Growth model

https://github.com/dudung/sk5003-02-2022-2

Sparisoma Viridi
Master Program in Computational Science, Nuclear Physics and Biophysics Research Division, Department of Physics, Faculty of Mathematics and Natural Sciences, Institut Teknologi Bandung, Bandung 40132, Indonesia

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Silakan berdiskusi untuk kuliah hari ini di
https://github.com/dudung/sk5003-02-2022-2/issues/10

Kerangka

SAP dan referensi

Minggu 9-10

Minggu	Topik	Subtopik	Capaian Belajar
9	Model komputasi fundamental dengan Python	Model komputasi dengan pertum-buhan aritmetik dan kuadratik	Kemampuan untuk membuat model komputasi dengan pertumbuhan aritmetik dan kuadratik
10	Model komputasi fundamental dengan Python	Model komputasi dengan pertum-buhan geometrik dan polinomial	Kemampuan untuk membuat model komputasi dengan pertumbuhan geometrik dan polynomial

Referensi utama

- Jose M. Garrido, "Introduction to Computational Models with Python", Routledge, 1st edition, 2020,

  url https://isbnsearch.org/isbn/9780367575533.

R1

C12

- Introduction
- Mathematical modeling
- Difference equations
- Functional equations
- Arithmetic growth

C13

- Introduction
- Differences of data
- Difference equations
- Functional equtions
- Quadratic growth

R1

C14

- Introduction
- Basic concepts
- Difference equations
- Functional equations
- Geometric growth

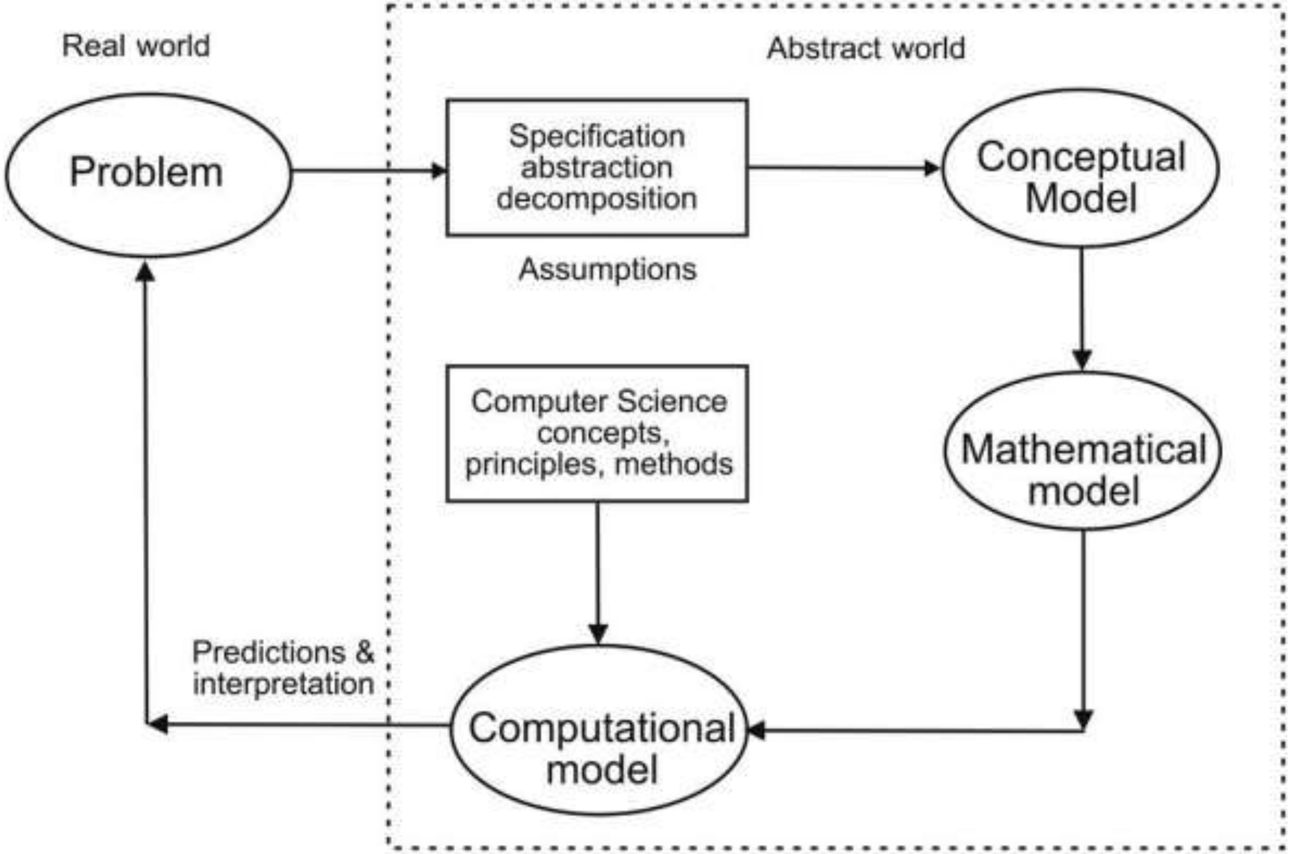
15

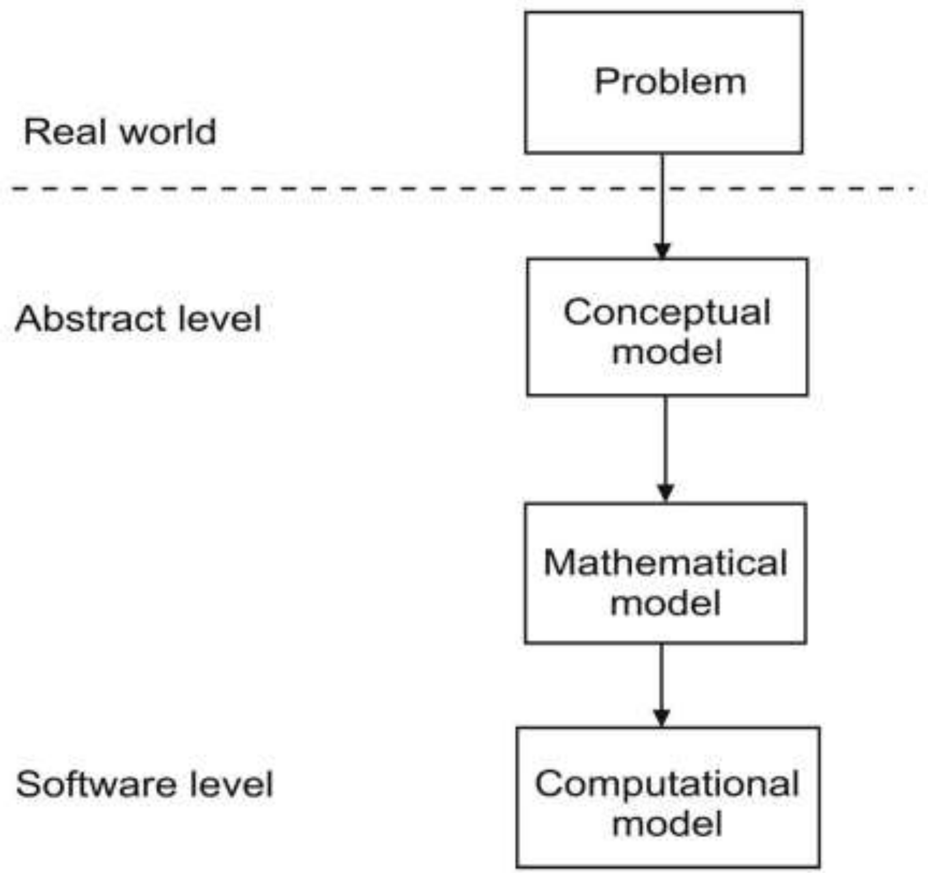
- Introduction
- Polynomial functions
- Numpy array, linspace
- Numpy polynomial
- Numpy polyroots

Model

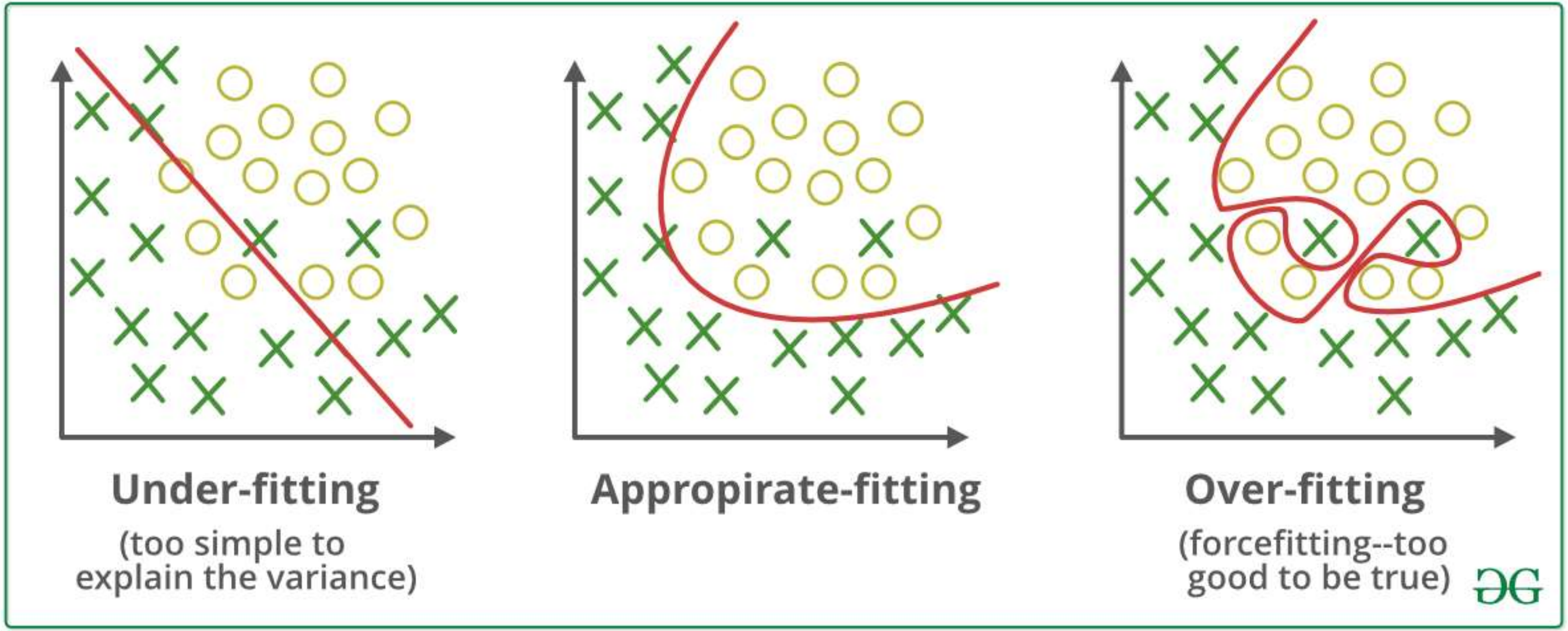
Alur

- Temukan suatu problem dalam dunia nyata
- Definisikan model matema-tis untuk masalah tersebut
- Formulasikan dalam varia-bel dan persamaan
- Rumuskan model komputasi
- Pilih dan gunakan satu atau beberapa teknik komputasi
- Implementasikan dalam bentuk program komputer
- Lakukan perhitungan
- Analisa hasil yang diperoleh
- Validasi dan ubah salah satu dari langkah sebelumnya

