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Institute of Education

Pedagogical Strategies, Approaches and Methodologies to Support Numeracy in Post-Primary Education

A Review of the Literature

Prepared by Paul Grimes, Thérèse Dooley and Siún Nic Mhuirí, Institute of Education, Dublin City University, for the Department of Education

Author Note

Paul Grimes https://orcid.org/0000-0001-6705-9566

Thérèse Dooley https://orcid.org/0000-0002-5671-4075

Siún Nic Mhuirí https://orcid.org/0000-0001-5008-5573

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Summary

The review was guided by the following research question:

What are the pedagogical strategies, approaches and methodologies in mathematics that support the development of numeracy at post-primary level?

A broad view of numeracy, encompassing the use of mathematical skills and understanding to solve complex problems and a critical perspective on the use of mathematics in the world, is taken in this report. It is recognised that numeracy can be developed across the curriculum. In this report, we focus on the development of numeracy within the subject of mathematics. There is a limited number of meta-analyses and systematic reviews investigating numeracy at post-primary level specifically. We focus on studies that investigate students' mathematical development that align with our view of numeracy, and report on the pedagogical strategies, approaches and methodologies that were shown to be effective. Upon screening of abstracts and full texts, 28 studies were selected for inclusion in this review as well as additional studies sourced from outside our main search. Four themes - in which some aspects overlap - arose from analysis of studies: *mathematical problem solving*, *classroom interactions, non-digital and digital tools,* and *metacognition and self-regulated learning.*

- **Mathematical problem solving**

Mathematical problems vary from structured to unstructured, and can stem from real-life contexts and more abstract mathematical situations. An emphasis on problem solving and problem posing in the teaching and learning of all strands of mathematics helps to improve post-primary students' achievement in and attitude towards mathematics as well as motivation to engage in the subject. There is evidence that this is true for all students, regardless of their level of attainment in mathematics - however, careful consideration needs to be given to the practices that support different learners.

- **Classroom interactions**

The available literature focuses on three interrelated aspects of classroom interactions: discussion (with particular focus on the role of the teacher), collaborative learning (with particular focus on group work), and peer and cross-age tutoring. Effective classroom discussions are those in which teaching elicits and responds to student thinking and appropriate feedback is provided. Collaboration has been found to have positive effects on both student achievement and student attitudes towards mathematics, particularly at post-primary level. While there is some evidence of positive effects of peer-tutoring, a specific type of collaboration, as of yet there is limited evidence in the context of post-primary mathematics.

- **Non-digital and digital tools**

Studies in the review indicate that non-digital and digital tools play an important role in supporting students' mathematical learning. When compared with teaching using only abstract mathematics symbols, the use of non-digital tools was found to have a positive impact on student learning. Effects were mediated by the levels of instructional guidance provided. Digital tools designed with a particular emphasis on mathematics were found to have a more positive impact on learning than more general tools (e.g., iPad use).

- **Metacognition and self-regulated learning**

A wide range of strategies associated with metacognition and self-regulated learning (SRL) is found to facilitate student learning of mathematics; however, reflections by students on their thinking through verbalization or writing emerge as particularly important. The development of metacognitive and SRL skills takes time, the role of the teacher is important, and in post-

primary schools, a focus on learner motivation may be more effective than a focus on cognitive strategies.

Recommendations

In order to support the development of numeracy within the subject of mathematics at post-primary level, we recommend the following (our recommendations are organized around the relevant pillars of the education system identified in the 2011-2020 NLNS):

Curriculum and the learning experience

- A range of problems, varying from unstructured to structured, should be embedded in classroom practice and should infuse all strands of the post-primary mathematics curriculum. (Cason et al., 2019; Hodgen et al., 2018; McDonald & Smith, 2020; Merritt et al., 2017)
- Through discussion, teachers should elicit, listen to and respond to the substance of students' mathematical thinking providing feedback as appropriate and adjusting teaching based on observations of their thinking. (Anthony & Walshaw, 2007; Apthorp et al., 2012; Dietrichson et al., 2017; Dietrichson et al., 2020; Hodgen et al., 2018)
- Teachers should spend time developing a classroom culture in mathematics that supports effective peer-to-peer collaboration; encouraging student explanations and questions, and teaching students how to listen and respond to each other's ideas while holding individuals accountable for their own learning. (Anthony & Walshaw, 2007; Baker et al., 2015; Hodgen et al., 2018)
- A wide range of appropriate tools - digital and non-digital - should be carefully selected and made available to students to develop and extend their mathematical understanding. Their use should be supported by appropriate instruction, and should allow students to explore mathematics, and facilitate them to see mathematical structures. When possible, non-digital and digital tools should be used in conjunction with each other. (Moyer-Packenham & Westenskow, 2013)
- A focus on metacognition and self-regulated learning should be built into students' engagement with mathematical tasks or problems. In particular, students should be given opportunities to elaborate on their thinking in their learning of mathematics. (Donker et al., 2014; Hodgen et al., 2018; Shin and Bryant, 2015)

School and ECEC practitioners' professional practice

- Mathematics teachers should explore and interrogate problems that are meaningful for their learners, including those that arise in other subject areas. (McDonald & Smith, 2020; Merritt et al., 2017)
- Sustained professional learning opportunities, with a focus on the pedagogical strategies, approaches and methodologies in mathematics that support the development of numeracy, should be provided to post-primary mathematics teachers. (McDonald & Smith, 2020; Slavin, 2013)

Building the capacity of school leadership

- Professional learning opportunities, with a focus on the pedagogical strategies, approaches and methodologies in mathematics that support the development of numeracy and the requisite resources should be provided to school leaders, including subject leaders. (Anthony & Walshaw, 2007; McDonald & Smith, 2020)

Students with Additional Learning Needs

- Pedagogical approaches in mathematics that support the development of numeracy - problem-solving/problem posing; interactive methodologies; use of digital and non-digital tools; and development of metacognitive and SRL skills - should underpin the mathematics learning experience of all learners including those with additional learning needs. (Donker et al., 2014; Hodgen et al., 2018)

Pedagogical Strategies, Approaches and Methodologies to

Support Numeracy in Post-primary Schools

As described by Geiger, Goos, and Forgasz (2015), the term “numeracy” can be traced back to the Crowther report published by the UK government in 1959. Although it remains a significant component of primary, post-primary and tertiary curricula internationally, it is open to a range of interpretations. For example, it is sometimes equated with acquisition of basic number skills and operations. More generally, as is the case in the *National Strategy: Literacy and Numeracy for Learning and Life 2011-2020* (DES, 2011), it is viewed as related to the ability to use mathematical skills and understanding to solve complex problems and meet the demands of day-to-day living (which we will refer to hereon as “problem-solving numeracy”.) The more radical “critical numeracy” engages learners in thinking critically about the use of mathematics in the world, the power inherent in mathematics, and the effect of this power on themselves and others (e.g., Askew, 2015; Frankenstein 2009). These interpretations are not necessarily discrete; for instance, many see critical numeracy as embodying problem-solving numeracy. The purpose of this review is to report on pedagogical strategies that support the development of numeracy within mathematics at post-primary level[1]. We reviewed systematic reviews and meta-analyses that were carried out between 2011 and 2021. The review was guided by the following research question:

Research Question:

What are the pedagogical strategies, approaches and methodologies in mathematics that support the development of numeracy at post-primary level?

(For further details of the research strategy, see appendix)

[1] A systematic review on integration across the curriculum has also been completed as part of this tender.

Here we present the detailed findings of our review. Our search yielded few studies specifically related to numeracy; most focused on the pedagogies that support students' understanding and development in mathematics more generally. Specifically given that numeracy concerns the capacity to use mathematics "to act in the real world" (Geiger et al., 2015, p.612), we included the search terms, "Science, Technology, Engineering and Mathematics (STEM) Education", and MATH* (to retrieve studies that looked at all areas of mathematics, including mathematisation[2]). Some studies reported on interventions that were specifically aimed at students who struggle to learn mathematics. These studies were deemed appropriate for inclusion in our review because research indicates that students with difficulties in mathematics (sometimes identified as students achieving at less than the 35th percentile) respond in similar ways to high-quality interventions as both their higher-achieving peers and students identified with specific disabilities (Lien et al., 20202; Marita & Hord, 2017; Zhang & Xin, 2015). In fact, it appears that teaching approaches designed to minimize barriers faced by students with disabilities or mathematics difficulties may be beneficial to all learners (Marita & Hord, 2017). As a result of our search process, four key overlapping themes emerged*:* *mathematical problem solving*, *classroom interactions, non-digital and digital tools* and *metacognition and self-regulated learning.* This section of the report is organised around these themes. A number of the studies reviewed spanned primary and post-primary levels; where possible, we report on the findings that are most relevant to post-primary students' development.

[2] This term derives from Freudenthal (1968) who sees mathematical activity as 'mathematizing', that is, organising reality from a mathematical perspective. A key aspect of this viewpoint for mathematics education is that students are given the opportunity to reinvent mathematics by solving problems that are real for them.

