Signatures of gluon saturation from structure-function measurements arXiv:2203.05846

DIS2022 WG6

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- In collinear factorization framework the Q^2 evolution comes from Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution equations
- To see saturation effects on experimental data we have to distinguish the genuine difference between DGLAP and BK dynamics
- Both frameworks require input which are fitted to the same experimental data $\longrightarrow$ The results do not deviate dramatically and the distinguishing DGLAP/BK evolution is difficult

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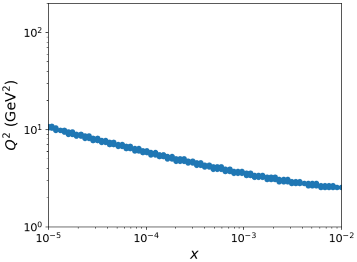

Matching line in (x, Q^2) plane

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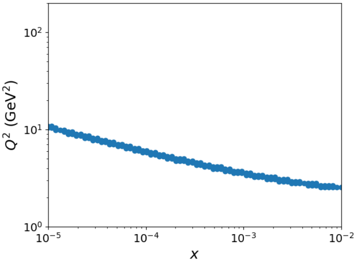

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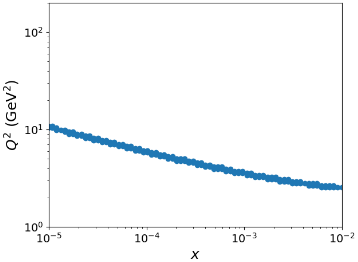
4. With differences we can approximate the accuracy of $F_{2,\mathrm{L}}$ saturation measurements in EIC and LHeC/FCC-he



Matching line in (x, Q^2) plane

$F_{2,\mathrm{L}}$ with collinear factorization vs. CGC

Collinear factorization:

- Collinear factorization $F_{2,\mathrm{L}}$ using APFEL [1] and LHAPDF [2] libraries
- `NNPDF31_nlo_as_0118_1000` as proton PDF set
- `nNNPDF20_nlo_as_0118_Au197` as nuclear PDF set
- Both PDF sets have 1000 Monte Carlo replicas

Color Glass Condensate (CGC):

- Dipole picture $F_{2,\mathrm{L}}$ fitted to HERA data
- Leading order total photon-nucleus cross sections
- Running coupling BK evolution [1]

- We match collinear factorization $F_{2,\mathrm{L}}$ to corresponding CGC structure functions in a line in (x, Q^2) plane

[1] T. Lappi and H. Mäntysaari. "Single inclusive particle production at high energy from HERA data to proton-nucleus collisions". In: *Phys. Rev. D* 88 (2013), p. 114020. arXiv: 1309.6963 [hep-ph]

PDF matching

Bayesian reweighting method [4, 5]:

For each PDF replica f_k we define

$$\chi_k^2 = \sum_{i=1}^{N_{\mathrm{data}}} \frac{(\mathcal{O}_i - \mathcal{O}_i[f_k])^2}{(\delta_{\mathrm{BK}}\mathcal{O}_i)^2}$$

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and so called **Giele-Keller** weights [6]

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Then we define reweighted observables as

$$\mathcal{O}^{\mathrm{Rew}} = \frac{1}{N_{\mathrm{rep}}}\sum_{k=1}^{N_{\mathrm{rep}}} \omega_k \mathcal{O}[f_k]$$

We also construct a PDF set matched to BK in (x, Q^2) line (Back up)

Fixing matching parameters

- We want to match the reweighted values to BK values as closely as possible
  - Finite number of replicas (1000) prevent the absolute match
- Effective number of replicas [4, 7]

$$N_{\text{eff}} = \exp \frac{1}{N_{\text{rep}}} \sum_{k=1}^{N_{\text{rep}}} \omega_k \ln \left(\frac{N_{\text{rep}}}{\omega_k} \right)$$

gives an approximation on how many PDF replicas have significant weight

- We adjust δ_{BK} in χ_k^2 in order to fix $N_{\text{eff}} \approx 10$

$$\chi_k^2 = \sum_{i=1}^{N_{\text{data}}} \frac{(y_i - y_i[f_k])^2}{(\delta_{\text{BK}} y_i)^2}$$

$$\omega_k = \frac{e^{-\frac{1}{2}\chi_k^2}}{\frac{1}{N_{\text{rep}}} \sum_{k=1}^{N_{\text{rep}}} e^{-\frac{1}{2}\chi_k^2}}$$

