

Ramanujan and the Pyramid of Venus at Giza

Dream dimensions

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Abstract

The Pyramid of Venus, located at the celestial north pole in the star map that inspired Giza, has dimensions that provide the key numbers in Ramanujan's famous formula approximating π . This is the most precise reference to π at Giza.

Keywords: Pyramids of Giza, History of Mathematics, π , φ , e

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1 Introduction

The Giza site plan was inspired by a stellar arrangement, which had interesting mathematical properties. The arrangement fitted neatly into a double square, as shown in Figure 1. (*Zep Tepi Mathematics 101* [1])

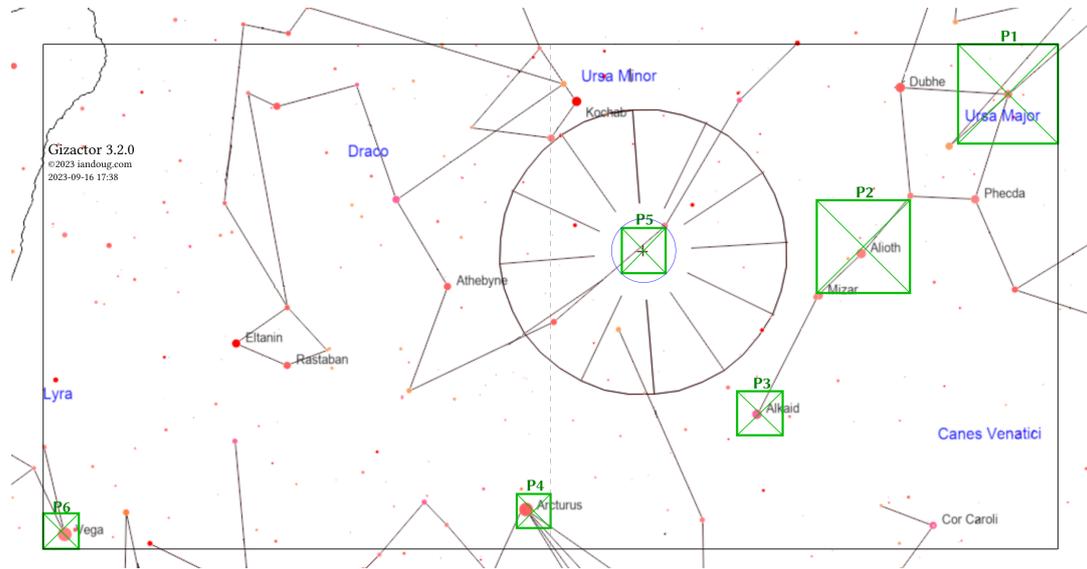


Figure 1: The star alignment, in 55.5k BCE

Evidence for the 4th pyramid, aligned with Arcturus, comes from Norden (*Voyage d'Égypte et de Nubie* [2]), as illustrated in Figure 2.

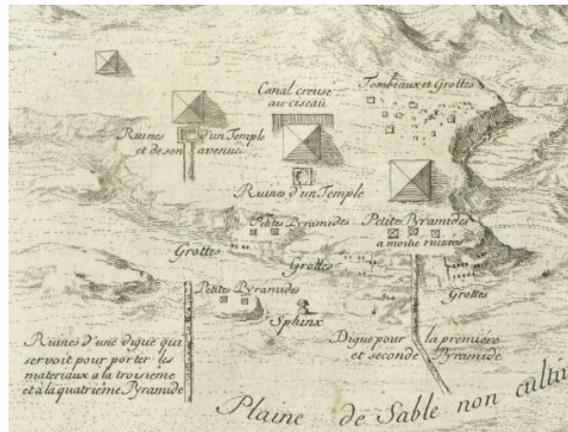


Figure 2: Norden's map showing four main pyramids

I rediscovered Norden's pyramid (P4) because the star map suggested it was there. Similarly for Thuban at the celestial North Pole (P5), and Vega (P6)... once you add them in the correct spots, everything falls into place.

The actual spacing between the pyramids on the right side of the plan, while "inspired by real constellations," was calculated mathematically, as shown in Figure 3. The designers rounded pyramid base sizes, and the spaces between them, to the nearest whole cubit.

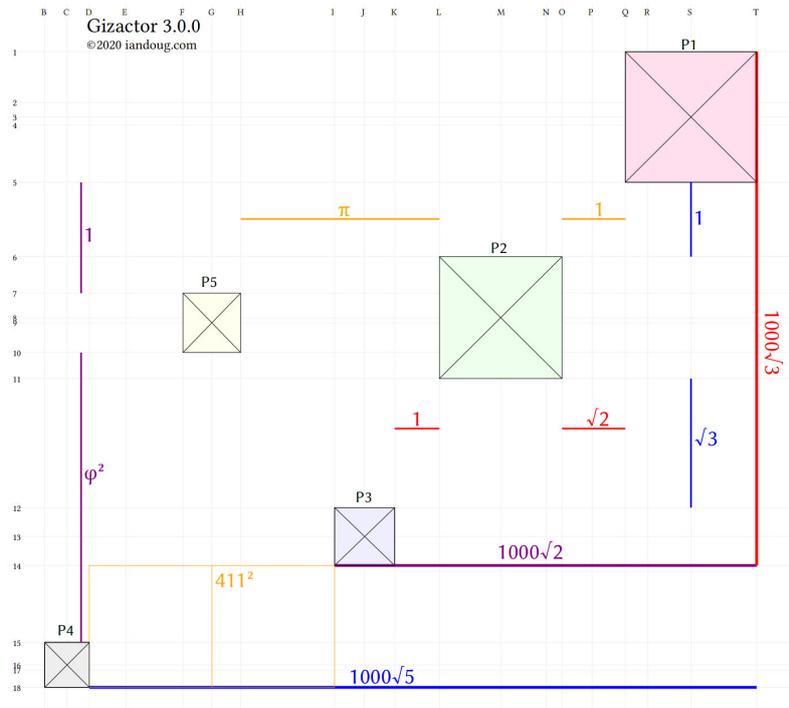


Figure 3: The irrational site plan

The pyramids also model the visible planets in our solar system, where their volume as a percentage of Khufu, matches the associated planet's diameter as a percentage of Jupiter, with Khufu as Jupiter. (*The six pyramids and planets at Giza* [3])