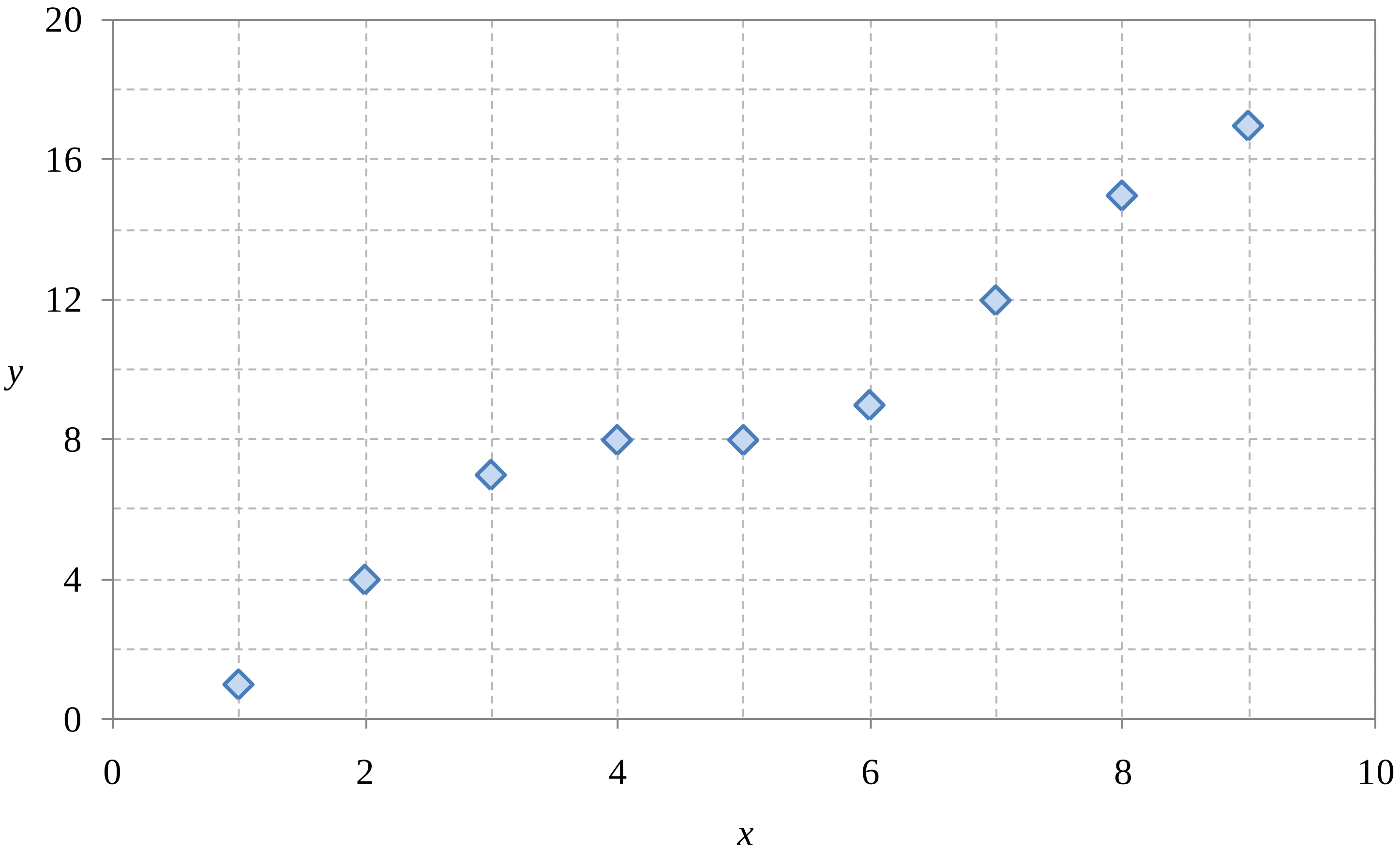

Real world
Abstract world
Problem
Specification
abstraction
decomposition
Assumptions
Conceptual
Model
Computer Science
concepts,
principles, methods
Mathematical
model
Predictions &
interpretation
Computational
model

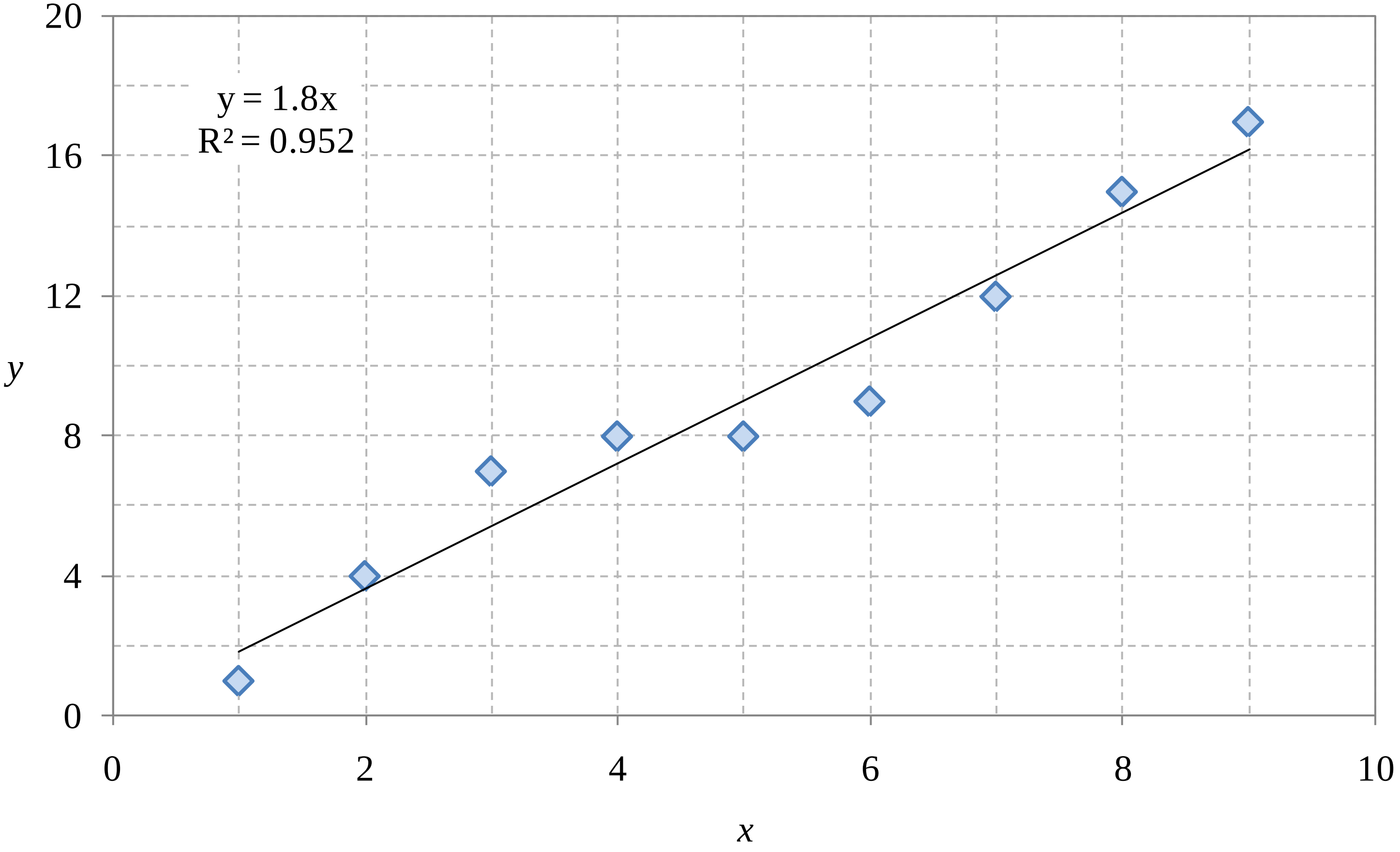

Problem
Real world
Abstract level
Conceptual model
Mathematical model
Software level
Computational model