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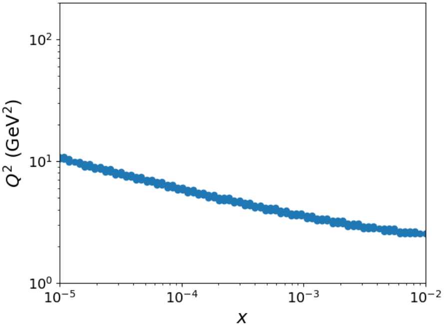
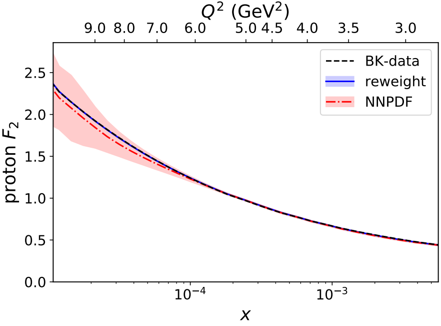
$\longrightarrow$ **We choose to do the matching on points** $Q^2(x) \approx 10 \times Q_s^2(x)$

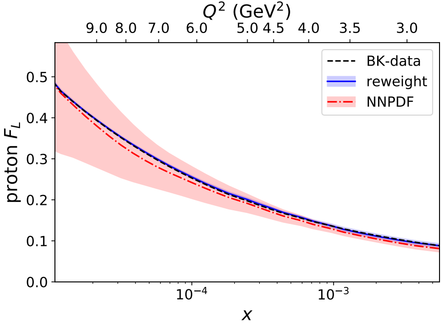


Matching line in (x, Q^2) plane

Proton matching



(a) F_2

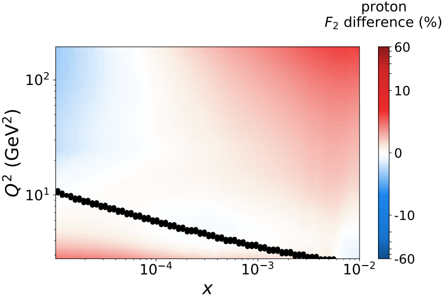


(b) F_L

The structure functions for proton as a function of x at $Q^2 \approx 10 Q_s^2(x)$

- Separate matching for proton F_2 and F_L are both almost perfect

Relative difference of proton F_2^{Rew} to F_2^{BK}



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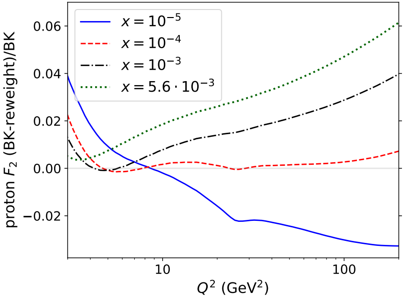
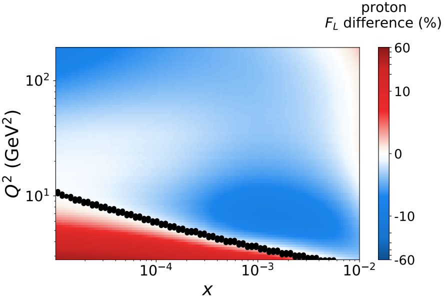


(b) F_2

Relative difference $(F_2^{\mathrm{BK}} - F_2^{\mathrm{Rew}})/F_2^{\mathrm{BK}}$

- For proton F_2 the relative difference is only a few percent
- Generically slower x dependence in BK evolution