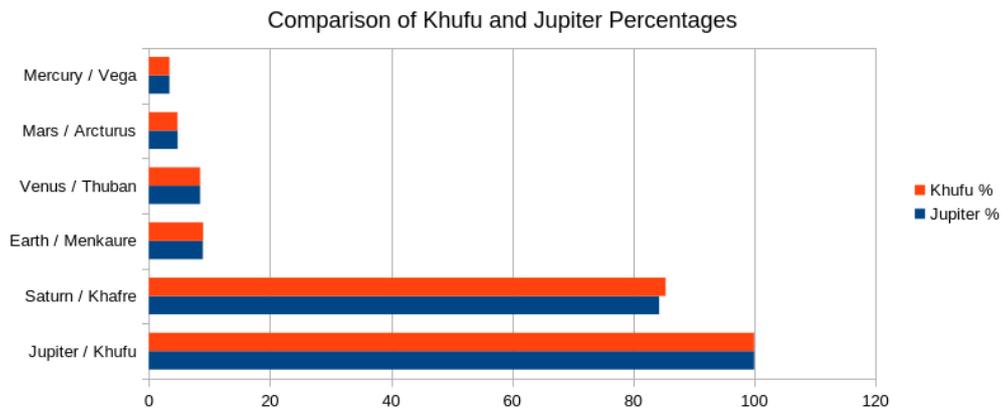


Figure 4: Planets and pyramids correlation

Khafre/Saturn is a little off, because Khafre needed to be a Pythagorean triangle, and the base sizes for Khufu, Khafre, and Vega/Mercury make a right-angled Douglas Triangle (*The Douglas Triangle, Khufu and Khafre* [4]), which has the sides in the ratio $1 : \varphi^2 : \varphi\sqrt{3}$. That gives us bases of $157 : 411 : 440$. Note that 157 is 50π , rounded.

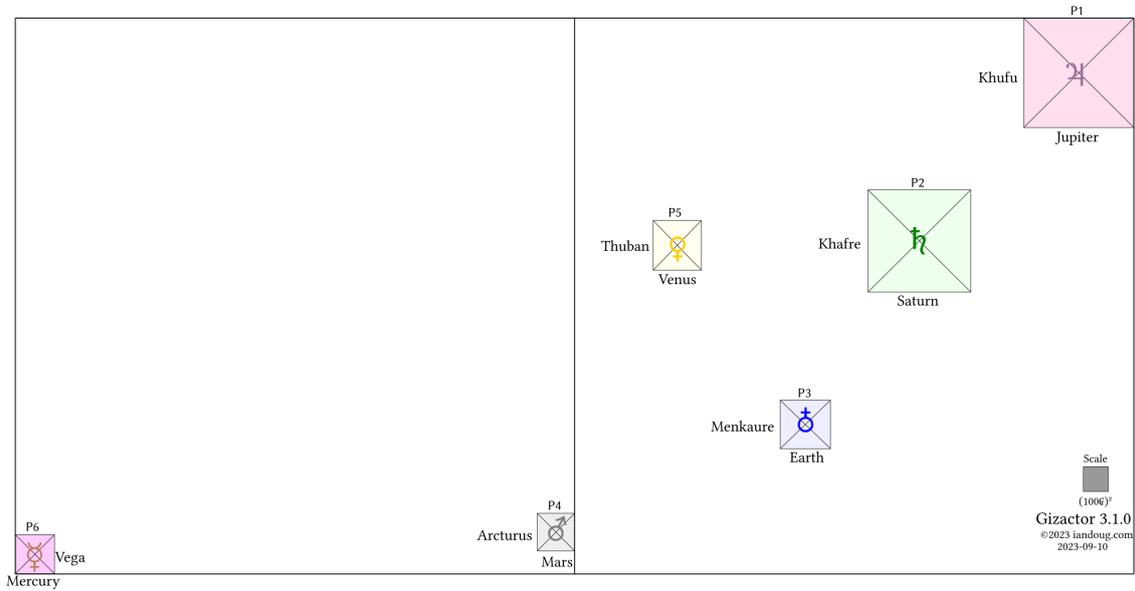


Figure 5: Pyramids and their planets

Figure 6 shows important dimensions.