Narrative Report

Mathematical Problem Solving

As described by Santos-Trigo (2007) and others, mathematics can be conceived as a set of problems or dilemmas that can be solved through the use of mathematical resources. Weber and Leikin (2016) outline how mathematical problem solving has evolved and expanded from being concerned with problems or tasks in which there is no obvious solution to those that have multiple solution pathways and to situations where students pose their own problems, deciding in the process what might be valuable to infer or deduce. Specifically, and of interest to the focus of this report, is the findings that emanate from studies related to problem solving and/or problem posing relevant to the problem-solving and critical perspectives on numeracy, described above.

In Cason et al.'s (2019) review, there is reference to the problem-solving perspective on numeracy. They address the question of the overall effect of numerical competency development on mathematics achievement, and describe numerical competence as having three foci: maths fact fluency (recall and mastery of basic facts), number sense (by which the importance of the underlying interconnectedness of basic number combinations is emphasized) and numeracy (working mathematics knowledge that involves the functional, social, and cultural dimensions of mathematics). While their meta-analysis concerned primary and post-primary students, very few studies involved the grade 9-12 (ages 14-18) cohort. They found an overall large effect size for the impact of numerical competency on student achievement. Disaggregating the different components of numerical competency revealed that the largest effect sizes were for number sense and numeracy, that is, these aspects of numerical competency had greater impact on mathematical achievement than did maths fact fluency. The greatest effects were found for children in Grades EC-2 (ages 3-8) and in non-intervention studies. The authors conclude that low-achieving children need continuous support to deal with gaps in their number sense and numeracy, support that might extend beyond the early years. However, they also argue that their meta-analysis reveals the need for numerical competency - especially number sense and numeracy - to be infused across all strands of the K-12 mathematics curriculum.

Merritt et al. (2017) conducted a systematic literature review on problem-based learning (PBL) which they describe as a pedagogical approach that fosters learning through the use of problems and dilemmas in real-world settings. They describe problem types used in PBL as ranging from highly unstructured to structured. They found that unstructured scenarios or case-based problems - where learners identify problems from a given context - were generally used in studies involving older primary or post-primary students, whereas more structured problems were used with younger children. Student-centred learning opportunities deriving from PBL helped to improve four skills: problem-solving, cooperative skills, communication skills and technology skills (with less emphasis in studies on the latter two). PBL was found to have positive effects on students' academic achievement, knowledge retention, conceptual development, and attitudes. However, the authors note that it was not always clear to them how "attitude" was conceptualised in the relevant studies, nor was there any study that addressed students' attitudes to PBL. Noteworthy is the fact that although Merritt et al. were interested in PBL as it related to mathematics and science, all studies that were screened by them focused on science. Moreover, their findings are limited by the small number of studies in their review. Hodgen et al. (2018), based on their review of mathematical problem solving, argue that teachers should provide learners with worked examples for comparison and analysis purposes and should encourage them to attend to the underlying mathematical structure. They also conclude that students should monitor, reflect on, and discuss the problem-solving experience so that the solution does not become the sole focus of the process.

Hodgen et al. (2018) also found that the cognitive load exerted by the exploration required to solve unstructured problems can interfere with efficient learning and that teachers therefore need to scaffold learners appropriately. They advise that problem solving is appropriate for learners at all levels of attainment and that learners should be exposed to many problem types, including those that do not have predefined solution methods. This suggestion is supported by Marita and Hord (2017) who synthesize findings across different studies to show that both lower and normal-achieving students benefit from teaching approaches which focus on problem-based learning, and which use strategies such as visual representations and systematic instruction, where tasks become gradually more challenging. They suggest that such approaches allow students time to develop problem-solving strategies while being appropriately challenged with high expectations (p. 38). Zhang and Xin (2012) who investigated word-problem interventions for students with learning difficulties in mathematics found that those targeting 'problem structure representation' techniques yielded the greatest effect size. Here, problem structure representation describes instruction that helps learners understand and represent the problem in schema-based diagrams or mathematical models (p. 306). This is also reported in a study by Lein et al. (2020). Based on a review of 31 studies, they recommend that interventions that focus on problem structure can accelerate growth in closing achievement gaps in mathematics.

The findings of a systematic review conducted by McDonald and Smith (2020) on the benefits for learners of using mathematical problem posing in the curriculum (specifically, for them, the Curriculum for Excellence in Scotland) corroborate and extend the findings of Merritt et al. (2017) in terms of mathematics. They draw on the definition of problem posing furnished by Stoyanova and Ellerton (1996, p. 518), that is "the process by which, on the basis of mathematical experience, students construct personal interpretations of concrete situations and formulate them as meaningful mathematical problems". As described above for PBL, problem posing can stem from very unstructured, semi-structured and structured situations. McDonald and Smith found that engagement in problem posing at post-primary level was linked with improvements in the areas of probability, geometry, general mathematical knowledge, and algebra. Furthermore, post-primary pupils reported more positive attitudes towards probability and also mathematics in general, as well as increased interest in mathematics and improved motivation to engage in mathematics. Mathematical problem posing is not without its challenges, however - some that were highlighted in the review were the inability to pose contextualised or sensible problems, semantic difficulties and the identification of salient information in posing a problem. Learners were also more inclined to engage in problems that they found relevant to their lives and interests. Group work and reflection were also found by the authors to be important to the problem-posing process. McDonald and Smith (2020) conclude that high-quality professional development (PD) is warranted in order to ensure that problem posing is embedded within mathematics teaching and learning by individual teachers and at whole-school level.

A strong message emerging from the reviews on problem solving and problem posing returned in our search is that a pedagogy based on the use of problems that are interesting to the learners concerned has a positive impact on achievement in and attitude towards mathematics as well as on motivation to engage in the subject. All students, regardless of their level of attainment in mathematics, should be exposed to a variety of problem types varying from structured to unstructured and should be encouraged to pose and develop solutions for their own problems. However, careful consideration needs to be given to the practices that support different learners.

Classroom Interactions

Interactions within the classroom are a key driver of student learning (Slavin et al., 2008). In this section, we present findings related to the use of discussion and collaborative learning.

Discussion

Mathematical discussion has been described in several different ways in the literature but is widely regarded as a key element of mathematics teaching and learning (e.g., Hodgen et al., 2018). Although there is a growing body of empirical research investigating the characteristics of effective discussion (for example, the impact of elaboration and querying previous contributions is described in Howe et al., (2019) and Hennesy et al., (2021), our search of systematic reviews and meta-analyses returned few/no results with a sole focus on classroom discussion/dialogue for the date range we are interested in. We therefore report on two older reviews (Kyriacouu & Kissitt, 2008; Anthony & Walshaw, 2007) and other more recent individual studies on mathematical discussions. Hodgen et al. (2018) report that while discussion is an important factor in the teaching and learning of mathematics, there is limited evidence reporting on the effectiveness of interventions aimed at improving discussion in mathematics classrooms. This is perhaps due to the difficulty of comparing experimental results in studies of this nature (Bakker et al., 2015; Howe et al., 2019).

Kyriacouu and Issitt (2008) carried out a review of 15 studies to investigate the characteristics of teacher-initiated teacher-pupil talk that promoted a conceptual understanding of mathematics. Although they found limited evidence that addressed their question, they report that the strongest evidence for the promotion of conceptual understanding in mathematics came from studies where students were taught how to use dialogue effectively (Bakker et al. (2015) also note the importance of students learning how to question and learn new things for themselves through dialogue). Anthony and Walshaw (2007) highlight the role of the teacher in developing skills in explanation, argumentation and justification through eliciting students' thinking, and pressing for reasoning. The importance of setting the expectation that students should listen to, and respect, the contributions of their peers (Anthony & Walshaw, 2007) as well as supporting students' capacities to listen and engage in discussions (Hodgen et al., 2018) is also noted. Anthony and Walshaw (2007) contend that establishing participation processes and responsibilities for class discussion is an important pedagogical strategy which will support the development of mathematical communities of practice within classrooms.