Relative difference of proton F_L^{Rew} to F_L^{BK}



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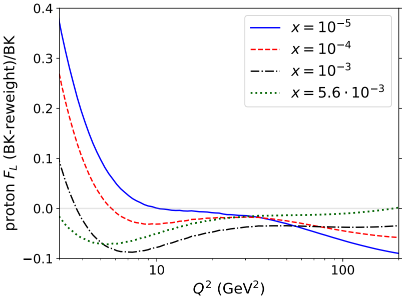
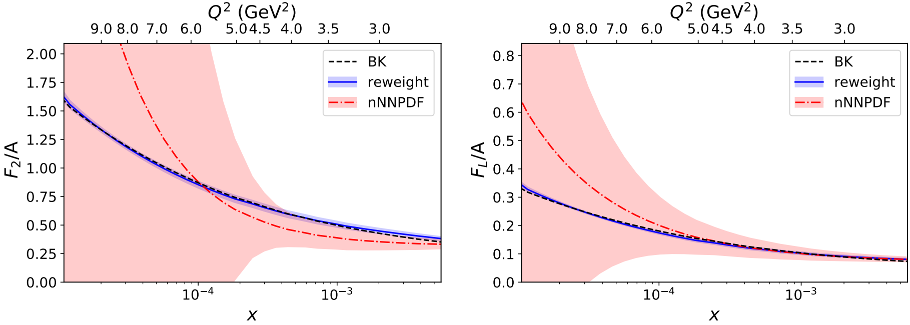


(b) F_L

The relative difference $(F_L^{\mathrm{BK}} - F_L^{\mathrm{Rew}})/F_L^{\mathrm{BK}}$.

- For proton F_L the relative difference is:
  - $\lesssim 10\%$ for $x = 10^{-3} ... 5.6 \times 10^{-3}$ (EIC)
  - $\lesssim 40\%$ for $x = 10^{-5} ... 10^{-4}$ (LHeC/FCC-he)
- F_L is much more sensitive to saturation than F_2

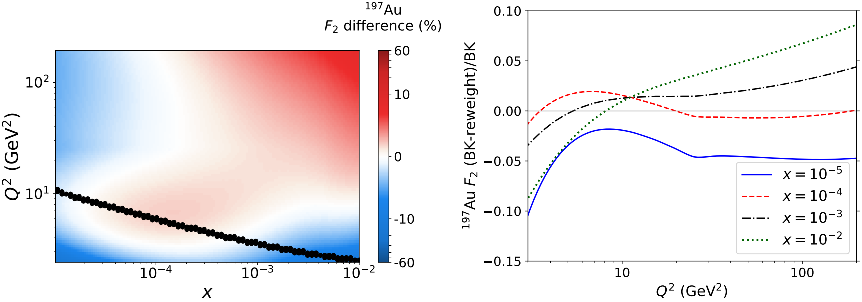


(a) F_2 (b) F_L

The structure functions for $^{197}\mathrm{Au}$ as a function of x at $Q^2 \approx 10Q_s^2(x)$.

- Nuclear reweight is not as successful as for proton since there are not enough Monte Carlo replicas to get a precise match

Relative difference of nuclear F_2 to F_2^{BK}

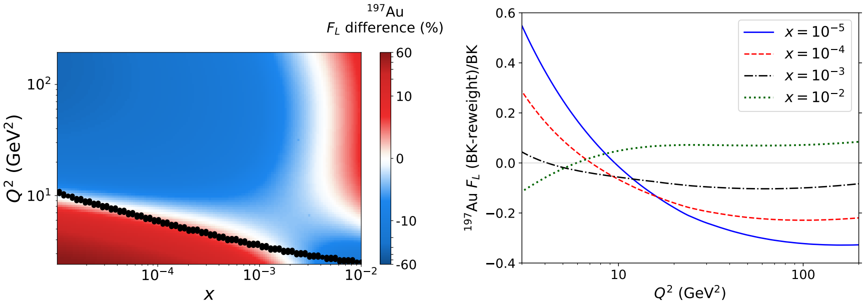


(a) F_2 (b) F_2

Relative difference $(F_2^{\mathrm{BK}} - F_2^{\mathrm{Rew}})/F_2^{\mathrm{BK}}$.

- For nuclear F_2 the relative difference is $\lesssim 10\%$
- The relative difference is much larger than in the proton case
  - It is expected since saturation effects are stronger in nuclei

Relative difference of nuclear $F_{\mathrm{L}}^{\mathrm{Rew}}$ to $F_{\mathrm{L}}^{\mathrm{BK}}$



(a) F_{L} (b) F_{L}

The relative difference $(F_{\mathrm{L}}^{\mathrm{BK}} - F_{\mathrm{L}}^{\mathrm{Rew}})/F_{\mathrm{L}}^{\mathrm{BK}}$.