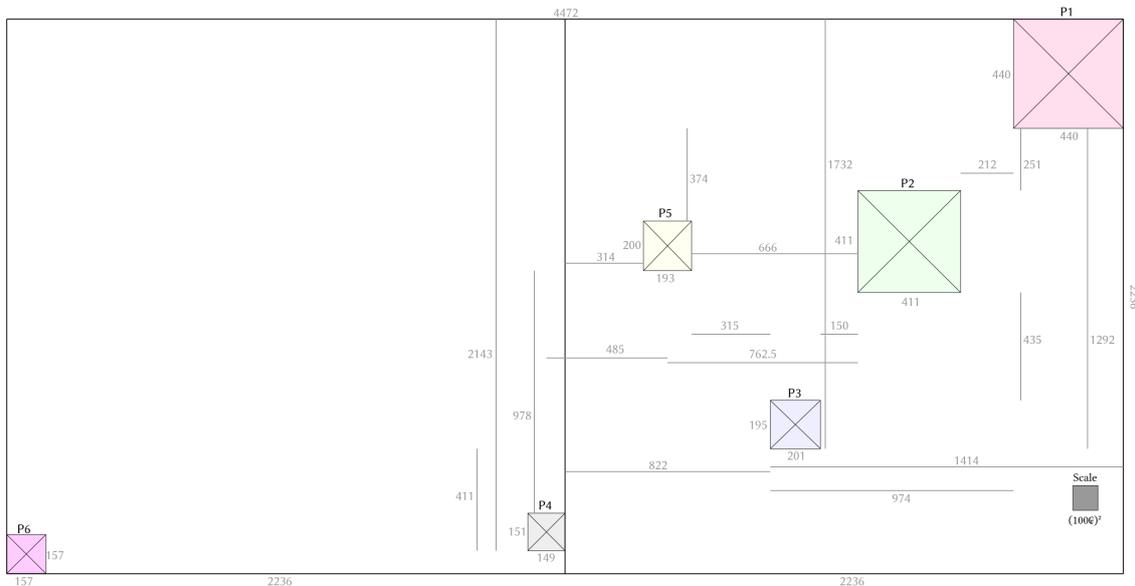


Figure 6: Plan with dimensions

The proof or validation for this arrangement comes from the mathematics inherent in the design. For further details, see *Zep Tepi Mathematics 101* [1]. Two examples will suffice here, first the Golden Cross in Figure 7, and Squaring the Circle, Giza style, in Figure 8.

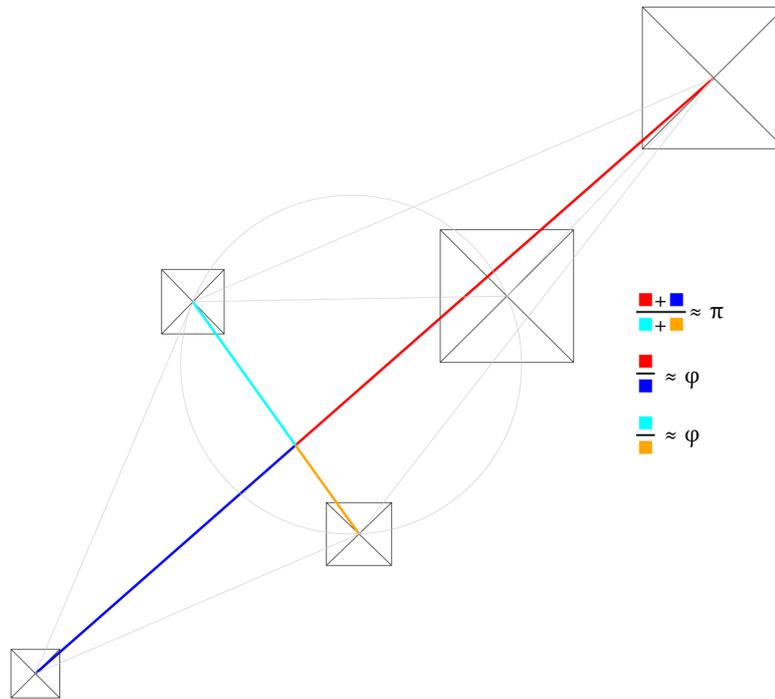


Figure 7: Golden Cross in Skeleton Blueprint

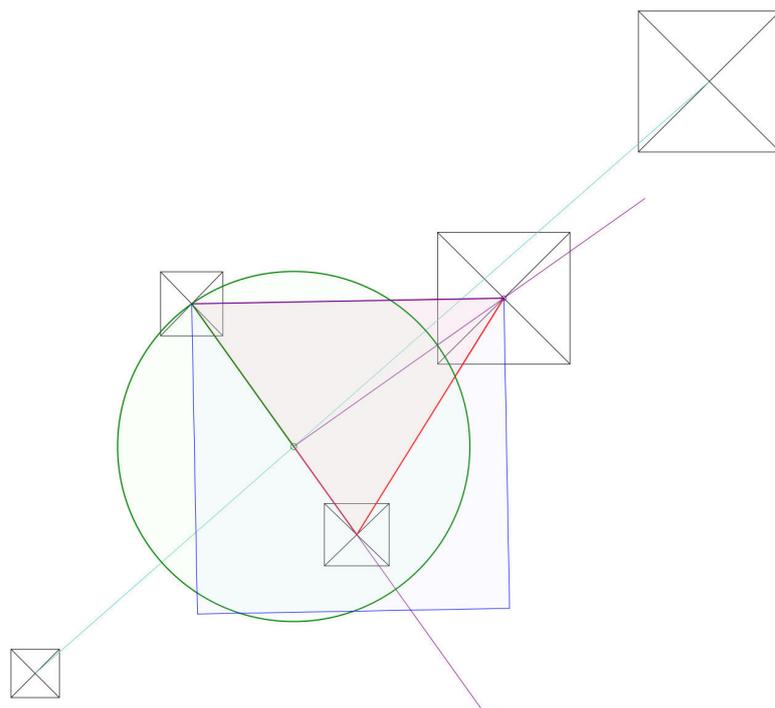


Figure 8: Squaring the circle using 1991 method by Robert Dixon [5]

The pyramid dimensions are shown in Table 1.

I will discuss Menkaure's height later.

We see that the outermost three have square bases, while the innermost three pyramids are rectangular.

The pyramid at Abu Rawash was about the same size as Menkaure, and has been totally dismantled, so it is quite possible that the similarly-sized P5 was also dismantled. I suspect (without evidence) that the granite casing stones around Menkaure were scavenged from P5. Menkaure was enlarged from 201×195 to its current size, in the process shifting the footprint slightly left. See *The Consequences of Legon's Rectangle: The Rational Giza Design* [6] for a discussion about this.

