An Application of Inductive *Anvaya*

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Abstract

In this note, the author uses a combination of mathematical induction and *anvaya* from Indian logic in order to deduce properties of a list of objects. The objects are of a specific, 'nested' type. For these objects, it is shown that for any object in the list, the object inheres in the previous one.

Nyaya is the Indian system of logic [1, 2, 3]. It has some features which are unique to it and not present in modern Western logic. In recent years there have been substantial developments in the understanding, interpretation and analysis of nyaya [4, 5, 6, 7]. In this note, we aim to produce a proof that sheds some light on certain types of objects. Specifically, we use a combination of mathematical induction and *anvaya* in order to deduce properties of a list of objects. The objects considered are of a 'nested' or 'layered' type. For these objects, it is shown that the n-th object inheres in the previous one, for any n.

Consider n objects $A_1, ..., A_n$ formed as unions of other objects. The objects are formed in a nested manner. This is clarified in the following definition:

Definition 1. (Nested Objects)

$$A_l = \xi_l \bigcup \left[\bigcup_{i=1}^{l-1} A_i\right] \tag{1}$$

Thus, if we write out the objects, they would look as follows,

$$A_1 = \xi_1, \tag{2}$$

$$A_2 = \xi_2 \bigcup A_1 \tag{3}$$

and the general object can be written as,

$$A_m = \xi_m \bigcup \xi_{m-1} \bigcup ... \bigcup \xi_1. \tag{4}$$

Using this definition, we are able to state and prove the following theorem.

Theorem 2. *If the objects A_2 and A_1 are such that the former inheres in the latter, then the general object A_n inheres in A_{n-1} for all n.*

Proof. Suppose we have n objects, $A_1, A_2, ..., A_n$. We first observe that A_2 inheres in A_1. We treat (A_1, A_2) as the base case. Consider next, the k-th case, (A_k, A_{k+1}) and assume that A_{k+1} inheres in A_k. Next, we want to show that whenever A_{k+1} inheres in A_k, A_{k+2} inheres in A_k.

That is we want to show that (A_k, A_{k+1}) is concomitant with (A_{k+1}, A_{k+2}). That is, we want to show a meta-*anvaya*. How does one show that an object inheres in another? One would need to know if they are concomitant, as per the *anvaya* rule. Remarkably, here we have to show only a meta-*anvaya*. And this is clear, since both (A_k, A_{k+1}) and (A_{k+1}, A_{k+2}) contain A_{k+1}, that is the link of concomitance. Thus the latter inheres in the former.

Further, if the pair (A_{k+1}, A_{k+2}) inheres in the pair (A_k, A_{k+1}), then every member of the former pair inheres in the latter pair. In particular, A_{k+2} inheres in (A_k, A_{k+1}). This means that A_{k+2} is dependent on the presence of both A_k and A_{k+1}. But A_{k+1} inheres in A_k. So it is sufficient to say that A_{k+2} inheres in A_k.

Coming to the specific form of the objects, the k-th case tells us that $\xi_{k+1} \bigcup \xi_k \bigcup ... \bigcup \xi_1$ inheres in $\xi_k \bigcup \xi_{k-1} \bigcup ... \bigcup \xi_1$. And we have shown that $\xi_{k+2} \bigcup \xi_{k+1} \bigcup ... \bigcup \xi_1$ inheres in $\xi_k \bigcup \xi_{k-1} \bigcup ... \bigcup \xi_1$. However, since $\xi_{k+1} \bigcup \xi_k \bigcup ... \bigcup \xi_1 = \xi_{k+1} \bigcup (\xi_k \bigcup ... \bigcup \xi_1)$, therefore by transitivity of inherence, A_{k+2} inheres in A_{k+1}. Thus, the general statement is proved for all n as required. □

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