# Magic Rectangles in Construction of Magic and Block Bordered Magic Squares

**Inder J. Taneja**[1]

## Abstract

*In this work, blocks of **magic rectangles** with **equal and unequal sums** are used to construct magic squares. The **equal sum blocks magic rectangles** lead us to **even order** magic squares, such as, of orders 8, 12, 16, 20, 24, 28, 32, 36, 40, 42, 44 and 48. These magic squares are constructed using **magic rectangles** of orders* $2 \times 4$, $2 \times 14$, $4 \times 6$, $6 \times 10$, $6 \times 14$, *etc. There is only one example of even order magic square with **different sums blocks of magic rectangles**, i.e., of order 18. The **odd order** magic squares are with **unequal sum blocks magic rectangles**. These are of orders 15, 21, 27, 33, 39 and 45. These are constructed using **magic rectangles** of orders* $3 \times 5$, $3 \times 7$, $3 \times 9$, $3 \times 11$, $3 \times 13$, $3 \times 15$, $5 \times 17$, *etc. The magic squares of orders 12, 20, 24, 28, 42, 45 and 48 are written in two different ways. These even and odd orders magic squares are used to bring **block bordered** magic squares. These **block bordered** are of orders 10, 14, 17, 19, 23, 26, 29, 31, 34, 37, 38, 41, 43, 46 and 47. Most of the magic squares from order 8 to 48 are studied in this work, except the orders 9, 11, 13 and 25. These are already known in the literature [14].*

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# 1 Introduction

In past author worked in magic squares in different situations, such as:

1. *Digital Fonts*;
2. *Two Digits Universal Magic Squares*;
3. *Different Digits Magic Squares*;
4. *Pythagorean Triples Magic Squares*;
5. *Block-Wise Magic Squares*;
6. *Selfie and Palindromic Type Magic Squares*;
7. *Block Bordered Magic Squares*;
8. *Block-Wise Bordered Magic Squares*;
9. *Magic Crosses, Letters and Numbers*;
10. *Area Representations*;
11. *Single Digit and Single Letter Representations*.

More details on these works can be seen in author's web-sites:

- *Magic Squares: https://inderjtaneja.com/2019/06/27/publications-magic-squares/*;
- *Magic Squares: https://numbers-magic.com/?p=668*.

The aim of this work is to extend the idea of block-wise magic squares. In the previous work [3, 4, 5, 6, 7, 10] the blocks are magic squares. In this work blocks are **magic rectangles**. The idea of magic rectangles is given in the Section 4 as an Appendix. The follow magic squares are studied in this work:

1. ***Even order magic squares*** *studied are of orders 8, 12, 16, 18, 20, 24, 28, 30, 32, 36, 40, 42, 44 and 48. Except of orders 18 and 42, the other even order magic squares are with blocks* ***equal sums magic rectangles***;
2. *Based on magic rectangle of order* $2 \times 4$*, a sequence of magic squares multiples of 4 starting from 8 are written. These are of orders 8, 12, 16, 20, 24, 28,32,36,40, 44 and 48. Magic rectangles of order* $4 \times 6$ *are used to calculate magic squares of orders 12, 24, 36 and 48. Higher orders shall be written in another work*;
3. *Based on even order magic squares, the* ***block bordered*** *magic squares studied are of orders 10, 14, 22, 26, 30, 34, 38 and 46*;

4. ***Odd order magic squares*** *studied are of orders 15, 21, 27, 33, 39 and 45. All these magic squares are with blocks of* ***different sums magic rectangles****. These are based on magic triangles of order multiples of 3*;
5. *Based on odd order magic squares, the* ***block bordered*** *magic squares studied are of orders 17, 19, 23, 29, 31, 37, 41 and 47*;
6. *Where it is not possible to write* ***block-wise*** *magic squares, we write them as* ***block bordered*** *magic squares. These orders are either of prime numbers or double of prime numbers.*
7. *This work contains magic squares constructed with the help of* ***magic rectangles*** *from orders 8 to 48, except the orders 9, 11, 13 and 25. The order 9 is already constructed at many place [14] by using magic or semi-magic squares of order 3 . The orders 11 and 13 are constructed as* ***block bordered*** *where the inner magic square is of order 9. The magic square of order 25 is already known in the literature [7] as* ***pandiagonal bimagic*** *square with equal sum* ***pandiagonal*** *magic squares of order 5.*

# 2 Even Orders Magic and Block Bordered Magic Squares

Below are few even order magic squares constructed with blocks of equal sums of **magic rectangles** except the orders 18 and 42.

## 2.1 Magic Square of Order $8 \times 8$

Magic square of order 8 is constructed based on equal sums magic triangles of order $2 \times 4$ given in Example 4.1.

**Example 2.1.** *A magic square of order 8 constructed based on equal sums magic triangles of order* $2 \times 4$ *is given by*

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | | 260 |
| 12 | 54 | 55 | 9 | 8 | 58 | 59 | 5 | 260 |
| 53 | 11 | 10 | 56 | 57 | 7 | 6 | 60 | 260 |
| 1 | 63 | 62 | 4 | 13 | 51 | 50 | 16 | 260 |
| 64 | 2 | 3 | 61 | 52 | 14 | 15 | 49 | 260 |
| 25 | 39 | 38 | 28 | 24 | 42 | 43 | 21 | 260 |
| 40 | 26 | 27 | 37 | 41 | 23 | 22 | 44 | 260 |
| 20 | 46 | 47 | 17 | 29 | 35 | 34 | 32 | 260 |
| 45 | 19 | 18 | 48 | 36 | 30 | 31 | 33 | 260 |
| 260 | 260 | 260 | 260 | 260 | 260 | 260 | 260 | 260 |

*The **magic square** sum is $S_{8\times 8} := 260$. The magic rectangles of order $2\times 4$ are of **equal magic sums** given by $R_{2\times 4} := (130, 65)$.*

**Remark 2.1.** *A magic square of order 8 can also be constructed as 4 equal sums **pandiagonal** magic squares of order 4. This can be seen in author's another works [6, 10].*

## 2.2 Block Bordered Magic Square of Order $10\times 10$

It is based on the Example 2.1. It is with single external border, and internal block is a magic square of order 8 given in Example 2.1.

**Example 2.2.** *A **block bordered** magic square of order 10 with inner block as magic square of order 10 is given by*

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | 505 |
| 91 | 86 | 16 | 84 | 18 | 14 | 4 | 98 | 2 | 92 | 505 |
| 13 | 30 | 72 | 73 | 27 | 26 | 76 | 77 | 23 | 88 | 505 |
| 89 | 71 | 29 | 28 | 74 | 75 | 25 | 24 | 78 | 12 | 505 |
| 11 | 19 | 81 | 80 | 22 | 31 | 69 | 68 | 34 | 90 | 505 |
| 96 | 82 | 20 | 21 | 79 | 70 | 32 | 33 | 67 | 5 | 505 |
| 1 | 43 | 57 | 56 | 46 | 42 | 60 | 61 | 39 | 100 | 505 |
| 93 | 58 | 44 | 45 | 55 | 59 | 41 | 40 | 62 | 8 | 505 |
| 7 | 38 | 64 | 65 | 35 | 47 | 53 | 52 | 50 | 94 | 505 |
| 95 | 63 | 37 | 36 | 66 | 54 | 48 | 49 | 51 | 6 | 505 |
| 9 | 15 | 85 | 17 | 83 | 87 | 97 | 3 | 99 | 10 | 505 |
| 505 | 505 | 505 | 505 | 505 | 505 | 505 | 505 | 505 | 505 | 505 |

*In this case the magic sums are $S_{10\times 10} := 505$ and $S_{8\times 8} := 404$. The magic rectangles of order $2\times 4$ are of **equal magic sums** given by $R_{2\times 4} := (202, 101)$.*

## 2.3 Magic Square of Order $12\times 12$

Magic square of order 12 is constructed in two different ways based on magic triangles of orders $2\times 4$ and $4\times 6$ given in Examples 4.1 and 4.7.

**Example 2.3.** *A magic square of order 12 constructed based on equal sums magic triangles of order $2\times 4$ is given by*

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | 870 |
| 28 | 118 | 119 | 25 | 5 | 139 | 138 | 8 | 24 | 123 | 122 | 21 | 870 |
| 117 | 27 | 26 | 120 | 140 | 6 | 7 | 137 | 121 | 22 | 23 | 124 | 870 |
| 13 | 131 | 130 | 16 | 9 | 135 | 134 | 12 | 20 | 126 | 127 | 17 | 870 |
| 132 | 14 | 15 | 129 | 136 | 10 | 11 | 133 | 125 | 19 | 18 | 128 | 870 |
| 1 | 143 | 142 | 4 | 29 | 115 | 114 | 32 | 33 | 111 | 110 | 36 | 870 |
| 144 | 2 | 3 | 141 | 116 | 30 | 31 | 113 | 112 | 34 | 35 | 109 | 870 |
| 49 | 95 | 94 | 52 | 41 | 103 | 102 | 44 | 45 | 99 | 98 | 48 | 870 |
| 96 | 50 | 51 | 93 | 104 | 42 | 43 | 101 | 100 | 46 | 47 | 97 | 870 |
| 69 | 74 | 75 | 72 | 37 | 107 | 106 | 40 | 57 | 87 | 86 | 60 | 870 |
| 76 | 71 | 70 | 73 | 108 | 38 | 39 | 105 | 88 | 58 | 59 | 85 | 870 |
| 64 | 82 | 83 | 61 | 65 | 79 | 78 | 68 | 53 | 91 | 90 | 56 | 870 |
| 81 | 63 | 62 | 84 | 80 | 66 | 67 | 77 | 92 | 54 | 55 | 89 | 870 |
| 870 | 870 | 870 | 870 | 870 | 870 | 870 | 870 | 870 | 870 | 870 | 870 | 870 |

*The **magic square** sum is* $S_{12\times 12} := 870$*. The magic rectangles of order* $2\times 4$ *are of **equal magic sums** given by* $R_{2\times 4} := (290, 145)$.

**Example 2.4.** *A magic square of order 12 constructed based on equal sums magic triangles of order* $4\times 6$ *is given by*

| | | | | | | | | | | | | 870 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 140 | 3 | 4 | 143 | 144 | 48 | 38 | 39 | 106 | 107 | 97 | 870 |
| 139 | 2 | 141 | 142 | 5 | 6 | 102 | 101 | 100 | 45 | 44 | 43 | 870 |
| 138 | 137 | 136 | 9 | 8 | 7 | 103 | 104 | 105 | 40 | 41 | 42 | 870 |
| 12 | 11 | 10 | 135 | 134 | 133 | 37 | 47 | 46 | 99 | 98 | 108 | 870 |
| 25 | 26 | 27 | 118 | 119 | 120 | 13 | 14 | 15 | 130 | 131 | 132 | 870 |
| 115 | 116 | 117 | 28 | 29 | 30 | 127 | 128 | 129 | 16 | 17 | 18 | 870 |
| 114 | 113 | 112 | 33 | 32 | 31 | 126 | 125 | 124 | 21 | 20 | 19 | 870 |
| 36 | 35 | 34 | 111 | 110 | 109 | 24 | 23 | 22 | 123 | 122 | 121 | 870 |
| 49 | 50 | 51 | 94 | 95 | 96 | 61 | 62 | 81 | 64 | 83 | 84 | 870 |
| 91 | 92 | 88 | 57 | 53 | 54 | 79 | 80 | 63 | 82 | 65 | 66 | 870 |
| 60 | 89 | 93 | 52 | 56 | 85 | 72 | 77 | 76 | 69 | 68 | 73 | 870 |
| 90 | 59 | 58 | 87 | 86 | 55 | 78 | 71 | 70 | 75 | 74 | 67 | 870 |
| 870 | 870 | 870 | 870 | 870 | 870 | 870 | 870 | 870 | 870 | 870 | 870 | 870 |

*The **magic square** sum is $S_{12\times 12} := 870$. The magic rectangles of order $4\times 6$ are of **equal magic sums** given by $R_{4\times 6} := (435, 290)$.*

## 2.4 Block Bordered Magic Square of Order $14 \times 14$

It is based on the Examples 2.3 2.4 and with single external border, and internal block is magic square of order 12.

**Example 2.5.** *A **block bordered** magic square of order 14 with inner block as magic square of order 12 is given by*

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | | 1379 |
| 183 | 172 | 174 | 176 | 192 | 194 | 26 | 24 | 22 | 20 | 6 | 4 | 2 | 184 | 1379 |
| 19 | 54 | 144 | 145 | 51 | 31 | 165 | 164 | 34 | 50 | 149 | 148 | 47 | 178 | 1379 |
| 17 | 143 | 53 | 52 | 146 | 166 | 32 | 33 | 163 | 147 | 48 | 49 | 150 | 180 | 1379 |
| 15 | 39 | 157 | 156 | 42 | 35 | 161 | 160 | 38 | 46 | 152 | 153 | 43 | 182 | 1379 |
| 11 | 158 | 40 | 41 | 155 | 162 | 36 | 37 | 159 | 151 | 45 | 44 | 154 | 186 | 1379 |
| 9 | 27 | 169 | 168 | 30 | 55 | 141 | 140 | 58 | 59 | 137 | 136 | 62 | 188 | 1379 |
| 1 | 170 | 28 | 29 | 167 | 142 | 56 | 57 | 139 | 138 | 60 | 61 | 135 | 196 | 1379 |
| 179 | 75 | 121 | 120 | 78 | 67 | 129 | 128 | 70 | 71 | 125 | 124 | 74 | 18 | 1379 |
| 181 | 122 | 76 | 77 | 119 | 130 | 68 | 69 | 127 | 126 | 72 | 73 | 123 | 16 | 1379 |
| 185 | 95 | 100 | 101 | 98 | 63 | 133 | 132 | 66 | 83 | 113 | 112 | 86 | 12 | 1379 |
| 187 | 102 | 97 | 96 | 99 | 134 | 64 | 65 | 131 | 114 | 84 | 85 | 111 | 10 | 1379 |
| 189 | 90 | 108 | 109 | 87 | 91 | 105 | 104 | 94 | 79 | 117 | 116 | 82 | 8 | 1379 |
| 190 | 107 | 89 | 88 | 110 | 106 | 92 | 93 | 103 | 118 | 80 | 81 | 115 | 7 | 1379 |
| 13 | 25 | 23 | 21 | 5 | 3 | 171 | 173 | 175 | 177 | 191 | 193 | 195 | 14 | 1379 |
| 1379 | 1379 | 1379 | 1379 | 1379 | 1379 | 1379 | 1379 | 1379 | 1379 | 1379 | 1379 | 1379 | 1379 | 1379 |

*In this case the magic sums are $S_{14\times 14} := 1379$ and $S_{12\times 12} := 1182$. The magic rectangles of order $2\times 4$ are of* ***equal magic sums*** *given by $R_{2\times 4} := (394, 197)$.*

**Example 2.6.** *A* ***block bordered*** *magic square of order 14 with inner block as magic square of order 12 is given by*

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| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | | 1379 |
| 183 | 172 | 174 | 176 | 192 | 194 | 26 | 24 | 22 | 20 | 6 | 4 | 2 | 184 | 1379 |
| 19 | 27 | 166 | 29 | 30 | 169 | 170 | 74 | 64 | 65 | 132 | 133 | 123 | 178 | 1379 |
| 17 | 165 | 28 | 167 | 168 | 31 | 32 | 128 | 127 | 126 | 71 | 70 | 69 | 180 | 1379 |
| 15 | 164 | 163 | 162 | 35 | 34 | 33 | 129 | 130 | 131 | 66 | 67 | 68 | 182 | 1379 |
| 11 | 38 | 37 | 36 | 161 | 160 | 159 | 63 | 73 | 72 | 125 | 124 | 134 | 186 | 1379 |
| 9 | 51 | 52 | 53 | 144 | 145 | 146 | 39 | 40 | 41 | 156 | 157 | 158 | 188 | 1379 |
| 1 | 141 | 142 | 143 | 54 | 55 | 56 | 153 | 154 | 155 | 42 | 43 | 44 | 196 | 1379 |
| 179 | 140 | 139 | 138 | 59 | 58 | 57 | 152 | 151 | 150 | 47 | 46 | 45 | 18 | 1379 |
| 181 | 62 | 61 | 60 | 137 | 136 | 135 | 50 | 49 | 48 | 149 | 148 | 147 | 16 | 1379 |
| 185 | 75 | 76 | 77 | 120 | 121 | 122 | 87 | 88 | 107 | 90 | 109 | 110 | 12 | 1379 |
| 187 | 117 | 118 | 114 | 83 | 79 | 80 | 105 | 106 | 89 | 108 | 91 | 92 | 10 | 1379 |
| 189 | 86 | 115 | 119 | 78 | 82 | 111 | 98 | 103 | 102 | 95 | 94 | 99 | 8 | 1379 |
| 190 | 116 | 85 | 84 | 113 | 112 | 81 | 104 | 97 | 96 | 101 | 100 | 93 | 7 | 1379 |
| 13 | 25 | 23 | 21 | 5 | 3 | 171 | 173 | 175 | 177 | 191 | 193 | 195 | 14 | 1379 |
| 1379 | 1379 | 1379 | 1379 | 1379 | 1379 | 1379 | 1379 | 1379 | 1379 | 1379 | 1379 | 1379 | 1379 | 1379 |

*In this case the magic sums are $S_{14\times14} := 1379$ and $S_{12\times12} := 1182$. The magic rectangles of order $4\times6$ are of* ***equal magic sums*** *given by $R_{4\times6} := (591, 394)$.*

## 2.5 Magic Square of Order $16\times16$

Magic square of order 16 is constructed in two different ways based on magic triangles of order $2\times4$ given in Example 4.1.

**Example 2.7.** *A magic square of order 16 constructed based on equal sums magic triangles of order $2\times4$ is given by*

| | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | | | | 2056 |
| 12 | 246 | 247 | 9 | 8 | 250 | 251 | 5 | 44 | 214 | 215 | 41 | 40 | 218 | 219 | 37 | 2056 |
| 245 | 11 | 10 | 248 | 249 | 7 | 6 | 252 | 213 | 43 | 42 | 216 | 217 | 39 | 38 | 220 | 2056 |
| 1 | 255 | 254 | 4 | 13 | 243 | 242 | 16 | 33 | 223 | 222 | 36 | 45 | 211 | 210 | 48 | 2056 |
| 256 | 2 | 3 | 253 | 244 | 14 | 15 | 241 | 224 | 34 | 35 | 221 | 212 | 46 | 47 | 209 | 2056 |
| 25 | 231 | 230 | 28 | 24 | 234 | 235 | 21 | 57 | 199 | 198 | 60 | 56 | 202 | 203 | 53 | 2056 |
| 232 | 26 | 27 | 229 | 233 | 23 | 22 | 236 | 200 | 58 | 59 | 197 | 201 | 55 | 54 | 204 | 2056 |
| 20 | 238 | 239 | 17 | 29 | 227 | 226 | 32 | 52 | 206 | 207 | 49 | 61 | 195 | 194 | 64 | 2056 |
| 237 | 19 | 18 | 240 | 228 | 30 | 31 | 225 | 205 | 51 | 50 | 208 | 196 | 62 | 63 | 193 | 2056 |
| 76 | 182 | 183 | 73 | 72 | 186 | 187 | 69 | 108 | 150 | 151 | 105 | 104 | 154 | 155 | 101 | 2056 |
| 181 | 75 | 74 | 184 | 185 | 71 | 70 | 188 | 149 | 107 | 106 | 152 | 153 | 103 | 102 | 156 | 2056 |
| 65 | 191 | 190 | 68 | 77 | 179 | 178 | 80 | 97 | 159 | 158 | 100 | 109 | 147 | 146 | 112 | 2056 |
| 192 | 66 | 67 | 189 | 180 | 78 | 79 | 177 | 160 | 98 | 99 | 157 | 148 | 110 | 111 | 145 | 2056 |
| 89 | 167 | 166 | 92 | 88 | 170 | 171 | 85 | 121 | 135 | 134 | 124 | 120 | 138 | 139 | 117 | 2056 |
| 168 | 90 | 91 | 165 | 169 | 87 | 86 | 172 | 136 | 122 | 123 | 133 | 137 | 119 | 118 | 140 | 2056 |
| 84 | 174 | 175 | 81 | 93 | 163 | 162 | 96 | 116 | 142 | 143 | 113 | 125 | 131 | 130 | 128 | 2056 |
| 173 | 83 | 82 | 176 | 164 | 94 | 95 | 161 | 141 | 115 | 114 | 144 | 132 | 126 | 127 | 129 | 2056 |
| 2056 | 2056 | 2056 | 2056 | 2056 | 2056 | 2056 | 2056 | 2056 | 2056 | 2056 | 2056 | 2056 | 2056 | 2056 | 2056 | 2056 |

*The **magic square** sum is $S_{16\times16} := 2056$. The magic rectangles of order $2\times4$ are of **equal magic sums** given by $R_{2\times4} := (514, 257)$. Moreover, each block of order $8\times8$ is a magic square with magic sums $S_{8\times8} := 1028$.*

**Remark 2.2.** *A magic square of order 16 can also be constructed as 16 equal sums **pandiagonal** magic squares of order 4. This can be seen in author's another works [6, 10].*

## 2.6 Magic Square of Order $18\times18$

Magic square of order 18 is constructed based on magic triangles of order $2\times6$ given in Example 4.2.