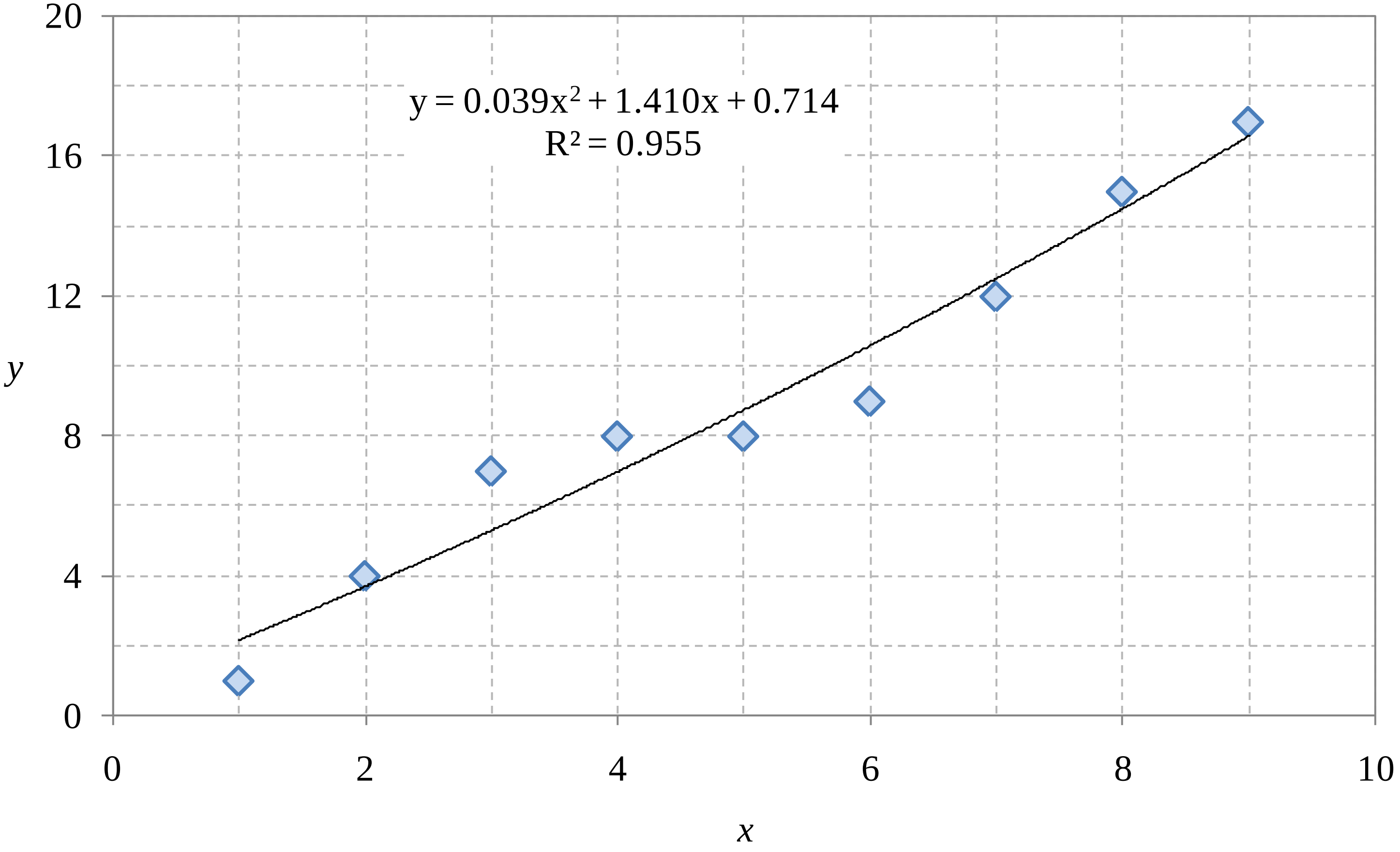
Fitting dengan kurva

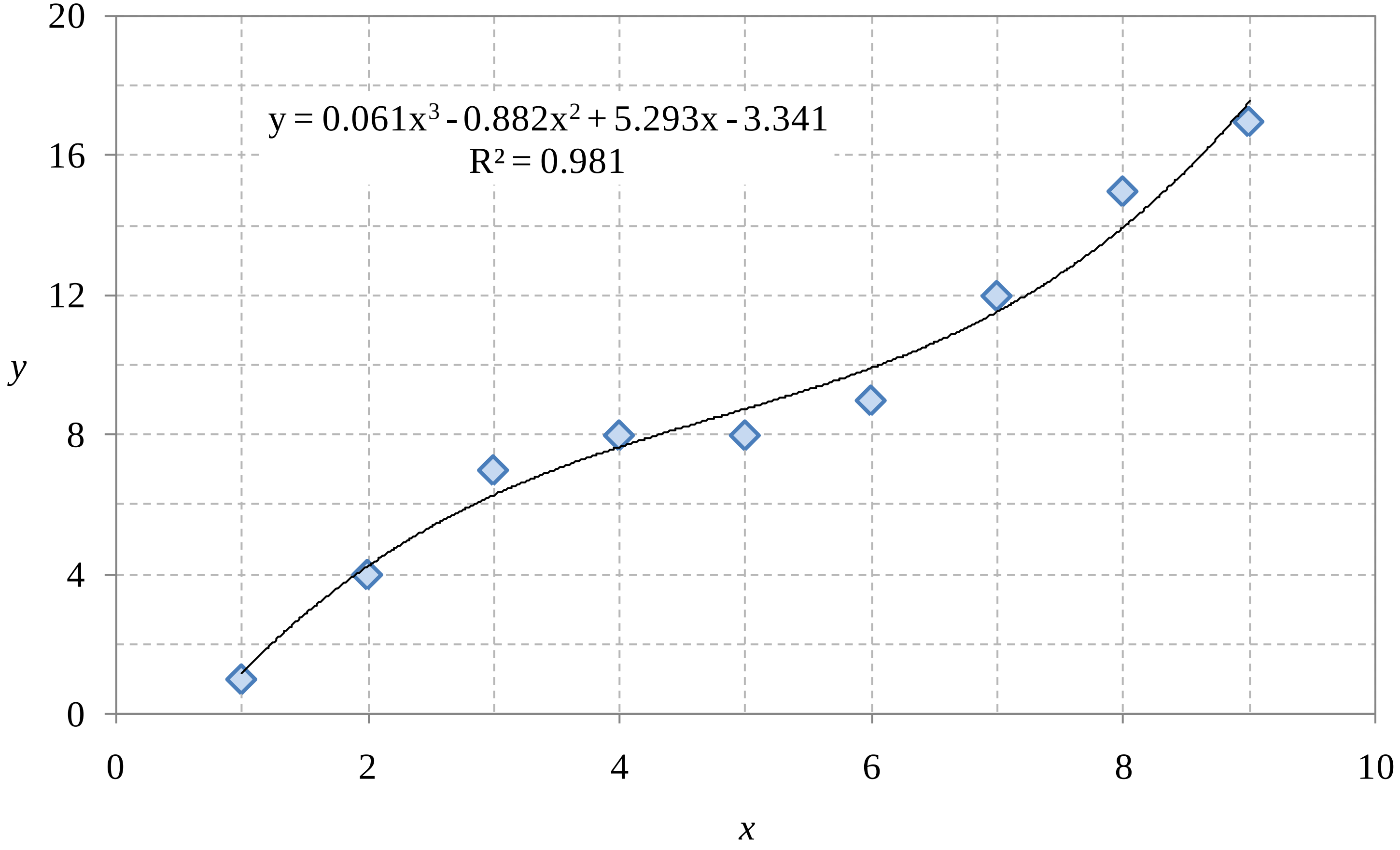


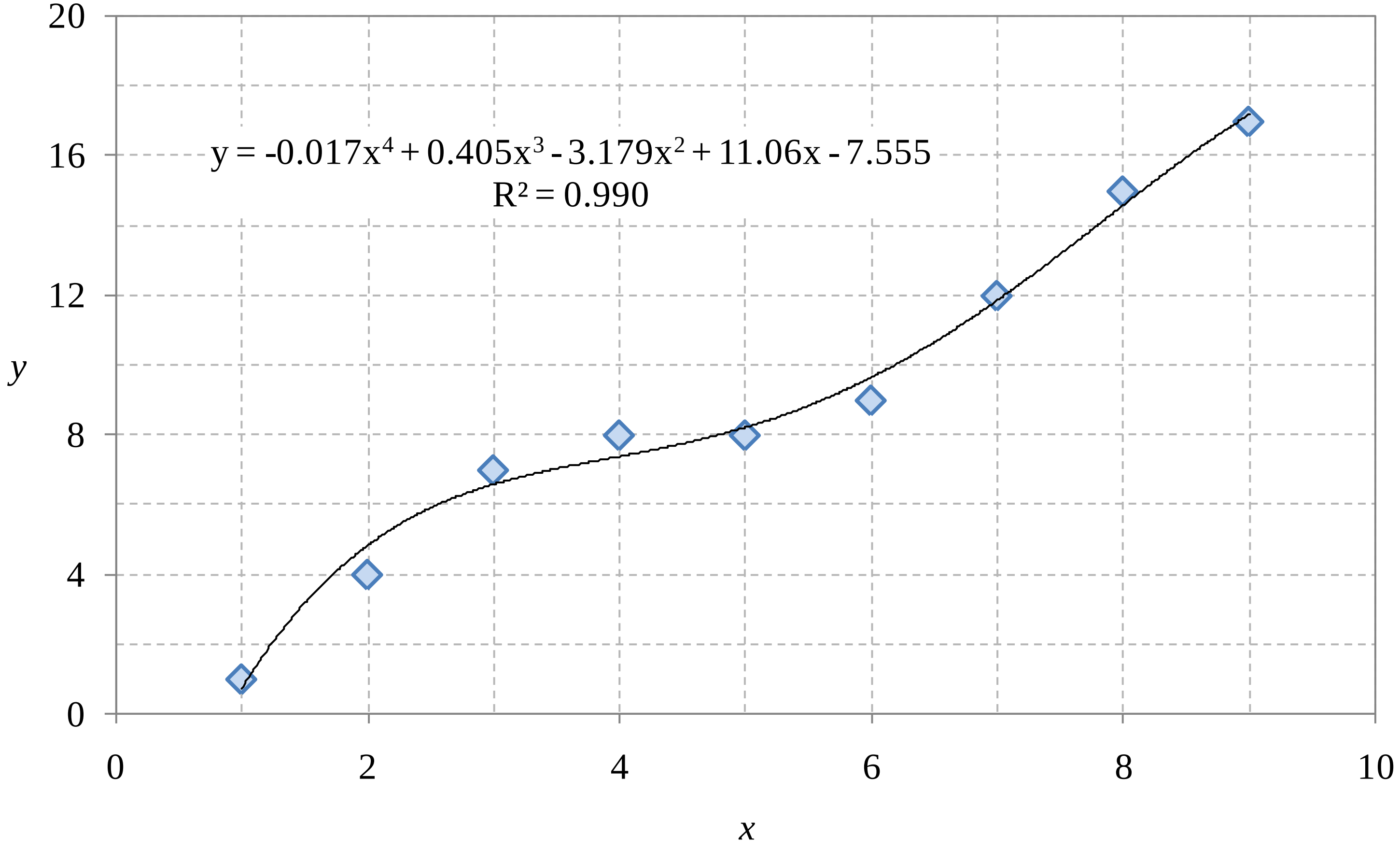
dewangNautiyal, “ML | Underfitting and Overfitting”, GeeksforGeeks, 20 Feb 2023, url https://www.geeksforgeeks.org/underfitting-and-overfitting-in-machine-learning/ [20230512].