Kyriacouu and Issitt (2008) report that effective classroom discussion requires the teacher to actively listen to students' contributions and explanations, rather than just listening for the purpose of evaluation. Anthony and Walshaw (2007) suggest that teacher questioning for the sole purpose of evaluation can lead to students' reticence to share ideas or ask questions of their own. They maintain that the value teachers place on students' thinking influences the way in which students view their relationship with mathematics and suggest that teachers need to listen actively in order to build dialogue that will develop mathematical ideas and positive mathematical identities. Hodgen et al. (2018) also note that teachers need to elicit and use student explanations as a basis for discussion. In the meta-analysis carried out by Apthorp et al. (2012), the most effective teaching interventions were those which included opportunities to assess and respond to student thinking on an ongoing basis. This included opportunities to self-assess and in the most effective interventions, teachers provided supportive cues and prompts for student reflection. Much of the discussion that occurs in classrooms to support learning can be considered as feedback (Anthony & Walshaw, 2007). Hodgen et al. (2018) report that it may be of particular importance to include feedback in formative assessment approaches in mathematics. While important for all students, feedback and progress monitoring were found to have significant positive effects on the achievement of elementary and middle school children from low SES backgrounds (Dietrichson et al., 2017); and on the achievement of at-risk (e.g., low achievement, disability, low SES, minority background) second-level students (Dietrichson et al., 2020). Hodgen et al. (2018) also note that formative assessment was found to be more effective where teachers had received professional development.[3]

Collaborative Learning

Collaborative or cooperative learning has been defined in different ways in the literature, ranging from very general ideas about students 'working together' to more specific definitions such as small groups of students working together toward a common goal (see Hodgen et al., 2018). Collaborative learning can also include peer-tutoring approaches such as the approach described below. It is often used in tandem with other teaching approaches. For example, as discussed above, the research synthesis of Merrit et al. (2017) which found positive effects for PBL on student achievement, also reports that all studies reviewed used small-groups and emphasized collaboration toward solving the problem. The importance of group work and reflection in the problem-solving process is also highlighted by Mc Donald

[3] See the systematic review on Formative Assessment completed as part of this tender.

and Smith (2020) and Hodgen et al. (2018). In their synthesis, Merrit et al. (2017) report that all studies used small groups and emphasized collaborative problem solving. Three of nine studies targeted communication skills explicitly, with groups expected to report findings back to the whole class. Anthony and Walshaw (2007) recommend that groups should be mixed in relation to academic achievement and any other relevant status characteristics. Furthermore, they draw attention to the need to set and maintain expectations around individual students' participation in groups, e.g., that all students will contribute ideas and aim to reach consensus about key mathematical ideas. Noting that students need some time and space to work independently to develop their thinking, Anthony and Walshaw (2007) also recommended a balance between group work and individual thinking time, and ongoing monitoring and reflection by the teacher, in order to enhance the inclusiveness and effectiveness of instruction.

Drawing only on research conducted in the US, the synthesis of findings presented in Hodgen et al. (2018) reports positive effects of collaborative learning interventions on the mathematics attainment and attitudes of students, with larger effects at secondary level. Replicable, structured programmes lasting 3 months or more have been shown to have the largest effects though Hodgen et al. (2018) note the difficulties involved in adapting programmes for local contexts, particularly when changes in classroom culture are necessary. In line with the comments on discussion above, their synthesis notes that time and teacher-effort is needed in teaching students how to collaborate effectively. Older research by Slavin et al. (2008) also highlights the importance of individual accountability within cooperative learning approaches. While Slavin et al. (2008) note that cooperative learning may be less effective for lower achieving students, it was found to have significant positive effects on the achievement of elementary and middle school children from low SES backgrounds (Dietrichson et al., 2017).

Peer and Cross-Age Tutoring

Peer-mediated instruction can involve strategies such as class-wide peer tutoring, or peer-assisted learning strategies such as reciprocal tutoring, where students alternate as tutor and tutee (Wexler et al., 2015). Peer-tutoring interventions have been shown to be cost-efficient and have benefits for tutors - who develop mastery of the material and a sense of responsibility, for tutees - who may be more comfortable asking questions of a peer, and for teachers - who free up time for other tasks (Hodgen et al., 2018). Though there has as yet been limited research conducted in the area of mathematics (Hodgen et al., 2018), Wexler et al.

(2015) report moderate to high effects for peer-mediated instruction with struggling second-level learners. Higher effect sizes are reported for cross-age tutoring, where an older student works with a younger learner, than for peer tutoring where students of the same age work together as tutor and tutee (Hodgen et al., 2018). Peer-tutoring interventions have been found to be effective particularly when students work in mixed-ability pairs and interventions include a peer-mediated feedback component (Wexler et al., 2015). Importantly, there is some evidence to suggest that these interventions have social validity amongst both adolescents and teachers, with both teachers and students describing peer tutoring as beneficial for learning. However, Hodgen et al. (2018) caution that while existing research highlights the importance of training for tutors, more research is needed on effective implementation practices.

Non-digital and Digital Tools

Tools can be regarded from a Vygotskian perspective as both psychological (e.g., language) and physical (i.e., those designed for a specific purpose such as a protractor) (Askew, 2016). Talk and discussion can therefore be regarded as psychological tools - however, since reviews returned in our search show they play such an important role in the teaching and learning of mathematics, they are dealt with in a separate section above. Physical tools encompass not only concrete manipulatives but also pictures, diagrams and mathematical symbols etc. Several of the studies identified in our search looked at the use of digital tools in the mathematics classroom. There were also a smaller number of studies that investigated the use of non-digital tools. While these studies did not specifically describe how they interpreted a mathematical tool they generally focused on the use of physical manipulatives in mathematics classrooms.

Carbonneau et al. (2013) reviewed 55 studies spanning kindergarten to college level in which instruction using manipulatives was compared to instruction that only used abstract mathematical symbols. Overall, they found small to moderate positive effects when manipulatives were used. Furthermore, when compared with teaching using only abstract mathematical symbols, using manipulatives was found to have moderate to large effects on students' retention and small effects on problem solving, transfer and justification. Their analysis showed that effects were mediated by instructional characteristics such as level of instructional guidance, mathematical topic, development status (operationalised as age/grade level), perceptual richness, and instructional time. Higher levels of instructional guidance were associated with higher student learning. Carbonneau et al. (2013) suggest that in general explicit teaching helps learners to establish connections between the concrete manipulatives and the intended mathematical ideas, which in turn facilitates comprehension and understanding. However, if the pedagogical objectives are for learners to transfer knowledge to other areas of mathematics, it may be important to reduce the extent of scaffolding provided for the use of the manipulatives. They suggest that more research is needed to investigate how the required level of instructional guidance may vary depending upon learning objective (e.g., learning for transfer vs. learning for retention). Studies targeting post-primary students were found to have smaller effects than those targeting younger students. Attending to the nature of the manipulatives themselves, Carbonneau et al. (2013) differentiate between bland or non-descript manipulatives and perceptually-rich materials which are either representative of real objects or the actual objects themselves. As for instructional guidance, the nature of the manipulatives was found to have different effects depending on the nature of the learning outcomes. Perceptually rich manipulatives had smaller effects for student retention and negative effects for problem solving when compared with bland manipulatives. In contrast, bland manipulatives were found to have lower negative mean effect sizes than perceptually rich manipulatives in the context of teaching for transfer.

Moyer-Packenham and Westenskow (2013) carried out a meta-analysis that synthesised the findings of 66 studies examining the effects of virtual manipulatives on student achievement. They also investigated the effects of virtual manipulatives on student achievement in comparison to using physical manipulatives. They found that the use of virtual manipulatives (compared to other instructional approaches) had a moderate effect on student achievement. Using virtual manipulatives only (compared to using physical manipulatives only) had a small positive effect. Using a combination of physical and virtual manipulatives had a moderate effect compared to other instructional treatments, but had a much larger effect when compared to classroom instruction with textbooks.

They also analysed the effect using virtual manipulatives to support student learning in particular mathematical topics. They looked at the effect of using virtual manipulatives for teaching various topics compared to other instructional treatments. They found that the use of virtual manipulatives had a moderate effect on student learning in the topics of fractions, numbers, operations, geometry, and measurement, with the largest effect being in the domain of fractions. The topic of fractions was also investigated by Shin and Byrant (2015). They reviewed 17 studies to investigate the features of fractions interventions, particularly interventions for students struggling to learn mathematics. 14 of the studies were based in settings with students at 5th grade or older. They found that there were highly positive outcomes for studies using concrete and visual representations in combination with explicit, systematic instruction. The use of visual models is also endorsed by Jitendra et al. (2018) and Myers et al. (2021). Both of these reviews found that interventions using visual models had positive effects for low-achieving students.

Moyer-Packenham and Westenskow (2013) also analysed each of the 66 studies to look at the effects by grade level. The majority of the studies they analysed were conducted in pre-kindergarten to grade 6 level (ages 11-12). For the studies carried out in settings above grade 6 (secondary school), five were in the context of grade 7-8 (ages 12-14). In studies at these grade levels, the use of virtual manipulatives was found to have no effect. At grades 9-12 (ages 14-18) the effects were found to be moderate (however, they do caution that two of the three studies carried out at this grade level were in the context of unique grouping/setting e.g., special education students and an individualised online algebra module). Finally, they outline 5 affordances (emerging from the studies) of the use of virtual manipulatives that promote student learning in mathematics:

- They focus and constrain student attention on mathematical objects and processes
- They encourage creativity and increase the variety of students' solutions
- They simultaneously link representations with each other and with students' actions
- They contain precise representations allowing accurate and efficient use
- They motivate students to persist at mathematical tasks

Holmes (2013) carried out a meta-analysis to investigate the effects of manipulative use on mathematical achievement for PK-12 students. Their study had two strands, one to compare the effects of physical manipulative use to instruction that does not involve manipulative use. They also looked at the effects of using digital manipulatives compared to using physical manipulatives. Overall, they found that mathematics achievement at all grade levels can be improved through the use of manipulatives. Students who used manipulatives during mathematics instruction had statistically significant higher mathematics achievement than students who did not use manipulatives during mathematics instruction. They also found that using virtual manipulatives compared to physical manipulatives had a small positive effect on student mathematical achievement. The effects of using manipulatives (physical and virtual) are lower at secondary school level compared to those at primary level.