Pyramid	Name	X \mathbb{G}	Y \mathbb{G}	Height \mathbb{G}	% Khufu	Planet	% Jupiter
P1	Khufu	440	440	280	100.00	Jupiter	100.00
P2	Khafre	411	411	274	85.38	Saturn	84.30
P3	Menkaure	201	195	124	8.97	Earth	8.92
P4	Arcturus	149	151	114 (114.38?)	8.47	Mars	8.47
P5	Thuban	193	200	119	4.73	Venus	4.75
P6	Vega	157	157	75	3.41	Mercury	3.41

Table 1: Pyramid names and dimensions

2 These are a few of my favourite things

Certain numbers keep popping up at Giza. I call these “their favourite numbers”. The list includes π , φ , φ^2 , e , c , $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, 1.87, and 137. Since the use case was construction, they used practical approximations. It appears that while they used 3.1416 for π when needed, they were happy to approximate that to simpler versions like 3.14 or 3.142 when showing intent rather than exactitude was sufficient. Even today, we still use $\frac{22}{7}$ or 3.14 in school. Similarly with the other irrationals. The numbers we need are shown in Table 2.

Symbol	Name	“Full” value	Practical value	Practical % Accuracy
π	Archimedes’ constant	3.1415926...	3.1416 or 3.142 or 3.14	99.9998, 99.9870 or 99.9493
η	Eagle’s constant ($\frac{\pi}{2}$)	1.570796327...	1.5708 or 1.571 or 1.57	99.9998, 99.9870 or 99.9493
e	Euler’s number	2.7182818...	2.7183 or 2.718 or 2.72	99.9993, 99.9896 or 99.9368
φ	Golden ratio	1.61803398...	1.618 $\varphi + 1 = \varphi^2 = 2.618$	99.9979
ρ	Plastic ratio	1.32471795...	1.3247 $\rho + 1 = \rho^3 = 2.3247$	99.9986 or 9.9458
α	Fine Structure Constant	0.0072973525...	0.007297 or α^{-1}	99.9952
α^{-1}	Inverse FSC	137.03599908...	137	99.9737
c	Speed of light	299792458 m/s or 572560080.2 \mathbb{G}/s		
$\sqrt{2}$	Root 2	1.414213562...	1.4142 or 1.414	99.9990 or 99.9849
$\sqrt{3}$	Root 3	1.732050807...	1.732	99.9971
$\sqrt{5}$	Root 5	2.236067977...	2.236	99.9970
\mathbb{G}	Royal cubit	0.5236 m	From $\frac{\pi}{6}$ or $\frac{\varphi^2}{5}$, rounded	
ϑ	Thoth’s Constant ($\frac{\pi\varphi}{e}$)	1.87000613...	1.87	99.9997

Table 2: Symbols, names and values

I use “cubit” for the Royal cubit (\mathbb{G}). Other researchers use other values, but 0.5236 works for me.

Equations shown below use a science/engineering approach, not pure mathematics, so $1.999 = 2.0$.

We should remember that we are dealing with a different culture, which may have had a different approach to accuracy and precision, compared to our modern scientific mindset. My Giza analysis suggests they typically worked to 3 or 4 decimal places. This may indicate that they used abacuses or counting tables, or possibly slide rules or logarithms, to calculate. Alternatively, they may just have used four decimals as anything more did not make sense in construction. $0.0001 \mathbb{G}$ is 0.05236 mm , which is about 50 microns, half the smallest distance that can be seen with the naked eye, and approaching the length of a human liver cell.

3 Thoth’s Constant

Thoth’s Constant ϑ (*Thoth’s Constant* [7]) appears in various places at Giza. For example, the digit is 1.87 cm, as $28 \times 1.87 = 52.36 \text{ cm}$. The likely width of the sarcophagus in Khufu’s King’s chamber is $1.87 \mathbb{G}$ (*Khufu’s Coffin* [8]). Depending on the approximations you use for π , φ , and e , then $\log_{\varphi} \vartheta$ times Khufu’s base of $440 \mathbb{G}$ gives a good approximation of the speed of light (*Another reference to the speed of light by the Great Pyramid* [9]).

We shall see ϑ again shortly.

I now want to run through the six pyramids, showing some of the mathematics in the designs, i.e. why those base sizes and heights, over and above the requirements for the site plan, and matching the pyramids

to the planets. The genius of Giza is how it does everything at the same time, and how it all works together. It's likely a finely-constructed mechanical watch with dozens of complications ... change one thing and it does not work.



Figure 9: Patek Philippe watch with complications (Wikipedia [10])

4 The square-base pyramids

Pyramids with square bases are amenable to simple ratio operations between the various dimensions.

4.1 Khufu

Khufu is based on the Kepler triangle (*Kepler triangle* [11]). I see that Wikipedia goes out of their way to dismiss this idea, which would be valid if Giza was built by the 4th Dynasty. Giza is not 4th Dynasty, and labelling people who point out the mathematics in the design as “pyramidologists” is anti-scientific name-calling. Ignoring the mathematics allows the myth of 4th dynasty to continue.

The particular dimensions chosen not only provide excellent approximations for π , φ , and e , but also c .

$$\frac{\text{twice base}}{\text{height}} = \frac{2 \times 440}{280} = 3.142857 \approx \pi$$

The side slope is $\sqrt{220^2 + 280^2} = 356.089876$.

$$\text{Then } \frac{\text{side slope}}{\text{half base}} = \frac{356.089876}{220} = 1.61859 \approx \varphi, \text{ and}$$

$$\frac{\text{base area}}{100 \times \text{twice side slope}} = \frac{440^2}{100 \times 2 \times 356.089876} = 2.7184 \approx e$$

For e , the 100 is a scaling factor. Instead of squaring the base, you could multiply the base by half the base, and dividing that by the side slope will give you approximately $100e$.

The error for π and φ is less than 0.1%, and less than 0.01% for e .

We can turn the problem around and ask what dimensions would give better approximations. We keep the base fixed at 440 €, and vary the height.