20
16
12
8
4
0
y
0
2
4
6
8
10
x

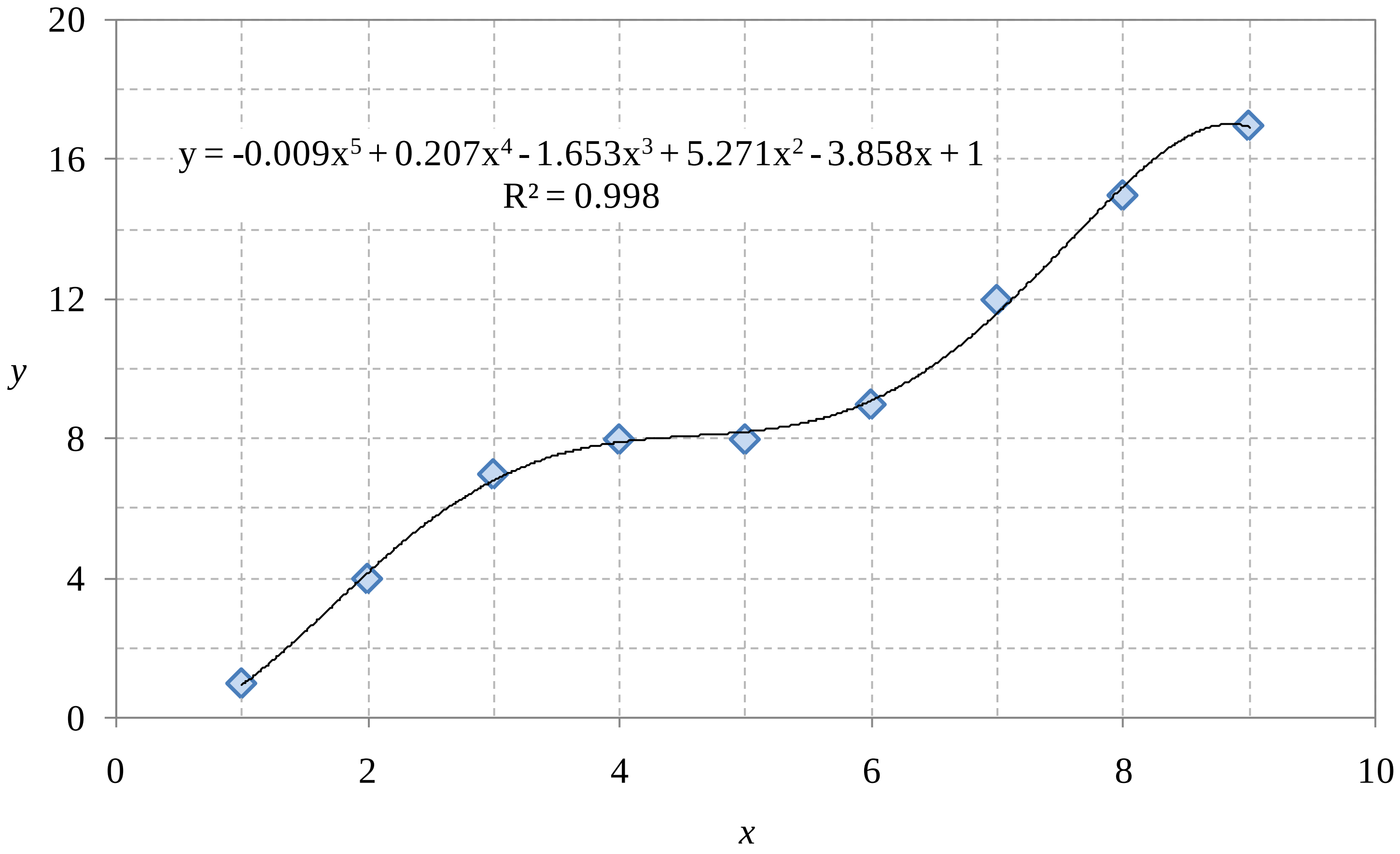

y = 1.8x
R² = 0.952
20
16
12
8
4
0
y
0
2
4
6
8
10
x


y = 0.039x² + 1.410x + 0.714
R² = 0.955
20
16
12
8
4
0
y
0
2
4
6
8
10
x

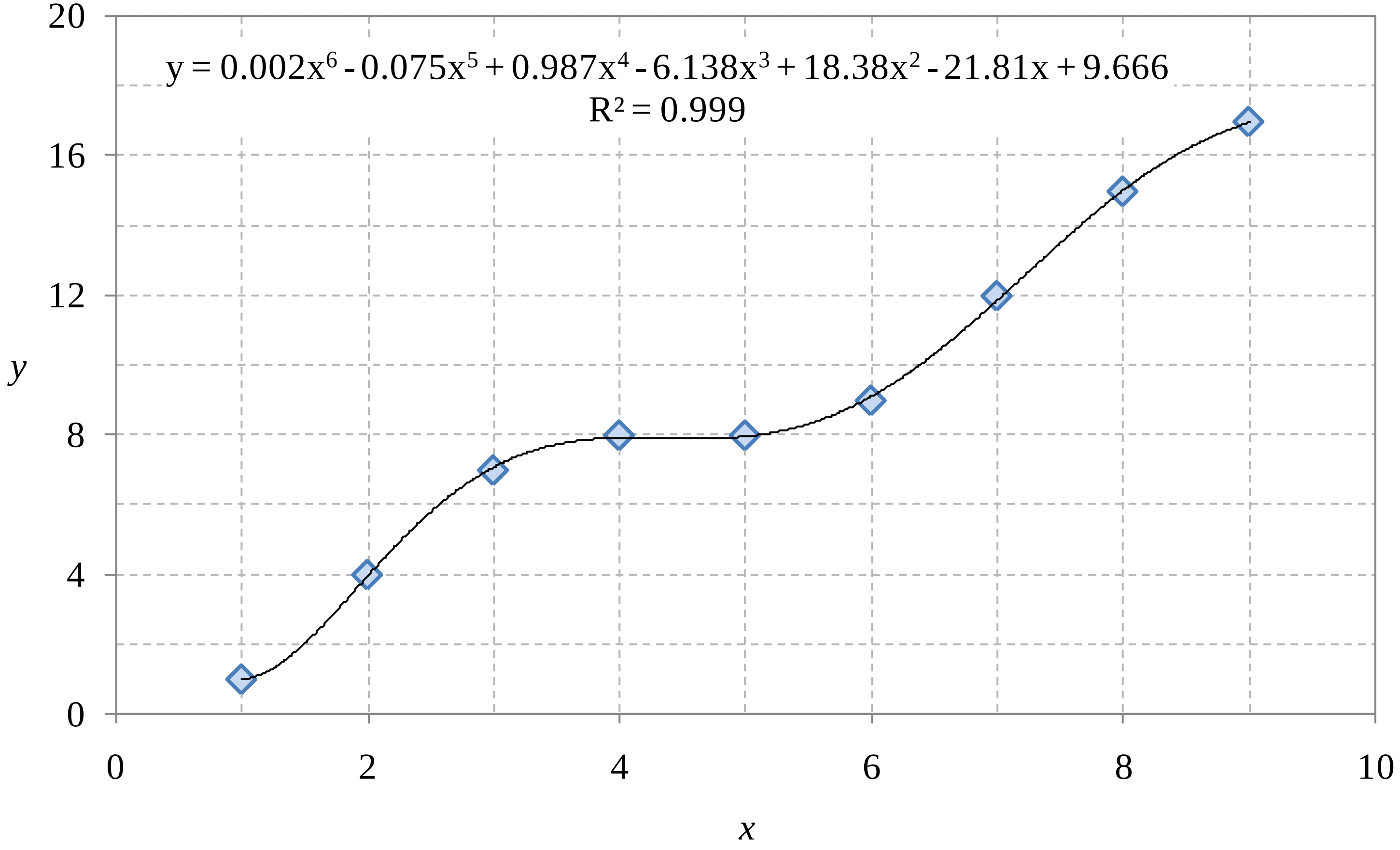

y = 0.061x³ - 0.882x² + 5.293x - 3.341
R² = 0.981
20
16
12
8
4
0
y
0
2
4
6
8
10
x

