How small is small x?
A perspective from the NLO CGC phenomenology

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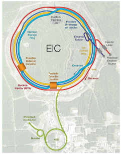
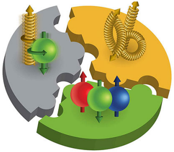
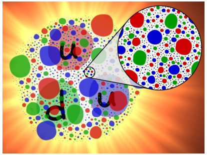
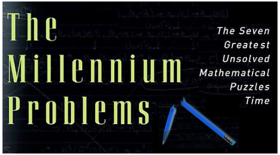
Ultimate Questions and Challenges in QCD

To understand our physical world, we have to understand QCD!





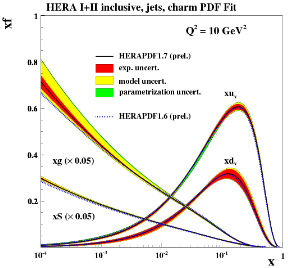
Three pillars of EIC Physics:

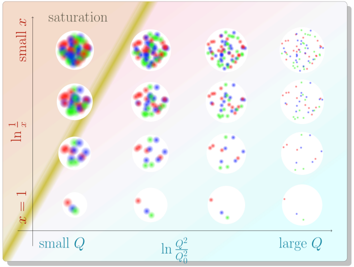
- How does the spin of proton arise? (Spin puzzle)
- What are the emergent properties of dense gluon system?
- How does proton mass arise? Mass gap: million dollar question.

EICs: keys to unlocking these mysteries! Many opportunities will be in front of us!

Saturation Physics (Color Glass Condensate)

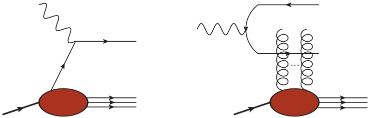
QCD matter at extremely high gluon density





- Gluon density grows rapidly as x gets small.
- Many gluons with fixed size packed in a confined hadron, gluons overlap and recombine ⇒ Non-linear QCD dynamics (BK/JIMWLK) ⇒ ultra-dense gluonic matter
- Multiple Scattering (MV model) + Small-x (high energy) evolution

Dual Descriptions of Deep Inelastic Scattering



Bjorken frame   Dipole frame

Bjorken frame $F_2(x, Q^2) = \sum_q e_q^2 x \left[f_q(x, Q^2) + f_{\bar{q}}(x, Q^2) \right]$.

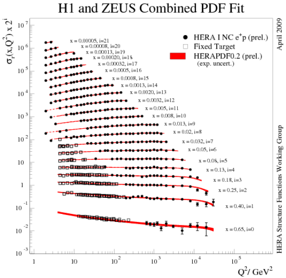
Dipole frame [A. Mueller, 01; Parton Saturation-An Overview]

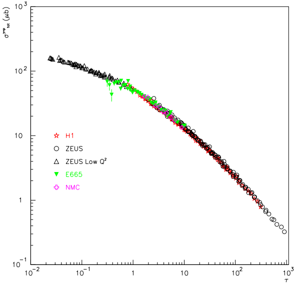
$$F_2(x, Q^2) = \sum_f e_f^2 \frac{Q^2}{4\pi^2 \alpha_{\rm em}} S_\perp \int_0^1 {\rm d}z \int {\rm d}^2 r_\perp \left| \psi\left(z, r_\perp, Q\right) \right|^2 \left[1 - S^{(2)}\left(Q_s r_\perp\right) \right]$$

- Bjorken: partonic picture is manifest. Saturation shows up as limit of number density.
- Dipole: the partonic picture is no longer manifest. Saturation appears as the unitarity limit for scattering. Convenient to resum the multiple gluon interactions.

Geometrical Scaling in DIS

[Golec-Biernat, Stasto, Kwiecinski; 01, Munier, Peschanski, 03]

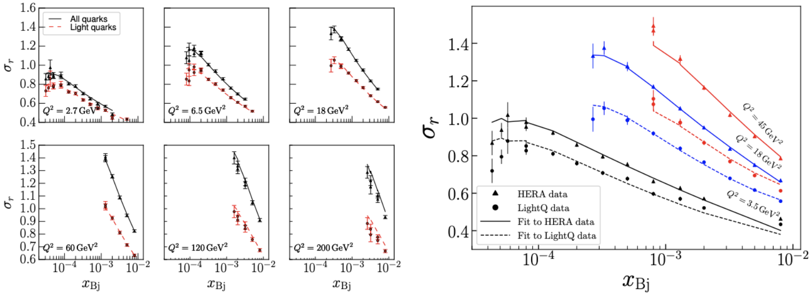




- Define $Q_s^2(x) = (x_0/x)^{\lambda}\mathrm{GeV}^2$ with $x_0 = 3.04 \times 10^{-3}$ and $\lambda = 0.288$. All low-x data with $x \leq 0.01$ and $Q^2 \leq 450 GeV^2$ is function of a single variable $\tau = Q^2/Q_s^2$.