- For π , the height needs to be 280.1126998 €. That is 5.9 cm more than 280 €.
- For φ , the height needs to be 279.8443229 €. That is 8.15 cm less than 280 €.
- For e , the height needs to be 280.0221571 €. That is 1.16 cm more than 280 €.

To do all three at once, we take the average, which is $\frac{280.1126998 + 279.8443229 + 280.0221571}{3} = 279.9930599$. That differs from 280 € by 0.006940082 €, which is 3.6338 mm, on a structure 146.6 m tall.

Another interesting way to get π is *twice base - height*, which is $880 - 280 = 600$ €. Then convert to metres, giving 100π metres.

* * *

The base size gives us 1000φ and $1000\sqrt{\pi}$, rounded, shown in Figure 10.

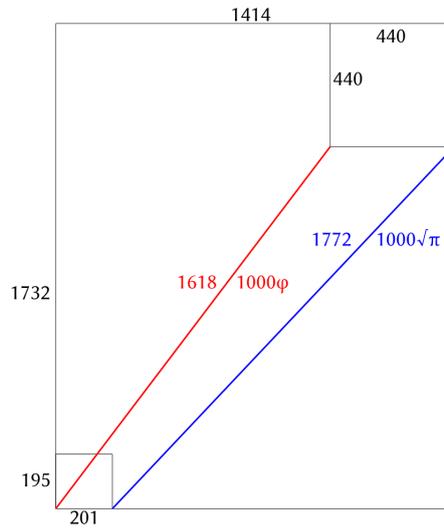


Figure 10: ϕ and $\sqrt{\pi}$

The classic approximation for the speed of light uses the circumcircle around Khufu, and the contained incircle. The red incircle has a diameter of 440 \mathcal{C} , while the blue surrounding circumcircle has a diameter of $440\sqrt{2}$ \mathcal{C} .

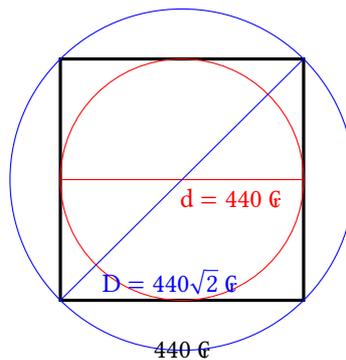


Figure 11: The circumcircle and incircle for Khufu

The circumference of the incircle is $\pi d = 440\pi$.
 The circumference of the circumcircle is $\pi D = 440\sqrt{2}\pi$.

The difference between them is

$$440\sqrt{2}\pi - 440\pi = 440\pi(\sqrt{2} - 1)$$

The question is, what are the values for π and $\sqrt{2}$?

We either use “full” values, in practice about 9 decimals, although modern calculators may work to 17 places internally, or a more pragmatic approach as in Table 2, using 3.1416 for π and 1.4142 for $\sqrt{2}$.

Using full values, the difference is 572.5677252 \mathcal{C} , while using rounded values, the difference is 572.5503168 \mathcal{C} . These values are close. We can compare them to the speed of light as shown in Table 3.

The earliest source I can find attributes this method to Jonathan Langdale [12].

I covered this method in more detail in *G1, c and C: Khufu’s pyramid, the speed of light, and the royal cubit, compared* [13]).

For a different, more accurate method using the base and Thoth’s Constant, see *Another reference to the speed of light by the Great Pyramid* [9].

	Full values	Rounded values
c in M€	572.5614191	572.5600802
Circles difference €	572.5677252	572.5503168
Difference to c	0.006306110	0.009763400
Scaled ($\times 10^6$)	6306.110	9763.400
Convert to m/s	3301.872	5112.116

Table 3: Circles differences

4.2 Khafre

Kepler famously said,

“Geometry has two great treasures: one is the theorem of Pythagoras, the other the division of a line into extreme and mean ratio. The first we may compare to a mass of gold, the second we may call a precious jewel..”

Johannes Kepler

Khufu is based on the golden ratio (“the division of a line into extreme and mean ratio”). The second biggest pyramid, next to Khufu, is based on the theorem of Pythagoras, using a scaled version of the simplest and best-known numbers. We have Kepler’s gold and the precious jewel next to each other at Giza.

Khafre’s proportions are the classic 3:4:5 right triangle, with a multiplier of 68.5. That gives us two back-to-back triangles of 205.5 : 274 : 342.5 €.

However, if we look at the entire base and height, then it is 3×137 by 2×137 . The number 137 is peculiar, as it is a common approximation for the inverse of the Fine Structure Constant (α), an important scientific constant.

If we accept that the reference to π as $\frac{22}{7}$ in Khufu is a hint to look for more precise values of π , then perhaps the same applies here, and we should look for more precise values of α or its inverse.

The difference in height between Khufu and Khafre is $280 - 274 = 6$ €. That is π metres.

Pythagorean 3:4:5 triangles provide an interesting approximation for e , as $\frac{3\sqrt{\text{hypotenuse}} - \sqrt{\text{long}}}{\sqrt{\text{short}}}$. In Khafre’s case, we can write

$$\frac{3\sqrt{342.5} - \sqrt{275}}{\sqrt{205.5}} = 2.718282808$$

Khafre, together with Khufu and Vega/Mercury, are the three legs of a Douglas triangle. The underlying Douglas triangle (*The Douglas Triangle, Prime Roots, and e* [14]) provides exactly the same formula, written in terms of φ , as $\frac{\varphi^2+5}{\varphi\sqrt{3}}$. See the paper for proof that $\frac{\varphi^2+5}{\varphi\sqrt{3}} = \frac{3\sqrt{5}-\sqrt{4}}{\sqrt{3}}$. In Giza terms, using the rounded values of the pyramid bases, it is

$$\frac{411 + (5 \times 157)}{440} = 2.718181818$$

The small difference comes from using the rounded pyramid bases.