For nuclear F_{L} the relative difference is:

- $\lesssim 15\%$ for $x = 10^{-3} \ldots 10^{-2}$ (EIC)
- $\lesssim 60\%$ for $x = 10^{-5} \ldots 10^{-4}$ (LHeC/FCC-he)

Summary

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- In LHeC/FCC-he the differences are a few times larger
- Saturation is stronger in heavy nuclei than in proton
- F_{L} is more sensitive to saturation than F_2

References

[1] Valerio Bertone, Stefano Carrazza, and Juan Rojo. "APFEL: A PDF Evolution Library with QED corrections". In: *Comput. Phys. Commun.* 185 (2014), pp. 1647–1668. DOI: 10.1016/j.cpc.2014.03.007. arXiv: 1310.1394 [hep-ph].

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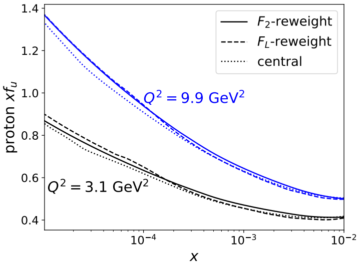
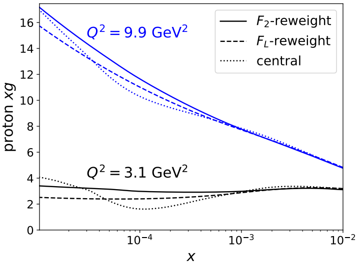
Back up: Weighted proton PDFs



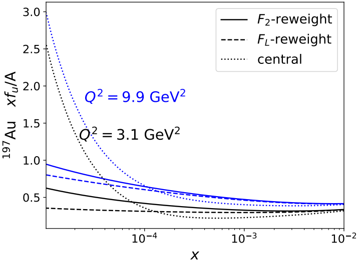
(a) Proton up quark



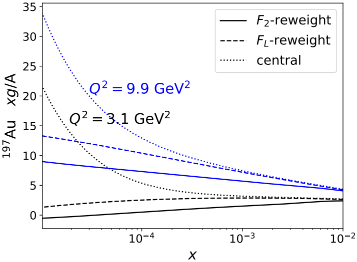
(b) Proton gluon

- Reweighting has slightly stronger effect on gluon distribution than on up quark
- Moderate effects expected since NNPDF3.1 PDFs are fitted to same HERA data as BK boundary conditions

Back up: Weighted nuclear PDFs



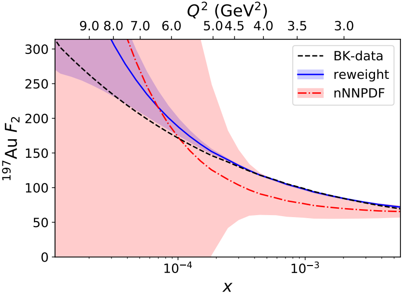
(c) Nuclear up quark



(d) Nuclear gluon

- Nuclear PDFs are affecter more than proton PDFs
- Reweighting has stronger effect on gluon distribution than on up quark

Back up: Reweight with smaller x region



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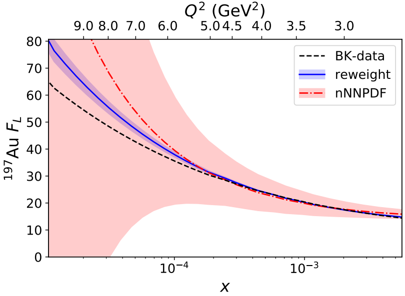
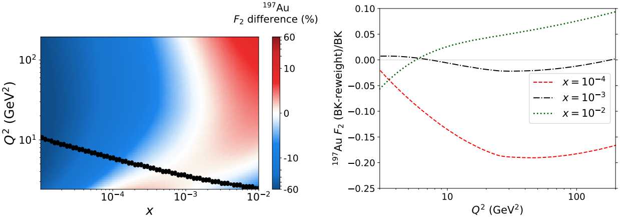


(b) F_L

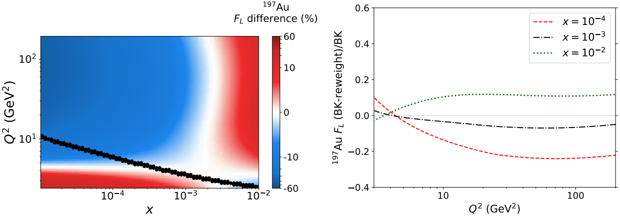
Nuclear reweight in region $x = 10^{-4} ... 10^{-2}$