NLO CGC meets HERA data

[Beuf, Hänninen, T. Lappi, and H. Mäntysaari, 20]



- Dipole-amplitude fits to HERA inclusive data using the full NLO impact factor combined with an improved BK evolution.
- Robust predictions for future deep inelastic scattering experiments.
- The needs for extension to heavy quark case at NLO. [Beuf, Lappi, Paatelainen, 22]

A Tale of Two Gluon Distributions

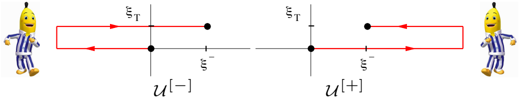
Two gauge invariant TMD operator def. [Bomhof, Mulders and Pijlman, 06] ▸ Link [Dominguez, Marquet, Xiao and Yuan, 11] ▸ Link

I. Weizsäcker Williams distribution: conventional density

$$xG_{\rm WW}(x,k_\perp)=2\int\frac{d\xi^- d\xi_\perp}{(2\pi)^3P^+}e^{ixP^+\xi^- -ik_\perp\cdot\xi_\perp}\,{\rm Tr}\langle P|F^{+i}(\xi^-,\xi_\perp)\mathcal{U}^{[+]\dagger}F^{+i}(0)\mathcal{U}^{[+]}|P\rangle .$$

II. Color Dipole gluon distributions:

$$xG_{\rm DP}(x,k_\perp)=2\int\frac{d\xi^- d\xi_\perp}{(2\pi)^3P^+}e^{ixP^+\xi^- -ik_\perp\cdot\xi_\perp}\,{\rm Tr}\langle P|F^{+i}(\xi^-,\xi_\perp)\mathcal{U}^{[-]\dagger}F^{+i}(0)\mathcal{U}^{[+]}|P\rangle .$$



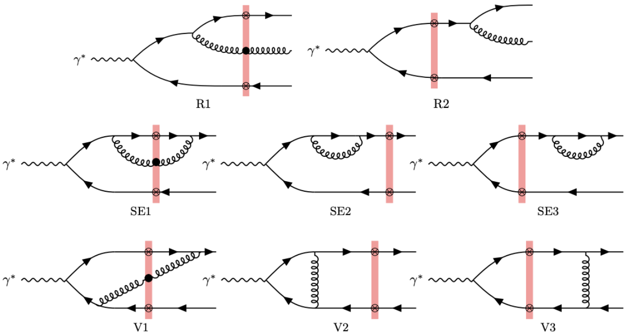
- Modified Universality for Gluon Distributions:

	Inclusive	Single Inc	DIS dijet	γ +jet	dijet in pA
$xG_{\rm WW}$	×	×	✓	×	✓
$xG_{\rm DP}$	✓	✓	×	✓	✓

✓ ⇒ Apppear. × ⇒ Do Not Appear.

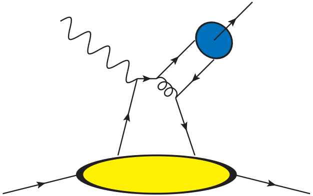
NLO CGC Computation for dijet in DIS

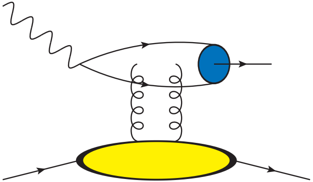
[Caucal, Salazar, and Venugopalan, 21]

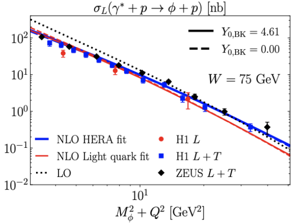


- First complete next-to-leading order computation of inclusive dijet production in DIS.
- Dijet photoproduction at low-x at NLO and its back-to-back limit. [Taels, Altinoluk, Beuf, Marquet, 22]