Our search identified just one study that reported on the use of tools that were designed specifically for mathematics. Other studies that looked more broadly at the use of technology in the mathematics classroom were also identified through our search. Overall, these studies report mixed results as to the effectiveness of digital tools to support mathematics achievement. One of the most promising technological interventions was the use of Dynamic Geometry Software (DGS). This type of software allows students to construct and manipulate geometric shapes in a computer-based environment. In a meta-analysis conducted to study the effectiveness of dynamic geometrical software on students' mathematical achievement, Chan and Leung (2014) found that DGS based instruction produced a positive and large effect size on mathematics achievement for students. Six of the nine studies included in this meta-analysis were in a middle or high school setting. Their findings indicate that DGS is more effective in improving students' mathematical achievement when compared to instruction using traditional (paper and ruler) based instruction.

Two studies reported more generally on the use of technology to support mathematics achievement, with a specific interest in the use of digital games. In these studies, digital games are broadly described as having interactive elements, specific rules, and a level of uncertainty that encourages engagement and provides various sensory stimuli for students. Byun and Joung (2018) investigated trends of using digital games-based learning and their effect on achievement in mathematics for K-12 students. They found that the overall weighted effect size represented a moderate effect of digital games-based learning on mathematics achievement. They conclude that the result implies that there may be other ways for students to learn mathematics that are more effective than digital games-based learning. Similarly, Tocak (2019) found a small but marginally significant effect of video games on the achievement of students in mathematics compared with traditional classroom instructional methods. They also report that the effect size was smaller for studies carried out in the context of grade 7 and above.

Some reviews reported on the use of specific digital tools and their use in the classroom. Boon et al. (2021) examined if iPad use supported learning in students aged 9-14 years. They conducted a systematic literature review to investigate this. Eight studies included in their review had a specific focus on mathematics. They found that the results were mixed in relation to mathematics. While some studies reported that iPad interventions supported mathematics learning, other studies showed either no significant difference in learning outcomes or inconsistent results in student learning outcomes.

It seems that, in general, there are mixed results for the effect of using technology in the classroom. Boon et al. (2021) identify several other factors that may influence student learning while using technology, including students and teachers' digital literacy skills, and teachers' management of learning while using technology. Byun and Joung (2018) also discuss different factors that contribute to the impact on student learning (including the purpose of different games). Both Boon et al. and Byun and Joung call for further studies into technology use in the eclassroom so that we can better understand their effects on student learning.

Hodgen et al. (2018) note that there is widespread acceptance of the importance of digital and non-digital manipulatives and representations within the learning of mathematics, and highlight that students need to learn to interpret, coordinate and use different mathematical representations to describe and analyse mathematical relationships. However, drawing on Ainsworth (2006), they also note that multiple representations can exert a heavy cognitive load and more research is needed to understand how and in what circumstances different representations are more or less effective. They suggest that manipulatives and representations should be used in principled ways - for example, the use of one tool or representation to support learning across a number of connected topics.

Metacognition and Self-regulated Learning

Metacognition and self-regulated learning (SRL) are highlighted as important for successful learning of mathematics (Donker et al., 2014; Hodgen et al., 2018). While there are variations in their interpretation in the literature, Muijs and Bokhove (2020, p.5) describe them in their recent research synthesis as follows:

> Essentially, self-regulation is about the extent to which learners are aware of their strengths and weaknesses, the strategies they use to learn, can motivate themselves to engage in learning, and can develop strategies and tactics to enhance learning. Metacognition, in turn, is specifically about the ways learners can monitor and purposefully direct their learning, for example by deciding that a particular strategy for memorisation is likely to be successful, monitor whether it has indeed been successful, and then deliberately change (or not change) their memorisation method based on that evidence.

SRL encompasses metacognition, cognition, and motivation. SRL and metacognition are usually considered in terms of their associated strategies. For example, Donker et al. (2014) argue that there is a wide range of learning strategies - cognitive (e.g., rehearsal, elaboration, and organisation), metacognitive (e.g., planning, monitoring, evaluation) and management (of effort, others, environment) - that can facilitate students' performance. However, they claim that also important in the learning process are metacognitive knowledge (knowledge of how, when and why to use learning strategies) and motivational aspects (self-efficacy, task value and goal orientation). Many individuals develop metacognitive and SRL skills and strategies spontaneously through maturation and interaction with others; however, some - especially those from lower SES backgrounds - need to be supported through instruction to develop them. Moreover, instruction can facilitate the development of efficient metacognitive and SRL skills and strategies for all learners. As discussed by Hodgen et al. (2018), there is considerable variation across studies and approaches to such instruction. Donker et al. (2014) found that, while a variety of learning strategies supported primary and post-primary students' performance in reading, writing and mathematics, instruction in metacognitive knowledge, task value and planning emerged as valuable in all subjects - with greater support in their studies for metacognitive knowledge than either task value or planning. Interestingly "rehearsal" was not found to have a significant impact on performance. In a more fine-grained analysis, they found that "elaboration" where students attach meaning to new material by connecting it with older information is particularly effective for the learning of mathematics. Students' explanation of mathematical processes to themselves or their identification of similarities across a number of tasks are examples of elaboration strategies. However, Apthorp et al. (2012) found that, while the effect size for use of similarities/differences was greater when compared to textbook instruction, it was similar to other interactive teaching approaches. As mentioned in the previous section, other effective teaching interventions included assessment of and response to student understanding, the establishment of clear learning targets, the provision of supportive cues, prompting students to reflect, providing feedback etc. - many of which are likely to prompt students to engage in metacognition and SRL (Muijs and Bokhove, 2020). Shin and Bryant's (2015) synthesis of fraction intervention studies for struggling learners also reports significant positive effects for interventions which emphasized reflective processes and student verbalisations. Likewise, Myers at al. (2021) found that cognitive-based instruction interventions were most effective for students with mathematical learning difficulties when compared to other interventions. In these interventions, students were, for example, supported to use a variety of cognitive and metacognitive techniques to formulate plans, select appropriate strategies, and reflect on their thought processes and solutions.

It can be seen that metacognition, SRL and their associated strategies are part of a complex interaction of pedagogical approaches that help to enhance the teaching of mathematics. Some findings on the effect of writing and of homework on mathematics achievement can be explained, at least to some extent, by the findings on metacognition and SRL above and thus, while significant in themselves, are dealt with in this section.

Graham et al. (2020) examined whether students writing about content material in science, social studies, and mathematics facilitated learning. “Writing to learn” which they defined as “the use of writing as a vehicle for strengthening, extending, and deepening students’ knowledge” (p.180) had similar effects at elementary, middle, and high school levels for science, social studies and mathematics. There were no statistically significant differences for moderators they examined such as features of writing activities or of instruction. However, the impact of argumentative writing was more than double that of graphical representation, and writing-to-learn treatments that involved metacognitive prompting, was more than double that of treatments that did not involve such prompting. The authors suggest that, as more studies become available, it should be possible to examine more closely the effects of potential moderators, and interactions among them. However, the relatively strong effect of argumentative writing which would involve an explanation and clarification of one’s thinking corroborates the findings for elaboration above. The effect of metacognitive prompts (described by Peters and Kitsantas (2010, p.27) as “cue(s) for learners to self-monitor and self-evaluate”) in writing-to-learn activities also supplements the findings on the significant link between metacognition and achievement.

A small and positive relationship between homework and academic achievement in math and science was found by Fan et al. (2017). The homework relationship in math/science was shown to be the strongest in the studies involving US students, whereas it was the weakest in the studies involving Asian students, suggesting that cultural context matters in terms of homework. The relationship in math/science was stronger for elementary and high school students than for middle school students. The authors posit that the decline in motivation and academic performance that is associated with the transition from primary school as well as decreased involvement by parents in students’ mathematics homework at the post-primary stage are possible explanatory factors for the findings regarding middle school students. For post-primary students “homework completion” and “homework effort” had more effect than, for example, “time spent on homework” or “frequency of homework”. Hodgen et al (2018) found a stronger effect of homework at secondary than the primary level - however, their evidence base is weak. Moreover, they did not differentiate between lower and upper post-primary students which may be the reason that their finding is different to that of Fan et al.

Notwithstanding this they found that homework is more important for attainment in older students. Active engagement in homework was revealed to be more important than procedural work or rote learning and, as emerged in Fan et al.'s review, students' effort had more effect than time or quantity. Hodgen et al. recommend that students should be given homework that is engaging for them and that encourages metacognitive activity. A focus on effort and motivation - both related to metacognition and SRL - appear to be among important considerations in homework for post-primary students, particularly the younger cohort.