**Example 2.8.** *A magic square of order 18 constructed based on magic triangles of order $2\times6$ is given by*

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| | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | | | | | | 2925 |
| 1 | 11 | 3 | 9 | 8 | 7 | 277 | 287 | 279 | 285 | 284 | 283 | 193 | 203 | 195 | 201 | 200 | 199 | 2925 |
| 12 | 2 | 10 | 4 | 5 | 6 | 288 | 278 | 286 | 280 | 281 | 282 | 204 | 194 | 202 | 196 | 197 | 198 | 2925 |
| 169 | 179 | 177 | 171 | 176 | 175 | 289 | 299 | 291 | 297 | 296 | 295 | 13 | 23 | 15 | 20 | 21 | 19 | 2925 |
| 180 | 170 | 172 | 178 | 173 | 174 | 300 | 290 | 298 | 292 | 293 | 294 | 24 | 14 | 22 | 17 | 16 | 18 | 2925 |
| 49 | 59 | 51 | 57 | 56 | 55 | 205 | 215 | 207 | 213 | 212 | 211 | 227 | 217 | 219 | 225 | 224 | 223 | 2925 |
| 60 | 50 | 58 | 52 | 53 | 54 | 216 | 206 | 214 | 208 | 209 | 210 | 218 | 228 | 226 | 220 | 221 | 222 | 2925 |
| 181 | 191 | 183 | 189 | 188 | 187 | 229 | 239 | 231 | 237 | 236 | 235 | 61 | 71 | 63 | 69 | 68 | 67 | 2925 |
| 192 | 182 | 190 | 184 | 185 | 186 | 240 | 230 | 238 | 232 | 233 | 234 | 72 | 62 | 70 | 64 | 65 | 66 | 2925 |
| 241 | 251 | 243 | 249 | 248 | 247 | 157 | 167 | 159 | 164 | 165 | 163 | 73 | 83 | 75 | 81 | 80 | 79 | 2925 |
| 252 | 242 | 250 | 244 | 245 | 246 | 168 | 158 | 166 | 161 | 160 | 162 | 84 | 74 | 82 | 76 | 77 | 78 | 2925 |
| 253 | 263 | 255 | 261 | 260 | 259 | 95 | 85 | 87 | 93 | 92 | 91 | 133 | 143 | 135 | 141 | 140 | 139 | 2925 |
| 264 | 254 | 262 | 256 | 257 | 258 | 86 | 96 | 94 | 88 | 89 | 90 | 144 | 134 | 142 | 136 | 137 | 138 | 2925 |
| 97 | 107 | 99 | 104 | 105 | 103 | 109 | 119 | 111 | 117 | 116 | 115 | 275 | 265 | 267 | 273 | 272 | 271 | 2925 |
| 108 | 98 | 106 | 101 | 100 | 102 | 120 | 110 | 118 | 112 | 113 | 114 | 266 | 276 | 274 | 268 | 269 | 270 | 2925 |
| 301 | 311 | 303 | 307 | 308 | 309 | 25 | 35 | 27 | 33 | 32 | 31 | 145 | 155 | 153 | 147 | 152 | 151 | 2925 |
| 312 | 302 | 310 | 306 | 305 | 304 | 36 | 26 | 34 | 28 | 29 | 30 | 156 | 146 | 148 | 154 | 149 | 150 | 2925 |
| 121 | 131 | 123 | 129 | 128 | 127 | 37 | 47 | 39 | 45 | 44 | 43 | 313 | 323 | 315 | 321 | 320 | 319 | 2925 |
| 132 | 122 | 130 | 124 | 125 | 126 | 48 | 38 | 46 | 40 | 41 | 42 | 324 | 314 | 322 | 316 | 317 | 318 | 2925 |
| 2925 | 2925 | 2925 | 2925 | 2925 | 2925 | 2925 | 2925 | 2925 | 2925 | 2925 | 2925 | 2925 | 2925 | 2925 | 2925 | 2925 | 2925 | 2925 |

*The **magic square** sum is $S_{18\times18} := 2925$. The magic rectangles of order $2\times6$ are of **different magic sums***

## 2.7 Magic Square of Order $20\times20$

**Example 2.9.** *A magic square of order $20\times20$ constructed based on magic rectangles of order $2\times4$ is given by*

| | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | | | | | | | | 4010 |
| 108 | 295 | 294 | 105 | 5 | 395 | 394 | 8 | 9 | 391 | 390 | 12 | 13 | 387 | 386 | 16 | 120 | 283 | 282 | 117 | 4010 |
| 293 | 106 | 107 | 296 | 396 | 6 | 7 | 393 | 392 | 10 | 11 | 389 | 388 | 14 | 15 | 385 | 281 | 118 | 119 | 284 | 4010 |
| 24 | 379 | 378 | 21 | 25 | 375 | 374 | 28 | 29 | 371 | 370 | 32 | 33 | 367 | 366 | 36 | 40 | 363 | 362 | 37 | 4010 |
| 377 | 22 | 23 | 380 | 376 | 26 | 27 | 373 | 372 | 30 | 31 | 369 | 368 | 34 | 35 | 365 | 361 | 38 | 39 | 364 | 4010 |
| 41 | 359 | 358 | 44 | 45 | 355 | 354 | 48 | 49 | 351 | 350 | 52 | 56 | 347 | 346 | 53 | 57 | 343 | 342 | 60 | 4010 |
| 360 | 42 | 43 | 357 | 356 | 46 | 47 | 353 | 352 | 50 | 51 | 349 | 345 | 54 | 55 | 348 | 344 | 58 | 59 | 341 | 4010 |
| 61 | 339 | 338 | 64 | 68 | 335 | 334 | 65 | 69 | 331 | 330 | 72 | 73 | 327 | 326 | 76 | 77 | 323 | 322 | 80 | 4010 |
| 340 | 62 | 63 | 337 | 333 | 66 | 67 | 336 | 332 | 70 | 71 | 329 | 328 | 74 | 75 | 325 | 324 | 78 | 79 | 321 | 4010 |
| 81 | 319 | 318 | 84 | 85 | 315 | 314 | 88 | 89 | 311 | 310 | 92 | 93 | 307 | 306 | 96 | 97 | 303 | 302 | 100 | 4010 |
| 320 | 82 | 83 | 317 | 316 | 86 | 87 | 313 | 312 | 90 | 91 | 309 | 308 | 94 | 95 | 305 | 304 | 98 | 99 | 301 | 4010 |
| 101 | 299 | 298 | 104 | 121 | 279 | 278 | 124 | 109 | 291 | 290 | 112 | 113 | 287 | 286 | 116 | 17 | 383 | 382 | 20 | 4010 |
| 300 | 102 | 103 | 297 | 280 | 122 | 123 | 277 | 292 | 110 | 111 | 289 | 288 | 114 | 115 | 285 | 384 | 18 | 19 | 381 | 4010 |
| 1 | 399 | 398 | 4 | 128 | 275 | 274 | 125 | 129 | 271 | 270 | 132 | 136 | 267 | 266 | 133 | 137 | 263 | 262 | 140 | 4010 |
| 400 | 2 | 3 | 397 | 273 | 126 | 127 | 276 | 272 | 130 | 131 | 269 | 265 | 134 | 135 | 268 | 264 | 138 | 139 | 261 | 4010 |
| 141 | 259 | 258 | 144 | 148 | 254 | 255 | 145 | 149 | 251 | 250 | 152 | 153 | 247 | 246 | 156 | 157 | 243 | 242 | 160 | 4010 |
| 260 | 142 | 143 | 257 | 253 | 147 | 146 | 256 | 252 | 150 | 151 | 249 | 248 | 154 | 155 | 245 | 244 | 158 | 159 | 241 | 4010 |
| 164 | 238 | 239 | 161 | 165 | 235 | 234 | 168 | 169 | 231 | 230 | 172 | 173 | 227 | 226 | 176 | 177 | 223 | 222 | 180 | 4010 |
| 237 | 163 | 162 | 240 | 236 | 166 | 167 | 233 | 232 | 170 | 171 | 229 | 228 | 174 | 175 | 225 | 224 | 178 | 179 | 221 | 4010 |
| 181 | 219 | 218 | 184 | 185 | 215 | 214 | 188 | 189 | 211 | 210 | 192 | 193 | 207 | 206 | 196 | 197 | 203 | 202 | 200 | 4010 |
| 220 | 182 | 183 | 217 | 216 | 186 | 187 | 213 | 212 | 190 | 191 | 209 | 208 | 194 | 195 | 205 | 204 | 198 | 199 | 201 | 4010 |
| 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 |

*The **magic square** sum is $S_{20\times 20} := 4010$. The magic rectangles of order $2\times 4$ are of **equal magic sums**, and the sum is given by $R_{2\times 4} := (802, 401)$.*

**Example 2.10.** *A magic square of order $20\times 20$ constructed based on magic rectangles of order $4\times 10$ is given by*

| | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | | | | | | | | 4010 |
| 1 | 2 | 3 | 4 | 5 | 396 | 397 | 398 | 399 | 400 | 21 | 22 | 23 | 24 | 25 | 376 | 377 | 378 | 379 | 380 | 4010 |
| 391 | 392 | 393 | 394 | 395 | 6 | 7 | 8 | 9 | 10 | 371 | 372 | 373 | 374 | 375 | 26 | 27 | 28 | 29 | 30 | 4010 |
| 390 | 389 | 388 | 387 | 386 | 15 | 14 | 13 | 12 | 11 | 370 | 369 | 368 | 367 | 366 | 35 | 34 | 33 | 32 | 31 | 4010 |
| 20 | 19 | 18 | 17 | 16 | 385 | 384 | 383 | 382 | 381 | 40 | 39 | 38 | 37 | 36 | 365 | 364 | 363 | 362 | 361 | 4010 |
| 41 | 42 | 43 | 44 | 45 | 356 | 357 | 358 | 359 | 360 | 61 | 62 | 63 | 64 | 65 | 336 | 337 | 338 | 339 | 340 | 4010 |
| 351 | 352 | 353 | 354 | 355 | 46 | 47 | 48 | 49 | 50 | 331 | 332 | 333 | 334 | 335 | 66 | 67 | 68 | 69 | 70 | 4010 |
| 350 | 349 | 348 | 347 | 346 | 55 | 54 | 53 | 52 | 51 | 330 | 329 | 328 | 327 | 326 | 75 | 74 | 73 | 72 | 71 | 4010 |
| 60 | 59 | 58 | 57 | 56 | 345 | 344 | 343 | 342 | 341 | 80 | 79 | 78 | 77 | 76 | 325 | 324 | 323 | 322 | 321 | 4010 |
| 81 | 82 | 83 | 84 | 315 | 316 | 317 | 318 | 319 | 90 | 291 | 102 | 103 | 104 | 105 | 296 | 297 | 298 | 299 | 110 | 4010 |
| 311 | 312 | 313 | 314 | 85 | 86 | 87 | 88 | 89 | 320 | 101 | 292 | 293 | 294 | 295 | 106 | 107 | 108 | 109 | 300 | 4010 |
| 310 | 309 | 308 | 307 | 306 | 95 | 94 | 93 | 92 | 91 | 290 | 289 | 288 | 287 | 286 | 115 | 114 | 113 | 112 | 111 | 4010 |
| 100 | 99 | 98 | 97 | 96 | 305 | 304 | 303 | 302 | 301 | 120 | 119 | 118 | 117 | 116 | 285 | 284 | 283 | 282 | 281 | 4010 |
| 121 | 122 | 123 | 124 | 125 | 276 | 277 | 278 | 279 | 280 | 141 | 142 | 143 | 144 | 145 | 256 | 257 | 258 | 259 | 260 | 4010 |
| 271 | 272 | 273 | 274 | 275 | 126 | 127 | 128 | 129 | 130 | 251 | 252 | 253 | 254 | 255 | 146 | 147 | 148 | 149 | 150 | 4010 |
| 270 | 269 | 268 | 267 | 266 | 135 | 134 | 133 | 132 | 131 | 250 | 249 | 248 | 247 | 246 | 155 | 154 | 153 | 152 | 151 | 4010 |
| 140 | 139 | 138 | 137 | 136 | 265 | 264 | 263 | 262 | 261 | 160 | 159 | 158 | 157 | 156 | 245 | 244 | 243 | 242 | 241 | 4010 |
| 161 | 162 | 163 | 164 | 165 | 236 | 237 | 238 | 239 | 240 | 181 | 182 | 183 | 184 | 185 | 216 | 217 | 218 | 219 | 220 | 4010 |
| 231 | 232 | 233 | 234 | 235 | 166 | 167 | 168 | 169 | 170 | 211 | 212 | 213 | 214 | 215 | 186 | 187 | 188 | 189 | 190 | 4010 |
| 180 | 229 | 228 | 227 | 226 | 225 | 174 | 173 | 172 | 171 | 210 | 209 | 208 | 207 | 196 | 195 | 194 | 193 | 192 | 201 | 4010 |
| 230 | 179 | 178 | 177 | 176 | 175 | 224 | 223 | 222 | 221 | 200 | 199 | 198 | 197 | 206 | 205 | 204 | 203 | 202 | 191 | 4010 |
| 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 |

*The **magic square** sum is $S_{20\times 20} := 4010$. The magic rectangles of order $4\times 10$ are of **equal magic sums**, and the sum is given by $R_{4\times 10} := (2005, 802)$.*

## 2.8 Block Bordered Magic Squares of Order $22\times 22$

It is based on the Examples 2.8, 2.9 and 2.10 with double and single external borders, and the internal blocks are magic squares of order 18 and 20.

**Example 2.11.** *A **block bordered** magic square of order 22 with inner block as a magic square of order 18 is given by*

| | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | | | | | | | | | | 5335 |
| 463 | 444 | 446 | 448 | 450 | 452 | 476 | 478 | 480 | 482 | 42 | 40 | 38 | 36 | 34 | 32 | 10 | 8 | 6 | 4 | 2 | 464 | 5335 |
| 31 | 423 | 406 | 408 | 410 | 412 | 435 | 437 | 439 | 441 | 80 | 78 | 76 | 74 | 72 | 51 | 49 | 47 | 45 | 43 | 424 | 454 | 5335 |
| 29 | 71 | 81 | 91 | 83 | 89 | 88 | 87 | 357 | 367 | 359 | 365 | 364 | 363 | 273 | 283 | 275 | 281 | 280 | 279 | 414 | 456 | 5335 |
| 27 | 69 | 92 | 82 | 90 | 84 | 85 | 86 | 368 | 358 | 366 | 360 | 361 | 362 | 284 | 274 | 282 | 276 | 277 | 278 | 416 | 458 | 5335 |
| 25 | 67 | 249 | 259 | 257 | 251 | 256 | 255 | 369 | 379 | 371 | 377 | 376 | 375 | 93 | 103 | 95 | 100 | 101 | 99 | 418 | 460 | 5335 |
| 23 | 64 | 260 | 250 | 252 | 258 | 253 | 254 | 380 | 370 | 378 | 372 | 373 | 374 | 104 | 94 | 102 | 97 | 96 | 98 | 421 | 462 | 5335 |
| 19 | 63 | 129 | 139 | 131 | 137 | 136 | 135 | 285 | 295 | 287 | 293 | 292 | 291 | 307 | 297 | 299 | 305 | 304 | 303 | 422 | 466 | 5335 |
| 17 | 58 | 140 | 130 | 138 | 132 | 133 | 134 | 296 | 286 | 294 | 288 | 289 | 290 | 298 | 308 | 306 | 300 | 301 | 302 | 427 | 468 | 5335 |
| 15 | 56 | 261 | 271 | 263 | 269 | 268 | 267 | 309 | 319 | 311 | 317 | 316 | 315 | 141 | 151 | 143 | 149 | 148 | 147 | 429 | 470 | 5335 |
| 13 | 54 | 272 | 262 | 270 | 264 | 265 | 266 | 320 | 310 | 318 | 312 | 313 | 314 | 152 | 142 | 150 | 144 | 145 | 146 | 431 | 472 | 5335 |
| 1 | 52 | 321 | 331 | 323 | 329 | 328 | 327 | 237 | 247 | 239 | 244 | 245 | 243 | 153 | 163 | 155 | 161 | 160 | 159 | 433 | 484 | 5335 |
| 455 | 415 | 332 | 322 | 330 | 324 | 325 | 326 | 248 | 238 | 246 | 241 | 240 | 242 | 164 | 154 | 162 | 156 | 157 | 158 | 70 | 30 | 5335 |
| 457 | 417 | 333 | 343 | 335 | 341 | 340 | 339 | 175 | 165 | 167 | 173 | 172 | 171 | 213 | 223 | 215 | 221 | 220 | 219 | 68 | 28 | 5335 |
| 459 | 419 | 344 | 334 | 342 | 336 | 337 | 338 | 166 | 176 | 174 | 168 | 169 | 170 | 224 | 214 | 222 | 216 | 217 | 218 | 66 | 26 | 5335 |
| 461 | 420 | 177 | 187 | 179 | 184 | 185 | 183 | 189 | 199 | 191 | 197 | 196 | 195 | 355 | 345 | 347 | 353 | 352 | 351 | 65 | 24 | 5335 |
| 465 | 425 | 188 | 178 | 186 | 181 | 180 | 182 | 200 | 190 | 198 | 192 | 193 | 194 | 346 | 356 | 354 | 348 | 349 | 350 | 60 | 20 | 5335 |
| 467 | 426 | 381 | 391 | 383 | 387 | 388 | 389 | 105 | 115 | 107 | 113 | 112 | 111 | 225 | 235 | 233 | 227 | 232 | 231 | 59 | 18 | 5335 |
| 469 | 428 | 392 | 382 | 390 | 386 | 385 | 384 | 116 | 106 | 114 | 108 | 109 | 110 | 236 | 226 | 228 | 234 | 229 | 230 | 57 | 16 | 5335 |
| 471 | 430 | 201 | 211 | 203 | 209 | 208 | 207 | 117 | 127 | 119 | 125 | 124 | 123 | 393 | 403 | 395 | 401 | 400 | 399 | 55 | 14 | 5335 |
| 473 | 432 | 212 | 202 | 210 | 204 | 205 | 206 | 128 | 118 | 126 | 120 | 121 | 122 | 404 | 394 | 402 | 396 | 397 | 398 | 53 | 12 | 5335 |
| 474 | 61 | 79 | 77 | 75 | 73 | 50 | 48 | 46 | 44 | 405 | 407 | 409 | 411 | 413 | 434 | 436 | 438 | 440 | 442 | 62 | 11 | 5335 |
| 21 | 41 | 39 | 37 | 35 | 33 | 9 | 7 | 5 | 3 | 443 | 445 | 447 | 449 | 451 | 453 | 475 | 477 | 479 | 481 | 483 | 22 | 5335 |
| 5335 | 5335 | 5335 | 5335 | 5335 | 5335 | 5335 | 5335 | 5335 | 5335 | 5335 | 5335 | 5335 | 5335 | 5335 | 5335 | 5335 | 5335 | 5335 | 5335 | 5335 | 5335 | 5335 |

*In this case the magic sums are* $S_{22\times22} := 5335$, $S_{20\times20} := 4850$, *and* $S_{18\times18} := 4365$. *The magic rectangles of order* $2\times6$ *are of* ***different magic sums***.