Diffractive and Exclusive processes in DIS





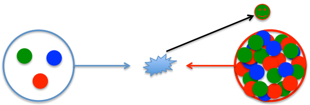


- LO [Brodsky, Frankfurt, Gunion, Mueller, Strikman, 94; Kowalski, Teaney, 03; Kowalski, Motyka, Watt, 06; Kowalski, Caldwell, 10; Berger, Stasto, 13]...
- Incoherent diffractive production for nucleon/nuclear targets [T. Lappi, H. Mantysaari, 11; Toll, Ullrich, 12; Lappi, Mantysaari, R. Venugopalan, 15]...;
- NLO[Boussarie, Grabovsky, Ivanov, Szymanowski, Wallon, 16] Link
- Numerical NLO results with light and heavy quarks [Mäntysaari and Penttala, 22]

Forward hadron production in pA collisions

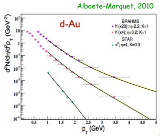
[Dumitru, Jalilian-Marian, 02] Dilute-dense factorization at forward rapidity

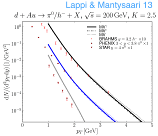
$$\frac{d\sigma_{\rm LO}^{pA\to hX}}{d^2p_\perp dy_h} = \int_\tau^1 \frac{dz}{z^2}\left[x_1 q_f(x_1,\mu)\mathcal{F}_{x_2}(k_\perp)D_{h/q}(z,\mu) + x_1 g(x_1,\mu)\tilde{\mathcal{F}}_{x_2}(k_\perp)D_{h/g}(z,\mu)\right].$$



projectile: $x_1 \sim \frac{p_\perp}{\sqrt{s}}e^{+y} \sim 1$ valence

target: $x_2 \sim \frac{p_\perp}{\sqrt{s}}e^{-y} \ll 1$ gluon

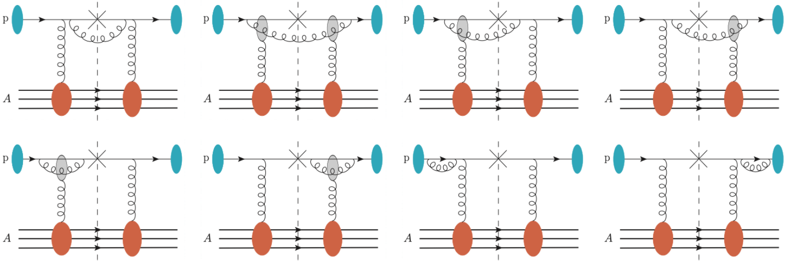




- Proton: Collinear PDFs and FFs (Strongly depends on μ^2).; Nucleus: Small-x gluon!
- Need NLO correction! IR cutoff: [Dumitru, Hayashigaki, Jalilian-Marian, 06; Altinoluk, Kovner 11] [Altinoluk, Armesto, Beuf, Kovner, Lublinsky, 14]; Full NLO [Chirilli, BX and Yuan, 12]
- Forward jets at LO and NLO [Mäntysaari, Paukkunen, 19; Liu, Xie, Kang, Liu, 22]

NLO diagrams in the $q \to q$ channel

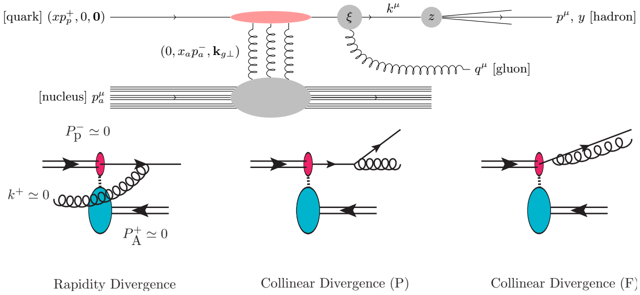
[Chirilli, BX and Yuan, 12]



- Take into account real (top) and virtual (bottom) diagrams together!
- Non-linear multiple interactions inside the grey blobs!
- Integrate over gluon phase space $\Rightarrow$Divergences!.

Factorization for single inclusive hadron productions

Factorization for the $p + A \to H + X$ process [Chirilli, BX and Yuan, 12]



- Include all real and virtual graphs in all channels $q \to q$, $q \to g$, $g \to q(\bar{q})$ and $g \to g$.
- 1. collinear to the target nucleus; $\Rightarrow$ BK evolution for UGD $\mathcal{F}(k_\perp)$.
- 2. collinear to the initial quark; $\Rightarrow$ DGLAP evolution for PDFs
- 3. collinear to the final quark. $\Rightarrow$ DGLAP evolution for FFs.