Hodgen et al. (2018) conclude that the development of metacognitive and SRL skills takes time, that the role of the teacher is important (e.g., asking learners to articulate their learning strategies or modelling strategies), and that it seems that in post-primary schools, a focus on learner motivation may be more effective than a focus on cognitive strategies. The importance of professional development in the area is highlighted in the findings of a systematic review of research on primary and post-primary mathematics (as well as reading and programmes for struggling readers conducted by Slavin (2013). He found that programmes providing extensive professional development (PD) in instructional approaches focusing, for example, on metacognitive strategy instruction, cooperative learning and student motivation had more effects on the teaching of mathematics (and indeed reading) than programmes evaluating either curricular reform or computer-assisted instruction.

Conclusions

In this report we focus on the development of numeracy within the subject of mathematics. We have adopted a broad view of numeracy, encompassing the use of mathematical skills and understanding to solve problems and a critical perspective on the use of mathematics in the world. The research question which guided our study was

What are the pedagogical strategies, approaches and methodologies in mathematics that support the development of numeracy at post-primary level?

Due to the lack of studies which investigate numeracy and numeracy development, we identified studies that reported on those pedagogical strategies, approaches and methodologies that have been shown to be effective for post-primary students' general mathematical development. The systematic search of the literature resulted in 28 studies for review, from which we identified pedagogical strategies that sit under four broad themes: *mathematical problem solving; classroom interactions; digital and non-digital tools; and metacognition and self-regulated learning*. Each of these was found to be effective for post-primary students' achievement in and/or attitude to mathematics and their broad mathematical development. In some cases, the effect of some strategies and approaches seemed to be stronger than others. For example, the impact of problem solving and problem posing on mathematical achievement seemed to be strong; however, this was based on a very limited number of meta-analyses and reviews. In the case of classroom interactions, the evidence available is sparse, and effect sizes were either very low or not available. That is not to say that classroom interactions do not play an important role in students' mathematical development. However, they are very difficult to measure experimentally, and are usually studied using qualitative approaches. There was also some variation and inconsistency of findings within each theme, and the strength of the effect on student learning varied. For example, studies focusing on the effects of digital approaches found mixed results. Digital tools designed with a particular emphasis on mathematics were found to have a more positive impact on students' mathematical development than more general tools such as iPads. However, almost all studies called for further research to study the effects of digital tools on mathematics learning. For all of the strategies identified, the active role of the learner in the learning process emerged as highly important. For example, there is strong evidence that metacognition and self-regulation are key to effective learning and, in particular, students should be given an opportunity to elaborate on their thinking (e.g., orally, through diagrams, or in writing) and to reflect on their learning processes. Moreover, the use of tasks and problems that are meaningful to learners is recommended - problems therefore can vary according to the social and cultural context of particular schools and students.

In general, practices that are effective for children who are considered to be "at-risk" due to a variety of factors (e.g., low achievement scores or low SES) were found to be effective for learners who are not at risk and vice-versa. However, a greater level of support might be required for struggling learners. Many of our studies spanned a wide age range (students 4 -18 years), and in some instances effects varied for different age cohorts. For example, concrete materials seem to have a greater impact at primary than at post-primary level. This may be due to the fact that there is a greater tradition of use of such materials with young children and that post-primary students and teachers are not accustomed to drawing on them as a tool for learning. Studies that focused on the use of virtual manipulatives seemed to hold promise, particularly for older post-primary students, but this finding is based on a small number of studies. It is expected that with a greater focus on the use of virtual manipulatives with post-primary students, further findings on the topic will emerge. In some cases, effects of interventions were lower for junior post-primary students than for primary or senior post-primary students, e.g., virtual manipulatives (mentioned above) and homework. This may be due to issues related to the primary/post-primary transition where there can be a decline in students' attitude towards mathematics (Deieso & Fraser, 2019), and a discontinuity in the educational experiences of students (O'Meara et al., 2020).

The review which we conducted highlights that there is no single approach to teaching mathematics at post-primary level that enhances students' capacity to be numerate. Rather pedagogical strategies interact across each of the themes -*mathematical problem solving, classroom interactions, non-digital and digital tools, and metacognition and self-regulated learning.* For example, effective problem solving/problem posing in mathematics requires a high level of interaction in the classroom, the use of various and appropriate non-digital and digital tools, and a focus on student metacognition and self-regulation. Within the themes themselves, strategies are not discrete. For example, it seems that the use of digital and non-digital tools in conjunction with each other is particularly effective for students' mathematical learning; problem-solving activities should give rise to problem posing and vice-versa; classroom discussion is an important aspect of the various collaborative approaches to learning mathematics and so on. The creation of learning environments underpinned by these four themes is undoubtedly challenging for teachers. It calls on a high level of teacher knowledge and professional judgement, and requires significant support at whole-school and systemic levels. However, the teaching of mathematics using the pedagogical strategies, approaches and methodologies identified in this review can support post-primary students to see mathematics as useful in their lives, to be confident in using mathematics as a tool to solve problems and to think critically about the use of mathematics in the world.

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Author Biographies

Dr Paul Grimes is *an Assistant Professor in Mathematics Education in the School of STEM Education, Innovation and Global Studies, Dublin City University and is a member of the CASTeL research centre. He is a qualified post-primary mathematics and physics teacher. He teaches at undergraduate and postgraduate levels in the areas of mathematics education and physics education. He is project coordinator and principal investigator on the Erasmus+ funded Supporting Transitions Across Mathematics and Physics Education (STAMPEd) project. He has published research in the areas of mathematics and physics education, and research interests include the use of design based research in education, discourse and sense making in mathematics and physics, and STEM education at the transition from primary to post-primary schools. More information available* *https://www.dcu.ie/researchsupport/research-profile?person_id=14555*

Dr Thérèse Dooley *is Emeritus Associate Professor in Mathematics Education at DCU Institute of Education. One of the lead authors of the NCCA desktop review of research on mathematics education at early-childhood level, she authored an addendum on primary mathematics. She was a member of the STEM Education Review Group that conducted a comprehensive review of STEM Education in Irish schools, and was Chair of the Tenth Conference of the European Society for Research in Mathematics Education held in Dublin in 2017. Passionate about learners' development of powerful mathematical thinking, she continues to work, research and publish in the area. More information available* *https://www.dcu.ie/researchsupport/research-profile?person_id=14624*

Dr Siún Nic Mhuirí *is an Assistant Professor in Mathematics Education, in the School of STEM Education, Innovation and Global Studies, Institute of Education, DCU. She teaches at undergraduate and postgraduate levels. Siún has been involved in research evaluations which support policy developments as both contributing author and principal investigator. She has conducted research in the area of teaching and learning of mathematics in early childhood and primary education, of specific mathematical strands in upper primary school and on the Maths4All project. Siún is also an active member of CASTeL, and co-leader of a thematic working group (TWG) of the European Society for Research in Mathematics Education. More information available* *https://www.dcu.ie/researchsupport/research-profile?person_id=14630*

Appendix

Research Strategy and Tabulation of Results

Research Question:

What are the pedagogical strategies, approaches and methodologies that support the development of numeracy at post primary level?

Key Data Sources Consulted

Three databases were searched:

(a) EBSCO Education Research Complete

(b) EBSCO ERIC

(c) Scopus.

Sample search strings

S1 DE "STEM education"; S2 STEM Education; S3 S1 OR S2; S4 DE "MATHEMATICS education"; S5 math* education; S6 S4 OR S5; S7 S3 OR S6; S8 math* teaching or numeracy or math* literacy or math*; S9 S7 OR S8; S10 meta-analysis or systematic review and S9; S10 (secondary education or high school or junior high or middle school or secondary school or k6-12 or ks3 or post primary or ks4) AND S9; S11 S10 NOT (higher education or tertiary education); S12 (meta-analysis or systematic review) AND S11

(TITLE-ABS-KEY ("math* education") OR TITLE-ABS-KEY ("numeracy" OR "math* literacy" OR "math*") AND TITLE-ABS-KEY ("secondary education" OR "high school" OR "junior high" OR "middle school" OR "secondary school" OR "k6-12" OR "k8-12" OR "ks3" OR "post primary" OR "ks4" OR "pre tertiary") AND TITLE-ABS-KEY ("meta analysis" OR "systematic review")) AND PUBYEAR > 2010

'Grey' literature was identified through hand searches, for example, Hodgen et al. (2018) and Baker et al. (2015). Reviews found in the grey literature that draw on a substantial number of studies are included in our tabulation.

Exclusion Criteria

Upon full text review, 38 studies were excluded due to the following criteria:

Insufficient connections to the research question, including:

- Insufficient focus on mathematics teaching and learning
- Insufficient focus on pedagogy/pedagogical approaches
- Insufficient focus on mathematics/numeracy
- Insufficient focus on post-primary

Inappropriate study design, including:

- Studies selected from a cultural setting that is too specific or not relevant to the Irish educational context
- Not a systematic review/meta-analysis/literature review

Other studies were excluded because there was insufficient information available about the study.

Summary of Papers Reviewed

Upon screening of abstracts and full texts, 28 studies were selected for inclusion in this review (Figure 1). A summary of these studies, and their key findings, is presented in Table 1.