No one can provide a simple reason why Khufu is 440 €, and Khafre is 411 €. A base of 411 € is hardly an “obvious” choice. I suggest that the Douglas Triangle, in addition to the reasons why Khufu is 440 €, provides a sane explanation for the base sizes, as well as hinting at the missing Mercury / Vega pyramid. This triangle suggests knowledge of φ , and may also have been used as the design paradigm for the unexplored Great Chamber in the Great Pyramid (*A preliminary analysis of the unexplored Great Chamber in the Great Pyramid* [15]).

4.3 Vega / Mercury

The now-gone sixth pyramid had dimensions of 157 € square by 75 € high. This length provides the short side for the 157 : 411 : 440 Douglas right-angled triangle linking the base sizes of the three square-

base pyramids, discussed above under Khufu. See *The Douglas Triangle, Khufu and Khafre* [4] for a fuller discussion.

157 is 50π , rounded.

The half-base is 78.5, then $\frac{78.5}{75} = 1.0466\dot{6} = \frac{3.14}{3}$.

That concludes the three square-base pyramids. We now look at the three rectangular-base pyramids.

5 The rectangular-base pyramids

Pyramids with rectangular bases need different approaches to see the design ideas. At Giza, lengths are not just lengths, but have purpose.

5.1 Menkaure

While writing *Zep Tepi Mathematics 101* [1], I came to the conclusion that Menkaure's base size was originally 201×195 ƒ. There is a fuller discussion about this in *The Consequences of Legon's Rectangle: The Rational Giza Design* [6].

Petrie (*The pyramids and temples of Gizeh*) [16], §81, estimated Menkaure's height at 2564 ± 15 inches, which is 124.38 ± 0.73 ƒ, or 123.65 to 125.11 ƒ. Petrie admits great difficulty in measuring the slope angle, and was working with the enlarged base, so it is a "best-guess". I previously took the height to be around 125 or 126 ƒ, and tried to make the case for a Kepler Triangle design like Khufu, but 124 ƒ seems to be the correct height. It gives the best correlation to Earth in the planetary correlation sequence.

We also have this relation, which is the best that can be done with whole-cubit dimensions.

$\frac{201 \times 195}{124} = 316.0887$, which is a good approximation for $100\sqrt{10}$.

$3.160887^2 = 9.9912$.

The "best fit" for $\sqrt{10}$ is 123.945 ƒ, a difference of 0.0545 ƒ or about 2.9 cm on a height of about 65 metres.

5.2 Arcturus / Mars

Norden's fourth pyramid had a base 149 by 151 ƒ, and was 114 ƒ high.

The perimeter is 600 ƒ, which is 100π metres.

Norden points out that the peak was different to other pyramids:

"As to the fourth pyramid, it is still one hundred feet less than the third. It is likewise without coating, closed, and resembles the others, but without any temple like the first. It has however one particular deserving remark; which is, that its summit is terminated by a single great stone, which seems to have served as a pedestal. Its summit is of a yellowish stone, and of the quality of that of Portland; and it is likewise the same kind of stone, that the other pyramids are built with. I shall speak elsewhere of its top, which terminates in a cube." (*Travels in Egypt and Nubia* [17]).

The existing mathematical papyri include example pyramid calculations where the height is not an integer. If these were not just academic exercises, then there may be pyramids that did not have integer heights.

In this case, if the height was 114.38 (an extra 20cm) then

$149 \times 151 \times 114.38 = 2573435.62$.

The cube root of that is

$\sqrt[3]{2573435.62} = 137.0369769$, which is 99.999295% correct for α^{-1} .

5.3 Thuban / Venus

That brings us to the main topic, Ramanujan and the Pyramid of Venus. The fifth pyramid is sadly missing, and the current Google Earth view shows some sort of construction taking place at its location.



Figure 12: Construction at celestial north pole. Image credit: Google Earth

5.3.1 π at Giza

Khufu's version of π as $\frac{22}{7}$ suggests we look for π elsewhere. We do not have to look far, as π is found all over Giza. The ratios in Figure 13 are all correct to 2 decimals.

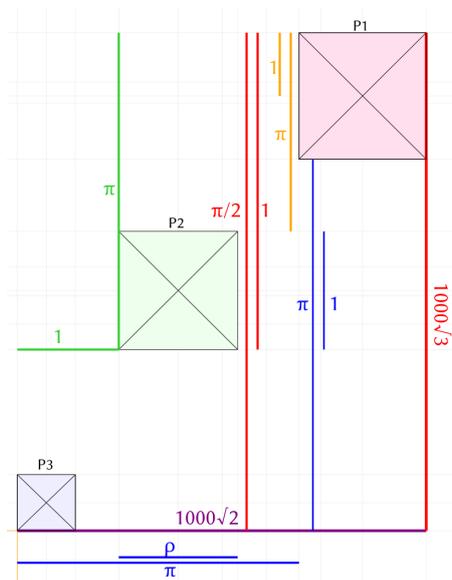


Figure 13: π ratios

The ratios in Figure 14 all round to 3.14.

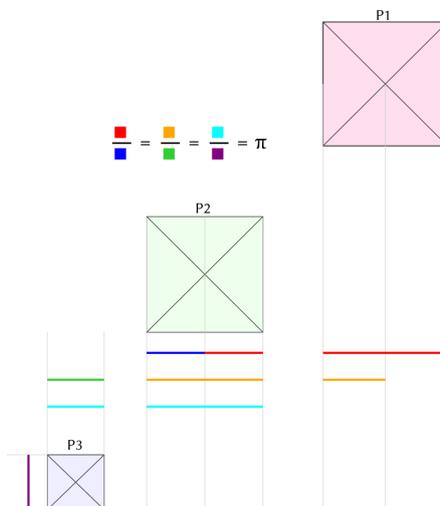


Figure 14: Further π ratios

Even Khufu’s incircle provides 3.14, as $\log_{10}440\pi = 3.14$, rounded to two decimals. See Figure 11.