**Example 2.12.** *A* ***block bordered*** *magic square of order 22 with inner block as a magic square of order 20 is given by*

| | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | | | | | | | | | | 5335 |
| 463 | 444 | 446 | 448 | 450 | 452 | 476 | 478 | 480 | 482 | 42 | 40 | 38 | 36 | 34 | 32 | 10 | 8 | 6 | 4 | 2 | 464 | 5335 |
| 31 | 150 | 337 | 336 | 147 | 47 | 437 | 436 | 50 | 51 | 433 | 432 | 54 | 55 | 429 | 428 | 58 | 162 | 325 | 324 | 159 | 454 | 5335 |
| 29 | 335 | 148 | 149 | 338 | 438 | 48 | 49 | 435 | 434 | 52 | 53 | 431 | 430 | 56 | 57 | 427 | 323 | 160 | 161 | 326 | 456 | 5335 |
| 27 | 66 | 421 | 420 | 63 | 67 | 417 | 416 | 70 | 71 | 413 | 412 | 74 | 75 | 409 | 408 | 78 | 82 | 405 | 404 | 79 | 458 | 5335 |
| 25 | 419 | 64 | 65 | 422 | 418 | 68 | 69 | 415 | 414 | 72 | 73 | 411 | 410 | 76 | 77 | 407 | 403 | 80 | 81 | 406 | 460 | 5335 |
| 23 | 83 | 401 | 400 | 86 | 87 | 397 | 396 | 90 | 91 | 393 | 392 | 94 | 98 | 389 | 388 | 95 | 99 | 385 | 384 | 102 | 462 | 5335 |
| 19 | 402 | 84 | 85 | 399 | 398 | 88 | 89 | 395 | 394 | 92 | 93 | 391 | 387 | 96 | 97 | 390 | 386 | 100 | 101 | 383 | 466 | 5335 |
| 17 | 103 | 381 | 380 | 106 | 110 | 377 | 376 | 107 | 111 | 373 | 372 | 114 | 115 | 369 | 368 | 118 | 119 | 365 | 364 | 122 | 468 | 5335 |
| 15 | 382 | 104 | 105 | 379 | 375 | 108 | 109 | 378 | 374 | 112 | 113 | 371 | 370 | 116 | 117 | 367 | 366 | 120 | 121 | 363 | 470 | 5335 |
| 13 | 123 | 361 | 360 | 126 | 127 | 357 | 356 | 130 | 131 | 353 | 352 | 134 | 135 | 349 | 348 | 138 | 139 | 345 | 344 | 142 | 472 | 5335 |
| 1 | 362 | 124 | 125 | 359 | 358 | 128 | 129 | 355 | 354 | 132 | 133 | 351 | 350 | 136 | 137 | 347 | 346 | 140 | 141 | 343 | 484 | 5335 |
| 455 | 143 | 341 | 340 | 146 | 163 | 321 | 320 | 166 | 151 | 333 | 332 | 154 | 155 | 329 | 328 | 158 | 59 | 425 | 424 | 62 | 30 | 5335 |
| 457 | 342 | 144 | 145 | 339 | 322 | 164 | 165 | 319 | 334 | 152 | 153 | 331 | 330 | 156 | 157 | 327 | 426 | 60 | 61 | 423 | 28 | 5335 |
| 459 | 43 | 441 | 440 | 46 | 170 | 317 | 316 | 167 | 171 | 313 | 312 | 174 | 178 | 309 | 308 | 175 | 179 | 305 | 304 | 182 | 26 | 5335 |
| 461 | 442 | 44 | 45 | 439 | 315 | 168 | 169 | 318 | 314 | 172 | 173 | 311 | 307 | 176 | 177 | 310 | 306 | 180 | 181 | 303 | 24 | 5335 |
| 465 | 183 | 301 | 300 | 186 | 190 | 296 | 297 | 187 | 191 | 293 | 292 | 194 | 195 | 289 | 288 | 198 | 199 | 285 | 284 | 202 | 20 | 5335 |
| 467 | 302 | 184 | 185 | 299 | 295 | 189 | 188 | 298 | 294 | 192 | 193 | 291 | 290 | 196 | 197 | 287 | 286 | 200 | 201 | 283 | 18 | 5335 |
| 469 | 206 | 280 | 281 | 203 | 207 | 277 | 276 | 210 | 211 | 273 | 272 | 214 | 215 | 269 | 268 | 218 | 219 | 265 | 264 | 222 | 16 | 5335 |
| 471 | 279 | 205 | 204 | 282 | 278 | 208 | 209 | 275 | 274 | 212 | 213 | 271 | 270 | 216 | 217 | 267 | 266 | 220 | 221 | 263 | 14 | 5335 |
| 473 | 223 | 261 | 260 | 226 | 227 | 257 | 256 | 230 | 231 | 253 | 252 | 234 | 235 | 249 | 248 | 238 | 239 | 245 | 244 | 242 | 12 | 5335 |
| 474 | 262 | 224 | 225 | 259 | 258 | 228 | 229 | 255 | 254 | 232 | 233 | 251 | 250 | 236 | 237 | 247 | 246 | 240 | 241 | 243 | 11 | 5335 |
| 21 | 41 | 39 | 37 | 35 | 33 | 9 | 7 | 5 | 3 | 443 | 445 | 447 | 449 | 451 | 453 | 475 | 477 | 479 | 481 | 483 | 22 | 5335 |
| 5335 | 5335 | 5335 | 5335 | 5335 | 5335 | 5335 | 5335 | 5335 | 5335 | 5335 | 5335 | 5335 | 5335 | 5335 | 5335 | 5335 | 5335 | 5335 | 5335 | 5335 | 5335 | 5335 |

*In this case the magic sums are* $S_{22\times22} := 5335$ *and* $S_{20\times20} := 4850$*. The magic rectangles of order* $2\times4$ *are of* ***equal magic sums*** *given by* $R_{2\times4} := (970, 485)$.

**Example 2.13.** *A* ***block bordered*** *magic square of order 22 with inner block as a magic square of order 20 is given by*

Magic Rectangles in Construction of Magic and Block-Bordered Magic Squares, June 07, 2022, pp. 1-70, https://doi.org/10.5281/zenodo.6621071

| | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | | | | | | | | | | 5335 |
| 463 | 444 | 446 | 448 | 450 | 452 | 476 | 478 | 480 | 482 | 42 | 40 | 38 | 36 | 34 | 32 | 10 | 8 | 6 | 4 | 2 | 464 | 5335 |
| 31 | 43 | 44 | 45 | 46 | 47 | 438 | 439 | 440 | 441 | 442 | 63 | 64 | 65 | 66 | 67 | 418 | 419 | 420 | 421 | 422 | 454 | 5335 |
| 29 | 433 | 434 | 435 | 436 | 437 | 48 | 49 | 50 | 51 | 52 | 413 | 414 | 415 | 416 | 417 | 68 | 69 | 70 | 71 | 72 | 456 | 5335 |
| 27 | 432 | 431 | 430 | 429 | 428 | 57 | 56 | 55 | 54 | 53 | 412 | 411 | 410 | 409 | 408 | 77 | 76 | 75 | 74 | 73 | 458 | 5335 |
| 25 | 62 | 61 | 60 | 59 | 58 | 427 | 426 | 425 | 424 | 423 | 82 | 81 | 80 | 79 | 78 | 407 | 406 | 405 | 404 | 403 | 460 | 5335 |
| 23 | 83 | 84 | 85 | 86 | 87 | 398 | 399 | 400 | 401 | 402 | 103 | 104 | 105 | 106 | 107 | 378 | 379 | 380 | 381 | 382 | 462 | 5335 |
| 19 | 393 | 394 | 395 | 396 | 397 | 88 | 89 | 90 | 91 | 92 | 373 | 374 | 375 | 376 | 377 | 108 | 109 | 110 | 111 | 112 | 466 | 5335 |
| 17 | 392 | 391 | 390 | 389 | 388 | 97 | 96 | 95 | 94 | 93 | 372 | 371 | 370 | 369 | 368 | 117 | 116 | 115 | 114 | 113 | 468 | 5335 |
| 15 | 102 | 101 | 100 | 99 | 98 | 387 | 386 | 385 | 384 | 383 | 122 | 121 | 120 | 119 | 118 | 367 | 366 | 365 | 364 | 363 | 470 | 5335 |
| 13 | 123 | 124 | 125 | 126 | 357 | 358 | 359 | 360 | 361 | 132 | 333 | 144 | 145 | 146 | 147 | 338 | 339 | 340 | 341 | 152 | 472 | 5335 |
| 1 | 353 | 354 | 355 | 356 | 127 | 128 | 129 | 130 | 131 | 362 | 143 | 334 | 335 | 336 | 337 | 148 | 149 | 150 | 151 | 342 | 484 | 5335 |
| 455 | 352 | 351 | 350 | 349 | 348 | 137 | 136 | 135 | 134 | 133 | 332 | 331 | 330 | 329 | 328 | 157 | 156 | 155 | 154 | 153 | 30 | 5335 |
| 457 | 142 | 141 | 140 | 139 | 138 | 347 | 346 | 345 | 344 | 343 | 162 | 161 | 160 | 159 | 158 | 327 | 326 | 325 | 324 | 323 | 28 | 5335 |
| 459 | 163 | 164 | 165 | 166 | 167 | 318 | 319 | 320 | 321 | 322 | 183 | 184 | 185 | 186 | 187 | 298 | 299 | 300 | 301 | 302 | 26 | 5335 |
| 461 | 313 | 314 | 315 | 316 | 317 | 168 | 169 | 170 | 171 | 172 | 293 | 294 | 295 | 296 | 297 | 188 | 189 | 190 | 191 | 192 | 24 | 5335 |
| 465 | 312 | 311 | 310 | 309 | 308 | 177 | 176 | 175 | 174 | 173 | 292 | 291 | 290 | 289 | 288 | 197 | 196 | 195 | 194 | 193 | 20 | 5335 |
| 467 | 182 | 181 | 180 | 179 | 178 | 307 | 306 | 305 | 304 | 303 | 202 | 201 | 200 | 199 | 198 | 287 | 286 | 285 | 284 | 283 | 18 | 5335 |
| 469 | 203 | 204 | 205 | 206 | 207 | 278 | 279 | 280 | 281 | 282 | 223 | 224 | 225 | 226 | 227 | 258 | 259 | 260 | 261 | 262 | 16 | 5335 |
| 471 | 273 | 274 | 275 | 276 | 277 | 208 | 209 | 210 | 211 | 212 | 253 | 254 | 255 | 256 | 257 | 228 | 229 | 230 | 231 | 232 | 14 | 5335 |
| 473 | 222 | 271 | 270 | 269 | 268 | 267 | 216 | 215 | 214 | 213 | 252 | 251 | 250 | 249 | 238 | 237 | 236 | 235 | 234 | 243 | 12 | 5335 |
| 474 | 272 | 221 | 220 | 219 | 218 | 217 | 266 | 265 | 264 | 263 | 242 | 241 | 240 | 239 | 248 | 247 | 246 | 245 | 244 | 233 | 11 | 5335 |
| 21 | 41 | 39 | 37 | 35 | 33 | 9 | 7 | 5 | 3 | 443 | 445 | 447 | 449 | 451 | 453 | 475 | 477 | 479 | 481 | 483 | 22 | 5335 |
| 5335 | 5335 | 5335 | 5335 | 5335 | 5335 | 5335 | 5335 | 5335 | 5335 | 5335 | 5335 | 5335 | 5335 | 5335 | 5335 | 5335 | 5335 | 5335 | 5335 | 5335 | 5335 | 5335 |

*In this case the magic sums are* $S_{22\times 22} := 5335$ *and* $S_{20\times 20} := 4850$*. The magic rectangles of order* $4\times 10$ *are of* ***equal magic sums*** *given by* $R_{4\times 10} := (2425, 970)$.

## 2.9 Magic Square of Order $24\times 24$

**Example 2.14.** *A magic square of order* $24\times 24$ *constructed based on equal sums magic rectangles of order* $2\times 4$ *is given by*

| | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | | | | | | | | | | | | 6924 |
| 12 | 566 | 567 | 9 | 8 | 570 | 571 | 5 | 44 | 534 | 535 | 41 | 40 | 538 | 539 | 37 | 76 | 502 | 503 | 73 | 72 | 506 | 507 | 69 | 6924 |
| 565 | 11 | 10 | 568 | 569 | 7 | 6 | 572 | 533 | 43 | 42 | 536 | 537 | 39 | 38 | 540 | 501 | 75 | 74 | 504 | 505 | 71 | 70 | 508 | 6924 |
| 1 | 575 | 574 | 4 | 13 | 563 | 562 | 16 | 33 | 543 | 542 | 36 | 45 | 531 | 530 | 48 | 65 | 511 | 510 | 68 | 77 | 499 | 498 | 80 | 6924 |
| 576 | 2 | 3 | 573 | 564 | 14 | 15 | 561 | 544 | 34 | 35 | 541 | 532 | 46 | 47 | 529 | 512 | 66 | 67 | 509 | 500 | 78 | 79 | 497 | 6924 |
| 25 | 551 | 550 | 28 | 24 | 554 | 555 | 21 | 57 | 519 | 518 | 60 | 56 | 522 | 523 | 53 | 89 | 487 | 486 | 92 | 88 | 490 | 491 | 85 | 6924 |
| 552 | 26 | 27 | 549 | 553 | 23 | 22 | 556 | 520 | 58 | 59 | 517 | 521 | 55 | 54 | 524 | 488 | 90 | 91 | 485 | 489 | 87 | 86 | 492 | 6924 |
| 20 | 558 | 559 | 17 | 29 | 547 | 546 | 32 | 52 | 526 | 527 | 49 | 61 | 515 | 514 | 64 | 84 | 494 | 495 | 81 | 93 | 483 | 482 | 96 | 6924 |
| 557 | 19 | 18 | 560 | 548 | 30 | 31 | 545 | 525 | 51 | 50 | 528 | 516 | 62 | 63 | 513 | 493 | 83 | 82 | 496 | 484 | 94 | 95 | 481 | 6924 |
| 108 | 470 | 471 | 105 | 104 | 474 | 475 | 101 | 140 | 438 | 439 | 137 | 136 | 442 | 443 | 133 | 172 | 406 | 407 | 169 | 168 | 410 | 411 | 165 | 6924 |
| 469 | 107 | 106 | 472 | 473 | 103 | 102 | 476 | 437 | 139 | 138 | 440 | 441 | 135 | 134 | 444 | 405 | 171 | 170 | 408 | 409 | 167 | 166 | 412 | 6924 |
| 97 | 479 | 478 | 100 | 109 | 467 | 466 | 112 | 129 | 447 | 446 | 132 | 141 | 435 | 434 | 144 | 161 | 415 | 414 | 164 | 173 | 403 | 402 | 176 | 6924 |
| 480 | 98 | 99 | 477 | 468 | 110 | 111 | 465 | 448 | 130 | 131 | 445 | 436 | 142 | 143 | 433 | 416 | 162 | 163 | 413 | 404 | 174 | 175 | 401 | 6924 |
| 121 | 455 | 454 | 124 | 120 | 458 | 459 | 117 | 153 | 423 | 422 | 156 | 152 | 426 | 427 | 149 | 185 | 391 | 390 | 188 | 184 | 394 | 395 | 181 | 6924 |
| 456 | 122 | 123 | 453 | 457 | 119 | 118 | 460 | 424 | 154 | 155 | 421 | 425 | 151 | 150 | 428 | 392 | 186 | 187 | 389 | 393 | 183 | 182 | 396 | 6924 |
| 116 | 462 | 463 | 113 | 125 | 451 | 450 | 128 | 148 | 430 | 431 | 145 | 157 | 419 | 418 | 160 | 180 | 398 | 399 | 177 | 189 | 387 | 386 | 192 | 6924 |
| 461 | 115 | 114 | 464 | 452 | 126 | 127 | 449 | 429 | 147 | 146 | 432 | 420 | 158 | 159 | 417 | 397 | 179 | 178 | 400 | 388 | 190 | 191 | 385 | 6924 |
| 204 | 374 | 375 | 201 | 200 | 378 | 379 | 197 | 236 | 342 | 343 | 233 | 232 | 346 | 347 | 229 | 268 | 310 | 311 | 265 | 264 | 314 | 315 | 261 | 6924 |
| 373 | 203 | 202 | 376 | 377 | 199 | 198 | 380 | 341 | 235 | 234 | 344 | 345 | 231 | 230 | 348 | 309 | 267 | 266 | 312 | 313 | 263 | 262 | 316 | 6924 |
| 193 | 383 | 382 | 196 | 205 | 371 | 370 | 208 | 225 | 351 | 350 | 228 | 237 | 339 | 338 | 240 | 257 | 319 | 318 | 260 | 269 | 307 | 306 | 272 | 6924 |
| 384 | 194 | 195 | 381 | 372 | 206 | 207 | 369 | 352 | 226 | 227 | 349 | 340 | 238 | 239 | 337 | 320 | 258 | 259 | 317 | 308 | 270 | 271 | 305 | 6924 |
| 217 | 359 | 358 | 220 | 216 | 362 | 363 | 213 | 249 | 327 | 326 | 252 | 248 | 330 | 331 | 245 | 281 | 295 | 294 | 284 | 280 | 298 | 299 | 277 | 6924 |
| 360 | 218 | 219 | 357 | 361 | 215 | 214 | 364 | 328 | 250 | 251 | 325 | 329 | 247 | 246 | 332 | 296 | 282 | 283 | 293 | 297 | 279 | 278 | 300 | 6924 |
| 212 | 366 | 367 | 209 | 221 | 355 | 354 | 224 | 244 | 334 | 335 | 241 | 253 | 323 | 322 | 256 | 276 | 302 | 303 | 273 | 285 | 291 | 290 | 288 | 6924 |
| 365 | 211 | 210 | 368 | 356 | 222 | 223 | 353 | 333 | 243 | 242 | 336 | 324 | 254 | 255 | 321 | 301 | 275 | 274 | 304 | 292 | 286 | 287 | 289 | 6924 |
| 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 |

*The **magic square** sum is $S_{24\times24} := 6924$. The magic rectangles of order $2\times4$ are of **equal magic sums** given by $R_{2\times4} := (1154, 577)$. Moreover, each block of order $8\times8$ is a magic square with magic sums $S_{8\times8} := 2308$.*

**Example 2.15.** *A magic square of order $24\times24$ constructed based on magic rectangles of order $4\times6$ is given by*

| | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | | | | | | | | | | | | 6924 |
| 1 | 2 | 3 | 574 | 575 | 576 | 217 | 218 | 219 | 358 | 359 | 360 | 205 | 206 | 207 | 370 | 371 | 372 | 133 | 134 | 135 | 442 | 443 | 444 | 6924 |
| 571 | 572 | 573 | 4 | 5 | 6 | 355 | 356 | 357 | 220 | 221 | 222 | 367 | 368 | 369 | 208 | 209 | 210 | 439 | 440 | 441 | 136 | 137 | 138 | 6924 |
| 570 | 11 | 568 | 567 | 8 | 7 | 354 | 353 | 352 | 225 | 224 | 223 | 366 | 365 | 364 | 213 | 212 | 211 | 438 | 437 | 436 | 141 | 140 | 139 | 6924 |
| 12 | 569 | 10 | 9 | 566 | 565 | 228 | 227 | 226 | 351 | 350 | 349 | 216 | 215 | 214 | 363 | 362 | 361 | 144 | 143 | 142 | 435 | 434 | 433 | 6924 |
| 13 | 14 | 15 | 562 | 563 | 564 | 229 | 230 | 231 | 346 | 347 | 348 | 193 | 194 | 195 | 382 | 383 | 384 | 121 | 452 | 123 | 454 | 455 | 126 | 6924 |
| 559 | 560 | 561 | 16 | 17 | 18 | 343 | 344 | 345 | 232 | 233 | 234 | 379 | 380 | 381 | 196 | 197 | 198 | 451 | 122 | 453 | 124 | 125 | 456 | 6924 |
| 558 | 557 | 556 | 21 | 20 | 19 | 342 | 341 | 340 | 237 | 236 | 235 | 204 | 377 | 376 | 201 | 200 | 373 | 450 | 449 | 448 | 129 | 128 | 127 | 6924 |
| 24 | 23 | 22 | 555 | 554 | 553 | 240 | 239 | 238 | 339 | 338 | 337 | 378 | 203 | 202 | 375 | 374 | 199 | 132 | 131 | 130 | 447 | 446 | 445 | 6924 |
| 25 | 26 | 27 | 550 | 551 | 552 | 241 | 242 | 243 | 334 | 335 | 336 | 181 | 182 | 183 | 394 | 395 | 396 | 109 | 110 | 111 | 466 | 467 | 468 | 6924 |
| 547 | 548 | 549 | 28 | 29 | 30 | 331 | 332 | 333 | 244 | 245 | 246 | 391 | 392 | 393 | 184 | 185 | 186 | 463 | 464 | 465 | 112 | 113 | 114 | 6924 |
| 546 | 545 | 544 | 33 | 32 | 31 | 330 | 329 | 328 | 249 | 248 | 247 | 390 | 389 | 388 | 189 | 188 | 187 | 462 | 461 | 460 | 117 | 116 | 115 | 6924 |
| 36 | 35 | 34 | 543 | 542 | 541 | 252 | 251 | 250 | 327 | 326 | 325 | 192 | 191 | 190 | 387 | 386 | 385 | 120 | 119 | 118 | 459 | 458 | 457 | 6924 |
| 277 | 296 | 279 | 280 | 299 | 300 | 37 | 38 | 39 | 538 | 539 | 540 | 97 | 98 | 99 | 478 | 479 | 480 | 169 | 170 | 171 | 406 | 407 | 408 | 6924 |
| 295 | 278 | 297 | 298 | 281 | 282 | 535 | 536 | 532 | 45 | 41 | 42 | 475 | 476 | 472 | 105 | 101 | 102 | 403 | 404 | 405 | 172 | 173 | 174 | 6924 |
| 288 | 293 | 292 | 285 | 290 | 283 | 534 | 533 | 537 | 40 | 44 | 43 | 474 | 473 | 477 | 100 | 104 | 103 | 402 | 401 | 400 | 177 | 176 | 175 | 6924 |
| 294 | 287 | 286 | 291 | 284 | 289 | 48 | 47 | 46 | 531 | 530 | 529 | 108 | 107 | 106 | 471 | 470 | 469 | 180 | 179 | 178 | 399 | 398 | 397 | 6924 |
| 253 | 254 | 255 | 322 | 323 | 324 | 49 | 50 | 51 | 526 | 527 | 528 | 85 | 86 | 87 | 490 | 491 | 492 | 157 | 158 | 159 | 418 | 419 | 420 | 6924 |
| 319 | 320 | 321 | 256 | 257 | 258 | 523 | 524 | 525 | 52 | 53 | 54 | 486 | 488 | 489 | 88 | 89 | 91 | 415 | 416 | 417 | 160 | 161 | 162 | 6924 |
| 318 | 317 | 316 | 261 | 260 | 259 | 522 | 521 | 520 | 57 | 56 | 55 | 487 | 485 | 484 | 93 | 92 | 90 | 414 | 413 | 412 | 165 | 164 | 163 | 6924 |
| 264 | 263 | 262 | 315 | 314 | 313 | 60 | 59 | 58 | 519 | 518 | 517 | 96 | 95 | 94 | 483 | 482 | 481 | 168 | 167 | 166 | 411 | 410 | 409 | 6924 |
| 265 | 266 | 267 | 310 | 311 | 312 | 61 | 62 | 63 | 514 | 515 | 516 | 73 | 74 | 75 | 502 | 503 | 504 | 145 | 146 | 147 | 430 | 431 | 432 | 6924 |
| 307 | 309 | 308 | 268 | 269 | 270 | 511 | 512 | 513 | 64 | 65 | 66 | 499 | 500 | 501 | 76 | 77 | 78 | 427 | 428 | 429 | 148 | 149 | 150 | 6924 |
| 276 | 305 | 304 | 273 | 272 | 301 | 510 | 509 | 508 | 69 | 68 | 67 | 498 | 497 | 496 | 81 | 80 | 79 | 426 | 425 | 424 | 153 | 152 | 151 | 6924 |
| 306 | 274 | 275 | 303 | 302 | 271 | 72 | 71 | 70 | 507 | 506 | 505 | 84 | 83 | 82 | 495 | 494 | 493 | 156 | 155 | 154 | 423 | 422 | 421 | 6924 |
| 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 |

*The **magic square** sum is $S_{24\times 24} := 6924$. The magic rectangles of order $4\times 6$ are of **equal magic sums** given by $R_{4\times 6} := (1731, 1154)$.*

## 2.10 Block Bordered Magic Squares of Order $26\times 26$

It is based on the Examples 2.14 and 2.15 and single external border. The internal blocks are magic squares of order 24.

