Appendix: Frequency response analysis with NLvib

Harmonic Balance formulation

solve $\boldsymbol{R}(\boldsymbol{X}) = \begin{bmatrix} \boldsymbol{R}_0 \\ \Re\{\boldsymbol{R}_1\} \\ \Im\{\boldsymbol{R}_1\} \\ \vdots \\ \Im\{\boldsymbol{R}_H\} \end{bmatrix} = \boldsymbol{0}$

where $\boldsymbol{R}_k = \left[-(k\omega)^2\boldsymbol{M} + \mathrm{i}k\omega\boldsymbol{D} + \boldsymbol{K}\right]\boldsymbol{Q}_k + \boldsymbol{F}_{\mathrm{nl},k} - \boldsymbol{F}_{\mathrm{ex},k}$

with respect to $\boldsymbol{X} = \begin{bmatrix} \boldsymbol{Q}_0^{\mathrm{T}} & \Re\{\boldsymbol{Q}_1^{\mathrm{T}}\} & \Im\{\boldsymbol{Q}_1^{\mathrm{T}}\} & \dots & \Im\{\boldsymbol{Q}_H^{\mathrm{T}}\} & \Omega \end{bmatrix}^{\mathrm{T}}$

in the interval $\Omega^{(\mathrm{s})} \leq \Omega \leq \Omega^{(\mathrm{e})}$

Appendix: Frequency response analysis with NLvib

Shooting formulation

solve $\boldsymbol{R}(\boldsymbol{X}) = \begin{bmatrix} (\boldsymbol{q}(T) - \boldsymbol{q}_0)\,\frac{1}{q_{\mathrm{scl}}} \\ (\boldsymbol{u}(T) - \boldsymbol{u}_0)\,\frac{1}{q_{\mathrm{scl}}} \end{bmatrix} = \boldsymbol{0}$

with respect to $\boldsymbol{X} = \begin{bmatrix} \boldsymbol{q}_0^{\mathrm{T}} & \frac{\boldsymbol{u}_0^{\mathrm{T}}}{\Omega} & \Omega \end{bmatrix}^{\mathrm{T}}$

in the interval $\Omega^{(\mathrm{s})} \leq \Omega \leq \Omega^{(\mathrm{e})}$

where $\boldsymbol{q}(T), \boldsymbol{u}(T)$ are determined by forward numerical integration

q_{scl} *positive real-valued scalar*

Rationale behind scaling of residual: achieve similar orders of magnitude for quite different vibration levels. Otherwise the solver might misinterpret e.g. a small value as a converged residual.

Appendix: Nonlinear modal analysis with NLvib

Harmonic Balance formulation

f_{scl} *positive real-valued scalar*

solve
$$\boldsymbol{R}(\boldsymbol{X}) = \begin{bmatrix} \boldsymbol{R}_0 \, \frac{f_{\mathrm{scl}}}{a} \\ \Re\{\boldsymbol{R}_1\} \, \frac{f_{\mathrm{scl}}}{a} \\ \Im\{\boldsymbol{R}_1\} \, \frac{f_{\mathrm{scl}}}{a} \\ \vdots \\ \Im\{\boldsymbol{R}_H\} \, \frac{f_{\mathrm{scl}}}{a} \\ \Re\{\sum\limits_{k=0}^{H} \boldsymbol{Q}_k^{\mathrm{H}} \boldsymbol{M} \boldsymbol{Q}_k\}/a^2 - 1 \\ \dot{q}_{i_{\mathrm{norm}}}(0)/(\omega a) \end{bmatrix} = \boldsymbol{0}$$

amplitude normalization

phase normalization

where
$$\boldsymbol{R}_k = \left[-(k\omega)^2 \boldsymbol{M} + \mathrm{i}k\omega \left(\boldsymbol{D} - 2\delta\omega \boldsymbol{M}\right) + \boldsymbol{K}\right] \boldsymbol{Q}_k + \boldsymbol{F}_{\mathrm{nl},k}$$

with respect to
$$\boldsymbol{X} = \begin{bmatrix} \frac{\boldsymbol{Q}_0^{\mathrm{T}}}{a} & \Re\{\frac{\boldsymbol{Q}_1^{\mathrm{T}}}{a}\} & \Im\{\frac{\boldsymbol{Q}_1^{\mathrm{T}}}{a}\} & \ldots & \Im\{\frac{\boldsymbol{Q}_H^{\mathrm{T}}}{a}\} & \omega & \delta & \log_{10} a \end{bmatrix}^{\mathrm{T}}$$

in the interval
$$\log_{10} a^{(\mathrm{s})} \leq \log_{10} a \leq \log_{10} a^{(\mathrm{e})}$$

Rationale behind scaling of residual: achieve similar orders of magnitude of typical values. Otherwise the dynamic force equilibrium or the normalization conditions would have unreasonably strong weight, which could have a negative influence the convergence of the solver.

Appendix: Nonlinear modal analysis with NLvib

Shooting formulation

solve $\boldsymbol{R}(\boldsymbol{X}) = \begin{bmatrix} (\boldsymbol{q}(T) - \boldsymbol{q}_0)\frac{1}{q_{\mathrm{scl}}} \\ (\boldsymbol{u}(T) - \boldsymbol{u}_0)\frac{1}{q_{\mathrm{scl}}} \end{bmatrix} = \boldsymbol{0}$

with respect to $\boldsymbol{X} = \begin{bmatrix} \frac{\boldsymbol{q}_{0-}^{\mathrm{T}}}{a} & \frac{\boldsymbol{u}_{0-}^{\mathrm{T}}}{\omega a} & \omega & D & \log_{10} a \end{bmatrix}^{\mathrm{T}}$

in the interval $\log_{10} a^{(\mathrm{s})} \leq \log_{10} a \leq \log_{10} a^{(\mathrm{e})}$

where $\boldsymbol{q}_0, \boldsymbol{u}_0$ are $\boldsymbol{q}_{0-}, \boldsymbol{u}_{0-}$ only with $q_{0,i_{\mathrm{norm}}} = a$ — *amplitude normalization*

$u_{0,i_{\mathrm{norm}}} = 0$ — *phase normalization*

where $\boldsymbol{q}(T), \boldsymbol{u}(T)$ are determined by forward numerical integration

q_{scl} *positive real-valued scalar*

Rationale behind scaling of residual: achieve similar orders of magnitude for quite different vibration levels. Otherwise the solver might misinterpret e.g. a small value as a converged residual.