Back up: Reweight with smaller x region



(a) F_2 (b) F_2

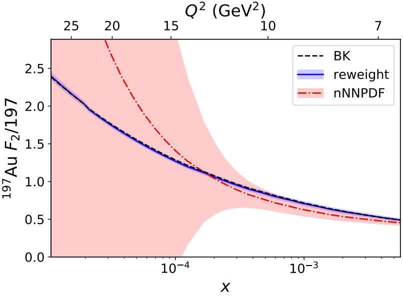
The relative difference $(F_2^{\mathrm{BK}} - F_2^{\mathrm{Rew}})/F_2^{\mathrm{BK}}$ with nuclear reweight in region $x = 10^{-4}...10^{-2}$.



(a) F_{L} (b) F_{L}

The relative difference $(F_{\mathrm{L}}^{\mathrm{BK}} - F_{\mathrm{L}}^{\mathrm{Rew}})/F_{\mathrm{L}}^{\mathrm{BK}}$ with nuclear reweight in region $x = 10^{-4}...10^{-2}$.

Back up: Reweight in line $Q^2(x) \approx 27 \times Q_s^2(x)$



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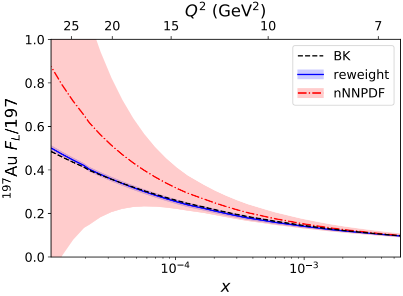
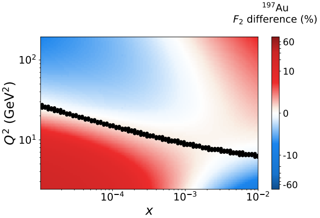


(b) F_L

Nuclear reweight in line $Q^2(x) \approx 27 \times Q_s^2(x)$.



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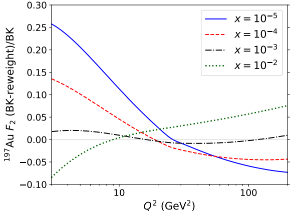
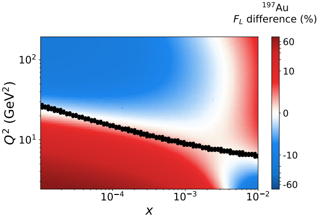
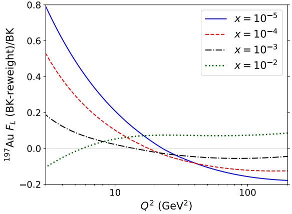


(b) F_2

The relative difference $(F_2^{\rm BK} - F_2^{\rm Rew})/F_2^{\rm BK}$ with nuclear reweight in line $Q^2(x) \approx 27 \times Q_s^2(x)$.



(a) $F_{\rm L}$



(b) $F_{\rm L}$

The relative difference $(F_{\rm L}^{\rm BK} - F_{\rm L}^{\rm Rew})/F_{\rm L}^{\rm BK}$ with nuclear reweight in line $Q^2(x) \approx 27 \times Q_s^2(x)$.

Back up: Weights

Giele-Keller weights which favor replicas with $\chi^2/N_{\rm data} \approx 0$

$$\omega_k = \frac{e^{-\frac{1}{2}\chi_k^2}}{\frac{1}{N_{\rm rep}}\sum_{k=1}^{N_{\rm rep}} e^{-\frac{1}{2}\chi_k^2}}$$

Weights used with experimental data favor replicas with $\chi^2/N_{\rm data} \approx 1$

$$\omega_k = \frac{\left(\chi_k^2\right)^{(N_{\rm data}-1)/2} e^{-\frac{1}{2}\chi_k^2}}{\frac{1}{N_{\rm rep}}\sum_{k=1}^{N_{\rm rep}}\left(\chi_k^2\right)^{(N_{\rm data}-1)/2} e^{-\frac{1}{2}\chi_k^2}}$$