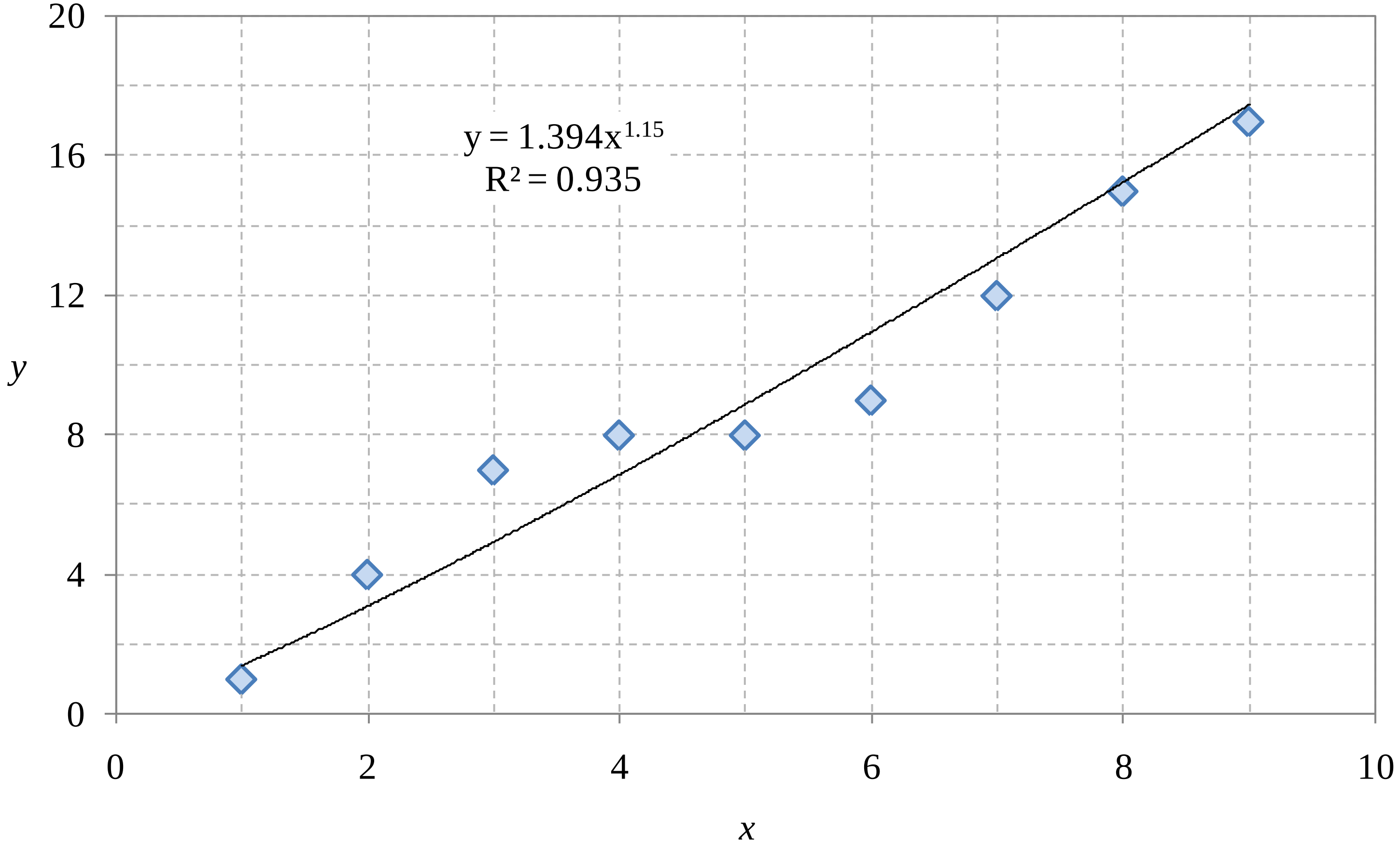


y = -0.017x4 + 0.405x3 - 3.179x2 + 11.06x - 7.555
R² = 0.990
20
16
12
8
4
0
y
0
2
4
6
8
10
x



y = -0.009x5 + 0.207x4 - 1.653x3 + 5.271x2 - 3.858x + 1
R² = 0.998
20
16
12
8
4
0
y
0
2
4
6
8
10
x

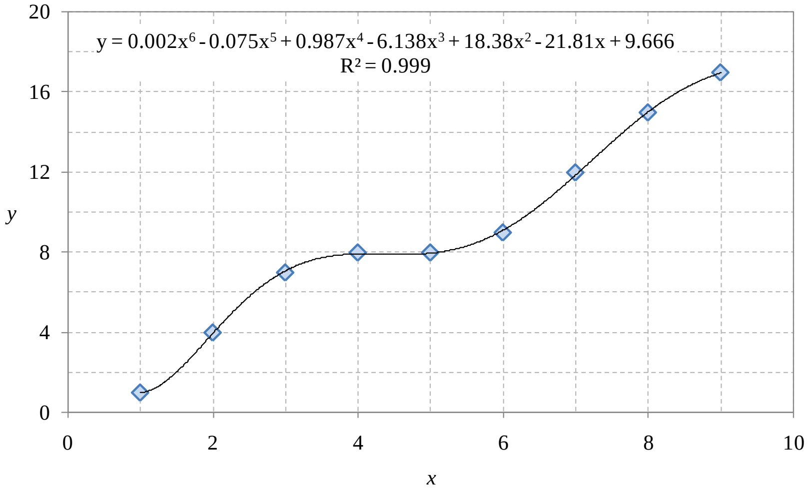

y = 0.002x6 - 0.075x5 + 0.987x4 - 6.138x3 + 18.38x2 - 21.81x + 9.666
R² = 0.999
20
16
12
8
4
0
y
0
2
4
6
8
10
x



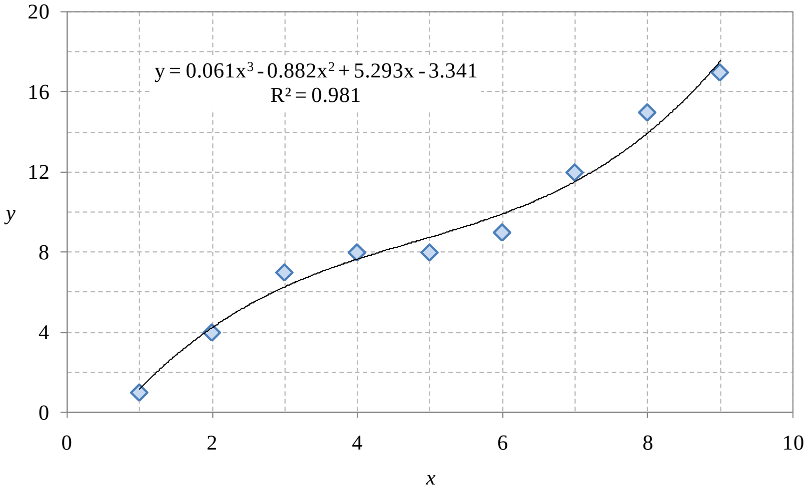
y = 1.394x^1.15
R² = 0.935
y
x
0
4
8
12
16
20
0
2
4
6
8
10

Model diperlukan untuk membatasi

Tanpa model (R = 0.999)



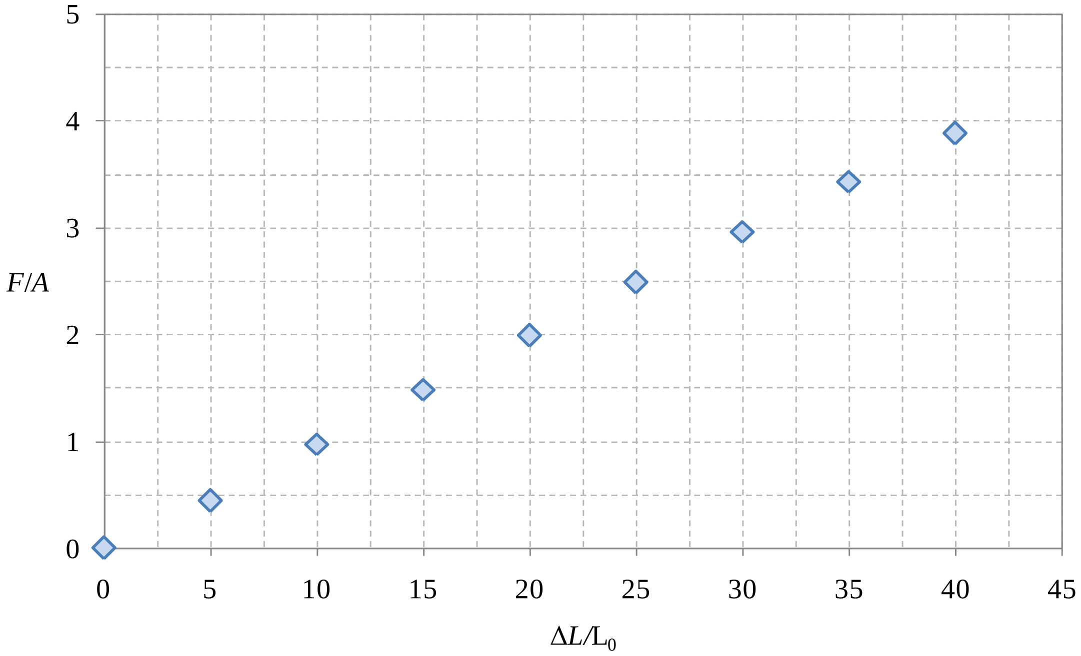
Model kubik (R = 0.981)



Contoh: Modulus tarik

Data

- Data asli seperti di sebelah kanan
- Data dimodifikasi agar terlihat lebih tidak linier
- Model mengharus-kan fungsi linier



-, "Use The Experimental Data To Calculate The Young's Modulus Of The Beam", Chegg, url https://www.chegg.com/homework-help/questions-and-answers/q69322020 [20230512].

