Entropy Changes Due to Impulses?

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Classical mechanics introduces both the concepts of work and impulse i.e.: Work= F dot dr and Impulse = Fdt= dp. As we have noted previously, there seems to be a tendency in classical mechanics to focus only on the work definition which is linked to energy and time. Thus one solves Newton's equation to find x(t) or writes a conservation of energy equation. This approach seems to be carried over into thermodynamics as the first law is written as: dE=TdS - dW.

We have argued in (1) that in the case of quantum mechanics, both the impulse-momentum picture and work-energy-time picture are needed. (In particular, the one dimensional description of a photon reflecting and refracting cannot be solved without the exp(ipx) impulse scenario.) Here we suggest that the notion of entropy should also be associated with impulses. For example, consider the classical case of a particle moving in a circle with a central inward force. The kinetic energy does not change and work is zero, but the momentum vector changes and so there is an impulse. Furthermore, information changes because one needs to continuously know the momentum coordinates. We suggest that this is an example of impulse being associated with entropy. A second example we suggest is a classical particle bouncing elastically off a wall. (One may argue, however, that these notions of entropy are hand-wavy.)

In the quantum world, however, we suggest that impulse plays a major role in creating entropy which is clearly defined. We suggest the example of a particle or photon with a fixed momentum passing through a tiny slit, or n-slits. In each case, one obtains a different pattern on a screen which leads to a different entropy expression. The photon or particle does not change the magnitude of its momentum i.e. there is no energy change or work, but there are impulses which are different in the various n-slit scenarios and lead to different entropies. Thus we suggest that impulses and information associated with them may lead to an increase in entropy. This suggests that the first law of thermodynamics is not a complete description of changes in entropy as it bypasses the idea of impulses and only considers work/energy changes.

Entropy and Probability

Shannon's entropy (and other entropies) are described in terms of probabilities. For example, Shannon's entropy is:

S (Shannon) = - Sum over i P(i) ln(P(i))   ((1))

A set of probabilities may be associated with the flipping of a coin or the tossing of a die etc. In classical mechanics, one has determinism and so originally there was no notion of entropy. Classical mechanics defines both work and impulse:

dW = change in work = F dot dr = change in energy ((2a)) and Impulse = dp = F dt ((2b))

In practice, however, the work scenario, which is linked with energy and hence velocity and changes in time, prevails as Newton's second law (which is equivalent to a conservation of energy equation) yields x(t). Lagrangian and Hamiltonian approaches lead to the same results. Thus, impulse does not seem to play a central role in calculations.

In (1) we argued that in quantum mechanics, one must consider both the idea of impulses and the energy-work-time picture. exp(ipx) we suggested, is linked with impulses and a wavefunction is W(x)=Sum over p a(p)exp(ipx). Thus even in a bound system which uses conservation of energy i.e. KE(x)+V(x) = E, impulses must be included i.e. one has the time-independent Schrodinger equation:

-1/2m d/dx dW/dx / W + V(x) = En ((3))

One cannot simply focus on the work-energy-time picture.

Entropy Changes Due to Impulses

We suggest that impulses may be responsible for changes in information and hence entropy already at the classical level. We consider two examples. The first is that of a particle moving at constant speed in a circle on a horizontal surface. There is no change in energy (kinetic energy) or work done (force is perpendicular to motion), but there is a force and a change in momentum (vector) hence there is an impulse. We suggest that information about the momentum vector changes with each impulse and that this might be equivalent to a change in entropy. Another example is that of a particle which elastically bounces off of a wall. Again there is no net work or change in energy. The work is: -F (dx) + (-F)(-dx) =0. There is a change in information i.e. the particle is originally moving to the right and is now moving to the left. We argue that a change in information may be related to a change in entropy. In these two examples, we do not precisely define entropy, but rather suggest that it is related to changes in information. Thus the above argument is somewhat hand-wavy.

In order to be more quantitative, one could perhaps assign a probability of p|cos(wt)| to the x component of the circular motion and treat this as a probability. The C|cos(wt)| would be a normalized probability in time and one could create a Shannon's entropy value. We have suggested in previous notes that both exp(-iEt) and exp(ipx) for a quantum free particle might have such associated entropies. In the quantum case there is uncertainty in the cos(wt) and cos(px) t and x values even though the particle moves forward in time and space in intervals of hbar/E and hbar/p. The classical case on the other hand is deterministic. If one were to assume that time increased in jumps of the period and that one could not follow time within a period, then |cos(wt)| might represent how much information of the total p one knows from just the x measurement i.e. p|cos(wt)|/p. For a die or coin, probability is: amount of information/total information. In each case, amount of information is 1, but for a coin total information is 2 and for a die 6.

We nevertheless suggest that even though energy and the magnitude of momentum are constant and work is zero, one should somehow create a measure which shows that two pieces of information, the x and y momentum components of the particle change in time.

There is, however, a precise example linking impulse changes to changes in entropy which occurs in quantum mechanics.

N-Slit Patterns and Entropy

A set of quantum particles or photons with momentum p may entropy an n-slit system and create different diffraction/interference patterns on a screen. (The slit widths and separations are of the order of the wavelength p=hbar/wavelength.)
The quantum impulse probability exp(ipx) is used and added for each path through a slit i.e. there is interference and one calculates a W(x). Finally one evaluates W*(x)W(x) to find the intensity profile and this value is used as a probability in Shannon's entropy. Thus entropy is defined clearly and is different for the various n-slit examples, Furthermore, the magnitude of p (momentum) does not change. No work is done by the n-slit apparatus, but impulses are delivered. In particular, the different n-slits have different impulse information sets associated with them and it is these which we argue lead to the final diffraction/interference patterns observed. Thus impulse (without work and the first law of thermodynamics) may lead to changes in entropy.

Extension of the First Law of Thermodynamics

Entropy originally appeared in the first law of thermodynamics, but this is an equation which includes work and energy (i.e. their changes) and not impulses. We suggest that based on what seems to be an accepted idea that n-slits create different entropies not based on work and energy, but on impulses, the notion of entropy changes must go beyond the idea of the first law of thermodynamics.

Conclusion

Entropy first appeared, it seems, in the first law of thermodynamics which is based on changes of energy and work. Work F dot dr, however, has a parallel in classical mechancis, namely impulse = dp = Fdt, although the latter seems to be the main focus in calculations. I.e. calculations yielding x(t) i.e. Newton's second law which is equivalent to an energy conservation statement seems to be the main focus. We have argued before that quantum mechanics makes use of both the impulse and energy-work-time picture i.e. one cannot simply discard the impulse scenario. Here we argue that impulses associated with n-slit interference/diffraction create entropy patterns which are not associated with changes in energy or work. As a result, we suggest the first law of thermodynamics needs to be extended in order that one has a statement that entropy may change due to impulses without changes in energy or work.

References

1. Ruggeri, Francesco R. Continuity Equation as One Part of Work-Impulse Dichotomy in Quantum Mechanics (preprint, zenodo, 2023)