance} \tag{5}$$

$$\mathbf{K}_k = \mathbf{P}_{k|k-1} \mathbf{H}_k^{\intercal} \mathbf{S}_k^{-1} \qquad \text{optimal Kalman gain} \tag{6}$$

$$\mathbf{x}_{k|k} = \mathbf{x}_{k|k-1} + \mathbf{K}_k \tilde{\mathbf{y}}_k \qquad \textit{a posteriori estimate} \tag{7}$$

$$\mathbf{P}_{k|k} = (\mathbf{I} - \mathbf{K}_k \mathbf{H}_k) \mathbf{P}_{k|k-1} \qquad \textit{a posteriori estimate covariance} \tag{8}$$

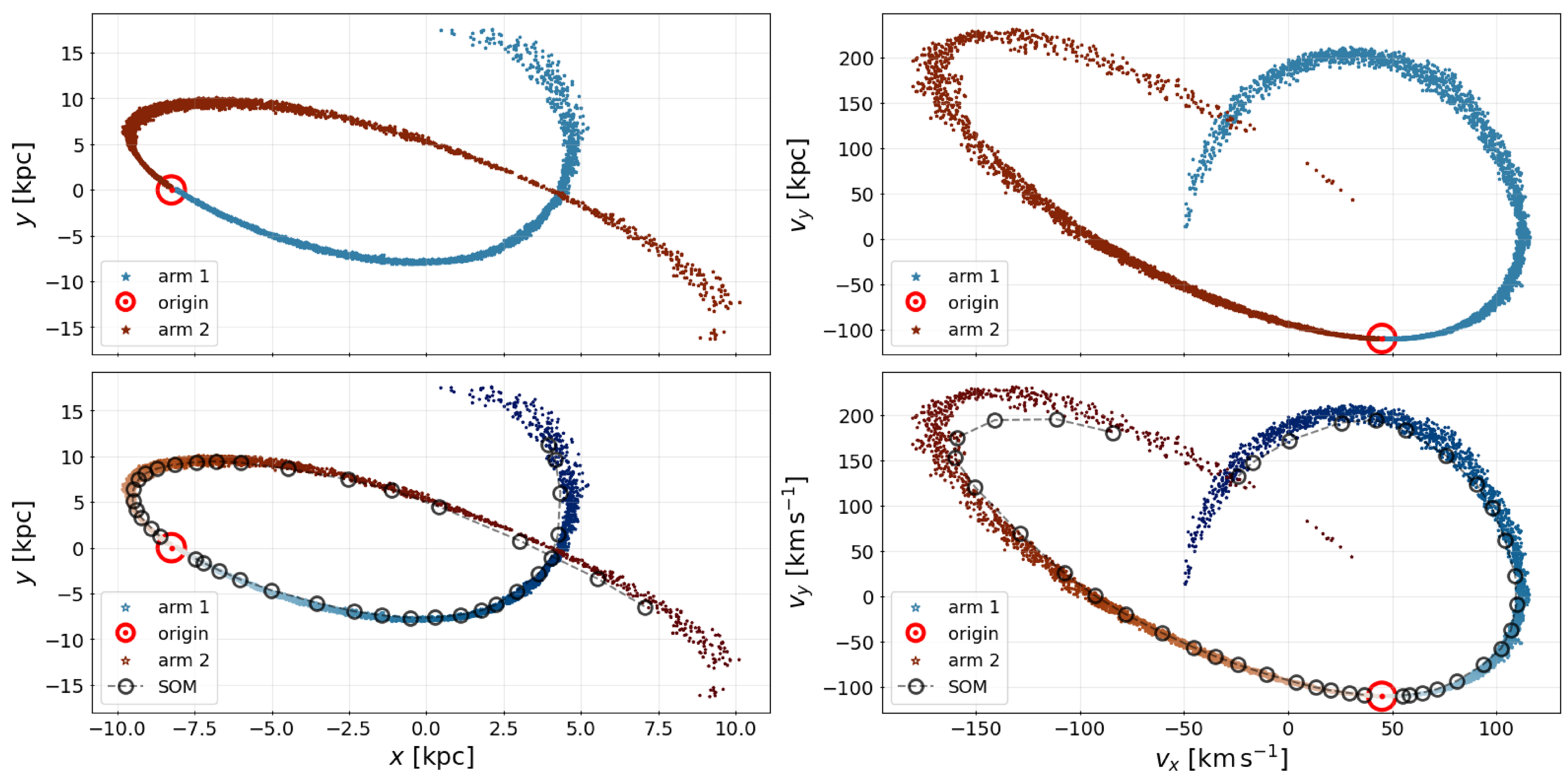
Subsequent steps are updated by previous steps, but not vice versa. To back-propagate information, the process is run in reverse by Rauch–Tung–Striebel (RTS) smoothing [4].