Numerical implementation of the NLO result

Single inclusive hadron production up to NLO

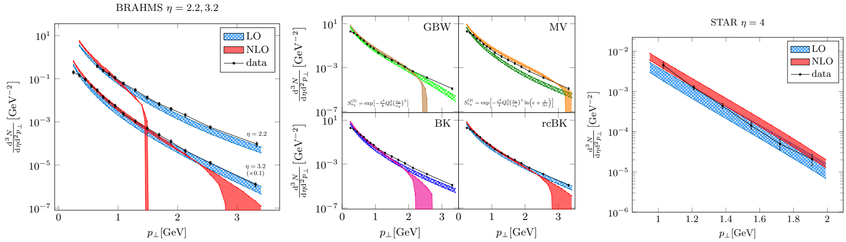
$$d\sigma = \int xf_a(x) \otimes D_a(z) \otimes \mathcal{F}_a^{x_g}(k_\perp) \otimes \mathcal{H}^{(0)} + \frac{\alpha_s}{2\pi} \int xf_a(x) \otimes D_b(z) \otimes \mathcal{F}_{(N)ab}^{x_g} \otimes \mathcal{H}_{ab}^{(1)} .$$

Consistent implementation should include all the NLO α_s corrections.

- NLO parton distributions. (MSTW or CTEQ)
- NLO fragmentation function. (DSS or others.)
- Use NLO hard factors. Partially by [Albacete, Dumitru, Fujii, Nara, 12]
- Use the one-loop approximation for the running coupling
- rcBK evolution equation for the dipole gluon distribution [Balitsky, Chirilli, 08; Kovchegov, Weigert, 07]. Full NLO BK evolution not available.
- Saturation physics at One Loop Order (SOLO). [Stasto, Xiao, Zaslavsky, 13]

Numerical implementation of the NLO result

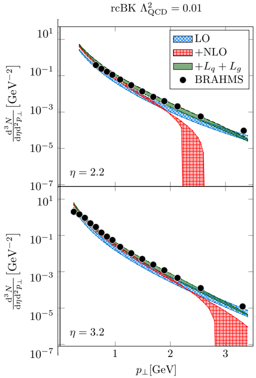
Saturation physics at One Loop Order (SOLO). [Stasto, Xiao, Zaslavsky, 13]



- Reduced factorization scale dependence!
- The abrupt drop at NLO when $p_T > Q_s$ was surprising and puzzling.
- Fixed order calculation in field theories is not guaranteed to be positive.

NLO hadron productions in pA collisions: An Odyssey

[Watanabe, Xiao, Yuan, Zaslavsky, 15] Rapidity subtraction! with kinematic constraints



- Originally assume the limit $s \to \infty$

$$\int_0^{1-\frac{q_\perp^2}{x_p s}} \frac{d\xi}{1-\xi} = \underbrace{\ln \frac{1}{x_g}}_{1-\xi<\frac{q_\perp^2}{k_\perp^2}} + \underbrace{\ln \frac{k_\perp^2}{q_\perp^2}}_{\text{missed earlier}} \Rightarrow$$

New terms: $L_q + L_g$ from $\quad q_\perp^2 \le (1-\xi)k_\perp^2.$

Related to threshold double logs!

- Negative when $p_T \gg Q_s$ at forward y ($x_p \to 1$)!
- Approach threshold at high $k_\perp$.

Extending the applicability of CGC calculation

Some Remarks:

- Towards a more complete framework. [Altinoluk, Armesto, Beuf, Kovner, Lublinsky, 14; Kang, Vitev, Xing, 14; Ducloue, Lappi and Zhu, 16, 17; Iancu, Mueller, Triantafyllopoulos, 16; Liu, Ma, Chao, 19; Kang, Liu, 19; Kang, Liu, Liu, 20;]
- Goal: find a solution within our current factorization (exactly resum $\alpha_s \ln 1/x_g$) to extend the applicability of CGC. Other scheme choices certainly is possible.
- More than just negativity problem. Need to work reliably (describe data) from RHIC to LHC, low p_T to high p_T.
- Demonstrate onset of saturation and visualize smooth transition to dilute regime.
- Add'l consideration: numerically challenging due to limited computing resources.
- A lot of logs occur in pQCD loop-calculations: DGLAP, small-x, threshold, Sudakov.
- Breakdown of α_s expansion occurs due to the appearance of logs in certain PS.

