In the following function, we construct the residual needed for time integration of second-order reduced system

$$\underbrace{\mathbf{V}^\top\mathbf{M}\mathbf{V}}_{\mathbf{M}_\mathbf{V}}\ddot{\mathbf{q}} + \underbrace{\mathbf{V}^\top\mathbf{C}\mathbf{V}}_{\mathbf{C}_\mathbf{V}}\dot{\mathbf{q}} + \underbrace{\mathbf{V}^\top\mathbf{K}\mathbf{V}}_{\mathbf{K}_\mathbf{V}}\mathbf{q} = \mathbf{V}^\top\mathbf{F}_{ext}(t),$$

where $\mathbf{V} \in \mathbb{R}^{n\times m}$ is the reduction basis. We use the residual is defined as

$$\mathbf{r}(\ddot{\mathbf{q}}, \dot{\mathbf{q}}, \mathbf{q}) = \mathbf{M}_\mathbf{V}\ddot{\mathbf{q}} + \mathbf{C}_\mathbf{V}\dot{\mathbf{q}} + \mathbf{K}_\mathbf{V}\mathbf{q} - \mathbf{V}^\top\mathbf{F}_{ext}(t).$$

A generic Residual function, whose *handle* is passed for performing Implicit Newmark and Generalized-*α* time integrations in this code has the following syntax for linear systems:

```
[r, drdqdd, drdqd, drdq] = Residual(q,qd,qdd,t);
```

where

$$\begin{aligned} \texttt{r} &= \mathbf{r}, \\ \texttt{drdqdd} &= \frac{\partial \mathbf{r}}{\partial \ddot{\mathbf{q}}}, \\ \texttt{drdqd} &= \frac{\partial \mathbf{r}}{\partial \dot{\mathbf{q}}}, \\ \texttt{drdq} &= \frac{\partial \mathbf{r}}{\partial \mathbf{q}} \end{aligned}$$

The extra arguments:

1. `Assembly`, which is an instance of ReducedAssembly class
2. `Fext`, which is a function handle for the external forcing on the full structure,
3. `V` is the reduction basis $\mathbf{V}$

are required for computing the residual in this case.

New residual functions that follow the above-mentioned syntax can be written according to user preference. This way, the same time integration class can be used to solve a variety of problems.

Please refer to the Mechanical directory in the examples folder to understand applications and usage.

```
function [ r, drdqdd, drdqd, drdq] = residual_reduced_linear( q, qd, qdd, t, Assembly, Fext)
```

In this function, it is assumed that the matrices $\mathbf{M}_\mathbf{V}, \mathbf{C}_\mathbf{V}, \mathbf{K}_\mathbf{V}$ for the finite element mesh were precomputed and stored in the `DATA` property of the `Assembly` object to avoid unnecessary assembly during each time-step. Note that this function may be modified appropriately to allow for adaptive selection. i.e., when the basis changes online during time integration.

```
M_V = Assembly.DATA.M;
C_V = Assembly.DATA.C;
K_V = Assembly.DATA.K;
F_V = Assembly.V.' * Fext(t);
```

Residual is computed according to the formula above:

```
r = M_V * qdd + C_V * qd + K_V * q - F_V ;

drdqdd = M_V;
drdqd = C_V;
drdq = K_V;
end
```