**Example 2.16.** *A **block bordered** magic square of order 26 with inner block as a magic square of order 24 is given by*

| | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | | | | | | | | | | | | | | 8801 |
| 651 | 628 | 630 | 632 | 634 | 636 | 638 | 666 | 668 | 670 | 672 | 674 | 50 | 48 | 46 | 44 | 42 | 40 | 38 | 12 | 10 | 8 | 6 | 4 | 2 | 652 | 8801 |
| 37 | 62 | 616 | 617 | 59 | 58 | 620 | 621 | 55 | 94 | 584 | 585 | 91 | 90 | 588 | 589 | 87 | 126 | 552 | 553 | 123 | 122 | 556 | 557 | 119 | 640 | 8801 |
| 35 | 615 | 61 | 60 | 618 | 619 | 57 | 56 | 622 | 583 | 93 | 92 | 586 | 587 | 89 | 88 | 590 | 551 | 125 | 124 | 554 | 555 | 121 | 120 | 558 | 642 | 8801 |
| 33 | 51 | 625 | 624 | 54 | 63 | 613 | 612 | 66 | 83 | 593 | 592 | 86 | 95 | 581 | 580 | 98 | 115 | 561 | 560 | 118 | 127 | 549 | 548 | 130 | 644 | 8801 |
| 31 | 626 | 52 | 53 | 623 | 614 | 64 | 65 | 611 | 594 | 84 | 85 | 591 | 582 | 96 | 97 | 579 | 562 | 116 | 117 | 559 | 550 | 128 | 129 | 547 | 646 | 8801 |
| 29 | 75 | 601 | 600 | 78 | 74 | 604 | 605 | 71 | 107 | 569 | 568 | 110 | 106 | 572 | 573 | 103 | 139 | 537 | 536 | 142 | 138 | 540 | 541 | 135 | 648 | 8801 |
| 27 | 602 | 76 | 77 | 599 | 603 | 73 | 72 | 606 | 570 | 108 | 109 | 567 | 571 | 105 | 104 | 574 | 538 | 140 | 141 | 535 | 539 | 137 | 136 | 542 | 650 | 8801 |
| 23 | 70 | 608 | 609 | 67 | 79 | 597 | 596 | 82 | 102 | 576 | 577 | 99 | 111 | 565 | 564 | 114 | 134 | 544 | 545 | 131 | 143 | 533 | 532 | 146 | 654 | 8801 |
| 21 | 607 | 69 | 68 | 610 | 598 | 80 | 81 | 595 | 575 | 101 | 100 | 578 | 566 | 112 | 113 | 563 | 543 | 133 | 132 | 546 | 534 | 144 | 145 | 531 | 656 | 8801 |
| 19 | 158 | 520 | 521 | 155 | 154 | 524 | 525 | 151 | 190 | 488 | 489 | 187 | 186 | 492 | 493 | 183 | 222 | 456 | 457 | 219 | 218 | 460 | 461 | 215 | 658 | 8801 |
| 17 | 519 | 157 | 156 | 522 | 523 | 153 | 152 | 526 | 487 | 189 | 188 | 490 | 491 | 185 | 184 | 494 | 455 | 221 | 220 | 458 | 459 | 217 | 216 | 462 | 660 | 8801 |
| 15 | 147 | 529 | 528 | 150 | 159 | 517 | 516 | 162 | 179 | 497 | 496 | 182 | 191 | 485 | 484 | 194 | 211 | 465 | 464 | 214 | 223 | 453 | 452 | 226 | 662 | 8801 |
| 1 | 530 | 148 | 149 | 527 | 518 | 160 | 161 | 515 | 498 | 180 | 181 | 495 | 486 | 192 | 193 | 483 | 466 | 212 | 213 | 463 | 454 | 224 | 225 | 451 | 676 | 8801 |
| 641 | 171 | 505 | 504 | 174 | 170 | 508 | 509 | 167 | 203 | 473 | 472 | 206 | 202 | 476 | 477 | 199 | 235 | 441 | 440 | 238 | 234 | 444 | 445 | 231 | 36 | 8801 |
| 643 | 506 | 172 | 173 | 503 | 507 | 169 | 168 | 510 | 474 | 204 | 205 | 471 | 475 | 201 | 200 | 478 | 442 | 236 | 237 | 439 | 443 | 233 | 232 | 446 | 34 | 8801 |
| 645 | 166 | 512 | 513 | 163 | 175 | 501 | 500 | 178 | 198 | 480 | 481 | 195 | 207 | 469 | 468 | 210 | 230 | 448 | 449 | 227 | 239 | 437 | 436 | 242 | 32 | 8801 |
| 647 | 511 | 165 | 164 | 514 | 502 | 176 | 177 | 499 | 479 | 197 | 196 | 482 | 470 | 208 | 209 | 467 | 447 | 229 | 228 | 450 | 438 | 240 | 241 | 435 | 30 | 8801 |
| 649 | 254 | 424 | 425 | 251 | 250 | 428 | 429 | 247 | 286 | 392 | 393 | 283 | 282 | 396 | 397 | 279 | 318 | 360 | 361 | 315 | 314 | 364 | 365 | 311 | 28 | 8801 |
| 653 | 423 | 253 | 252 | 426 | 427 | 249 | 248 | 430 | 391 | 285 | 284 | 394 | 395 | 281 | 280 | 398 | 359 | 317 | 316 | 362 | 363 | 313 | 312 | 366 | 24 | 8801 |
| 655 | 243 | 433 | 432 | 246 | 255 | 421 | 420 | 258 | 275 | 401 | 400 | 278 | 287 | 389 | 388 | 290 | 307 | 369 | 368 | 310 | 319 | 357 | 356 | 322 | 22 | 8801 |
| 657 | 434 | 244 | 245 | 431 | 422 | 256 | 257 | 419 | 402 | 276 | 277 | 399 | 390 | 288 | 289 | 387 | 370 | 308 | 309 | 367 | 358 | 320 | 321 | 355 | 20 | 8801 |
| 659 | 267 | 409 | 408 | 270 | 266 | 412 | 413 | 263 | 299 | 377 | 376 | 302 | 298 | 380 | 381 | 295 | 331 | 345 | 344 | 334 | 330 | 348 | 349 | 327 | 18 | 8801 |
| 661 | 410 | 268 | 269 | 407 | 411 | 265 | 264 | 414 | 378 | 300 | 301 | 375 | 379 | 297 | 296 | 382 | 346 | 332 | 333 | 343 | 347 | 329 | 328 | 350 | 16 | 8801 |
| 663 | 262 | 416 | 417 | 259 | 271 | 405 | 404 | 274 | 294 | 384 | 385 | 291 | 303 | 373 | 372 | 306 | 326 | 352 | 353 | 323 | 335 | 341 | 340 | 338 | 14 | 8801 |
| 664 | 415 | 261 | 260 | 418 | 406 | 272 | 273 | 403 | 383 | 293 | 292 | 386 | 374 | 304 | 305 | 371 | 351 | 325 | 324 | 354 | 342 | 336 | 337 | 339 | 13 | 8801 |
| 25 | 49 | 47 | 45 | 43 | 41 | 39 | 11 | 9 | 7 | 5 | 3 | 627 | 629 | 631 | 633 | 635 | 637 | 639 | 665 | 667 | 669 | 671 | 673 | 675 | 26 | 8801 |
| 8801 | 8801 | 8801 | 8801 | 8801 | 8801 | 8801 | 8801 | 8801 | 8801 | 8801 | 8801 | 8801 | 8801 | 8801 | 8801 | 8801 | 8801 | 8801 | 8801 | 8801 | 8801 | 8801 | 8801 | 8801 | 8801 | 8801 |

*In this case the magic sums are* $S_{26\times 26} := 8801$ *and* $S_{24\times 24} := 8124$*. The magic rectangles of order* $2\times 4$ *are of* ***equal magic sums*** *given by* $R_{2\times 4} := (1354, 677)$.

**Example 2.17.** *A* ***block bordered*** *magic square of order 26 with inner block as a magic square of order 24 is given by*

| | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | | | | | | | | | | | | | | 8801 |
| 651 | 628 | 630 | 632 | 634 | 636 | 638 | 666 | 668 | 670 | 672 | 674 | 50 | 48 | 46 | 44 | 42 | 40 | 38 | 12 | 10 | 8 | 6 | 4 | 2 | 652 | 8801 |
| 37 | 51 | 52 | 53 | 624 | 625 | 626 | 267 | 268 | 269 | 408 | 409 | 410 | 255 | 256 | 257 | 420 | 421 | 422 | 183 | 184 | 185 | 492 | 493 | 494 | 640 | 8801 |
| 35 | 621 | 622 | 623 | 54 | 55 | 56 | 405 | 406 | 407 | 270 | 271 | 272 | 417 | 418 | 419 | 258 | 259 | 260 | 489 | 490 | 491 | 186 | 187 | 188 | 642 | 8801 |
| 33 | 620 | 61 | 618 | 617 | 58 | 57 | 404 | 403 | 402 | 275 | 274 | 273 | 416 | 415 | 414 | 263 | 262 | 261 | 488 | 487 | 486 | 191 | 190 | 189 | 644 | 8801 |
| 31 | 62 | 619 | 60 | 59 | 616 | 615 | 278 | 277 | 276 | 401 | 400 | 399 | 266 | 265 | 264 | 413 | 412 | 411 | 194 | 193 | 192 | 485 | 484 | 483 | 646 | 8801 |
| 29 | 63 | 64 | 65 | 612 | 613 | 614 | 279 | 280 | 281 | 396 | 397 | 398 | 243 | 244 | 245 | 432 | 433 | 434 | 171 | 502 | 173 | 504 | 505 | 176 | 648 | 8801 |
| 27 | 609 | 610 | 611 | 66 | 67 | 68 | 393 | 394 | 395 | 282 | 283 | 284 | 429 | 430 | 431 | 246 | 247 | 248 | 501 | 172 | 503 | 174 | 175 | 506 | 650 | 8801 |
| 23 | 608 | 607 | 606 | 71 | 70 | 69 | 392 | 391 | 390 | 287 | 286 | 285 | 254 | 427 | 426 | 251 | 250 | 423 | 500 | 499 | 498 | 179 | 178 | 177 | 654 | 8801 |
| 21 | 74 | 73 | 72 | 605 | 604 | 603 | 290 | 289 | 288 | 389 | 388 | 387 | 428 | 253 | 252 | 425 | 424 | 249 | 182 | 181 | 180 | 497 | 496 | 495 | 656 | 8801 |
| 19 | 75 | 76 | 77 | 600 | 601 | 602 | 291 | 292 | 293 | 384 | 385 | 386 | 231 | 232 | 233 | 444 | 445 | 446 | 159 | 160 | 161 | 516 | 517 | 518 | 658 | 8801 |
| 17 | 597 | 598 | 599 | 78 | 79 | 80 | 381 | 382 | 383 | 294 | 295 | 296 | 441 | 442 | 443 | 234 | 235 | 236 | 513 | 514 | 515 | 162 | 163 | 164 | 660 | 8801 |
| 15 | 596 | 595 | 594 | 83 | 82 | 81 | 380 | 379 | 378 | 299 | 298 | 297 | 440 | 439 | 438 | 239 | 238 | 237 | 512 | 511 | 510 | 167 | 166 | 165 | 662 | 8801 |
| 1 | 86 | 85 | 84 | 593 | 592 | 591 | 302 | 301 | 300 | 377 | 376 | 375 | 242 | 241 | 240 | 437 | 436 | 435 | 170 | 169 | 168 | 509 | 508 | 507 | 676 | 8801 |
| 641 | 327 | 346 | 329 | 330 | 349 | 350 | 87 | 88 | 89 | 588 | 589 | 590 | 147 | 148 | 149 | 528 | 529 | 530 | 219 | 220 | 221 | 456 | 457 | 458 | 36 | 8801 |
| 643 | 345 | 328 | 347 | 348 | 331 | 332 | 585 | 586 | 582 | 95 | 91 | 92 | 525 | 526 | 522 | 155 | 151 | 152 | 453 | 454 | 455 | 222 | 223 | 224 | 34 | 8801 |
| 645 | 338 | 343 | 342 | 335 | 340 | 333 | 584 | 583 | 587 | 90 | 94 | 93 | 524 | 523 | 527 | 150 | 154 | 153 | 452 | 451 | 450 | 227 | 226 | 225 | 32 | 8801 |
| 647 | 344 | 337 | 336 | 341 | 334 | 339 | 98 | 97 | 96 | 581 | 580 | 579 | 158 | 157 | 156 | 521 | 520 | 519 | 230 | 229 | 228 | 449 | 448 | 447 | 30 | 8801 |
| 649 | 303 | 304 | 305 | 372 | 373 | 374 | 99 | 100 | 101 | 576 | 577 | 578 | 135 | 136 | 137 | 540 | 541 | 542 | 207 | 208 | 209 | 468 | 469 | 470 | 28 | 8801 |
| 653 | 369 | 370 | 371 | 306 | 307 | 308 | 573 | 574 | 575 | 102 | 103 | 104 | 536 | 538 | 539 | 138 | 139 | 141 | 465 | 466 | 467 | 210 | 211 | 212 | 24 | 8801 |
| 655 | 368 | 367 | 366 | 311 | 310 | 309 | 572 | 571 | 570 | 107 | 106 | 105 | 537 | 535 | 534 | 143 | 142 | 140 | 464 | 463 | 462 | 215 | 214 | 213 | 22 | 8801 |
| 657 | 314 | 313 | 312 | 365 | 364 | 363 | 110 | 109 | 108 | 569 | 568 | 567 | 146 | 145 | 144 | 533 | 532 | 531 | 218 | 217 | 216 | 461 | 460 | 459 | 20 | 8801 |
| 659 | 315 | 316 | 317 | 360 | 361 | 362 | 111 | 112 | 113 | 564 | 565 | 566 | 123 | 124 | 125 | 552 | 553 | 554 | 195 | 196 | 197 | 480 | 481 | 482 | 18 | 8801 |
| 661 | 357 | 359 | 358 | 318 | 319 | 320 | 561 | 562 | 563 | 114 | 115 | 116 | 549 | 550 | 551 | 126 | 127 | 128 | 477 | 478 | 479 | 198 | 199 | 200 | 16 | 8801 |
| 663 | 326 | 355 | 354 | 323 | 322 | 351 | 560 | 559 | 558 | 119 | 118 | 117 | 548 | 547 | 546 | 131 | 130 | 129 | 476 | 475 | 474 | 203 | 202 | 201 | 14 | 8801 |
| 664 | 356 | 324 | 325 | 353 | 352 | 321 | 122 | 121 | 120 | 557 | 556 | 555 | 134 | 133 | 132 | 545 | 544 | 543 | 206 | 205 | 204 | 473 | 472 | 471 | 13 | 8801 |
| 25 | 49 | 47 | 45 | 43 | 41 | 39 | 11 | 9 | 7 | 5 | 3 | 627 | 629 | 631 | 633 | 635 | 637 | 639 | 665 | 667 | 669 | 671 | 673 | 675 | 26 | 8801 |
| 8801 | 8801 | 8801 | 8801 | 8801 | 8801 | 8801 | 8801 | 8801 | 8801 | 8801 | 8801 | 8801 | 8801 | 8801 | 8801 | 8801 | 8801 | 8801 | 8801 | 8801 | 8801 | 8801 | 8801 | 8801 | 8801 | 8801 |

*In this case the magic sums are $S_{26\times26} := 8801$ and $S_{24\times24} := 8124$. The magic rectangles of order $4\times6$ are of* ***equal magic sums*** *given by $R_{2\times4} := (2031, 1354)$.*

## 2.11 Magic Square of Order $28\times28$

**Example 2.18.** *A magic square of order $28\times28$ constructed based on blocks of magic rectangles of order $2\times4$ is given by*