Threshold Logarithms

[Watanabe, Xiao, Yuan, Zaslavsky, 15; Shi, Wang, Wei, Xiao, 21] ▸ 2112.06975 [hep-ph]

- Numerical integration (8-d in total) is notoriously hard in $r_\perp$ space. Go to $k_\perp$ space.
- In the coordinate space, we can identify two types of logarithms

$$\text{single log: } \ln\frac{k_\perp^2}{\mu_r^2} \to \ln\frac{k_\perp^2}{\Lambda^2}, \quad \ln\frac{\mu^2}{\mu_r^2} \to \ln\frac{\mu^2}{\Lambda^2}; \quad \text{double log: } \ln^2\frac{k_\perp^2}{\mu_r^2} \to \ln^2\frac{k_\perp^2}{\Lambda^2},$$

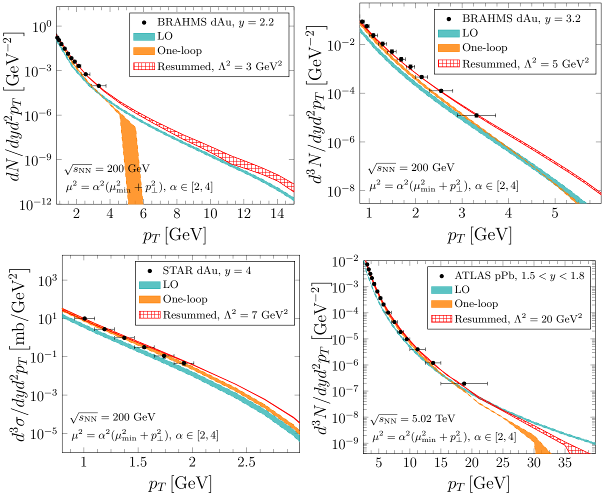
with $\mu_r \equiv c_0/r_\perp$ with $c_0 = 2e^{-\gamma_E}$. Performing Fouier transformations

$$\int \frac{d^2 r_\perp}{(2\pi)^2} S(r_\perp) \ln\frac{\mu^2}{\mu_r^2} e^{-ik_\perp \cdot r_\perp} = -\int \frac{d^2 l_\perp}{\pi l_\perp^2}\left[F(k_\perp + l_\perp) - J_0(\frac{c_0}{\mu} l_\perp) F(k_\perp)\right]$$

$$= \quad -\frac{1}{\pi}\int \frac{d^2 l_\perp}{(l_\perp - k_\perp)^2}\left[F(l_\perp) - \frac{\Lambda^2}{\Lambda^2 + (l_\perp - k_\perp)^2} F(k_\perp)\right] + F(k_\perp)\ln\frac{\mu^2}{\Lambda^2}.$$

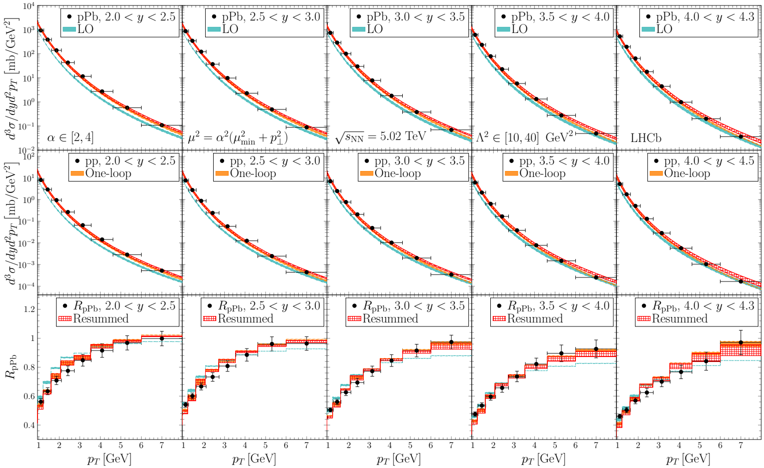
- Introduce a semi-hard auxiliary scale $\Lambda^2 \sim \mu_r^2 \gg \Lambda_{QCD}^2$. Identify dominant $r_\perp$!
- Dependences on μ^2, Λ^2 cancel order by order. Choose “natural" values at fixed order.