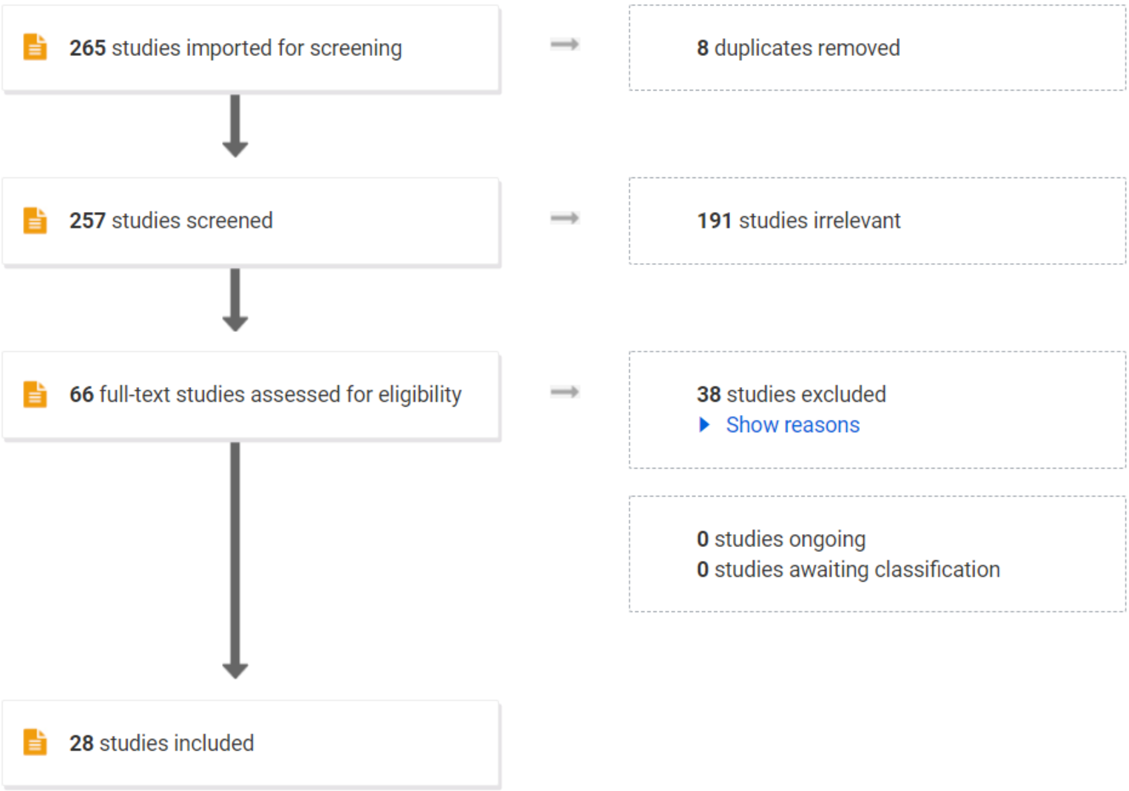

Figure 1. Prisma chart summarising the review process

Table 1.

Overview of studies

Theme	Review	N	Focus	Effect Size	Findings
Mathematical problem solving					
	Cason, M., Young, J., & Kuehnert, E. (2019). A meta-analysis of the effects of numerical competency development on achievement: Recommendations for mathematics educators. Investigations in Mathematics Learning, 11(2), 134-147.	17	Effect of numerical competence on mathematical achievement (K-12)	Mean random ES: 0.881 Numeracy ES: 0.954 Number sense ES: 0.977	Numerical competence classified as math fact fluency, number sense, and numeracy. Effect sizes significantly larger for numeracy and number sense outcome measures compared to math fact fluency. Large effect size for non-intervention studies, moderate effect size for intervention studies. Effect sizes for numerical competence development greatest for grades EC–2 (compared to grades 3–5 and middle grades). Continuous intervention necessary to close gaps that linger beyond the EC–2 grade span.
	Hodgen, J., Foster, C., Marks, R., & Brown, M. (2018). Improving Mathematics in Key Stages Two and Three: Evidence Review. Education Endowment Foundation.	9	The evidence regarding problem solving, inquiry-based learning and related approaches in mathematics (KS2/KS3)	N/A	Need for teachers to provide learners with worked examples; Importance of monitoring, reflecting and discussing problem-solving for students highlighted; Appropriate scaffolding of learners required to support them to solve unstructured problems; Problem solving appropriate for learners at all levels of attainment; Learners to be exposed to many problem types, including those that do not have predefined solution methods.
	Lein, A. E., Jitendra, A. K., & Harwell, M. R. (2020). Effectiveness of mathematical word problem solving interventions for students with learning disabilities and/or mathematics difficulties: A	33	Effect of interventions on mathematical word problem solving skills for students with learning disabilities and/or mathematics difficulties (K-12)	0.56	Mathematics word problem solving interventions for students with learning disabilities (LD) and mathematics difficulties (MD)-students scoring below the 35th percentile- found to have a moderate positive mean effect size; Intervention effects larger for elementary grades than secondary grades; No statistically significant difference based on LD/MD; Interventions focused on underlying problem structure and directly targeted teaching for transfer found to be most effective.

	meta-analysis. Journal of Educational Psychology, 112(7), 1388.				
	Marita, S., & Hord, C. (2017). Review of mathematics interventions for secondary students with learning disabilities. Learning Disability Quarterly, 40(1), 29-40.	12	Mathematics interventions for students with learning disabilities (Middle and High School)	N/A	Interventions involving systematic instruction, problem-based instruction, and visual representation found to be effective for all learners. A variety of interventions found to have positive outcomes for students with LD. Many successful strategies also beneficial for students without LD and more research needs to be conducted on strategies to support secondary-level students with LD in both the middle and high schools.
	McDonald, P. A., & Smith, J. M. (2020). Improving mathematical learning in Scotland's Curriculum for Excellence through problem posing: an integrative review. The Curriculum Journal, 31(3), 398-435.	20	Effect of mathematical problem-posing on mathematics achievement (primary, post-primary and initial teacher education)	N/A	Improvement in mathematical achievement reported for post-primary (and primary) students For post-primary students, improvement in areas of probability, geometry, general mathematical knowledge, and algebra. Effect sizes for students at this level was greater for academic achievement than for engagement, motivation or creativity. Problem-posing interventions found to improve both problem-posing and problem-solving skills Importance of group work and reflection noted Some difficulties encountered, e.g., inability to pose contextualised or sensible problems. Need for teacher professional learning and dialogue highlighted
	Merritt, J., Lee, M. Y., Rillero, P., & Kinach, B. M. (2017). Problem-based learning in K–8 mathematics and science education: A literature review. Interdisciplinary Journal of Problem-Based Learning, 11(2), 3.	9	Effectiveness of problem-based and project-based learning in mathematics and science (K - 8)	N/A	Problem-based learning (PBL) interventions designed in all nine studies seem to be divided into two perspectives: “students as active learners” and “teachers as facilitators.” PBL found to have positive effects on students’ academic achievement, knowledge retention, conceptual development, and attitudes. Limitations: some inconsistency in how PBL is conceptualised in studies; most studies involved students in K6-8; all studies focused on science and not mathematics

	Zhang, D., & Xin, Y. P. (2012). A follow-up meta-analysis for word-problem-solving interventions for students with mathematics difficulties. The Journal of educational research, 105(5), 303-318.	39	K-12	Overall effect for all treatment conditions: 1.848	Interventions implemented in special education settings found to have a smaller effect size than those implemented in inclusive classrooms; No statistically significant difference found between the effect of interventions on normal-achieving compared to students with learning problems; Factors mediating effect sizes included nature of measurement (higher for researcher-designed than for standardised test), and Interventions (‘problem structure representation’ ahead of cognitive strategy training which was assistive technology. Factors not mediating intervention effect size were problem-solving approach, whether algebraic- or arithmetically oriented, and nature of problem tasks (simple-structured vs. real-world problems).
Classroom interactions					
	Anthony, G., & Walshaw, M. (2007). Effective pedagogy in mathematics/Pàngarau: Best evidence synthesis iteration. Wellington, New Zealand: Ministry of Education.	660	Mathematical Communities of Practice (Preschool, primary, secondary, teacher education)	N/A	Teachers must work to develop relationships which support learning and positive mathematical identities. Classroom discussions should involve opportunities for students to articulate their thinking, to explain and justify their solutions, and to receive constructive feedback. Valuing student contributions, establishing norms of participation and revoicing identified as important practices. Groups of four to five tend to be most effective, and should be mixed in relation to academic achievement and any status characteristic. A balance between collaborative and individual work necessary. Explicit instruction a necessary part of mathematics teaching. Possible to promote students' agency for making sense of mathematics within explicit teaching approaches by encouraging them to reflect on the mathematics content and processes. Teacher reflection-in-action, e.g., listening, observing, and questioning for understanding and clarification, central to effective discussions. Robust teacher knowledge (content and