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | 10990 |
| 208 | 579 | 578 | 205 | 5 | 779 | 778 | 8 | 9 | 775 | 774 | 12 | 13 | 771 | 770 | 16 | 17 | 767 | 766 | 20 | 21 | 763 | 762 | 24 | 216 | 571 | 570 | 213 | 10990 |
| 577 | 206 | 207 | 580 | 780 | 6 | 7 | 777 | 776 | 10 | 11 | 773 | 772 | 14 | 15 | 769 | 768 | 18 | 19 | 765 | 764 | 22 | 23 | 761 | 569 | 214 | 215 | 572 | 10990 |
| 32 | 755 | 754 | 29 | 33 | 751 | 750 | 36 | 37 | 747 | 746 | 40 | 41 | 743 | 742 | 44 | 45 | 739 | 738 | 48 | 49 | 735 | 734 | 52 | 56 | 731 | 730 | 53 | 10990 |
| 753 | 30 | 31 | 756 | 752 | 34 | 35 | 749 | 748 | 38 | 39 | 745 | 744 | 42 | 43 | 741 | 740 | 46 | 47 | 737 | 736 | 50 | 51 | 733 | 729 | 54 | 55 | 732 | 10990 |
| 57 | 727 | 726 | 60 | 64 | 723 | 722 | 61 | 65 | 719 | 718 | 68 | 69 | 715 | 714 | 72 | 73 | 711 | 710 | 76 | 77 | 707 | 706 | 80 | 81 | 703 | 702 | 84 | 10990 |
| 728 | 58 | 59 | 725 | 721 | 62 | 63 | 724 | 720 | 66 | 67 | 717 | 716 | 70 | 71 | 713 | 712 | 74 | 75 | 709 | 708 | 78 | 79 | 705 | 704 | 82 | 83 | 701 | 10990 |
| 85 | 699 | 698 | 88 | 92 | 695 | 694 | 89 | 93 | 691 | 690 | 96 | 97 | 687 | 686 | 100 | 101 | 683 | 682 | 104 | 105 | 679 | 678 | 108 | 109 | 675 | 674 | 112 | 10990 |
| 700 | 86 | 87 | 697 | 693 | 90 | 91 | 696 | 692 | 94 | 95 | 689 | 688 | 98 | 99 | 685 | 684 | 102 | 103 | 681 | 680 | 106 | 107 | 677 | 676 | 110 | 111 | 673 | 10990 |
| 113 | 671 | 670 | 116 | 117 | 667 | 666 | 120 | 121 | 663 | 662 | 124 | 125 | 659 | 658 | 128 | 129 | 655 | 654 | 132 | 133 | 651 | 650 | 136 | 137 | 647 | 646 | 140 | 10990 |
| 672 | 114 | 115 | 669 | 668 | 118 | 119 | 665 | 664 | 122 | 123 | 661 | 660 | 126 | 127 | 657 | 656 | 130 | 131 | 653 | 652 | 134 | 135 | 649 | 648 | 138 | 139 | 645 | 10990 |
| 141 | 643 | 642 | 144 | 145 | 639 | 638 | 148 | 149 | 635 | 634 | 152 | 153 | 631 | 630 | 156 | 157 | 627 | 626 | 160 | 161 | 623 | 622 | 164 | 165 | 619 | 618 | 168 | 10990 |
| 644 | 142 | 143 | 641 | 640 | 146 | 147 | 637 | 636 | 150 | 151 | 633 | 632 | 154 | 155 | 629 | 628 | 158 | 159 | 625 | 624 | 162 | 163 | 621 | 620 | 166 | 167 | 617 | 10990 |
| 169 | 615 | 614 | 172 | 173 | 611 | 610 | 176 | 177 | 607 | 606 | 180 | 181 | 603 | 602 | 184 | 185 | 599 | 598 | 188 | 189 | 595 | 594 | 192 | 193 | 591 | 590 | 196 | 10990 |
| 616 | 170 | 171 | 613 | 612 | 174 | 175 | 609 | 608 | 178 | 179 | 605 | 604 | 182 | 183 | 601 | 600 | 186 | 187 | 597 | 596 | 190 | 191 | 593 | 592 | 194 | 195 | 589 | 10990 |
| 197 | 587 | 586 | 200 | 201 | 583 | 582 | 204 | 1 | 783 | 782 | 4 | 209 | 575 | 574 | 212 | 25 | 759 | 758 | 28 | 217 | 567 | 566 | 220 | 221 | 563 | 562 | 224 | 10990 |
| 588 | 198 | 199 | 585 | 584 | 202 | 203 | 581 | 784 | 2 | 3 | 781 | 576 | 210 | 211 | 573 | 760 | 26 | 27 | 757 | 568 | 218 | 219 | 565 | 564 | 222 | 223 | 561 | 10990 |
| 225 | 559 | 558 | 228 | 229 | 555 | 554 | 232 | 233 | 551 | 550 | 236 | 237 | 547 | 546 | 240 | 241 | 543 | 542 | 244 | 245 | 539 | 538 | 248 | 249 | 535 | 534 | 252 | 10990 |
| 560 | 226 | 227 | 557 | 556 | 230 | 231 | 553 | 552 | 234 | 235 | 549 | 548 | 238 | 239 | 545 | 544 | 242 | 243 | 541 | 540 | 246 | 247 | 537 | 536 | 250 | 251 | 533 | 10990 |
| 253 | 531 | 530 | 256 | 257 | 527 | 526 | 260 | 261 | 523 | 522 | 264 | 265 | 519 | 518 | 268 | 269 | 515 | 514 | 272 | 273 | 511 | 510 | 276 | 277 | 507 | 506 | 280 | 10990 |
| 532 | 254 | 255 | 529 | 528 | 258 | 259 | 525 | 524 | 262 | 263 | 521 | 520 | 266 | 267 | 517 | 516 | 270 | 271 | 513 | 512 | 274 | 275 | 509 | 508 | 278 | 279 | 505 | 10990 |
| 281 | 503 | 502 | 284 | 285 | 499 | 498 | 288 | 289 | 495 | 494 | 292 | 293 | 491 | 490 | 296 | 297 | 487 | 486 | 300 | 301 | 483 | 482 | 304 | 305 | 479 | 478 | 308 | 10990 |
| 504 | 282 | 283 | 501 | 500 | 286 | 287 | 497 | 496 | 290 | 291 | 493 | 492 | 294 | 295 | 489 | 488 | 298 | 299 | 485 | 484 | 302 | 303 | 481 | 480 | 306 | 307 | 477 | 10990 |
| 309 | 475 | 474 | 312 | 313 | 471 | 470 | 316 | 317 | 467 | 466 | 320 | 321 | 463 | 462 | 324 | 325 | 459 | 458 | 328 | 329 | 455 | 454 | 332 | 333 | 451 | 450 | 336 | 10990 |
| 476 | 310 | 311 | 473 | 472 | 314 | 315 | 469 | 468 | 318 | 319 | 465 | 464 | 322 | 323 | 461 | 460 | 326 | 327 | 457 | 456 | 330 | 331 | 453 | 452 | 334 | 335 | 449 | 10990 |
| 340 | 447 | 446 | 337 | 341 | 443 | 442 | 344 | 345 | 439 | 438 | 348 | 349 | 435 | 434 | 352 | 353 | 431 | 430 | 356 | 357 | 427 | 426 | 360 | 361 | 423 | 422 | 364 | 10990 |
| 445 | 338 | 339 | 448 | 444 | 342 | 343 | 441 | 440 | 346 | 347 | 437 | 436 | 350 | 351 | 433 | 432 | 354 | 355 | 429 | 428 | 358 | 359 | 425 | 424 | 362 | 363 | 421 | 10990 |
| 368 | 419 | 418 | 365 | 369 | 415 | 414 | 372 | 373 | 411 | 410 | 376 | 377 | 407 | 406 | 380 | 381 | 403 | 402 | 384 | 385 | 399 | 398 | 388 | 389 | 395 | 394 | 392 | 10990 |
| 417 | 366 | 367 | 420 | 416 | 370 | 371 | 413 | 412 | 374 | 375 | 409 | 408 | 378 | 379 | 405 | 404 | 382 | 383 | 401 | 400 | 386 | 387 | 397 | 396 | 390 | 391 | 393 | 10990 |
| 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 |

*The **magic square** sum is $S_{28\times 28} := 10990$. The magic rectangles of order $2\times 14$ are of **equal magic sums** given by $R_{2\times 4} := (1570, 785)$.*

**Example 2.19.** *A magic square of order $28\times 28$ constructed based on blocks of magic rectangles of order $4\times 14$ is given by*

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | 10990 |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 778 | 779 | 780 | 781 | 782 | 783 | 784 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 750 | 751 | 752 | 753 | 754 | 755 | 756 | 10990 |
| 771 | 772 | 773 | 774 | 775 | 776 | 777 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 743 | 744 | 745 | 746 | 747 | 748 | 749 | 36 | 37 | 38 | 39 | 40 | 41 | 42 | 10990 |
| 770 | 769 | 768 | 767 | 766 | 765 | 764 | 21 | 20 | 19 | 18 | 17 | 16 | 15 | 742 | 741 | 740 | 739 | 738 | 737 | 736 | 49 | 48 | 47 | 46 | 45 | 44 | 43 | 10990 |
| 28 | 27 | 26 | 25 | 24 | 23 | 22 | 763 | 762 | 761 | 760 | 759 | 758 | 757 | 56 | 55 | 54 | 53 | 52 | 51 | 50 | 735 | 734 | 733 | 732 | 731 | 730 | 729 | 10990 |
| 57 | 58 | 59 | 60 | 61 | 62 | 63 | 722 | 723 | 724 | 725 | 726 | 727 | 728 | 85 | 86 | 87 | 88 | 89 | 90 | 91 | 694 | 695 | 696 | 697 | 698 | 699 | 700 | 10990 |
| 715 | 716 | 717 | 718 | 719 | 720 | 721 | 64 | 65 | 66 | 67 | 68 | 69 | 70 | 687 | 688 | 689 | 690 | 691 | 692 | 693 | 98 | 93 | 94 | 95 | 96 | 97 | 92 | 10990 |
| 714 | 713 | 712 | 711 | 710 | 709 | 708 | 77 | 76 | 75 | 74 | 73 | 72 | 71 | 686 | 685 | 684 | 683 | 682 | 681 | 680 | 99 | 104 | 103 | 102 | 101 | 100 | 105 | 10990 |
| 84 | 83 | 82 | 81 | 80 | 79 | 78 | 707 | 706 | 705 | 704 | 703 | 702 | 701 | 112 | 111 | 110 | 109 | 108 | 107 | 106 | 679 | 678 | 677 | 676 | 675 | 674 | 673 | 10990 |
| 659 | 114 | 115 | 116 | 117 | 118 | 119 | 666 | 121 | 668 | 669 | 670 | 671 | 672 | 141 | 142 | 143 | 144 | 145 | 146 | 147 | 638 | 639 | 640 | 641 | 642 | 643 | 644 | 10990 |
| 113 | 660 | 661 | 662 | 663 | 664 | 665 | 120 | 667 | 122 | 123 | 124 | 125 | 126 | 631 | 632 | 633 | 634 | 635 | 636 | 637 | 148 | 149 | 150 | 151 | 152 | 153 | 154 | 10990 |
| 658 | 657 | 656 | 655 | 654 | 653 | 652 | 133 | 132 | 131 | 130 | 129 | 128 | 127 | 630 | 629 | 628 | 627 | 626 | 625 | 624 | 161 | 160 | 159 | 158 | 157 | 156 | 155 | 10990 |
| 140 | 139 | 138 | 137 | 136 | 135 | 134 | 651 | 650 | 649 | 648 | 647 | 646 | 645 | 168 | 167 | 166 | 165 | 164 | 163 | 162 | 623 | 622 | 621 | 620 | 619 | 618 | 617 | 10990 |
| 169 | 170 | 171 | 172 | 173 | 174 | 175 | 610 | 611 | 612 | 613 | 614 | 615 | 616 | 197 | 576 | 199 | 200 | 201 | 202 | 203 | 582 | 583 | 584 | 585 | 586 | 587 | 210 | 10990 |
| 603 | 604 | 605 | 606 | 607 | 608 | 609 | 176 | 177 | 178 | 179 | 180 | 181 | 182 | 575 | 198 | 577 | 578 | 579 | 580 | 581 | 204 | 205 | 206 | 207 | 208 | 209 | 588 | 10990 |
| 602 | 601 | 600 | 599 | 598 | 597 | 596 | 189 | 188 | 187 | 186 | 185 | 184 | 183 | 574 | 573 | 572 | 571 | 570 | 569 | 568 | 217 | 216 | 215 | 214 | 213 | 212 | 211 | 10990 |
| 196 | 195 | 194 | 193 | 192 | 191 | 190 | 595 | 594 | 593 | 592 | 591 | 590 | 589 | 224 | 223 | 222 | 221 | 220 | 219 | 218 | 567 | 566 | 565 | 564 | 563 | 562 | 561 | 10990 |
| 225 | 226 | 227 | 228 | 229 | 230 | 231 | 558 | 555 | 556 | 557 | 554 | 559 | 560 | 253 | 254 | 255 | 256 | 257 | 258 | 259 | 526 | 527 | 528 | 529 | 530 | 531 | 532 | 10990 |
| 547 | 548 | 549 | 550 | 551 | 552 | 553 | 232 | 233 | 234 | 235 | 236 | 237 | 238 | 519 | 520 | 521 | 522 | 523 | 524 | 525 | 260 | 261 | 262 | 263 | 264 | 265 | 266 | 10990 |
| 546 | 545 | 544 | 543 | 542 | 541 | 540 | 245 | 244 | 243 | 242 | 241 | 240 | 239 | 518 | 517 | 516 | 515 | 514 | 513 | 512 | 273 | 272 | 271 | 270 | 269 | 268 | 267 | 10990 |
| 252 | 251 | 250 | 249 | 248 | 247 | 246 | 535 | 538 | 537 | 536 | 539 | 534 | 533 | 280 | 279 | 278 | 277 | 276 | 275 | 274 | 511 | 510 | 509 | 508 | 507 | 506 | 505 | 10990 |
| 281 | 282 | 283 | 284 | 285 | 286 | 287 | 498 | 499 | 500 | 501 | 502 | 503 | 504 | 309 | 310 | 311 | 312 | 313 | 314 | 315 | 470 | 471 | 472 | 473 | 474 | 475 | 476 | 10990 |
| 491 | 492 | 497 | 494 | 495 | 496 | 493 | 288 | 289 | 290 | 291 | 292 | 293 | 294 | 463 | 464 | 465 | 466 | 467 | 468 | 469 | 316 | 317 | 318 | 319 | 320 | 321 | 322 | 10990 |
| 490 | 489 | 484 | 487 | 486 | 485 | 488 | 301 | 300 | 299 | 298 | 297 | 296 | 295 | 336 | 461 | 460 | 459 | 458 | 457 | 456 | 329 | 454 | 327 | 326 | 325 | 324 | 323 | 10990 |
| 308 | 307 | 306 | 305 | 304 | 303 | 302 | 483 | 482 | 481 | 480 | 479 | 478 | 477 | 462 | 335 | 334 | 333 | 332 | 331 | 330 | 455 | 328 | 453 | 452 | 451 | 450 | 449 | 10990 |
| 337 | 338 | 339 | 340 | 341 | 342 | 343 | 442 | 443 | 444 | 445 | 446 | 447 | 448 | 407 | 366 | 367 | 368 | 369 | 370 | 371 | 414 | 415 | 416 | 375 | 418 | 419 | 420 | 10990 |
| 435 | 436 | 437 | 438 | 439 | 440 | 441 | 344 | 345 | 346 | 347 | 348 | 349 | 350 | 365 | 408 | 409 | 410 | 411 | 412 | 413 | 372 | 373 | 374 | 417 | 376 | 377 | 378 | 10990 |
| 434 | 433 | 432 | 431 | 430 | 429 | 428 | 357 | 356 | 355 | 354 | 353 | 352 | 351 | 392 | 405 | 404 | 403 | 402 | 401 | 400 | 385 | 384 | 383 | 382 | 381 | 380 | 393 | 10990 |
| 364 | 363 | 362 | 361 | 360 | 359 | 358 | 427 | 426 | 425 | 424 | 423 | 422 | 421 | 406 | 391 | 390 | 389 | 388 | 387 | 386 | 399 | 398 | 397 | 396 | 395 | 394 | 379 | 10990 |
| 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 |

*The **magic square** sum is* $S_{28\times 28} := 10990$. *The magic rectangles of order* $4 \times 14$ *are of **equal magic sums** given by* $R_{4\times 14} := (5495, 1570)$.

## 2.12 Magic Square of Order $30 \times 30$

**Example 2.20.** *A magic square of order* $30 \times 30$ *constructed based on blocks of magic rectangles of order* $6 \times 10$ *is given by*

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 13515 |
| 1 | 895 | 894 | 12 | 13 | 887 | 886 | 885 | 14 | 18 | 31 | 865 | 864 | 42 | 43 | 857 | 856 | 855 | 44 | 48 | 241 | 655 | 654 | 252 | 253 | 647 | 646 | 645 | 254 | 258 | 13515 |
| 2 | 896 | 893 | 11 | 882 | 20 | 880 | 21 | 23 | 877 | 32 | 866 | 863 | 41 | 852 | 50 | 850 | 51 | 53 | 847 | 242 | 656 | 653 | 251 | 642 | 260 | 640 | 261 | 263 | 637 | 13515 |
| 3 | 897 | 892 | 10 | 876 | 875 | 27 | 28 | 872 | 25 | 33 | 867 | 862 | 40 | 846 | 845 | 57 | 58 | 842 | 55 | 243 | 657 | 652 | 250 | 636 | 635 | 267 | 268 | 632 | 265 | 13515 |
| 898 | 4 | 9 | 891 | 30 | 26 | 873 | 874 | 29 | 871 | 868 | 34 | 39 | 861 | 60 | 56 | 843 | 844 | 59 | 841 | 658 | 244 | 249 | 651 | 270 | 266 | 633 | 634 | 269 | 631 | 13515 |
| 899 | 5 | 8 | 890 | 19 | 878 | 22 | 879 | 881 | 24 | 869 | 35 | 38 | 860 | 49 | 848 | 52 | 849 | 851 | 54 | 659 | 245 | 248 | 650 | 259 | 638 | 262 | 639 | 641 | 264 | 13515 |
| 900 | 6 | 7 | 889 | 883 | 17 | 15 | 16 | 884 | 888 | 870 | 36 | 37 | 859 | 853 | 47 | 45 | 46 | 854 | 858 | 660 | 246 | 247 | 649 | 643 | 257 | 255 | 256 | 644 | 648 | 13515 |
| 91 | 805 | 804 | 102 | 103 | 797 | 796 | 795 | 104 | 108 | 121 | 775 | 774 | 132 | 133 | 767 | 766 | 765 | 134 | 138 | 163 | 737 | 736 | 735 | 164 | 168 | 151 | 745 | 744 | 162 | 13515 |
| 92 | 806 | 803 | 101 | 792 | 110 | 790 | 111 | 113 | 787 | 122 | 776 | 773 | 131 | 762 | 140 | 760 | 141 | 143 | 757 | 732 | 170 | 730 | 171 | 173 | 727 | 152 | 746 | 743 | 161 | 13515 |
| 93 | 807 | 802 | 100 | 786 | 785 | 117 | 118 | 782 | 115 | 123 | 777 | 772 | 130 | 756 | 755 | 147 | 148 | 752 | 145 | 726 | 725 | 177 | 178 | 722 | 175 | 153 | 747 | 742 | 160 | 13515 |
| 808 | 94 | 99 | 801 | 120 | 116 | 783 | 784 | 119 | 781 | 778 | 124 | 129 | 771 | 150 | 146 | 753 | 754 | 149 | 751 | 180 | 176 | 723 | 724 | 179 | 721 | 748 | 154 | 159 | 741 | 13515 |
| 809 | 95 | 98 | 800 | 109 | 788 | 112 | 789 | 791 | 114 | 779 | 125 | 128 | 770 | 139 | 758 | 142 | 759 | 761 | 144 | 169 | 728 | 172 | 729 | 731 | 174 | 749 | 155 | 158 | 740 | 13515 |
| 810 | 96 | 97 | 799 | 793 | 107 | 105 | 106 | 794 | 798 | 780 | 126 | 127 | 769 | 763 | 137 | 135 | 136 | 764 | 768 | 733 | 167 | 165 | 166 | 734 | 738 | 750 | 156 | 157 | 739 | 13515 |
| 181 | 715 | 714 | 192 | 193 | 707 | 706 | 705 | 194 | 198 | 211 | 684 | 685 | 222 | 223 | 677 | 676 | 675 | 224 | 228 | 61 | 835 | 834 | 72 | 73 | 827 | 826 | 825 | 74 | 78 | 13515 |
| 182 | 716 | 713 | 191 | 702 | 200 | 700 | 201 | 203 | 697 | 212 | 686 | 683 | 221 | 672 | 230 | 670 | 231 | 233 | 667 | 62 | 836 | 833 | 71 | 822 | 80 | 820 | 81 | 83 | 817 | 13515 |
| 183 | 717 | 712 | 190 | 696 | 695 | 207 | 208 | 692 | 205 | 213 | 687 | 220 | 682 | 666 | 665 | 237 | 238 | 662 | 235 | 63 | 837 | 832 | 70 | 816 | 815 | 87 | 88 | 812 | 85 | 13515 |
| 718 | 184 | 189 | 711 | 210 | 206 | 693 | 694 | 209 | 691 | 688 | 214 | 219 | 681 | 240 | 236 | 663 | 664 | 239 | 661 | 838 | 64 | 69 | 831 | 90 | 86 | 813 | 814 | 89 | 811 | 13515 |
| 719 | 185 | 188 | 710 | 199 | 698 | 202 | 699 | 701 | 204 | 689 | 215 | 680 | 218 | 229 | 668 | 232 | 669 | 671 | 234 | 839 | 65 | 68 | 830 | 79 | 818 | 82 | 819 | 821 | 84 | 13515 |
| 720 | 186 | 187 | 709 | 703 | 197 | 195 | 196 | 704 | 708 | 690 | 217 | 216 | 679 | 673 | 227 | 225 | 226 | 674 | 678 | 840 | 66 | 67 | 829 | 823 | 77 | 75 | 76 | 824 | 828 | 13515 |
| 391 | 505 | 504 | 402 | 403 | 497 | 496 | 495 | 404 | 408 | 595 | 301 | 594 | 312 | 313 | 587 | 586 | 585 | 314 | 318 | 331 | 565 | 564 | 342 | 343 | 557 | 556 | 555 | 344 | 348 | 13515 |
| 392 | 506 | 503 | 401 | 492 | 410 | 490 | 411 | 413 | 487 | 302 | 596 | 593 | 311 | 582 | 320 | 580 | 321 | 323 | 577 | 332 | 566 | 563 | 341 | 552 | 350 | 550 | 351 | 353 | 547 | 13515 |
| 393 | 507 | 502 | 400 | 486 | 485 | 417 | 418 | 482 | 415 | 303 | 597 | 592 | 310 | 576 | 575 | 327 | 328 | 572 | 325 | 333 | 567 | 562 | 340 | 546 | 545 | 357 | 358 | 542 | 355 | 13515 |
| 508 | 394 | 399 | 501 | 420 | 416 | 483 | 484 | 419 | 481 | 598 | 304 | 309 | 591 | 330 | 326 | 573 | 574 | 329 | 571 | 569 | 334 | 339 | 560 | 360 | 356 | 543 | 544 | 359 | 541 | 13515 |
| 509 | 395 | 398 | 500 | 409 | 488 | 412 | 489 | 491 | 414 | 599 | 305 | 308 | 590 | 319 | 578 | 322 | 579 | 581 | 324 | 568 | 335 | 338 | 561 | 349 | 548 | 352 | 549 | 551 | 354 | 13515 |
| 510 | 396 | 397 | 499 | 493 | 407 | 405 | 406 | 494 | 498 | 306 | 600 | 307 | 589 | 583 | 317 | 315 | 316 | 584 | 588 | 570 | 336 | 337 | 559 | 553 | 347 | 345 | 346 | 554 | 558 | 13515 |
| 535 | 361 | 534 | 372 | 373 | 527 | 526 | 525 | 374 | 378 | 271 | 625 | 624 | 282 | 283 | 617 | 616 | 615 | 284 | 288 | 421 | 475 | 474 | 432 | 433 | 467 | 466 | 465 | 434 | 438 | 13515 |
| 362 | 536 | 533 | 371 | 522 | 380 | 520 | 381 | 383 | 517 | 272 | 626 | 623 | 281 | 612 | 290 | 610 | 291 | 293 | 607 | 422 | 476 | 473 | 431 | 462 | 440 | 460 | 441 | 443 | 457 | 13515 |
| 363 | 537 | 532 | 370 | 516 | 515 | 387 | 388 | 512 | 385 | 273 | 627 | 622 | 280 | 606 | 605 | 297 | 298 | 602 | 295 | 423 | 477 | 472 | 430 | 456 | 455 | 447 | 448 | 452 | 445 | 13515 |
| 538 | 364 | 369 | 531 | 390 | 386 | 513 | 514 | 389 | 511 | 628 | 274 | 279 | 621 | 300 | 296 | 603 | 604 | 299 | 601 | 478 | 424 | 429 | 471 | 450 | 446 | 453 | 454 | 449 | 451 | 13515 |
| 539 | 365 | 368 | 530 | 379 | 518 | 382 | 519 | 521 | 384 | 629 | 275 | 278 | 620 | 289 | 608 | 292 | 609 | 611 | 294 | 479 | 425 | 428 | 470 | 439 | 458 | 442 | 459 | 461 | 444 | 13515 |
| 366 | 540 | 367 | 529 | 523 | 377 | 375 | 376 | 524 | 528 | 630 | 276 | 277 | 619 | 613 | 287 | 285 | 286 | 614 | 618 | 480 | 426 | 427 | 469 | 463 | 437 | 435 | 436 | 464 | 468 | 13515 |
| 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 |

*The **magic square** sum is* $S_{30\times 30} := 13515$. *The magic rectangles of order* $6 \times 10$ *are of **equal magic sums** given by* $R_{6\times 10} := (4505, 2703)$.