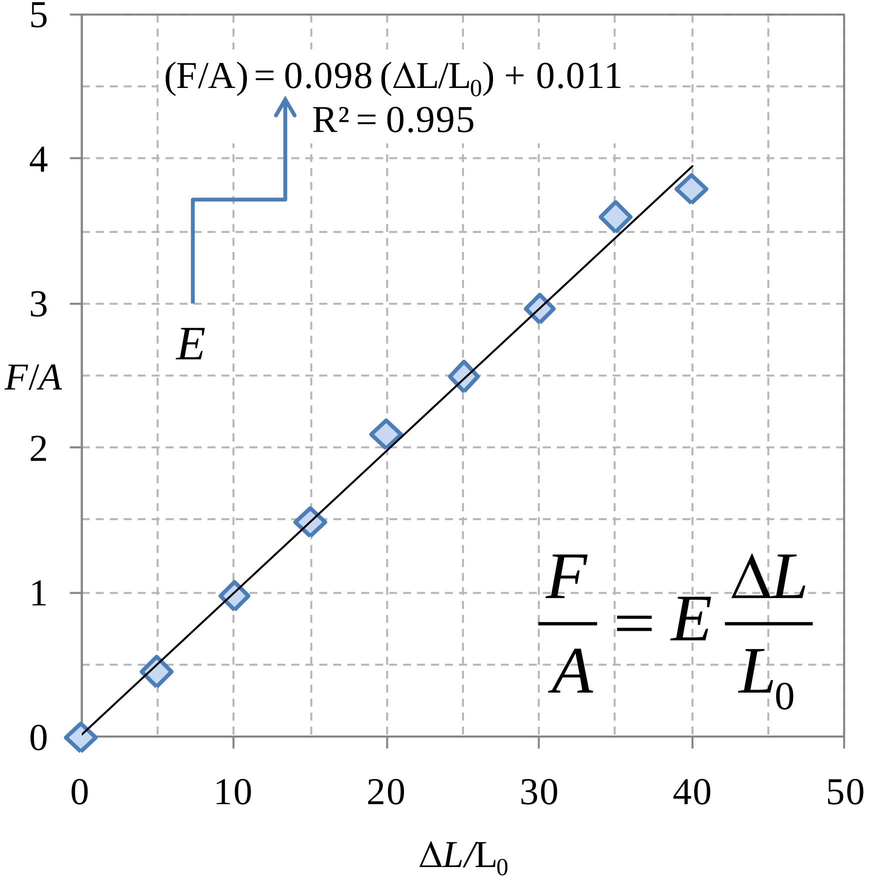
Fitting

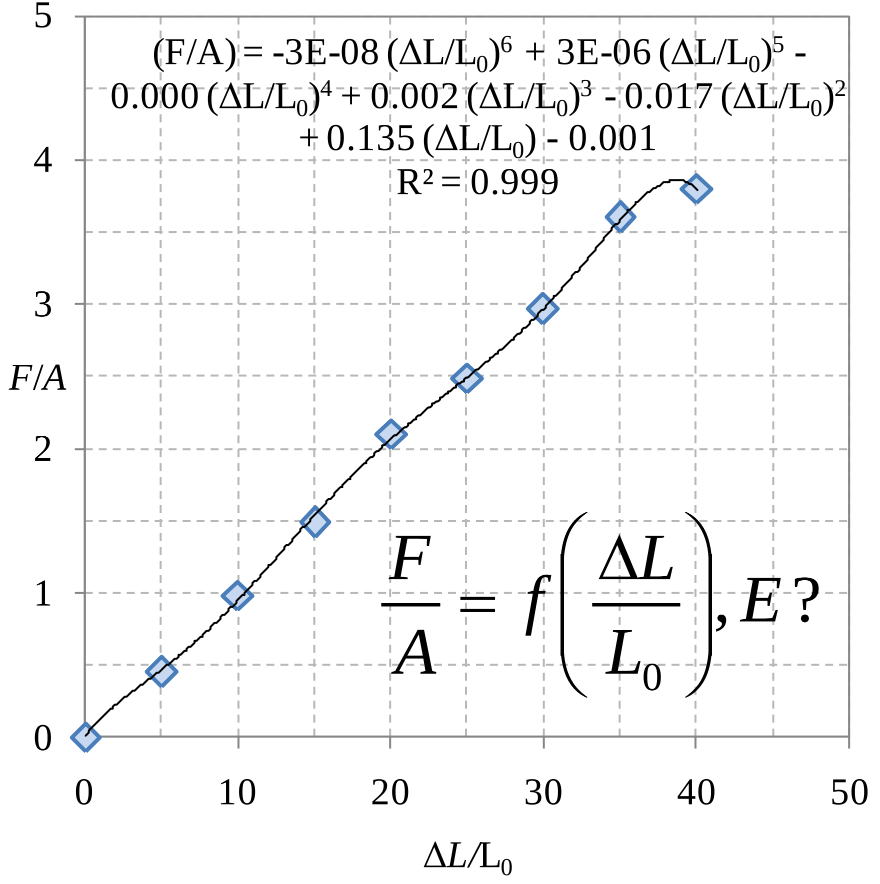
- Digunakan fungsi linier agar modulus tarik (modulus Young) E dapat diperoleh

$$\frac{F}{A} = E \frac{\Delta L}{L_0}$$

- Digunakan polinomial dengan derajat tertinggi yang tersedia (tidak jelas E terdapat pada parameter mana)

$$\frac{F}{A} = f\left(\frac{\Delta L}{L_0}\right), E\,?$$


(F/A) = 0.098 (ΔL/L0) + 0.011
R² = 0.995
E
F/A
ΔL/L0
$\frac{F}{A} = E\frac{\Delta L}{L_0}$
0
1
2
3
4
5
0
10
20
30
40
50


(F/A) = -3E-08 (ΔL/L0)^6 + 3E-06 (ΔL/L0)^5 - 0.000 (ΔL/L0)^4 + 0.002 (ΔL/L0)^3 - 0.017 (ΔL/L0)^2 + 0.135 (ΔL/L0) - 0.001
R² = 0.999
F/A
ΔL/L0
$\frac{F}{A} = f\left(\frac{\Delta L}{L_0}\right), E\,?$
0
1
2
3
4
5
0
10
20
30
40
50

Hasil

- Fungsi linier memberikan nilai $E = 0.098$

$$\frac{F}{A} = E \frac{\Delta L}{L_0}$$

- Fungsi polinomial tidak jelas suku yang memberikan nilai E
- Koefisien pada suku linier memberikan $E = 0.135$

$$\frac{F}{A} = f\left(\frac{\Delta L}{L_0}\right), E\,?$$

Pengertian lain model

Kutipan

- “All models are wrong but some are useful.” (George Box, 1976) dalam suatu makalah yang diterbitkan oleh Journal of the American Statistical Association.

George E. P. Box, “Science and Statistics”, Journal of the American Statistical Association, Vol. 71, No. 356. (Dec., 1976), pp. 791-799, url https://doi.org/10.1080/01621459.1976.10480949.

What is model?

- Dalam dunia fisis model secara umum model merupakan simplifikasi sesuatu dalam dunia nyata yang masih membawa esensi sesuatu yang dimodelkan tersebut.
- Maket suatu gedung menggambarkan struktur bangunan tersebut walaupun lebih kecil dan ringan dibandingkan obyek sebenarnya.
- Model suatu sel jauh lebih besar dari ukuran sel sebenarnya.

Russell A. Poldrack, “Chapter 5 Fitting models to data”, in Statistical Thinking for the 21st Century, 22 Dec 2022, url https://statsthinking21.github.io/statsthinking21-core-site/fitting-models.html [20230512].

Model statistik

- Dalam statistik berlaku yang sama, model menyediakan deskripsi terpadatkan yang mirip, akan tetapi lebih ke arah data dibandingkan dengan struktur fisis.
- Model statistik secara umum lebih sederhana dari data yang dideskripsikan.
- Model tersebut bertujuan untuk menangkap struktur data sesederhana mungkin.

Struktur dasar

- Suatu model statistik memiliki struktur dasar

  $$d = m + e$$

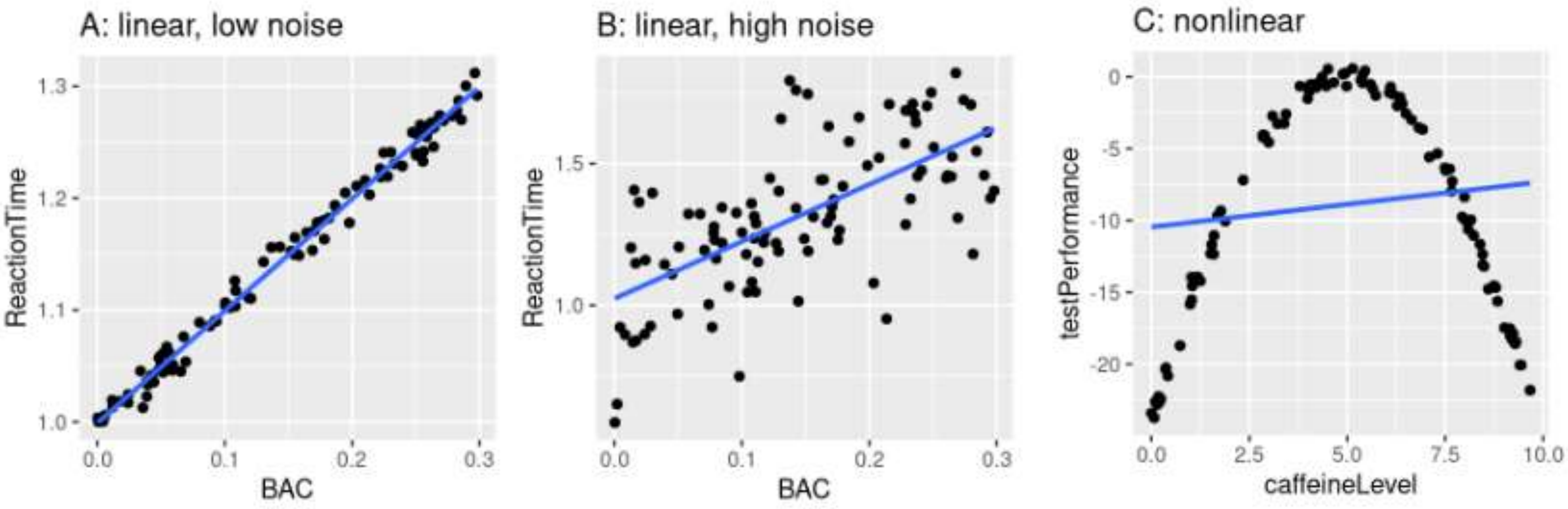
  dengan data d, model m, dan error e
- Prediksi oleh model untuk data ke i adalah

  $$\hat{d}_i = m_i$$
- Error menjadi

  $$e_i = d_i - \hat{d}_i$$

$\hat{d}_i$ bukan data sebenarnya

Contoh model linier



- Model memiliki makna pada setiap parameternya, bukan hanya sekedar fitting data dengan suatu fungsi

Model pertumbuhan

Types of methods

Graphical methods apply visualization of the data to help understand the data. Various types of graphs can be used; the most common one is the line graph.