					pedagogical) underpins quality mathematics teaching for diverse learners.
	Apthorp, H. S., Igel, C., & Dean, C. (2012). Using similarities and differences: A meta-analysis of its effects and emergent patterns. School Science and Mathematics, 112(4), 204-216.	12	Effectiveness of using similarities and differences as an instructional strategy in mathematics (K-12)	N/A	Using similarities and differences found to have positive influence on student mathematical achievement; Larger effect sizes associated with control groups receiving textbook-guided instruction versus those receiving interactive instruction; Under other conditions, the effect of facilitating student use of similarities and differences was no greater than that of creating other intellectually engaging, interactive conditions of instruction. Emergent patterns were observed for the positive influence of long-term instruction, systematic instruction, supportive cuing, and opportunity for reflection and discussion. Results support recommendations to guide students through analogical reasoning about, and classification of, important concepts and relationships in content-area instruction. Effect size greater when compared to textbook instruction, but similar to other interactive methodologies.
	Dietrichson, J., Bøg, M., Filges, T., & Klint Jørgensen, A. M. (2017). Academic interventions for elementary and middle school students with low socioeconomic status: A systematic review and meta-analysis. Review of Educational Research, 87(2), 243-282.	101 (25 with a focus on mathematics)	Effective academic interventions (reading and mathematics) for students with low socioeconomic status (SES) (K-8)	0.1 (Mathematics Only)	Many interventions found to result in substantial positive gains in achievement of low SES students on standardised tests. Similar effect sizes for interventions in reading and mathematics. Analysis across reading and mathematics interventions indicates statistically significant, robust effects for tutoring (0.36), feedback and progress monitoring (0.32), and cooperative learning (0.22). Tutoring interventions involve pedagogical support from an instructor (one-to-one or in a small group); Feedback and progress monitoring involve customizing teaching to the individual student's needs; Cooperative learning involves students working together in pairs or small groups in a systematic , structured manner.
	Hodgen, J., Foster, C., Marks, R., & Brown, M. (2018). Improving Mathematics in Key Stages Two and Three: Evidence	3 research syntheses	Evidence regarding the effective use of discussion in teaching	N/A	Strength of evidence low. Much classroom discourse follows the initiation-response-evaluation (IRE) pattern. Increased wait time, the time a teacher pauses after asking a question before accepting learner responses, found to increase quality of discussion. A need for teachers to move from questioning only

	Review. Education Endowment Foundation.		and learning mathematics (KS2/KS3)		for evaluation, to focusing discussion on exploration of mathematical ideas. Need to teach students how to listen to others and how to engage in discussion.
	Hodgen, J., Foster, C., Marks, R., & Brown, M. (2018). Improving Mathematics in Key Stages Two and Three: Evidence Review. Education Endowment Foundation.	6 meta-analyses	Evidence regarding the effective use of collaboration in teaching and learning mathematics (KS2/KS3)	N/A	Strength of evidence high but most studies US-based and often associated with particular intervention programmes. Variability in how collaborative learning is defined. Some studies report higher effect sizes for second level students than elementary or middle grades, and lower effects for students with learning difficulties. Analysis indicates that students need to learn how to collaborate effectively.
	Hodgen, J., Foster, C., Marks, R., & Brown, M. (2018). Improving Mathematics in Key Stages Two and Three: Evidence Review. Education Endowment Foundation.	4 meta-analyses	Evidence regarding the effective use of formative assessment and feedback in teaching and learning mathematics (KS2/KS3)	N/A	Strength of evidence high. Findings generally indicate feedback has a large effect on learning, but some studies show negative effects. Formative assessment found to have more modest effects, and found to more effective when teachers receive professional development or feedback is delivered through computer-assisted instruction. Mathematics-specific studies indicate that aspects of formative assessment that involve feedback particularly important. Recommendation that (i)existing literature on learners' understandings and misconceptions be used to inform assessment and (ii) feedback is reserved for use in complex tasks to support students' perseverance.
	Hodgen, J., Foster, C., Marks, R., & Brown, M. (2018). Improving Mathematics in Key Stages Two and Three: Evidence Review. Education Endowment Foundation.	10 meta-analyses	Evidence regarding the effective use of peer and cross-age tutoring in teaching and learning mathematics (KS2/KS3)	N/A	Strength of evidence medium. Evidence of benefits to tutors, tutees and teachers. Variable, non-negative effects found for peer-tutoring. Higher effects found for cross-age tutoring, where an older student works with a younger student, but a more limited evidence-base than for peer-tutoring. Caution needed for peer-tutoring interventions with very low achievers or students with learning disabilities. Structure of tutoring and support for tutors important with suggestion that mathematics-focussed training, rather than general approach, may be of benefit.
	Kyriacou, C., and J. Issitt. 2008. What Characterises	15	Characteristics of effective teacher-pupil	N/A	Strongest evidence in interventions where teachers taught pupils how to make use of dialogue. Other characteristics of effective

	Effective Teacher-Initiated Teacher-Pupil Dialogue to Promote Conceptual Understanding in Mathematics Lessons in England in Key Stages 2 and 3: A Systematic Review. London: EPPI-Centre, Social Science Research Unit, Institute of Education, University of London		dialogue for the promotion of conceptual understanding (KS2 and KS3)		dialogue proposed but as of yet limited evidence of their effects: moving beyond Initiation-Response-Feedback patterns; working collaboratively with students; transformative listening; scaffolding; enhancing pupils' self-knowledge of how to make use of teacher-pupil dialogue as a learning experience; encouraging high quality pupil dialogue; and inclusive teaching.
	Merritt, J., Lee, M. Y., Rillero, P., & Kinach, B. M. (2017). Problem-based learning in K–8 mathematics and science education: A literature review. Interdisciplinary Journal of Problem-Based Learning, 11(2), 3.	9	Effectiveness of problem-based and project-based learning in mathematics and science (K - 8)	N/A	Problem-based learning (PBL) found to have positive effects on students' academic achievement, knowledge retention, conceptual development, and attitudes. Components of PBL interventions included: collaborative problem-solving in small groups (all studies), and communication of their findings to whole class (3 out of 9 studies). Limitations: some inconsistency in how PBL is conceptualised in studies; most studies involved students in K6-8; all studies focused on science and not mathematics
	Wexler, J., Reed, D. K., Pyle, N., Mitchell, M., & Barton, E. E. (2015). A synthesis of peer-mediated academic interventions for secondary struggling learners. Journal of Learning Disabilities, 48(5), 451-470.	13 (1 focused on mathematics)	Effects of peer-mediated reading and mathematics interventions for low-achieving adolescents (Grades 6-12)	N/A	A variety of intervention types described-no identifiable associations with effectiveness identified across duration, frequency, and session length. Some similarities across interventions noted, e.g., reciprocal tutoring where students alternate roles as tutor and tutee, used in many interventions. Peer mediation found to have mostly moderate to high effects, particularly when implementing a peer-mediated feedback component. Calls for further research on peer-mediated mathematics interventions, peer-mediated interventions in alternative settings, and effective ways to pair students so as to incorporate a structured feedback component.
Non-digital and digital tools					

	Boon, H. J., Boon, L., & Bartle, T. (2021). Does iPad use support learning in students aged 9–14 years? A systematic review. The Australian Educational Researcher, 48(3), 525-541.	43	Ages 9-14 years	N/A	43 studies were reviewed, however information in the study indicates that 8 of these had a focus on mathematics. Findings related to mathematics were mixed. Participants in studies generally reported that iPad use improved mathematics learning. The findings of 2 quasi-experimental studies did not support iPad use for mathematics
	Byun, J., & Joung, E. (2018). Digital game-based learning for K–12 mathematics education: A meta-analysis. School Science and Mathematics, 118(3-4), 113-126.	17	K-12	0.37	The overall weighted effect size is 0.37. This result implies that there may be other ways for students to learn mathematics more effectively than DGBL, although the DGBL studies have shown statistically positive effects on students' learning mathematics
	Carbonneau, K. J., Marley, S. C., & Selig, J. P. (2013). A meta-analysis of the efficacy of teaching mathematics with concrete manipulatives. Journal of Educational Psychology, 105(2), 380.	55	K-College	Aggregated mean ES 0.37	The average effect of using concrete manipulatives in mathematics instruction was found to be small to moderate. The Level of instructional guidance, mathematical topic, development status, perceptual richness, and instructional time were statistically significant moderators of the effects of using concrete manipulatives
	Chan, K. K., & Leung, S. W. (2014). Dynamic geometry software improves mathematical achievement: Systematic review and meta-analysis. Journal of Educational Computing Research, 51(3), 311-325.	9	Elementary-High School	1.02	The findings of the review indicate that DGS based instruction produces a positive and large effect size on mathematics achievement (1.02). Higher effect size than effect sizes found in other studies looking at technology based programmes or educational technology