## 2.13 Magic Square of Order $32 \times 32$

**Example 2.21.** *A magic square of order* $32 \times 32$ *constructed based on blocks of magic rectangles of order* $2 \times 4$ *is given by*

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 16400 |
| 12 | 1014 | 1015 | 9 | 8 | 1018 | 1019 | 5 | 44 | 982 | 983 | 41 | 40 | 986 | 987 | 37 | 76 | 950 | 951 | 73 | 72 | 954 | 955 | 69 | 108 | 918 | 919 | 105 | 104 | 922 | 923 | 101 | 16400 |
| 1013 | 11 | 10 | 1016 | 1017 | 7 | 6 | 1020 | 981 | 43 | 42 | 984 | 985 | 39 | 38 | 988 | 949 | 75 | 74 | 952 | 953 | 71 | 70 | 956 | 917 | 107 | 106 | 920 | 921 | 103 | 102 | 924 | 16400 |
| 1 | 1023 | 1022 | 4 | 13 | 1011 | 1010 | 16 | 33 | 991 | 990 | 36 | 45 | 979 | 978 | 48 | 65 | 959 | 958 | 68 | 77 | 947 | 946 | 80 | 97 | 927 | 926 | 100 | 109 | 915 | 914 | 112 | 16400 |
| 1024 | 2 | 3 | 1021 | 1012 | 14 | 15 | 1009 | 992 | 34 | 35 | 989 | 980 | 46 | 47 | 977 | 960 | 66 | 67 | 957 | 948 | 78 | 79 | 945 | 928 | 98 | 99 | 925 | 916 | 110 | 111 | 913 | 16400 |
| 25 | 999 | 998 | 28 | 24 | 1002 | 1003 | 21 | 57 | 967 | 966 | 60 | 56 | 970 | 971 | 53 | 89 | 935 | 934 | 92 | 88 | 938 | 939 | 85 | 121 | 903 | 902 | 124 | 120 | 906 | 907 | 117 | 16400 |
| 1000 | 26 | 27 | 997 | 1001 | 23 | 22 | 1004 | 968 | 58 | 59 | 965 | 969 | 55 | 54 | 972 | 936 | 90 | 91 | 933 | 937 | 87 | 86 | 940 | 904 | 122 | 123 | 901 | 905 | 119 | 118 | 908 | 16400 |
| 20 | 1006 | 1007 | 17 | 29 | 995 | 994 | 32 | 52 | 974 | 975 | 49 | 61 | 963 | 962 | 64 | 84 | 942 | 943 | 81 | 93 | 931 | 930 | 96 | 116 | 910 | 911 | 113 | 125 | 899 | 898 | 128 | 16400 |
| 1005 | 19 | 18 | 1008 | 996 | 30 | 31 | 993 | 973 | 51 | 50 | 976 | 964 | 62 | 63 | 961 | 941 | 83 | 82 | 944 | 932 | 94 | 95 | 929 | 909 | 115 | 114 | 912 | 900 | 126 | 127 | 897 | 16400 |
| 140 | 886 | 887 | 137 | 136 | 890 | 891 | 133 | 172 | 854 | 855 | 169 | 168 | 858 | 859 | 165 | 204 | 822 | 823 | 201 | 200 | 826 | 827 | 197 | 236 | 790 | 791 | 233 | 232 | 794 | 795 | 229 | 16400 |
| 885 | 139 | 138 | 888 | 889 | 135 | 134 | 892 | 853 | 171 | 170 | 856 | 857 | 167 | 166 | 860 | 821 | 203 | 202 | 824 | 825 | 199 | 198 | 828 | 789 | 235 | 234 | 792 | 793 | 231 | 230 | 796 | 16400 |
| 129 | 895 | 894 | 132 | 141 | 883 | 882 | 144 | 161 | 863 | 862 | 164 | 173 | 851 | 850 | 176 | 193 | 831 | 830 | 196 | 205 | 819 | 818 | 208 | 225 | 799 | 798 | 228 | 237 | 787 | 786 | 240 | 16400 |
| 896 | 130 | 131 | 893 | 884 | 142 | 143 | 881 | 864 | 162 | 163 | 861 | 852 | 174 | 175 | 849 | 832 | 194 | 195 | 829 | 820 | 206 | 207 | 817 | 800 | 226 | 227 | 797 | 788 | 238 | 239 | 785 | 16400 |
| 153 | 871 | 870 | 156 | 152 | 874 | 875 | 149 | 185 | 839 | 838 | 188 | 184 | 842 | 843 | 181 | 217 | 807 | 806 | 220 | 216 | 810 | 811 | 213 | 249 | 775 | 774 | 252 | 248 | 778 | 779 | 245 | 16400 |
| 872 | 154 | 155 | 869 | 873 | 151 | 150 | 876 | 840 | 186 | 187 | 837 | 841 | 183 | 182 | 844 | 808 | 218 | 219 | 805 | 809 | 215 | 214 | 812 | 776 | 250 | 251 | 773 | 777 | 247 | 246 | 780 | 16400 |
| 148 | 878 | 879 | 145 | 157 | 867 | 866 | 160 | 180 | 846 | 847 | 177 | 189 | 835 | 834 | 192 | 212 | 814 | 815 | 209 | 221 | 803 | 802 | 224 | 244 | 782 | 783 | 241 | 253 | 771 | 770 | 256 | 16400 |
| 877 | 147 | 146 | 880 | 868 | 158 | 159 | 865 | 845 | 179 | 178 | 848 | 836 | 190 | 191 | 833 | 813 | 211 | 210 | 816 | 804 | 222 | 223 | 801 | 781 | 243 | 242 | 784 | 772 | 254 | 255 | 769 | 16400 |
| 268 | 758 | 759 | 265 | 264 | 762 | 763 | 261 | 300 | 726 | 727 | 297 | 296 | 730 | 731 | 293 | 332 | 694 | 695 | 329 | 328 | 698 | 699 | 325 | 364 | 662 | 663 | 361 | 360 | 666 | 667 | 357 | 16400 |
| 757 | 267 | 266 | 760 | 761 | 263 | 262 | 764 | 725 | 299 | 298 | 728 | 729 | 295 | 294 | 732 | 693 | 331 | 330 | 696 | 697 | 327 | 326 | 700 | 661 | 363 | 362 | 664 | 665 | 359 | 358 | 668 | 16400 |
| 257 | 767 | 766 | 260 | 269 | 755 | 754 | 272 | 289 | 735 | 734 | 292 | 301 | 723 | 722 | 304 | 321 | 703 | 702 | 324 | 333 | 691 | 690 | 336 | 353 | 671 | 670 | 356 | 365 | 659 | 658 | 368 | 16400 |
| 768 | 258 | 259 | 765 | 756 | 270 | 271 | 753 | 736 | 290 | 291 | 733 | 724 | 302 | 303 | 721 | 704 | 322 | 323 | 701 | 692 | 334 | 335 | 689 | 672 | 354 | 355 | 669 | 660 | 366 | 367 | 657 | 16400 |
| 281 | 743 | 742 | 284 | 280 | 746 | 747 | 277 | 313 | 711 | 710 | 316 | 312 | 714 | 715 | 309 | 345 | 679 | 678 | 348 | 344 | 682 | 683 | 341 | 377 | 647 | 646 | 380 | 376 | 650 | 651 | 373 | 16400 |
| 744 | 282 | 283 | 741 | 745 | 279 | 278 | 748 | 712 | 314 | 315 | 709 | 713 | 311 | 310 | 716 | 680 | 346 | 347 | 677 | 681 | 343 | 342 | 684 | 648 | 378 | 379 | 645 | 649 | 375 | 374 | 652 | 16400 |
| 276 | 750 | 751 | 273 | 285 | 739 | 738 | 288 | 308 | 718 | 719 | 305 | 317 | 707 | 706 | 320 | 340 | 686 | 687 | 337 | 349 | 675 | 674 | 352 | 372 | 654 | 655 | 369 | 381 | 643 | 642 | 384 | 16400 |
| 749 | 275 | 274 | 752 | 740 | 286 | 287 | 737 | 717 | 307 | 306 | 720 | 708 | 318 | 319 | 705 | 685 | 339 | 338 | 688 | 676 | 350 | 351 | 673 | 653 | 371 | 370 | 656 | 644 | 382 | 383 | 641 | 16400 |
| 396 | 630 | 631 | 393 | 392 | 634 | 635 | 389 | 428 | 598 | 599 | 425 | 424 | 602 | 603 | 421 | 460 | 566 | 567 | 457 | 456 | 570 | 571 | 453 | 492 | 534 | 535 | 489 | 488 | 538 | 539 | 485 | 16400 |
| 629 | 395 | 394 | 632 | 633 | 391 | 390 | 636 | 597 | 427 | 426 | 600 | 601 | 423 | 422 | 604 | 565 | 459 | 458 | 568 | 569 | 455 | 454 | 572 | 533 | 491 | 490 | 536 | 537 | 487 | 486 | 540 | 16400 |
| 385 | 639 | 638 | 388 | 397 | 627 | 626 | 400 | 417 | 607 | 606 | 420 | 429 | 595 | 594 | 432 | 449 | 575 | 574 | 452 | 461 | 563 | 562 | 464 | 481 | 543 | 542 | 484 | 493 | 531 | 530 | 496 | 16400 |
| 640 | 386 | 387 | 637 | 628 | 398 | 399 | 625 | 608 | 418 | 419 | 605 | 596 | 430 | 431 | 593 | 576 | 450 | 451 | 573 | 564 | 462 | 463 | 561 | 544 | 482 | 483 | 541 | 532 | 494 | 495 | 529 | 16400 |
| 409 | 615 | 614 | 412 | 408 | 618 | 619 | 405 | 441 | 583 | 582 | 444 | 440 | 586 | 587 | 437 | 473 | 551 | 550 | 476 | 472 | 554 | 555 | 469 | 505 | 519 | 518 | 508 | 504 | 522 | 523 | 501 | 16400 |
| 616 | 410 | 411 | 613 | 617 | 407 | 406 | 620 | 584 | 442 | 443 | 581 | 585 | 439 | 438 | 588 | 552 | 474 | 475 | 549 | 553 | 471 | 470 | 556 | 520 | 506 | 507 | 517 | 521 | 503 | 502 | 524 | 16400 |
| 404 | 622 | 623 | 401 | 413 | 611 | 610 | 416 | 436 | 590 | 591 | 433 | 445 | 579 | 578 | 448 | 468 | 558 | 559 | 465 | 477 | 547 | 546 | 480 | 500 | 526 | 527 | 497 | 509 | 515 | 514 | 512 | 16400 |
| 621 | 403 | 402 | 624 | 612 | 414 | 415 | 609 | 589 | 435 | 434 | 592 | 580 | 446 | 447 | 577 | 557 | 467 | 466 | 560 | 548 | 478 | 479 | 545 | 525 | 499 | 498 | 528 | 516 | 510 | 511 | 513 | 16400 |
| 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 |

*The **magic square** sum is $S_{32\times 32} := 16400$. The magic rectangles of order $6\times 10$ are of **equal magic sums** given by $R_{2\times 4} := (2050, 1025)$. Moreover, each block of order $8\times 8$ is a magic square with magic sums $S_{8\times 8} := 4100$.*

## 2.14 Block Bordered Magic Squares of Order $34\times 34$

It is based on the Examples 2.20 and 2.21. The internal blocks are magic squares of order 30 and 32.

**Example 2.22.** *A **block bordered** magic square of order 34 with inner block as a magic square of order 30 is given by*

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 19669 |
| 1123 | 1092 | 1094 | 1096 | 1098 | 1100 | 1102 | 1104 | 1106 | 1142 | 1144 | 1146 | 1148 | 1150 | 1152 | 1154 | 66 | 64 | 62 | 60 | 58 | 56 | 54 | 52 | 50 | 16 | 14 | 12 | 10 | 8 | 6 | 4 | 2 | 1124 | 19669 |
| 49 | 1059 | 1030 | 1032 | 1034 | 1036 | 1038 | 1040 | 1042 | 1077 | 1079 | 1081 | 1083 | 1085 | 1087 | 1089 | 128 | 126 | 124 | 122 | 120 | 118 | 116 | 114 | 81 | 79 | 77 | 75 | 73 | 71 | 69 | 67 | 1060 | 1108 | 19669 |
| 47 | 113 | 129 | 1023 | 1022 | 140 | 141 | 1015 | 1014 | 1013 | 142 | 146 | 159 | 993 | 992 | 170 | 171 | 985 | 984 | 983 | 172 | 176 | 369 | 783 | 782 | 380 | 381 | 775 | 774 | 773 | 382 | 386 | 1044 | 1110 | 19669 |
| 45 | 111 | 130 | 1024 | 1021 | 139 | 1010 | 148 | 1008 | 149 | 151 | 1005 | 160 | 994 | 991 | 169 | 980 | 178 | 978 | 179 | 181 | 975 | 370 | 784 | 781 | 379 | 770 | 388 | 768 | 389 | 391 | 765 | 1046 | 1112 | 19669 |
| 43 | 109 | 131 | 1025 | 1020 | 138 | 1004 | 1003 | 155 | 156 | 1000 | 153 | 161 | 995 | 990 | 168 | 974 | 973 | 185 | 186 | 970 | 183 | 371 | 785 | 780 | 378 | 764 | 763 | 395 | 396 | 760 | 393 | 1048 | 1114 | 19669 |
| 41 | 107 | 1026 | 132 | 137 | 1019 | 158 | 154 | 1001 | 1002 | 157 | 999 | 996 | 162 | 167 | 989 | 188 | 184 | 971 | 972 | 187 | 969 | 786 | 372 | 377 | 779 | 398 | 394 | 761 | 762 | 397 | 759 | 1050 | 1116 | 19669 |
| 39 | 105 | 1027 | 133 | 136 | 1018 | 147 | 1006 | 150 | 1007 | 1009 | 152 | 997 | 163 | 166 | 988 | 177 | 976 | 180 | 977 | 979 | 182 | 787 | 373 | 376 | 778 | 387 | 766 | 390 | 767 | 769 | 392 | 1052 | 1118 | 19669 |
| 37 | 103 | 1028 | 134 | 135 | 1017 | 1011 | 145 | 143 | 144 | 1012 | 1016 | 998 | 164 | 165 | 987 | 981 | 175 | 173 | 174 | 982 | 986 | 788 | 374 | 375 | 777 | 771 | 385 | 383 | 384 | 772 | 776 | 1054 | 1120 | 19669 |
| 35 | 100 | 219 | 933 | 932 | 230 | 231 | 925 | 924 | 923 | 232 | 236 | 249 | 903 | 902 | 260 | 261 | 895 | 894 | 893 | 262 | 266 | 291 | 865 | 864 | 863 | 292 | 296 | 279 | 873 | 872 | 290 | 1057 | 1122 | 19669 |
| 31 | 99 | 220 | 934 | 931 | 229 | 920 | 238 | 918 | 239 | 241 | 915 | 250 | 904 | 901 | 259 | 890 | 268 | 888 | 269 | 271 | 885 | 860 | 298 | 858 | 299 | 301 | 855 | 280 | 874 | 871 | 289 | 1058 | 1126 | 19669 |
| 29 | 94 | 221 | 935 | 930 | 228 | 914 | 913 | 245 | 246 | 910 | 243 | 251 | 905 | 900 | 258 | 884 | 883 | 275 | 276 | 880 | 273 | 854 | 853 | 305 | 306 | 850 | 303 | 281 | 875 | 870 | 288 | 1063 | 1128 | 19669 |
| 27 | 92 | 936 | 222 | 227 | 929 | 248 | 244 | 911 | 912 | 247 | 909 | 906 | 252 | 257 | 899 | 278 | 274 | 881 | 882 | 277 | 879 | 308 | 304 | 851 | 852 | 307 | 849 | 876 | 282 | 287 | 869 | 1065 | 1130 | 19669 |
| 25 | 90 | 937 | 223 | 226 | 928 | 237 | 916 | 240 | 917 | 919 | 242 | 907 | 253 | 256 | 898 | 267 | 886 | 270 | 887 | 889 | 272 | 297 | 856 | 300 | 857 | 859 | 302 | 877 | 283 | 286 | 868 | 1067 | 1132 | 19669 |
| 23 | 88 | 938 | 224 | 225 | 927 | 921 | 235 | 233 | 234 | 922 | 926 | 908 | 254 | 255 | 897 | 891 | 265 | 263 | 264 | 892 | 896 | 861 | 295 | 293 | 294 | 862 | 866 | 878 | 284 | 285 | 867 | 1069 | 1134 | 19669 |
| 21 | 86 | 309 | 843 | 842 | 320 | 321 | 835 | 834 | 833 | 322 | 326 | 339 | 812 | 813 | 350 | 351 | 805 | 804 | 803 | 352 | 356 | 189 | 963 | 962 | 200 | 201 | 955 | 954 | 953 | 202 | 206 | 1071 | 1136 | 19669 |
| 19 | 84 | 310 | 844 | 841 | 319 | 830 | 328 | 828 | 329 | 331 | 825 | 340 | 814 | 811 | 349 | 800 | 358 | 798 | 359 | 361 | 795 | 190 | 964 | 961 | 199 | 950 | 208 | 948 | 209 | 211 | 945 | 1073 | 1138 | 19669 |
| 1 | 82 | 311 | 845 | 840 | 318 | 824 | 823 | 335 | 336 | 820 | 333 | 341 | 815 | 348 | 810 | 794 | 793 | 365 | 366 | 790 | 363 | 191 | 965 | 960 | 198 | 944 | 943 | 215 | 216 | 940 | 213 | 1075 | 1156 | 19669 |
| 1109 | 1045 | 846 | 312 | 317 | 839 | 338 | 334 | 821 | 822 | 337 | 819 | 816 | 342 | 347 | 809 | 368 | 364 | 791 | 792 | 367 | 789 | 966 | 192 | 197 | 959 | 218 | 214 | 941 | 942 | 217 | 939 | 112 | 48 | 19669 |
| 1111 | 1047 | 847 | 313 | 316 | 838 | 327 | 826 | 330 | 827 | 829 | 332 | 817 | 343 | 808 | 346 | 357 | 796 | 360 | 797 | 799 | 362 | 967 | 193 | 196 | 958 | 207 | 946 | 210 | 947 | 949 | 212 | 110 | 46 | 19669 |
| 1113 | 1049 | 848 | 314 | 315 | 837 | 831 | 325 | 323 | 324 | 832 | 836 | 818 | 345 | 344 | 807 | 801 | 355 | 353 | 354 | 802 | 806 | 968 | 194 | 195 | 957 | 951 | 205 | 203 | 204 | 952 | 956 | 108 | 44 | 19669 |
| 1115 | 1051 | 519 | 633 | 632 | 530 | 531 | 625 | 624 | 623 | 532 | 536 | 723 | 429 | 722 | 440 | 441 | 715 | 714 | 713 | 442 | 446 | 459 | 693 | 692 | 470 | 471 | 685 | 684 | 683 | 472 | 476 | 106 | 42 | 19669 |
| 1117 | 1053 | 520 | 634 | 631 | 529 | 620 | 538 | 618 | 539 | 541 | 615 | 430 | 724 | 721 | 439 | 710 | 448 | 708 | 449 | 451 | 705 | 460 | 694 | 691 | 469 | 680 | 478 | 678 | 479 | 481 | 675 | 104 | 40 | 19669 |
| 1119 | 1055 | 521 | 635 | 630 | 528 | 614 | 613 | 545 | 546 | 610 | 543 | 431 | 725 | 720 | 438 | 704 | 703 | 455 | 456 | 700 | 453 | 461 | 695 | 690 | 468 | 674 | 673 | 485 | 486 | 670 | 483 | 102 | 38 | 19669 |
| 1121 | 1056 | 636 | 522 | 527 | 629 | 548 | 544 | 611 | 612 | 547 | 609 | 726 | 432 | 437 | 719 | 458 | 454 | 701 | 702 | 457 | 699 | 697 | 462 | 467 | 688 | 488 | 484 | 671 | 672 | 487 | 669 | 101 | 36 | 19669 |
| 1125 | 1061 | 637 | 523 | 526 | 628 | 537 | 616 | 540 | 617 | 619 | 542 | 727 | 433 | 436 | 718 | 447 | 706 | 450 | 707 | 709 | 452 | 696 | 463 | 466 | 689 | 477 | 676 | 480 | 677 | 679 | 482 | 96 | 32 | 19669 |
| 1127 | 1062 | 638 | 524 | 525 | 627 | 621 | 535 | 533 | 534 | 622 | 626 | 434 | 728 | 435 | 717 | 711 | 445 | 443 | 444 | 712 | 716 | 698 | 464 | 465 | 687 | 681 | 475 | 473 | 474 | 682 | 686 | 95 | 30 | 19669 |
| 1129 | 1064 | 663 | 489 | 662 | 500 | 501 | 655 | 654 | 653 | 502 | 506 | 399 | 753 | 752 | 410 | 411 | 745 | 744 | 743 | 412 | 416 | 549 | 603 | 602 | 560 | 561 | 595 | 594 | 593 | 562 | 566 | 93 | 28 | 19669 |
| 1131 | 1066 | 490 | 664 | 661 | 499 | 650 | 508 | 648 | 509 | 511 | 645 | 400 | 754 | 751 | 409 | 740 | 418 | 738 | 419 | 421 | 735 | 550 | 604 | 601 | 559 | 590 | 568 | 588 | 569 | 571 | 585 | 91 | 26 | 19669 |
| 1133 | 1068 | 491 | 665 | 660 | 498 | 644 | 643 | 515 | 516 | 640 | 513 | 401 | 755 | 750 | 408 | 734 | 733 | 425 | 426 | 730 | 423 | 551 | 605 | 600 | 558 | 584 | 583 | 575 | 576 | 580 | 573 | 89 | 24 | 19669 |
| 1135 | 1070 | 666 | 492 | 497 | 659 | 518 | 514 | 641 | 642 | 517 | 639 | 756 | 402 | 407 | 749 | 428 | 424 | 731 | 732 | 427 | 729 | 606 | 552 | 557 | 599 | 578 | 574 | 581 | 582 | 577 | 579 | 87 | 22 | 19669 |
| 1137 | 1072 | 667 | 493 | 496 | 658 | 507 | 646 | 510 | 647 | 649 | 512 | 757 | 403 | 406 | 748 | 417 | 736 | 420 | 737 | 739 | 422 | 607 | 553 | 556 | 598 | 567 | 586 | 570 | 587 | 589 | 572 | 85 | 20 | 19669 |
| 1139 | 1074 | 494 | 668 | 495 | 657 | 651 | 505 | 503 | 504 | 652 | 656 | 758 | 404 | 405 | 747 | 741 | 415 | 413 | 414 | 742 | 746 | 608 | 554 | 555 | 597 | 591 | 565 | 563 | 564 | 592 | 596 | 83 | 18 | 19669 |
| 1140 | 97 | 127 | 125 | 123 | 121 | 119 | 117 | 115 | 80 | 78 | 76 | 74 | 72 | 70 | 68 | 1029 | 1031 | 1033 | 1035 | 1037 | 1039 | 1041 | 1043 | 1076 | 1078 | 1080 | 1082 | 1084 | 1086 | 1088 | 1090 | 98 | 17 | 19669 |
| 33 | 65 | 63 | 61 | 59 | 57 | 55 | 53 | 51 | 15 | 13 | 11 | 9 | 7 | 5 | 3 | 1091 | 1093 | 1095 | 1097 | 1099 | 1101 | 1103 | 1105 | 1107 | 1141 | 1143 | 1145 | 1147 | 1149 | 1151 | 1153 | 1155 | 34 | 19669 |
| 19669 | 19669 | 19669 | 19669 | 19669 | 19669 | 19669 | 19669 | 19669 | 19669 | 19669 | 19669 | 19669 | 19669 | 19669 | 19669 | 19669 | 19669 | 19669 | 19669 | 19669 | 19669 | 19669 | 19669 | 19669 | 19669 | 19669 | 19669 | 19669 | 19669 | 19669 | 19669 | 19669 | 19669 | 19669 |