Numerical methods directly manipulate the data of the problem to compute various quantities of interest, such as the average change of the population size in a year.

Analytical methods use various forms of relations and equations to allow computation of the various quantities of interest.

Difference equations

- Penjumlahan

$$x_{n+1} = x_n + f\left(x_n, x_{n+1}\right)$$

- Perkalian

$$x_{n+1} = x_n \cdot f\left(x_n, x_{n+1}\right)$$

Functional equations

- Fungsi dari sequence ke n

$$x_n = x_1 + f(n)$$

atau

$$x_n = x_1 \cdot f(n)$$

Arithmetic growth

Selisih antar data

i	x	Δx
1	3	2
2	5	2
3	7	2
4	9	2
5	11	2
6	13	2
7	15	

$$x_{i+1} = x_i + \Delta x$$

$$\begin{aligned} x_2 &= x_1 + \Delta x \\ &= x_1 + (2-1)\Delta x \end{aligned}$$

$$\begin{aligned} x_3 &= x_2 + \Delta x \\ &= (x_1 + \Delta x) + \Delta x \\ &= x_1 + 2\Delta x \\ &= x_1 + (3-1)\Delta x \end{aligned}$$

$$\begin{aligned} x_4 &= x_3 + \Delta x \\ &= (x_2 + \Delta x) + \Delta x \\ &= x_2 + 2\Delta x \\ &= (x_1 + \Delta x) + 2\Delta x \\ &= x_1 + 3\Delta x \\ &= x_1 + (4-1)\Delta x \end{aligned}$$

Persamaan

$$x_{i+1} = x_i + \Delta x, \quad i = 1,\ 2,\ 3,\ \ldots,\ n \qquad (1)$$

$$x_i = x_1 + (i-1)\Delta x, \quad i = 1,\ 2,\ 3,\ \ldots,\ n \qquad (2)$$

$$\Delta x \geq 0 \qquad (3)$$

$$\delta x_i = x_{i+1} - x_i \qquad (4)$$

$$\Delta x = \overline{\delta x} = \frac{1}{n-1} \sum_{i=1}^{n-1} \delta x_i \qquad (5)$$

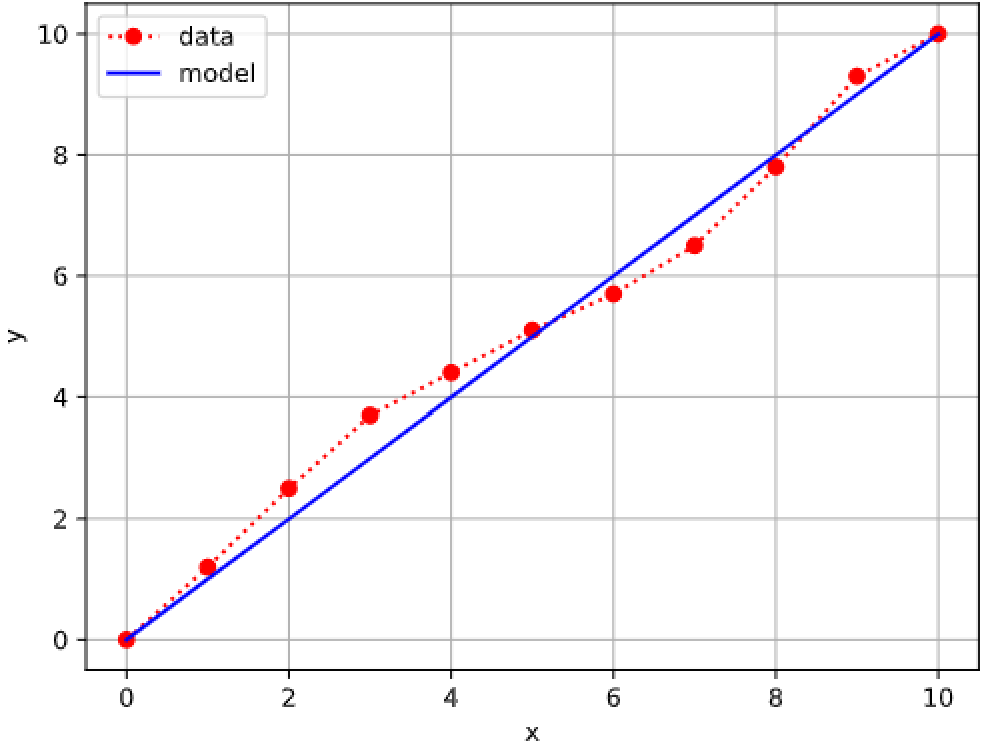


Fig 1. Arithmetic growth: model (—) dan data (·).

```python
import statistics as s
import matplotlib.pyplot as plt

def diff(x):
  n = len(x)
  d = [(x[i+1] - x[i]) for i in range(n-1)]
  return d

xdata = [0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10]
ydata = [0, 1.2, 2.5, 3.7, 4.4, 5.1, 5.7, 6.5, 7.8, 9.3, 10]
n = len(ydata)

dy = diff(ydata)
Dy = s.mean(dy)
ymodel = [ydata[0] + i * Dy for i in range(n)]

print("x", "ydata", "ymodel", sep='\t')
for i in range(n):
  print(xdata[i], end='\t')
  print(ydata[i], end='\t')
  print(ymodel[i])

plt.plot(xdata, ydata, 'ro:', xdata, ymodel, 'b-')
plt.xlabel("x")
plt.ylabel("y")
plt.grid()
plt.legend(["data", "model"])
plt.show()
```

Quadratic growth

Selisih antar data

i	x	Δx	$\Delta^2 x$
1	3		
2	7	4	
3	13	6	2
4	21	8	2
5	31	10	2
6	43	12	2
7	57	14	2

$$x_{i+1} = x_i + \Delta x_{i+1}$$

$$\Delta x_{i+1} = \Delta x_i + \Delta^2 x$$

$$x_{i+1} = x_i + \Delta x_i + \Delta^2 x$$

$$\begin{aligned} x_2 &= x_1 + \Delta x_1 \\ &= x_1 + \Delta x_1 + 0\Delta^2 x \end{aligned}$$

$$\begin{aligned} x_3 &= x_2 + \Delta x_2 \\ &= (x_1 + \Delta x_1) + (\Delta x_1 + \Delta^2 x) \\ &= x_1 + 2\Delta x_1 + \Delta^2 x \\ &= x_1 + 2\Delta x_1 + 1\Delta^2 x \end{aligned}$$

$$\begin{aligned} x_4 &= x_3 + \Delta x_3 \\ &= (x_2 + \Delta x_2) + (\Delta x_2 + \Delta^2 x) \\ &= x_2 + 2\Delta x_2 + \Delta^2 x \\ &= (x_1 + \Delta x_1) + 2(\Delta x_1 + \Delta^2 x) + \Delta^2 x \\ &= x_1 + 3\Delta x_1 + 3\Delta^2 x \end{aligned}$$

$$\begin{aligned} x_5 &= (x_1 + 3\Delta x_1 + 3\Delta^2 x) + (\Delta x_3 + \Delta^2 x) \\ &= x_1 + 3\Delta x_1 + \Delta x_3 + 4\Delta^2 x \\ &= x_1 + 3\Delta x_1 + (\Delta x_2 + \Delta^2 x) + 4\Delta^2 x \\ &= x_1 + 3\Delta x_1 + \Delta x_2 + 5\Delta^2 x \\ &= x_1 + 3\Delta x_1 + (\Delta x_1 + \Delta^2 x) + 5\Delta^2 x \\ &= x_1 + 4\Delta x_1 + 6\Delta^2 x \end{aligned}$$

Persamaan

$$x_{i+1} = x_i + \Delta x_{i+1} \tag{1}$$

$$\Delta x_{i+1} = \Delta x_i + \Delta^2 x \tag{2}$$

$$x_{i+1} = x_i + \Delta x_i + \Delta^2 x \tag{3}$$

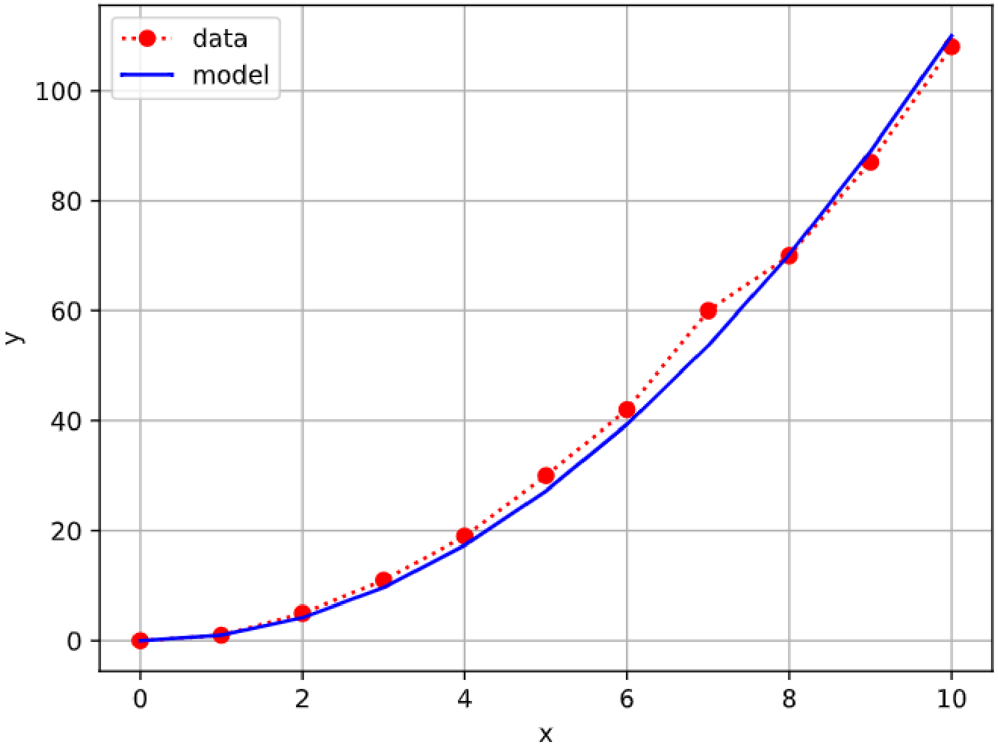
$$x_i = x_1 + (i-1)\Delta x_1 + \tfrac{1}{2}(i-1)(i-2)\Delta^2 x \tag{4}$$