Numerical Results for pA spectra



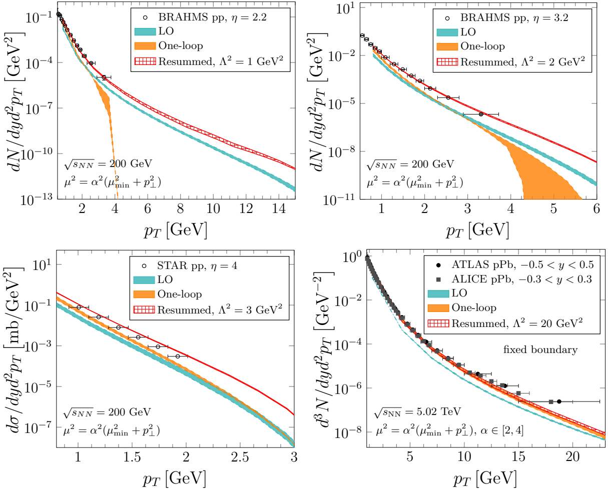
- $\mu^2 = \alpha^2(\mu^2_{\min} + p_T^2)$ & $\alpha \in [2, 4]$;
- RHIC: $\Lambda^2 \sim Q_s^2$; LHC, larger Λ^2.
- $\mu \sim Q \geq 2k_\perp$ ($\alpha > 2$) at high p_T. $2 \to 2$ hard scattering.
- Estimate higher order correction by varying μ and Λ.
- Threshold enhancement for σ.
- Nice agreement with data across many orders of magnitudes for different energies and p_T ranges!

Comparison with the new LHCb data



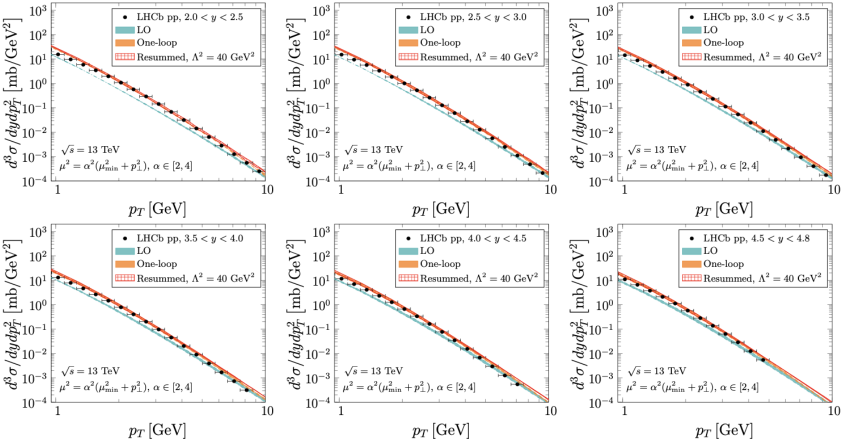
- LHCb data: 2108.13115
- Data Link DIS2021
- $\mu \sim (2 \sim 4)p_T$ with proper choice of Λ
- Threshold effect is not important at low p_T for LHCb data. Saturation effects are still dominant.
- Predictions are improved from LO to NLO.

Numerical Results for forward *pp* spectra and central rapidity *pA*



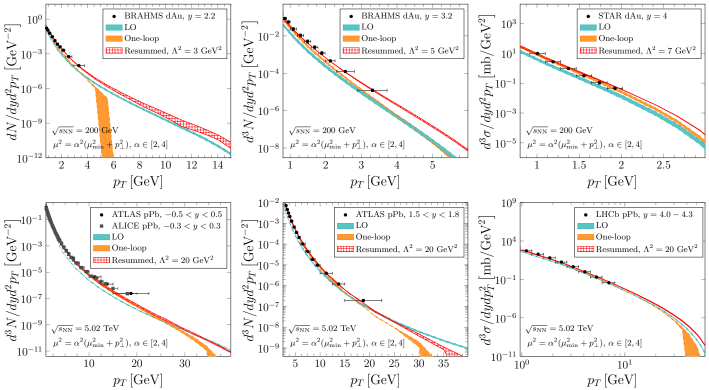
- Set $\mu^2 = \alpha^2(\mu^2_{\text{min}} + p_T^2)$ with $\alpha = 2 \to 4$
- $\mu \sim Q \geq 2k_\perp$ $(\alpha > 2)$ in the high p_T region. $2 \to 2$ hard scattering.
- Nice agreement with data for *pp* collisions and central rapidity *pA*!
- For large p_T data in *pA*, events with $x_g > 0.01$ starts to contribute.