*In this case the magic sums are* $S_{34\times 34} := 19669$, $S_{32\times 32} := 18512$ *and* $S_{30\times 30} := 17355$. *The magic rectangles of order* $6\times 10$ *are of **equal magic sums** given by* $R_{6\times 10} := (5785, 3471)$.

**Example 2.23.** *A **block bordered** magic square of order 34 with inner block as a magic square of order 32 is given by*

Magic Rectangles in Construction of Magic and Block-Bordered Magic Squares, June 07, 2022, pp. 1-70, https://doi.org/10.5281/zenodo.6621071

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 19669 |
| 1123 | 1092 | 1094 | 1096 | 1098 | 1100 | 1102 | 1104 | 1106 | 1142 | 1144 | 1146 | 1148 | 1150 | 1152 | 1154 | 66 | 64 | 62 | 60 | 58 | 56 | 54 | 52 | 50 | 16 | 14 | 12 | 10 | 8 | 6 | 4 | 2 | 1124 | 19669 |
| 49 | 78 | 1080 | 1081 | 75 | 74 | 1084 | 1085 | 71 | 110 | 1048 | 1049 | 107 | 106 | 1052 | 1053 | 103 | 142 | 1016 | 1017 | 139 | 138 | 1020 | 1021 | 135 | 174 | 984 | 985 | 171 | 170 | 988 | 989 | 167 | 1108 | 19669 |
| 47 | 1079 | 77 | 76 | 1082 | 1083 | 73 | 72 | 1086 | 1047 | 109 | 108 | 1050 | 1051 | 105 | 104 | 1054 | 1015 | 141 | 140 | 1018 | 1019 | 137 | 136 | 1022 | 983 | 173 | 172 | 986 | 987 | 169 | 168 | 990 | 1110 | 19669 |
| 45 | 67 | 1089 | 1088 | 70 | 79 | 1077 | 1076 | 82 | 99 | 1057 | 1056 | 102 | 111 | 1045 | 1044 | 114 | 131 | 1025 | 1024 | 134 | 143 | 1013 | 1012 | 146 | 163 | 993 | 992 | 166 | 175 | 981 | 980 | 178 | 1112 | 19669 |
| 43 | 1090 | 68 | 69 | 1087 | 1078 | 80 | 81 | 1075 | 1058 | 100 | 101 | 1055 | 1046 | 112 | 113 | 1043 | 1026 | 132 | 133 | 1023 | 1014 | 144 | 145 | 1011 | 994 | 164 | 165 | 991 | 982 | 176 | 177 | 979 | 1114 | 19669 |
| 41 | 91 | 1065 | 1064 | 94 | 90 | 1068 | 1069 | 87 | 123 | 1033 | 1032 | 126 | 122 | 1036 | 1037 | 119 | 155 | 1001 | 1000 | 158 | 154 | 1004 | 1005 | 151 | 187 | 969 | 968 | 190 | 186 | 972 | 973 | 183 | 1116 | 19669 |
| 39 | 1066 | 92 | 93 | 1063 | 1067 | 89 | 88 | 1070 | 1034 | 124 | 125 | 1031 | 1035 | 121 | 120 | 1038 | 1002 | 156 | 157 | 999 | 1003 | 153 | 152 | 1006 | 970 | 188 | 189 | 967 | 971 | 185 | 184 | 974 | 1118 | 19669 |
| 37 | 86 | 1072 | 1073 | 83 | 95 | 1061 | 1060 | 98 | 118 | 1040 | 1041 | 115 | 127 | 1029 | 1028 | 130 | 150 | 1008 | 1009 | 147 | 159 | 997 | 996 | 162 | 182 | 976 | 977 | 179 | 191 | 965 | 964 | 194 | 1120 | 19669 |
| 35 | 1071 | 85 | 84 | 1074 | 1062 | 96 | 97 | 1059 | 1039 | 117 | 116 | 1042 | 1030 | 128 | 129 | 1027 | 1007 | 149 | 148 | 1010 | 998 | 160 | 161 | 995 | 975 | 181 | 180 | 978 | 966 | 192 | 193 | 963 | 1122 | 19669 |
| 31 | 206 | 952 | 953 | 203 | 202 | 956 | 957 | 199 | 238 | 920 | 921 | 235 | 234 | 924 | 925 | 231 | 270 | 888 | 889 | 267 | 266 | 892 | 893 | 263 | 302 | 856 | 857 | 299 | 298 | 860 | 861 | 295 | 1126 | 19669 |
| 29 | 951 | 205 | 204 | 954 | 955 | 201 | 200 | 958 | 919 | 237 | 236 | 922 | 923 | 233 | 232 | 926 | 887 | 269 | 268 | 890 | 891 | 265 | 264 | 894 | 855 | 301 | 300 | 858 | 859 | 297 | 296 | 862 | 1128 | 19669 |
| 27 | 195 | 961 | 960 | 198 | 207 | 949 | 948 | 210 | 227 | 929 | 928 | 230 | 239 | 917 | 916 | 242 | 259 | 897 | 896 | 262 | 271 | 885 | 884 | 274 | 291 | 865 | 864 | 294 | 303 | 853 | 852 | 306 | 1130 | 19669 |
| 25 | 962 | 196 | 197 | 959 | 950 | 208 | 209 | 947 | 930 | 228 | 229 | 927 | 918 | 240 | 241 | 915 | 898 | 260 | 261 | 895 | 886 | 272 | 273 | 883 | 866 | 292 | 293 | 863 | 854 | 304 | 305 | 851 | 1132 | 19669 |
| 23 | 219 | 937 | 936 | 222 | 218 | 940 | 941 | 215 | 251 | 905 | 904 | 254 | 250 | 908 | 909 | 247 | 283 | 873 | 872 | 286 | 282 | 876 | 877 | 279 | 315 | 841 | 840 | 318 | 314 | 844 | 845 | 311 | 1134 | 19669 |
| 21 | 938 | 220 | 221 | 935 | 939 | 217 | 216 | 942 | 906 | 252 | 253 | 903 | 907 | 249 | 248 | 910 | 874 | 284 | 285 | 871 | 875 | 281 | 280 | 878 | 842 | 316 | 317 | 839 | 843 | 313 | 312 | 846 | 1136 | 19669 |
| 19 | 214 | 944 | 945 | 211 | 223 | 933 | 932 | 226 | 246 | 912 | 913 | 243 | 255 | 901 | 900 | 258 | 278 | 880 | 881 | 275 | 287 | 869 | 868 | 290 | 310 | 848 | 849 | 307 | 319 | 837 | 836 | 322 | 1138 | 19669 |
| 1 | 943 | 213 | 212 | 946 | 934 | 224 | 225 | 931 | 911 | 245 | 244 | 914 | 902 | 256 | 257 | 899 | 879 | 277 | 276 | 882 | 870 | 288 | 289 | 867 | 847 | 309 | 308 | 850 | 838 | 320 | 321 | 835 | 1156 | 19669 |
| 1109 | 334 | 824 | 825 | 331 | 330 | 828 | 829 | 327 | 366 | 792 | 793 | 363 | 362 | 796 | 797 | 359 | 398 | 760 | 761 | 395 | 394 | 764 | 765 | 391 | 430 | 728 | 729 | 427 | 426 | 732 | 733 | 423 | 48 | 19669 |
| 1111 | 823 | 333 | 332 | 826 | 827 | 329 | 328 | 830 | 791 | 365 | 364 | 794 | 795 | 361 | 360 | 798 | 759 | 397 | 396 | 762 | 763 | 393 | 392 | 766 | 727 | 429 | 428 | 730 | 731 | 425 | 424 | 734 | 46 | 19669 |
| 1113 | 323 | 833 | 832 | 326 | 335 | 821 | 820 | 338 | 355 | 801 | 800 | 358 | 367 | 789 | 788 | 370 | 387 | 769 | 768 | 390 | 399 | 757 | 756 | 402 | 419 | 737 | 736 | 422 | 431 | 725 | 724 | 434 | 44 | 19669 |
| 1115 | 834 | 324 | 325 | 831 | 822 | 336 | 337 | 819 | 802 | 356 | 357 | 799 | 790 | 368 | 369 | 787 | 770 | 388 | 389 | 767 | 758 | 400 | 401 | 755 | 738 | 420 | 421 | 735 | 726 | 432 | 433 | 723 | 42 | 19669 |
| 1117 | 347 | 809 | 808 | 350 | 346 | 812 | 813 | 343 | 379 | 777 | 776 | 382 | 378 | 780 | 781 | 375 | 411 | 745 | 744 | 414 | 410 | 748 | 749 | 407 | 443 | 713 | 712 | 446 | 442 | 716 | 717 | 439 | 40 | 19669 |
| 1119 | 810 | 348 | 349 | 807 | 811 | 345 | 344 | 814 | 778 | 380 | 381 | 775 | 779 | 377 | 376 | 782 | 746 | 412 | 413 | 743 | 747 | 409 | 408 | 750 | 714 | 444 | 445 | 711 | 715 | 441 | 440 | 718 | 38 | 19669 |
| 1121 | 342 | 816 | 817 | 339 | 351 | 805 | 804 | 354 | 374 | 784 | 785 | 371 | 383 | 773 | 772 | 386 | 406 | 752 | 753 | 403 | 415 | 741 | 740 | 418 | 438 | 720 | 721 | 435 | 447 | 709 | 708 | 450 | 36 | 19669 |
| 1125 | 815 | 341 | 340 | 818 | 806 | 352 | 353 | 803 | 783 | 373 | 372 | 786 | 774 | 384 | 385 | 771 | 751 | 405 | 404 | 754 | 742 | 416 | 417 | 739 | 719 | 437 | 436 | 722 | 710 | 448 | 449 | 707 | 32 | 19669 |
| 1127 | 462 | 696 | 697 | 459 | 458 | 700 | 701 | 455 | 494 | 664 | 665 | 491 | 490 | 668 | 669 | 487 | 526 | 632 | 633 | 523 | 522 | 636 | 637 | 519 | 558 | 600 | 601 | 555 | 554 | 604 | 605 | 551 | 30 | 19669 |
| 1129 | 695 | 461 | 460 | 698 | 699 | 457 | 456 | 702 | 663 | 493 | 492 | 666 | 667 | 489 | 488 | 670 | 631 | 525 | 524 | 634 | 635 | 521 | 520 | 638 | 599 | 557 | 556 | 602 | 603 | 553 | 552 | 606 | 28 | 19669 |
| 1131 | 451 | 705 | 704 | 454 | 463 | 693 | 692 | 466 | 483 | 673 | 672 | 486 | 495 | 661 | 660 | 498 | 515 | 641 | 640 | 518 | 527 | 629 | 628 | 530 | 547 | 609 | 608 | 550 | 559 | 597 | 596 | 562 | 26 | 19669 |
| 1133 | 706 | 452 | 453 | 703 | 694 | 464 | 465 | 691 | 674 | 484 | 485 | 671 | 662 | 496 | 497 | 659 | 642 | 516 | 517 | 639 | 630 | 528 | 529 | 627 | 610 | 548 | 549 | 607 | 598 | 560 | 561 | 595 | 24 | 19669 |
| 1135 | 475 | 681 | 680 | 478 | 474 | 684 | 685 | 471 | 507 | 649 | 648 | 510 | 506 | 652 | 653 | 503 | 539 | 617 | 616 | 542 | 538 | 620 | 621 | 535 | 571 | 585 | 584 | 574 | 570 | 588 | 589 | 567 | 22 | 19669 |
| 1137 | 682 | 476 | 477 | 679 | 683 | 473 | 472 | 686 | 650 | 508 | 509 | 647 | 651 | 505 | 504 | 654 | 618 | 540 | 541 | 615 | 619 | 537 | 536 | 622 | 586 | 572 | 573 | 583 | 587 | 569 | 568 | 590 | 20 | 19669 |
| 1139 | 470 | 688 | 689 | 467 | 479 | 677 | 676 | 482 | 502 | 656 | 657 | 499 | 511 | 645 | 644 | 514 | 534 | 624 | 625 | 531 | 543 | 613 | 612 | 546 | 566 | 592 | 593 | 563 | 575 | 581 | 580 | 578 | 18 | 19669 |
| 1140 | 687 | 469 | 468 | 690 | 678 | 480 | 481 | 675 | 655 | 501 | 500 | 658 | 646 | 512 | 513 | 643 | 623 | 533 | 532 | 626 | 614 | 544 | 545 | 611 | 591 | 565 | 564 | 594 | 582 | 576 | 577 | 579 | 17 | 19669 |
| 33 | 65 | 63 | 61 | 59 | 57 | 55 | 53 | 51 | 15 | 13 | 11 | 9 | 7 | 5 | 3 | 1091 | 1093 | 1095 | 1097 | 1099 | 1101 | 1103 | 1105 | 1107 | 1141 | 1143 | 1145 | 1147 | 1149 | 1151 | 1153 | 1155 | 34 | 19669 |
| 19669 | 19669 | 19669 | 19669 | 19669 | 19669 | 19669 | 19669 | 19669 | 19669 | 19669 | 19669 | 19669 | 19669 | 19669 | 19669 | 19669 | 19669 | 19669 | 19669 | 19669 | 19669 | 19669 | 19669 | 19669 | 19669 | 19669 | 19669 | 19669 | 19669 | 19669 | 19669 | 19669 | 19669 | 19669 |

*In this case the magic sums are* $S_{34\times34} := 19669$ *and* $S_{32\times32} := 18512$*. The magic rectangles of order* $2\times4$ *are of **equal magic sums** given by* $R_{2\times4} := (2314, 1157)$.