Reconstructing Stream Paths

A first-order Newtonian-dynamics, hidden-variable Kalman filter is used to construct the stream path, propagating both measurement and sampling-sparsity-induced uncertainty into the path. The technique is injective and self-affine parameterizing.

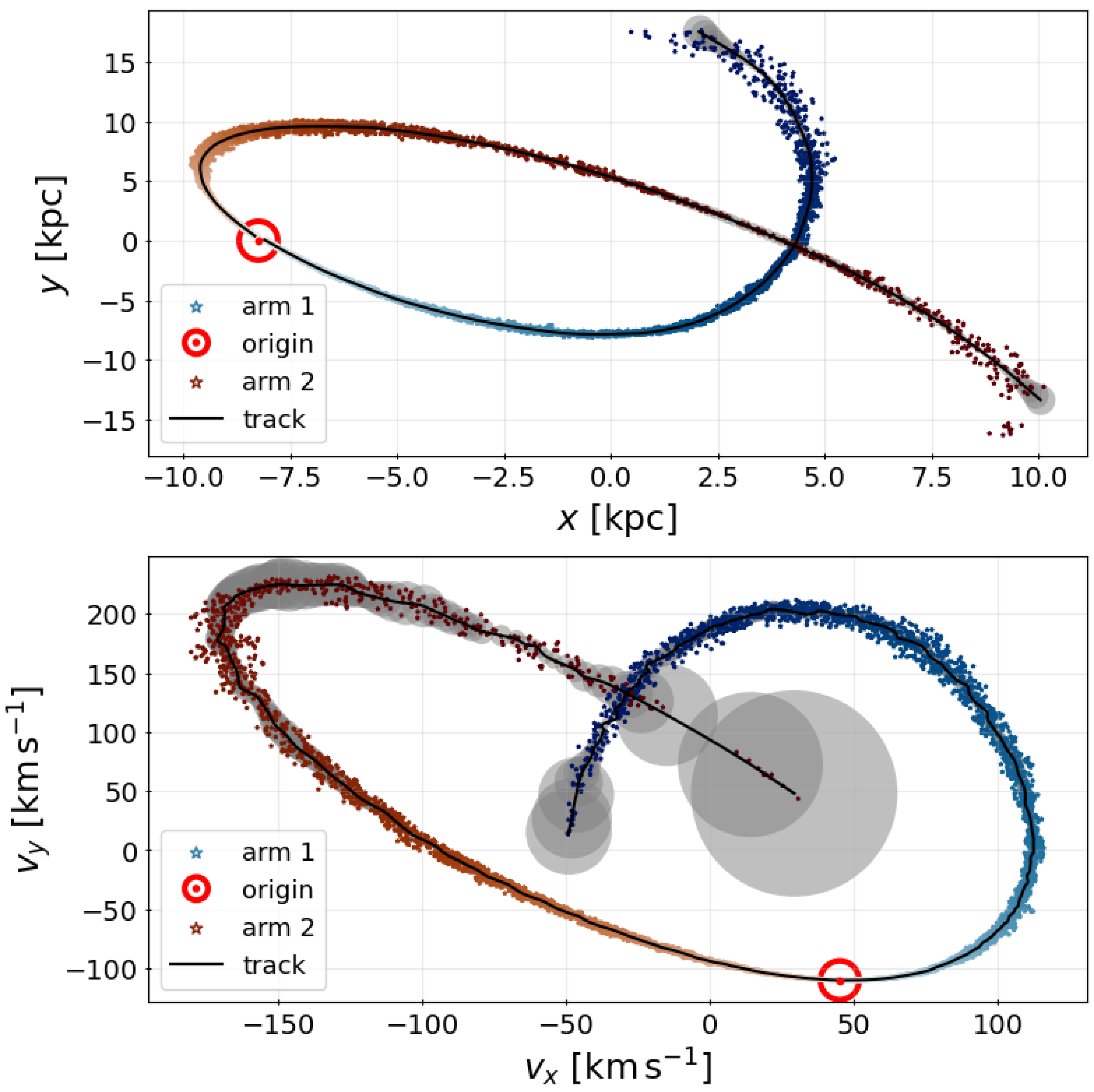
We start with the raw data (top panels) -- here an N-body simulation of Palomar 5 from [5]. The SOMs (bottom panels) map the 6D astrometric data to 1D.

Figure: N-Body of Palomar 5 stream



The Kalman filter uses the SOMs-ordered data to fit a track, with uncertainty, to the data.

Figure: Stream track fit to Palomar 5 mock stream



Conclusions

We reconstruct the mean path of stellar streams (e.g. center figure) from sparse and noisy data, respecting both as sources of error. By using reference-frame transformations in conjunction with Self-Organizing Maps, we treat stellar-stream data as a pseudo time-series, to which first-order Kalman filters can be applied. The path reconstruction properly propagates measurement errors and data sparsity into a path error, allowing for equal treatment and more precise comparison of data and simulation.

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