Comparison with the new LHCb *pp* data at 13 TeV



NLO is important and Resummed results overlap with One-loop!

Why the threshold resummation works?

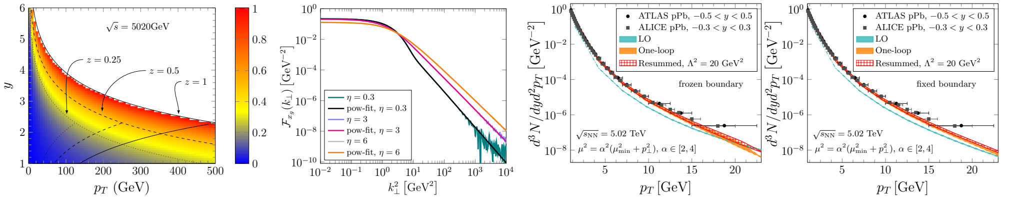


At low p_T, saturation dominates; At high p_T, threshold wins!

- At one-loop, negativity appears under two conditions:
  1. Need $p_T \gg Q_s$ for the threshold logarithmic terms to take over.
  2. Need to go to sufficiently forward rapidity to reach the kinematic boundary.
- At RHIC, negativity does not appear at $y = 4$ due to lack of phase space.
- Maybe counter-intuitive, but p_T expansion is key.

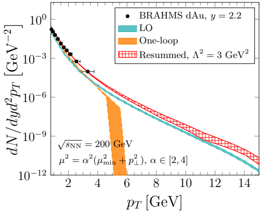
Applicability of CGC and Initial Condition

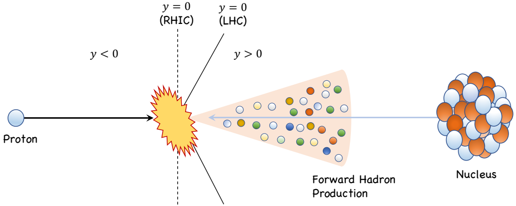
Kinematics: constraint $\tau/z = \frac{p_T e^y}{z\sqrt{s}} \leq 1$ and CGC constraint $x_g \equiv \frac{p_T e^{-y}}{z\sqrt{s}} \leq 10^{-2}$.

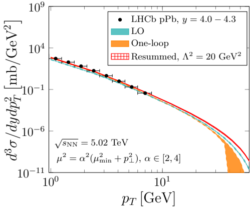


- Small-x gluon: [Albacete, Armesto, Milhano, Quiroga-Arias and Salgado, 11] Link
- Initial condition set at $x_g \equiv \frac{p_\perp e^{-y}}{z\sqrt{s}} = 10^{-2}$ + running coupling BK evolution.
- Applicability of CGC: rapidity y sufficiently large and $p_T = k_\perp z$ not too large.
- At high p_T, events with $x_g > 0.01$ start to contribute. $y = 0$ and $k_\perp > 50$ GeV.

Summary







- Ten-Year Odyssey in NLO hadron productions in pA collisions in CGC.
- Towards the precision test of saturation physics (CGC) at RHIC and LHC. Key!.
- Next Goal:Global analysis for CGC combining data from pA and DIS.
- A lot of remarkably difficult NLO calculations have been accomplished in CGC in the last couple of years.
- Entering an exciting time of NLO CGC phenomenology with the upcoming EIC and tremendous interesting physics results ahead.

Threshold resummation in the CGC formalism

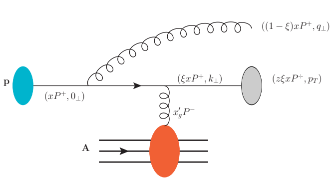
Threshold logarithms: Sudakov soft gluon part and Collinear (plus-distribution) part.