We can get more digits by paying a visit to the King’s Chamber, where we find 3.142, shown in Figure 15.

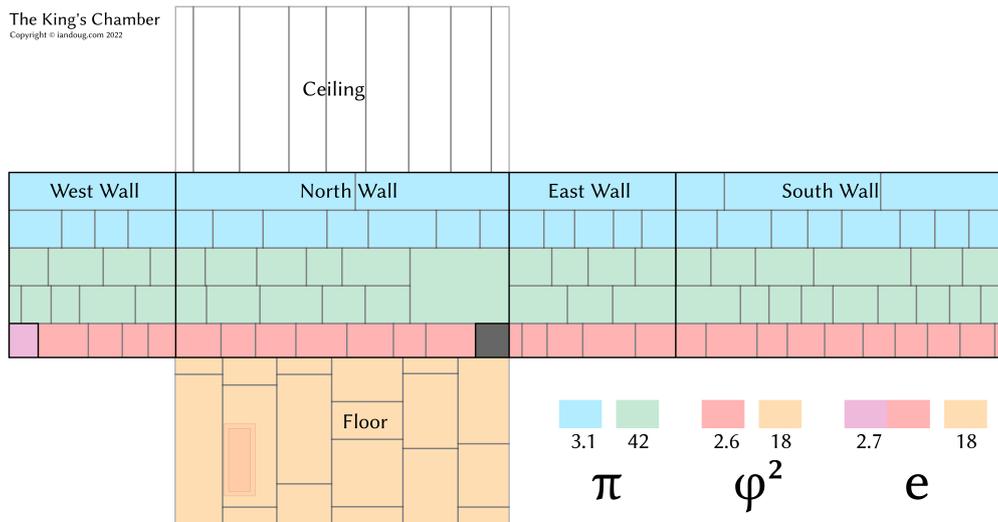


Figure 15: The three famous irrationals on the walls and floor of the King’s Chamber

Figure 16 shows 3.14159, the last digits needed to be swapped to make everything else work.

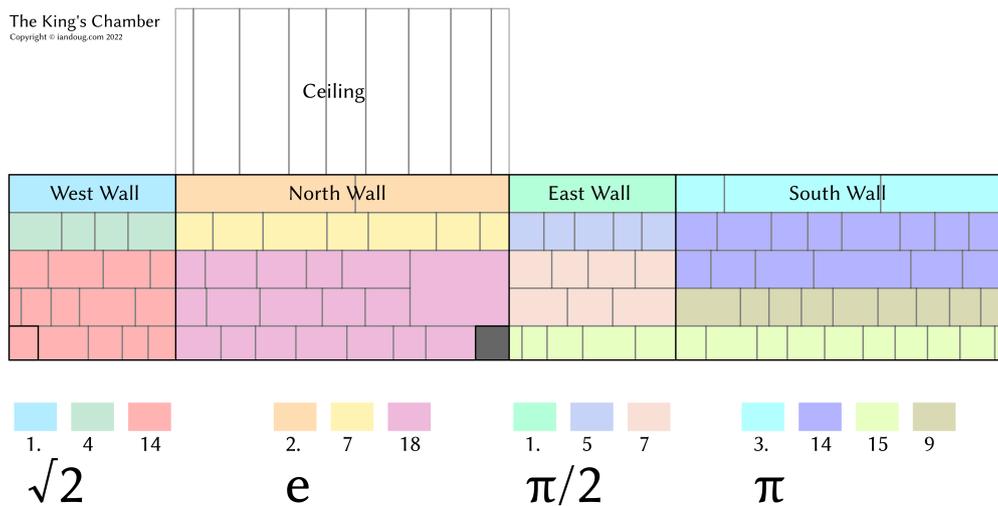


Figure 16: The main vertical patterns

For further examples, see *The Writing is on the Wall: The King’s Chamber Game* [18].

Using Egyptian-style unit fractions, we can get $\sqrt{31.416}$, which in 5.605 to three places.

Treat the number of blocks in each row as the divisor in a unit fraction, and sum them. See *$\sqrt{10\pi}$ on Khufu’s King’s Chamber Walls* [19] for more details and the progressive summation to 5.605, to three decimals.

This is illustrated in Figure 17.

West Wall	North Wall	East Wall	South Wall
$\frac{1}{1}$	$\frac{1}{2}$	$\frac{1}{1}$	$\frac{1}{3}$
$\frac{1}{4}$	$\frac{1}{7}$	$\frac{1}{5}$	$\frac{1}{8}$
$\frac{1}{4}$	$\frac{1}{10}$	$\frac{1}{4}$	$\frac{1}{6}$
$\frac{1}{5}$	$\frac{1}{7}$	$\frac{1}{3}$	$\frac{1}{9}$
$\frac{1}{5}$	$\frac{1}{7}$	$\frac{1}{5}$	$\frac{1}{10}$

Figure 17: Reading the rows as unit fractions.

There are only four good simple integer fraction approximations for π :

- $\frac{22}{7} = 3.142857143\dots$
- $\frac{333}{106} = 3.141509434\dots$
- $\frac{355}{113} = 3.14159292\dots$
- $\frac{377}{120} = 3.14166\dot{6}$

The first is done by Khufu, as discussed above. The second is done between Khufu, Khafre and Venus, as $\frac{666}{212}$. See Figure 3 for the locations and Figure 6 for the values.

The third and fourth are a little more complicated, and are illustrated in Figure 18, where the last is done as $\frac{754}{240}$.