$$\Delta x \geq 0, \quad \Delta^2 x \geq 0 \tag{5}$$

$$\Delta x_1 = x_2 - x_1 \tag{6}$$

$$\delta^2 x_i = x_{i+2} - 2x_{i+1} + x_i \tag{7}$$

$$\Delta^2 x = \overline{\delta^2 x} = \frac{1}{n-2} \sum_{i=1}^{n-2} \delta x_i \tag{8}$$




```python
import statistics as s
import matplotlib.pyplot as plt

def diff(x):
  n = len(x)
  d = [(x[i+1] - x[i]) for i in range(n-1)]
  return d

xdata = [0, 1, 2,  3,  4,  5,  6,  7,  8,  9,  10]
ydata = [0, 1, 5, 11, 19, 30, 42, 60, 70, 87, 108]
n = len(ydata)

dy = diff(ydata)
Dy = dy[0]
ddy = diff(dy)
DDy = s.mean(ddy)
ymodel = [ydata[0] + i*Dy + 0.5*(i)*(i-1)*DDy for i in range(n)]

print("x", "ydata", "ymodel", sep='\t')
for i in range(n):
  print(xdata[i], end='\t')
  print(ydata[i], end='\t')
  print(ymodel[i])

plt.plot(xdata, ydata, 'ro:', xdata, ymodel, 'b-')
plt.xlabel("x")
plt.ylabel("y")
plt.grid()
plt.legend(["data", "model"])
plt.show()
```

Geometric growth

Selisih (rasio) antar data

i	x	x_{i+1}/x_i
1	3	2
2	6	2
3	12	2
4	24	2
5	48	2
6	96	2
7	192	2
8	384	

$$x_{i+1} = c \cdot x_i$$

$$x_2 = c \cdot x_1$$

$$\begin{aligned} x_3 &= c \cdot x_2 \\ &= c \cdot (c \cdot x_1) \\ &= c^2 \cdot x_1 \end{aligned}$$

$$\begin{aligned} x_4 &= c \cdot x_3 \\ &= c \cdot (c \cdot x_2) \\ &= c^2 \cdot x_2 \\ &= c^2 \cdot (c \cdot x_1) \\ &= c^3 \cdot x_1 \end{aligned}$$

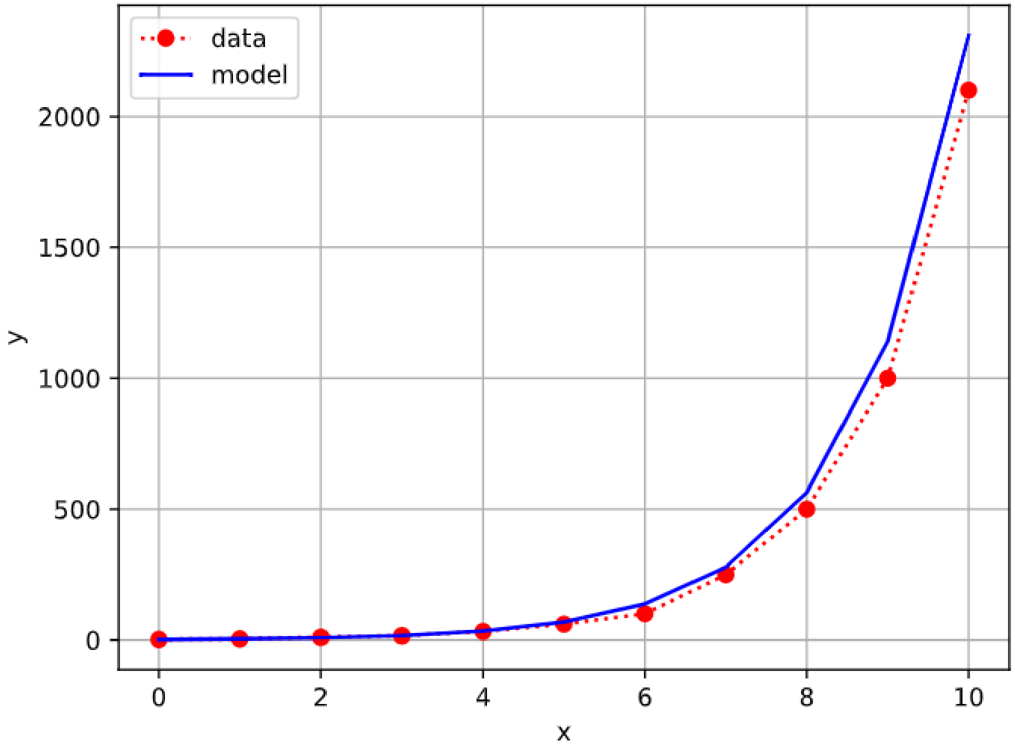
Persamaan

$$x_{i+1} = c \cdot x_i \tag{1}$$

$$x_i = c^{i-1} x_1 \tag{2}$$

$$\gamma_i = \frac{x_{i+1}}{x_i} \tag{3}$$

$$c = \bar{\gamma} = \frac{1}{n-1} \sum_{i=1}^{n-1} \gamma_i \tag{4}$$




```python
import statistics as s
import matplotlib.pyplot as plt

def ratio(x):
  n = len(x)
  d = [(x[i+1] / x[i]) for i in range(n-1)]
  return d

xdata = [0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10]
ydata = [2, 4, 10, 16, 32, 60, 100, 250, 500, 1000, 2100]
n = len(ydata)

gamma = ratio(ydata)
c = s.mean(gamma)
ymodel = [ydata[0] * c**i for i in range(n)]

print("x", "ydata", "ymodel", sep='\t')
for i in range(n):
  print(xdata[i], end='\t')
  print(ydata[i], end='\t')
  print(ymodel[i])

plt.plot(xdata, ydata, 'ro:', xdata, ymodel, 'b-')
plt.xlabel("x")
plt.ylabel("y")
plt.grid()
plt.legend(["data", "model"])
plt.show()
```

Cubic growth

Selisih antar data

i	x	Δx	$\Delta^2 x$	$\Delta^3 x$
0	1	3	8	6
1	4	11	14	6
2	15	25	20	6
3	40	45	26	6
4	85	71	32	6
5	156	103	38	6
6	259	141	44	6
7	400	185	50	6
8	585	235	56	
9	820	291		
10	1111			

$$
\begin{aligned}
x_2 &= x_1 + \Delta x_1 \\
\Delta x_2 &= \Delta x_1 + \Delta^2 x_1 \\
x_3 &= x_2 + \Delta x_2 \\
x_3 &= (x_1 + \Delta x_1) + (\Delta x_1 + \Delta^2 x_1) \\
&= x_1 + 2\Delta x_1 + \Delta^2 x_1
\end{aligned}
$$

$$
\begin{aligned}
\Delta^2 x_2 &= \Delta^2 x_1 + \Delta^3 x \\
\Delta x_3 &= \Delta x_2 + \Delta^2 x_2 \\
x_4 &= x_3 + \Delta x_3 \\
&= (x_1 + 2\Delta x_1 + \Delta^2 x_1) \\
&\quad + (\Delta x_2 + \Delta^2 x_2) \\
&= x_1 + 2\Delta x_1 + \Delta^2 x_1 \\
&\quad + \Delta x_2 + \Delta^2 x_2 \\
&= x_1 + 2\Delta x_1 + \Delta^2 x_1 \\
&\quad + (\Delta x_1 + \Delta^2 x_1) + (\Delta^2 x_1 + \Delta^3 x) \\
&= x_1 + 3\Delta x_1 + 3\Delta^2 x_1 + \Delta x^3
\end{aligned}
$$

$$
\begin{aligned}
\Delta^2 x_3 &= \Delta^2 x_2 + \Delta^3 x \\
\Delta x_4 &= \Delta x_3 + \Delta^2 x_3 \\
x_5 &= x_4 + \Delta x_4 \\
&= \cdots
\end{aligned}
$$

Persamaan

- Sudah terpikirkan bagaimana persamaan-persamaannya? Ada ide? Prosedur pencariannya sudah dimengerti?
- Perhatikan pola sebelumnya

$$x_{i+1} = x_i + \Delta x, \quad i = 1,\, 2,\, 3,\, \ldots,\, n \tag{1}$$

$$x_i = x_1 + (i-1)\Delta x, \quad i = 1,\, 2,\, 3,\, \ldots,\, n \tag{2}$$

$$x_{i+1} = x_i + \Delta x_{i+1} \tag{1}$$

$$\Delta x_{i+1} = \Delta x_i + \Delta^2 x \tag{2}$$

$$x_{i+1} = x_i + \Delta x_i + \Delta^2 x \tag{3}$$

$$x_i = x_1 + (i-1)\Delta x_1 + \tfrac{1}{2}(i-1)(i-2)\Delta^2 x \tag{4}$$

Dugaan

- Apakah akan menjadi seperti ini?

$$x_i = x_1 + (i-1)\Delta x_1 + \tfrac{1}{2}(i-1)(i-2)\Delta^2 x_1 + \tfrac{1}{6}(i-1)(i-2)(i-3)\Delta^2 x$$

- Buktikan!
- Belum ada pada referensi utama yang digunakan.

Diskusi dan latihan

Tugas sebelum dan setelah kuliah

- Isi kehadiran di SIX.
- Kerjakan tugas yang tersedia di Issue 10.

Diskusi

- Silakan mengajukan pertanyaan saat kuliah berlangsung
- Setelah kuliah pertanyaan dapat diajukan secara asinkron di url https://github.com/dudung/sk5003-02-2022-2/issues/10

Tugas

- Periksa jadwal dan tempat UTS untuk 20-21 Mei 2023
- Persiapkan perjalanan dan akomodasi melalui koordinasi dengan unit kerja dan program terkait
- Pelajari kembali materi minggu 1-7 yang merupakan bahan UTS

Terima kasih

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