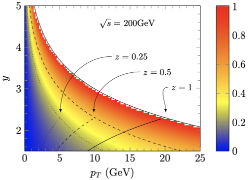
- Soft single and double logs ($\ln k_\perp^2/\Lambda^2$, $\ln^2 k_\perp^2/\Lambda^2$) are resummed via Sudakov factor.
- Two equivalent methods to resum the collinear part ($P_{ab}(\xi)\ln\Lambda^2/\mu^2$):
  1. Reverse DGLAP evolution; 2. RGE method (threshold limit $\xi \to 1$).
- Introduce forward threshold quark jet function $\Delta^q(\Lambda^2, \mu^2, \omega)$, which satisfies

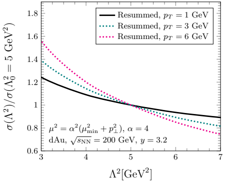
$$\frac{\mathrm{d}\Delta^q(\omega)}{\mathrm{d}\ln\mu^2} = -\frac{\mathrm{d}\Delta^q(\omega)}{\mathrm{d}\ln\Lambda^2} = -\frac{\alpha_s C_F}{\pi}\left[\ln\omega + \frac{3}{4}\right]\Delta^q(\omega) + \frac{\alpha_s C_F}{\pi}\int_0^\omega \mathrm{d}\omega' \frac{\Delta^q(\omega) - \Delta^q(\omega')}{\omega - \omega'}.$$

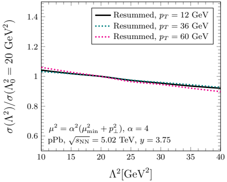
- Consistent with the threshold resummation in SCET[Becher, Neubert, 06]!
  Physically, the auxiliary scale Λ^2 is analogous to the intermediate scale μ_i^2 in SCET.
- Two formulations. [Xiao, Yuan, 18; Kang, Liu, 19; Liu, Kang, Liu, 20]

Natural Choice of the Auxiliary Scale









- At threshold: radiated gluon is soft! $\tau = \frac{p_T e^y}{\sqrt{s}} = x\xi z \leq 1$ with large $k_\perp$ (p_T).
- Intuitively, semi-hard cutoff $\Lambda^2 \sim (1-\xi)k_\perp^2 \sim (1-\tau)p_T^2 \gg \Lambda_{QCD}^2$ at fixed coupling.
- Saddle point approximation for $r_\perp$ integration at fixed and running coupling. $\Lambda^2 \sim \mu_r^2$
- For running coupling, $\Lambda^2 = \Lambda_{QCD}^2 \left[\frac{(1-\xi)k_\perp^2}{\Lambda_{QCD}^2}\right]^{C_R/[C_R+\beta_1]}$. Akin to CSS & Catani *et al*.
- When saturation momentum is large, $\Lambda^2 \sim Q_s^2$. (competing mechanism)
- Enhancement at high-p_T; Mild Λ dependence at low p_T far away from boundary.

Numerical Setup

[Xiao, Yuan, 18; Shi, Wang, Wei, Xiao, 2112.06975 [hep-ph]]

$$
\begin{aligned}
d\sigma &= \int x f_a(x,\mu) \otimes D_a(z,\mu) \otimes \mathcal{F}_a^{x_g}(k_\perp) \otimes \mathcal{H}^{(0)} \otimes \Delta(\mu,\Lambda) \otimes S_{\mathrm{Sud}}(\mu,\Lambda) \\
&\quad + \frac{\alpha_s}{2\pi} \int x f_a(x,\ \mu) \otimes D_b(z,\ \mu) \otimes \mathcal{F}_{(N)ab}^{x_g} \otimes \mathcal{H}_{ab}^{(1)}(\mu,\Lambda), \\
&= \int x f_a(x,\ \Lambda) \otimes D_a(z,\ \Lambda) \otimes \mathcal{F}_a^{x_g}(k_\perp) \otimes \mathcal{H}^{(0)} \otimes S_{\mathrm{Sud}}(\mu,\Lambda) \qquad \leftarrow \mu = \mu_b \ \text{TMD} \\
&\quad + \frac{\alpha_s}{2\pi} \int x f_a(x,\ \mu) \otimes D_b(z,\ \mu) \otimes \mathcal{F}_{(N)ab}^{x_g} \otimes \mathcal{H}_{ab}^{(1)}(\mu,\Lambda).
\end{aligned}
$$

- Natural choice of Λ^2: Competition between saturation and Sudakov $\Lambda \sim c_0/r_\perp$.
- Two implementation methods give similar numerical results.
- $\Delta(\mu,\Lambda)$ and $S_{\mathrm{Sud}}(\mu,\Lambda)$ satisfy collinear and Sudakov (soft) RGEs. $\Delta(\mu,\mu) = 1$