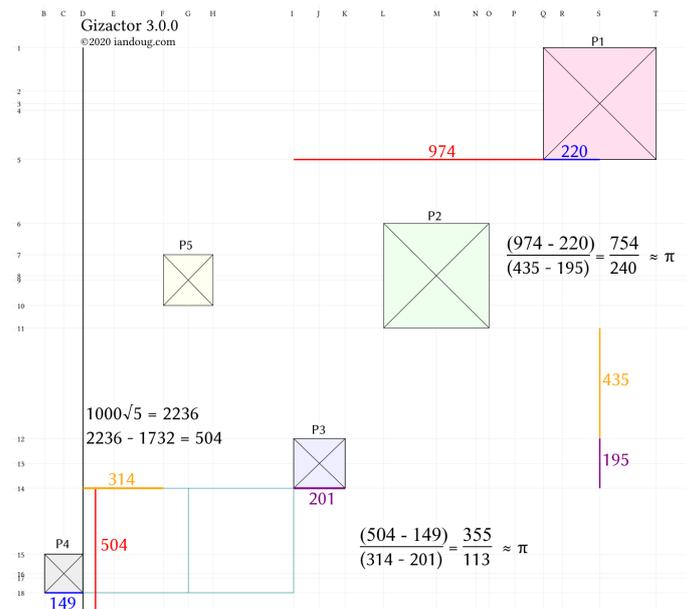


Figure 18: Integer approximations for π

5.3.2 Ramanujan's version

There is a better approximation for π , which Ramanujan says he received in a dream (*Ramanujan's Notebooks: Part IV* [20])(*Approximations of π* [21]). There are two occurrences at Giza of part of the formula, but not all the parts.

However, I had been advised that it did indeed exist, it was just a matter of finding it.

Many cultures have linked the planet Venus to a goddess of beauty, so it is appropriate that the pyramid of Venus also encapsulates a reference to arguably the most important (beautiful?) number in mathematics.

The number is well hidden... so it is an Easter Egg. A very precise and delicious Easter Egg.

I did the following calculation with the pyramid of Venus' dimensions:

$length \times breadth \times height = 193 \times 200 \times 119 = 4593400$.

Then I took the square root, and started laughing and shaking my head in astonishment. Then I looked at the answer properly, and laughed and shook my head even more.

The answer on my calculator was 2143.221874. Initially I only saw the 2143.22 part, which I recognised immediately.

The actual answer should be 2143.221873721897... but my calculator rounded it to 6 digits, which is fine.

So what does it mean, and why was I laughing?

We break it up into 4 parts: 2143 22 187 4. The decimal point is one separator, and 187 functions both as a separator and the author's signature.

The 187 is 100 times Thoth's Constant $\left(\frac{\pi\phi}{e}\right)$ which has the three most important numbers in mathematics, at least as far as they were concerned.

That leaves us with 2143 22 4 ... which are exactly the numbers from Ramanujan's approximation for π :

$$\sqrt[4]{\frac{2143}{22}} = 3.14159265258264612520...$$

which is 99.9999996794...% accurate for π .

6 Conclusion

The three existing pyramids have interesting mathematical properties. Egyptologists will say these are accidental, but to me it feels like they ignore the idea that it could be deliberate, because that forces a re-examination of the time line. That leads to conflict with any religious beliefs related to the 4004 BCE date of creation proposed by Bishop Usher [22] and still accepted by some branches of the Abrahamic religions.

My suggestions for the dimensions of the three missing pyramids, and Menkaure's original measurements, are impossible to prove, unless we find the site blueprint somewhere.

However, I arrived at those numbers by "following the mathematics", and many things "just work" with those dimensions. So either I was extremely lucky in creating a fictitious design, or I (with considerable guidance) have managed to reverse-engineer at least the basics of the original design.

The more I delve into Giza, the more impressed I become with the mind(s) that created it. A simple example is the walls in the King's Chamber... (see §4.1) where they have π twice, and even $\sqrt{31.416}$ at the same time, using a totally different way of handling numbers less than 1. Is it really possible that that was all "pure co-incidence", which is the only other explanation?

Venus' dimensions were calculated by a "cascading search" program, that checked various base sizes for one that would satisfy certain conditions:

1. The general location aligned with the celestial north pole on the star map.
2. The base would fit inside Thuban's orbit. Thuban was the pole star at the time.
3. The eastern edge would make the 1 : π ratio between Venus, Khafre, and Khufu work. See Figure 3.
4. Similarly, the northern and southern edges would make the 1 : ϕ^2 ratio between Khufu, Venus, and Mars work. See Figure 3.
5. Lastly, the location was optimised to make the "Squaring the Circle" diagram work. See Figure 8.

The exact location and base dimensions need to work together. For example, changing the width moves the centre, in order to maintain the 1 : π ratio. That in turn breaks the "Squaring the Circle" construction.

Finally, the height was calculated by making the ratio of volume to Khufu's volume the same as the ratio of Venus' diameter to Jupiter, to the nearest cubit.

So the size was determined by different mathematical processes, and aiming at a particular square root value was not even considered. I didn't even know it existed at the time.

We can ask what other similar pyramid dimensions would give the same three numbers (2143, 22, 4).

I wrote a program that checked all base sizes from 180 to 250, and heights from 50 to 200. Results were rounded to 6 digits, and checked for 2143 and 22.

There are only six combinations, but it's actually only three with the X and Y dimensions swapped.

They are:

- 193 200 119, product 4593400, root 2143.221874, Khufu% 8.47
- 198 209 111, product 4593402, root 2143.22234, Khufu% 8.47
- 211 213 115, product 5168445, root 2273.42143, Khufu% 9.53

Of these, only the first two match the volume correctly. The second would be slightly too large for Thuban's orbit, and break the five mathematical requirements listed above. That leaves only the current pyramid to satisfy all the requirements.

Is this a remarkable black swan event, or the result of skilful and ingenious design? The part that impresses me is how 187 is incorporated, both as a separator and "Designer's signature".

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