	Holmes, A. B. (2013). Effects of Manipulative Use on PK-12 Mathematics Achievement: A Meta-Analysis. Society for Research on Educational Effectiveness.	21	Pk-12	0.22 (the effect of manipulative use to non-use); 0.20 (the effect of digital manipulative use compared to physical manipulatives)	Overall, the use of manipulatives was found to have a small to moderate effect. The effect of using manipulatives compare to their non-use (14 studies) was found to be 0.22. When comparing the use of digital manipulatives to physical manipulatives (7 studies) the effect was found to be 0.20.
	Moyer-Packenham, P. S., & Westenskow, A. (2013). Effects of virtual manipulatives on student achievement and mathematics learning. International Journal of Virtual and Personal Learning Environments, 4(3), 35-50.	32	Pk-University	See findings	The review investigated the effects of manipulative use compared to other instructional treatments. The effect sizes found were: Virtual manipulatives (used alone or in combination) vs. all other instructional treatments: 0.35; Virtual manipulatives (only) vs. other instructional treatments: 0.34; Virtual manipulatives (only) vs. physical manipulatives: 0.15; Virtual manipulatives (only) vs. classroom instruction using textbooks: 0.75; Virtual and physical manipulatives (VM/PM combined) vs. other instructional treatments: 0.33; VM/PM combined vs. virtual manipulatives (only): 0.26; VM/PM combined vs. physical manipulatives (only): 0.20; VM/PM combined vs. classroom instruction using textbooks yielded: 0.69
	Shin, M., & Bryant, D. P. (2015). Fraction interventions for students struggling to learn mathematics: A research synthesis. Remedial and Special Education, 36(6), 374-387.	17	Research synthesis of effectiveness of fraction interventions for students struggling to learn mathematics (Grade 3- 12)	N/A	Interventions consisting of evidence-based instructional components (e.g., concrete and visual representations; explicit, systematic instruction; range and sequence of examples; heuristic strategies; and use of real-world problems) led to improved performance on measures with fraction concepts and skills. Large effect for visual representations with some studies providing evidence of greater effects for a Concrete-Representational- Abstract approach when compared to only representational-abstract. Highly positive outcomes for studies using concrete and visual representations in combination with explicit, systematic instruction involving guided practice,

					corrective feedback and cumulative review. Improved problem-solving and fraction computation skills observed in studies where heuristic strategies were used (promoting mathematical verbalisation with “think-aloud” approach or in combination with virtual manipulatives). Some evidence that interventions of longer durations may be beneficial for struggling students to better learn fraction concepts.
	Tokac, U., Novak, E., & Thompson, C. G. (2019). Effects of game-based learning on students' mathematics achievement: A meta-analysis. Journal of Computer Assisted Learning, 35(3), 407-420.	24	Meta-analysis investigated the effects of learning video games on mathematics achievement of PreK-12th-grade students compared with traditional classroom instructional methods.	0.13	A small but marginally significant effect size may suggs that video games may contribute to higher learning games compared to transitional instruction. The authors call for more research to further understand how video games can enhance mathematics learning.
Metacognition and self-regulated learning					
	Apthorp, H. S., Igel, C., & Dean, C. (2012). Using similarities and differences: A meta-analysis of its effects and emergent patterns. School Science and Mathematics, 112(4), 204-216.	12	Effectiveness of using similarities and differences as an instructional strategy in mathematics (K-12)	0.65	Using similarities and differences found to have positive influence on student mathematical achievement; Larger effect sizes associated with control groups receiving textbook-guided instruction versus those receiving interactive instruction; emergent patterns observed for the positive influence of long-term instruction, systematic instruction, supportive cuing, and opportunity for reflection and discussion.
	Donker, A. S., De Boer, H., Kostons, D., Van Ewijk, C. D., & van der Werf, M. P. (2014). Effectiveness of learning strategy instruction	58 articles (including 95 strategy-interventions)	Effect of learning strategy instruction which focused on improving self-regulated learning on	0.66	Interventions which included ‘general metacognitive knowledge’, ‘planning’ or ‘task value’ most effective in enhancing student performance; In mathematics elaboration, a substrategy of cognitive strategies and which involves students explaining strategy to themselves, or finding similarities, the

	on academic performance: A meta-analysis. Educational Research Review, 11, 1-26.		academic performance in comprehensive reading, writing texts, mathematics and science (Primary and post-primary)		only substrategy which improved student performance significantly more than other methods; learning strategy interventions found to benefit all students (regardless of age or level of academic achievement); For mathematics, no significant difference in type of test (self-developed or intervention-dependent) used
	Fan, H., Xu, J., Cai, Z., He, J., & Fan, X. (2017). Homework and students' achievement in math and science: A 30-year meta-analysis, 1986–2015. Educational Research Review, 20, 35-54.	28	Homework/achievement relationship in mathematics and science (Elementary , middle and high school)	0.221	Larger effect size for US studies than studies emanating from Asia; Effect strongest for elementary and weakest for middle school students; Effect sizes for 'homework completion', 'homework effort' and 'homework grade' larger than 'time spent on homework' or 'frequency of homework'.
	Graham, S., Kiuhara, S. A., & MacKay, M. (2020). The effects of writing on learning in science, social studies, and mathematics: A meta-analysis. Review of Educational Research, 90(2), 179-226.	56 studies in 53 documents	Effect of 'writing-to-learn' on student learning (Grades 1- 12)	0.30 (.31, science; .31 social studies, .32 mathematics)	Positive effect found at elementary, middle and high school levels; No statistically significant difference by content area (science, social studies, and mathematics), grade (elementary, middle, and high school school), features of writing activities (e.g., promotion of analysis and interpretation), features of instruction (e.g., professional development), features of assessments (e.g., type of test), or study quality; Impact of argumentative writing was more than double at least one of the other types of writing, graphical representation - graphical representation and no metacognitive prompting failed to produce statistically significant effects.
	Hodgen, J., Foster, C., Marks, R., & Brown, M. (2018). Improving Mathematics in Key Stages Two and Three: Evidence Review. Education Endowment Foundation.	6 meta-analyses (233 studies)	Effect of teaching thinking skills, metacognition and/or self-regulation on mathematics learning (KS2/KS3)	N/A	Teaching thinking skills, metacognition and self-regulation can be effective in mathematics; A great deal of variation is found across studies; development of thinking skills, metacognition and self-regulation takes time; Role of the teacher is important; Strategies that encourage self-explanation and elaboration appear to be beneficial; Some evidence that , in secondary, focus on learner motivation is more important than focus on cognitive strategies.

	Myers, J. A., Brownell, M. T., Griffin, C. C., Hughes, E. M., Witzel, B. S., Gage, N. A., Peyton, D., Acosta, K., & Wang, J. (2021). Mathematics Interventions for Adolescents with Mathematics Difficulties: A Meta-Analysis. Learn. Disabil. Res. Pract., 36(2), 145–166.	45	Effect of mathematics interventions for students with mathematics difficulties (Grades 6 - 12)	0.52	Interventions for students with mathematics difficulties (e.g., low achievement and/or learning disabilities) found to have moderately large effect on achievement, but substantial between-study heterogeneity noted. 21 studies conducted in general education classrooms, the remainder in alternate locations. Fraction interventions associated with significantly smaller effects than studies focused on students' performance in multiple domains. Larger effects for long-term (more than 30 sessions) than short-term interventions (less than seven sessions). Significant mean effects found for cognitive-based instruction (CBI), technology-based interventions (TBI), and visual representations (VR), with CBI yielding the largest mean effect size. CBI interventions involved supporting students to use cognitive and metacognitive strategies to formulate plans, select appropriate techniques, create visual representations, reflect on their thought process, and evaluate their solutions in solving problems involving numbers and operations.
	Shin, M., & Bryant, D. P. (2015). Fraction interventions for students struggling to learn mathematics: A research synthesis. Remedial and Special Education, 36(6), 374-387.	17	Research synthesis of effectiveness of fraction interventions for students with low achievement and/or learning disabilities (Grade 3- 12)	N/A	Interventions consisting of evidence-based instructional components (e.g., concrete and visual representations; explicit, systematic instruction; range and sequence of examples; heuristic strategies; and use of real-world problems) led to improved performance on measures with fraction concepts and skills. Large effect for visual representations with some studies providing evidence of greater effects for a Concrete-Representational- Abstract approach when compared to only representational-abstract. Highly positive outcomes for studies using concrete and visual representations in combination with explicit, systematic instruction involving guided practice, corrective feedback and cumulative review. Improved problem-solving and fraction computation skills observed in studies where heuristic strategies were used (promoting mathematical verbalisation with "think-aloud" approach or in combination with virtual manipulatives). Some evidence that interventions of

					longer durations may be beneficial for struggling students to better learn fraction concepts.
	Slavin, R. E. (2013). Effective programmes in reading and mathematics: lessons from the Best Evidence Encyclopaedia. School Effectiveness and School Improvement, 24(4), 383-391.	346	Principles of effective practice in reading and mathematics (Primary/Secondary)	0.26	Programmes providing extensive professional development in well-structured methods such as cooperative learning and teaching of metacognitive skills produced much more positive effect sizes than those evaluating either curricular reforms or computer-assisted instruction; Latter two can support or supplement changes in teaching practices, but do not have important effects on learning in themselves