## 2.15 Magic Square of Order $36\times36$

**Example 2.24.** *A magic square of order* $36\times36$ *constructed based on blocks of magic rectangles of order* $2\times4$ *is given by*

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 23346 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 28 | 1270 | 1271 | 25 | 5 | 1291 | 1290 | 8 | 24 | 1275 | 1274 | 21 | 100 | 1198 | 1199 | 97 | 77 | 1219 | 1218 | 80 | 96 | 1203 | 1202 | 93 | 172 | 1126 | 1127 | 169 | 149 | 1147 | 1146 | 152 | 168 | 1131 | 1130 | 165 | 23346 |
| 1269 | 27 | 26 | 1272 | 1292 | 6 | 7 | 1289 | 1273 | 22 | 23 | 1276 | 1197 | 99 | 98 | 1200 | 1220 | 78 | 79 | 1217 | 1201 | 94 | 95 | 1204 | 1125 | 171 | 170 | 1128 | 1148 | 150 | 151 | 1145 | 1129 | 166 | 167 | 1132 | 23346 |
| 13 | 1283 | 1282 | 16 | 9 | 1287 | 1286 | 12 | 20 | 1278 | 1279 | 17 | 85 | 1211 | 1210 | 88 | 81 | 1215 | 1214 | 84 | 92 | 1206 | 1207 | 89 | 157 | 1139 | 1138 | 160 | 153 | 1143 | 1142 | 156 | 164 | 1134 | 1135 | 161 | 23346 |
| 1284 | 14 | 15 | 1281 | 1288 | 10 | 11 | 1285 | 1277 | 19 | 18 | 1280 | 1212 | 86 | 87 | 1209 | 1216 | 82 | 83 | 1213 | 1205 | 91 | 90 | 1208 | 1140 | 158 | 159 | 1137 | 1144 | 154 | 155 | 1141 | 1133 | 163 | 162 | 1136 | 23346 |
| 1 | 1295 | 1294 | 4 | 29 | 1267 | 1266 | 32 | 33 | 1263 | 1262 | 36 | 73 | 1223 | 1222 | 76 | 101 | 1195 | 1194 | 104 | 105 | 1191 | 1190 | 108 | 145 | 1151 | 1150 | 148 | 173 | 1123 | 1122 | 176 | 177 | 1119 | 1118 | 180 | 23346 |
| 1296 | 2 | 3 | 1293 | 1268 | 30 | 31 | 1265 | 1264 | 34 | 35 | 1261 | 1224 | 74 | 75 | 1221 | 1196 | 102 | 103 | 1193 | 1192 | 106 | 107 | 1189 | 1152 | 146 | 147 | 1149 | 1124 | 174 | 175 | 1121 | 1120 | 178 | 179 | 1117 | 23346 |
| 49 | 1247 | 1246 | 52 | 41 | 1255 | 1254 | 44 | 45 | 1251 | 1250 | 48 | 121 | 1175 | 1174 | 124 | 113 | 1183 | 1182 | 116 | 117 | 1179 | 1178 | 120 | 193 | 1103 | 1102 | 196 | 185 | 1111 | 1110 | 188 | 189 | 1107 | 1106 | 192 | 23346 |
| 1248 | 50 | 51 | 1245 | 1256 | 42 | 43 | 1253 | 1252 | 46 | 47 | 1249 | 1176 | 122 | 123 | 1173 | 1184 | 114 | 115 | 1181 | 1180 | 118 | 119 | 1177 | 1104 | 194 | 195 | 1101 | 1112 | 186 | 187 | 1109 | 1108 | 190 | 191 | 1105 | 23346 |
| 69 | 1226 | 1227 | 72 | 37 | 1259 | 1258 | 40 | 57 | 1239 | 1238 | 60 | 141 | 1154 | 1155 | 144 | 109 | 1187 | 1186 | 112 | 129 | 1167 | 1166 | 132 | 213 | 1082 | 1083 | 216 | 181 | 1115 | 1114 | 184 | 201 | 1095 | 1094 | 204 | 23346 |
| 1228 | 71 | 70 | 1225 | 1260 | 38 | 39 | 1257 | 1240 | 58 | 59 | 1237 | 1156 | 143 | 142 | 1153 | 1188 | 110 | 111 | 1185 | 1168 | 130 | 131 | 1165 | 1084 | 215 | 214 | 1081 | 1116 | 182 | 183 | 1113 | 1096 | 202 | 203 | 1093 | 23346 |
| 64 | 1234 | 1235 | 61 | 65 | 1231 | 1230 | 68 | 53 | 1243 | 1242 | 56 | 136 | 1162 | 1163 | 133 | 137 | 1159 | 1158 | 140 | 125 | 1171 | 1170 | 128 | 208 | 1090 | 1091 | 205 | 209 | 1087 | 1086 | 212 | 197 | 1099 | 1098 | 200 | 23346 |
| 1233 | 63 | 62 | 1236 | 1232 | 66 | 67 | 1229 | 1244 | 54 | 55 | 1241 | 1161 | 135 | 134 | 1164 | 1160 | 138 | 139 | 1157 | 1172 | 126 | 127 | 1169 | 1089 | 207 | 206 | 1092 | 1088 | 210 | 211 | 1085 | 1100 | 198 | 199 | 1097 | 23346 |
| 244 | 1054 | 1055 | 241 | 221 | 1075 | 1074 | 224 | 240 | 1059 | 1058 | 237 | 316 | 982 | 983 | 313 | 293 | 1003 | 1002 | 296 | 312 | 987 | 986 | 309 | 388 | 910 | 911 | 385 | 365 | 931 | 930 | 368 | 384 | 915 | 914 | 381 | 23346 |
| 1053 | 243 | 242 | 1056 | 1076 | 222 | 223 | 1073 | 1057 | 238 | 239 | 1060 | 981 | 315 | 314 | 984 | 1004 | 294 | 295 | 1001 | 985 | 310 | 311 | 988 | 909 | 387 | 386 | 912 | 932 | 366 | 367 | 929 | 913 | 382 | 383 | 916 | 23346 |
| 229 | 1067 | 1066 | 232 | 225 | 1071 | 1070 | 228 | 236 | 1062 | 1063 | 233 | 301 | 995 | 994 | 304 | 297 | 999 | 998 | 300 | 308 | 990 | 991 | 305 | 373 | 923 | 922 | 376 | 369 | 927 | 926 | 372 | 380 | 918 | 919 | 377 | 23346 |
| 1068 | 230 | 231 | 1065 | 1072 | 226 | 227 | 1069 | 1061 | 235 | 234 | 1064 | 996 | 302 | 303 | 993 | 1000 | 298 | 299 | 997 | 989 | 307 | 306 | 992 | 924 | 374 | 375 | 921 | 928 | 370 | 371 | 925 | 917 | 379 | 378 | 920 | 23346 |
| 217 | 1079 | 1078 | 220 | 245 | 1051 | 1050 | 248 | 249 | 1047 | 1046 | 252 | 289 | 1007 | 1006 | 292 | 317 | 979 | 978 | 320 | 321 | 975 | 974 | 324 | 361 | 935 | 934 | 364 | 389 | 907 | 906 | 392 | 393 | 903 | 902 | 396 | 23346 |
| 1080 | 218 | 219 | 1077 | 1052 | 246 | 247 | 1049 | 1048 | 250 | 251 | 1045 | 1008 | 290 | 291 | 1005 | 980 | 318 | 319 | 977 | 976 | 322 | 323 | 973 | 936 | 362 | 363 | 933 | 908 | 390 | 391 | 905 | 904 | 394 | 395 | 901 | 23346 |
| 265 | 1031 | 1030 | 268 | 257 | 1039 | 1038 | 260 | 261 | 1035 | 1034 | 264 | 337 | 959 | 958 | 340 | 329 | 967 | 966 | 332 | 333 | 963 | 962 | 336 | 409 | 887 | 886 | 412 | 401 | 895 | 894 | 404 | 405 | 891 | 890 | 408 | 23346 |
| 1032 | 266 | 267 | 1029 | 1040 | 258 | 259 | 1037 | 1036 | 262 | 263 | 1033 | 960 | 338 | 339 | 957 | 968 | 330 | 331 | 965 | 964 | 334 | 335 | 961 | 888 | 410 | 411 | 885 | 896 | 402 | 403 | 893 | 892 | 406 | 407 | 889 | 23346 |
| 285 | 1010 | 1011 | 288 | 253 | 1043 | 1042 | 256 | 273 | 1023 | 1022 | 276 | 357 | 938 | 939 | 360 | 325 | 971 | 970 | 328 | 345 | 951 | 950 | 348 | 429 | 866 | 867 | 432 | 397 | 899 | 898 | 400 | 417 | 879 | 878 | 420 | 23346 |
| 1012 | 287 | 286 | 1009 | 1044 | 254 | 255 | 1041 | 1024 | 274 | 275 | 1021 | 940 | 359 | 358 | 937 | 972 | 326 | 327 | 969 | 952 | 346 | 347 | 949 | 868 | 431 | 430 | 865 | 900 | 398 | 399 | 897 | 880 | 418 | 419 | 877 | 23346 |
| 280 | 1018 | 1019 | 277 | 281 | 1015 | 1014 | 284 | 269 | 1027 | 1026 | 272 | 352 | 946 | 947 | 349 | 353 | 943 | 942 | 356 | 341 | 955 | 954 | 344 | 424 | 874 | 875 | 421 | 425 | 871 | 870 | 428 | 413 | 883 | 882 | 416 | 23346 |
| 1017 | 279 | 278 | 1020 | 1016 | 282 | 283 | 1013 | 1028 | 270 | 271 | 1025 | 945 | 351 | 350 | 948 | 944 | 354 | 355 | 941 | 956 | 342 | 343 | 953 | 873 | 423 | 422 | 876 | 872 | 426 | 427 | 869 | 884 | 414 | 415 | 881 | 23346 |
| 460 | 838 | 839 | 457 | 437 | 859 | 858 | 440 | 456 | 843 | 842 | 453 | 532 | 766 | 767 | 529 | 509 | 787 | 786 | 512 | 528 | 771 | 770 | 525 | 604 | 694 | 695 | 601 | 581 | 715 | 714 | 584 | 600 | 699 | 698 | 597 | 23346 |
| 837 | 459 | 458 | 840 | 860 | 438 | 439 | 857 | 841 | 454 | 455 | 844 | 765 | 531 | 530 | 768 | 788 | 510 | 511 | 785 | 769 | 526 | 527 | 772 | 693 | 603 | 602 | 696 | 716 | 582 | 583 | 713 | 697 | 598 | 599 | 700 | 23346 |
| 445 | 851 | 850 | 448 | 441 | 855 | 854 | 444 | 452 | 846 | 847 | 449 | 517 | 779 | 778 | 520 | 513 | 783 | 782 | 516 | 524 | 774 | 775 | 521 | 589 | 707 | 706 | 592 | 585 | 711 | 710 | 588 | 596 | 702 | 703 | 593 | 23346 |
| 852 | 446 | 447 | 849 | 856 | 442 | 443 | 853 | 845 | 451 | 450 | 848 | 780 | 518 | 519 | 777 | 784 | 514 | 515 | 781 | 773 | 523 | 522 | 776 | 708 | 590 | 591 | 705 | 712 | 586 | 587 | 709 | 701 | 595 | 594 | 704 | 23346 |
| 433 | 863 | 862 | 436 | 461 | 835 | 834 | 464 | 465 | 831 | 830 | 468 | 505 | 791 | 790 | 508 | 533 | 763 | 762 | 536 | 537 | 759 | 758 | 540 | 577 | 719 | 718 | 580 | 605 | 691 | 690 | 608 | 609 | 687 | 686 | 612 | 23346 |
| 864 | 434 | 435 | 861 | 836 | 462 | 463 | 833 | 832 | 466 | 467 | 829 | 792 | 506 | 507 | 789 | 764 | 534 | 535 | 761 | 760 | 538 | 539 | 757 | 720 | 578 | 579 | 717 | 692 | 606 | 607 | 689 | 688 | 610 | 611 | 685 | 23346 |
| 481 | 815 | 814 | 484 | 473 | 823 | 822 | 476 | 477 | 819 | 818 | 480 | 553 | 743 | 742 | 556 | 545 | 751 | 750 | 548 | 549 | 747 | 746 | 552 | 625 | 671 | 670 | 628 | 617 | 679 | 678 | 620 | 621 | 675 | 674 | 624 | 23346 |
| 816 | 482 | 483 | 813 | 824 | 474 | 475 | 821 | 820 | 478 | 479 | 817 | 744 | 554 | 555 | 741 | 752 | 546 | 547 | 749 | 748 | 550 | 551 | 745 | 672 | 626 | 627 | 669 | 680 | 618 | 619 | 677 | 676 | 622 | 623 | 673 | 23346 |
| 501 | 794 | 795 | 504 | 469 | 827 | 826 | 472 | 489 | 807 | 806 | 492 | 573 | 722 | 723 | 576 | 541 | 755 | 754 | 544 | 561 | 735 | 734 | 564 | 645 | 650 | 651 | 648 | 613 | 683 | 682 | 616 | 633 | 663 | 662 | 636 | 23346 |
| 796 | 503 | 502 | 793 | 828 | 470 | 471 | 825 | 808 | 490 | 491 | 805 | 724 | 575 | 574 | 721 | 756 | 542 | 543 | 753 | 736 | 562 | 563 | 733 | 652 | 647 | 646 | 649 | 684 | 614 | 615 | 681 | 664 | 634 | 635 | 661 | 23346 |
| 496 | 802 | 803 | 493 | 497 | 799 | 798 | 500 | 485 | 811 | 810 | 488 | 568 | 730 | 731 | 565 | 569 | 727 | 726 | 572 | 557 | 739 | 738 | 560 | 640 | 658 | 659 | 637 | 641 | 655 | 654 | 644 | 629 | 667 | 666 | 632 | 23346 |
| 801 | 495 | 494 | 804 | 800 | 498 | 499 | 797 | 812 | 486 | 487 | 809 | 729 | 567 | 566 | 732 | 728 | 570 | 571 | 725 | 740 | 558 | 559 | 737 | 657 | 639 | 638 | 660 | 656 | 642 | 643 | 653 | 668 | 630 | 631 | 665 | 23346 |
| 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 |

*The **magic square** sum is $S_{36\times 36} := 23346$. The magic rectangles of order $2\times 4$ are of **equal magic sums** given by $R_{2\times 4} := (2594, 1297)$. Moreover, each block of order $12\times 12$ is a magic square with magic sums $S_{12\times 12} := 7782$.*

**Example 2.25.** *A magic square of order $36\times 36$ constructed based on blocks of magic rectangles of order $4\times 6$ is given by*

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 23346 |
| 1 | 1292 | 3 | 4 | 1295 | 1296 | 48 | 38 | 39 | 1258 | 1259 | 1249 | 73 | 1220 | 75 | 76 | 1223 | 1224 | 120 | 110 | 111 | 1186 | 1187 | 1177 | 145 | 1148 | 147 | 148 | 1151 | 1152 | 1105 | 182 | 183 | 1114 | 1115 | 192 | 23346 |
| 1291 | 2 | 1293 | 1294 | 5 | 6 | 1254 | 1253 | 1252 | 45 | 44 | 43 | 1219 | 74 | 1221 | 1222 | 77 | 78 | 1182 | 1181 | 1180 | 117 | 116 | 115 | 1147 | 146 | 1149 | 1150 | 149 | 150 | 187 | 1109 | 1108 | 189 | 188 | 1110 | 23346 |
| 1290 | 1289 | 1288 | 9 | 8 | 7 | 1255 | 1256 | 1257 | 40 | 41 | 42 | 1218 | 1217 | 1216 | 81 | 80 | 79 | 1183 | 1184 | 1185 | 112 | 113 | 114 | 1146 | 1145 | 1144 | 153 | 152 | 151 | 186 | 1112 | 1113 | 184 | 185 | 1111 | 23346 |
| 12 | 11 | 10 | 1287 | 1286 | 1285 | 37 | 47 | 46 | 1251 | 1250 | 1260 | 84 | 83 | 82 | 1215 | 1214 | 1213 | 109 | 119 | 118 | 1179 | 1178 | 1188 | 156 | 155 | 154 | 1143 | 1142 | 1141 | 1116 | 191 | 190 | 1107 | 1106 | 181 | 23346 |
| 25 | 26 | 27 | 1270 | 1271 | 1272 | 13 | 14 | 15 | 1282 | 1283 | 1284 | 97 | 98 | 99 | 1198 | 1199 | 1200 | 85 | 86 | 87 | 1210 | 1211 | 1212 | 169 | 170 | 171 | 1126 | 1127 | 1128 | 1140 | 158 | 159 | 1138 | 1139 | 157 | 23346 |
| 1267 | 1268 | 1269 | 28 | 29 | 30 | 1279 | 1280 | 1281 | 16 | 17 | 18 | 1195 | 1196 | 1197 | 100 | 101 | 102 | 1207 | 1208 | 1209 | 88 | 89 | 90 | 1123 | 1124 | 1125 | 172 | 174 | 173 | 162 | 1136 | 1137 | 160 | 161 | 1135 | 23346 |
| 1266 | 1265 | 1264 | 33 | 32 | 31 | 1278 | 1277 | 1276 | 21 | 20 | 19 | 1194 | 1193 | 1192 | 105 | 104 | 103 | 1206 | 1205 | 1204 | 93 | 92 | 91 | 1122 | 1121 | 1120 | 177 | 175 | 176 | 163 | 1133 | 1132 | 165 | 164 | 1134 | 23346 |
| 36 | 35 | 34 | 1263 | 1262 | 1261 | 24 | 23 | 22 | 1275 | 1274 | 1273 | 108 | 107 | 106 | 1191 | 1190 | 1189 | 96 | 95 | 94 | 1203 | 1202 | 1201 | 180 | 179 | 178 | 1119 | 1118 | 1117 | 1129 | 167 | 166 | 1131 | 1130 | 168 | 23346 |
| 49 | 50 | 51 | 1246 | 1247 | 1248 | 61 | 62 | 1233 | 64 | 1235 | 1236 | 121 | 122 | 123 | 1174 | 1175 | 1176 | 133 | 134 | 1161 | 136 | 1163 | 1164 | 193 | 194 | 195 | 1102 | 1103 | 1104 | 205 | 206 | 1089 | 208 | 1091 | 1092 | 23346 |
| 1243 | 1244 | 1240 | 57 | 53 | 54 | 1231 | 1232 | 63 | 1234 | 65 | 66 | 1171 | 1172 | 1168 | 129 | 125 | 126 | 1159 | 1160 | 135 | 1162 | 137 | 138 | 1099 | 1100 | 1096 | 201 | 197 | 198 | 1087 | 1088 | 207 | 1090 | 209 | 210 | 23346 |
| 60 | 1241 | 1245 | 52 | 56 | 1237 | 72 | 1229 | 1228 | 69 | 68 | 1225 | 132 | 1169 | 1173 | 124 | 128 | 1165 | 144 | 1157 | 1156 | 141 | 140 | 1153 | 204 | 1097 | 1101 | 196 | 200 | 1093 | 216 | 1085 | 1084 | 213 | 212 | 1081 | 23346 |
| 1242 | 59 | 58 | 1239 | 1238 | 55 | 1230 | 71 | 70 | 1227 | 1226 | 67 | 1170 | 131 | 130 | 1167 | 1166 | 127 | 1158 | 143 | 142 | 1155 | 1154 | 139 | 1098 | 203 | 202 | 1095 | 1094 | 199 | 1086 | 215 | 214 | 1083 | 1082 | 211 | 23346 |
| 217 | 1076 | 219 | 220 | 1079 | 1080 | 264 | 254 | 255 | 1042 | 1043 | 1033 | 289 | 1004 | 291 | 292 | 1007 | 1008 | 336 | 326 | 327 | 970 | 971 | 961 | 361 | 932 | 363 | 364 | 935 | 936 | 408 | 398 | 399 | 898 | 899 | 889 | 23346 |
| 1075 | 218 | 1077 | 1078 | 221 | 222 | 1038 | 1037 | 1036 | 261 | 260 | 259 | 1003 | 290 | 1005 | 1006 | 293 | 294 | 966 | 965 | 964 | 333 | 332 | 331 | 931 | 362 | 933 | 934 | 365 | 366 | 894 | 893 | 892 | 405 | 404 | 403 | 23346 |
| 1074 | 1073 | 1072 | 225 | 224 | 223 | 1039 | 1040 | 1041 | 256 | 257 | 258 | 1002 | 1001 | 1000 | 297 | 296 | 295 | 967 | 968 | 969 | 328 | 329 | 330 | 930 | 929 | 928 | 369 | 368 | 367 | 895 | 896 | 897 | 400 | 401 | 402 | 23346 |
| 228 | 227 | 226 | 1071 | 1070 | 1069 | 253 | 263 | 262 | 1035 | 1034 | 1044 | 300 | 299 | 298 | 999 | 998 | 997 | 325 | 335 | 334 | 963 | 962 | 972 | 372 | 371 | 370 | 927 | 926 | 925 | 397 | 407 | 406 | 891 | 890 | 900 | 23346 |
| 241 | 242 | 243 | 1054 | 1055 | 1056 | 229 | 230 | 231 | 1066 | 1067 | 1068 | 313 | 314 | 315 | 982 | 983 | 984 | 301 | 302 | 303 | 994 | 995 | 996 | 385 | 386 | 387 | 910 | 911 | 912 | 373 | 374 | 375 | 922 | 923 | 924 | 23346 |
| 1051 | 1052 | 1053 | 244 | 245 | 246 | 1063 | 1064 | 1065 | 232 | 233 | 234 | 979 | 980 | 981 | 316 | 317 | 318 | 991 | 992 | 993 | 304 | 305 | 306 | 907 | 908 | 909 | 388 | 389 | 390 | 919 | 920 | 921 | 376 | 377 | 378 | 23346 |
| 1050 | 1049 | 1048 | 249 | 248 | 247 | 1062 | 1061 | 1060 | 237 | 236 | 235 | 978 | 977 | 976 | 321 | 320 | 319 | 990 | 989 | 988 | 309 | 308 | 307 | 906 | 905 | 904 | 393 | 392 | 391 | 918 | 917 | 916 | 381 | 380 | 379 | 23346 |
| 252 | 251 | 250 | 1047 | 1046 | 1045 | 240 | 239 | 238 | 1059 | 1058 | 1057 | 324 | 323 | 322 | 975 | 974 | 973 | 312 | 311 | 310 | 987 | 986 | 985 | 396 | 395 | 394 | 903 | 902 | 901 | 384 | 383 | 382 | 915 | 914 | 913 | 23346 |
| 265 | 266 | 267 | 1030 | 1031 | 1032 | 277 | 278 | 1017 | 280 | 1019 | 1020 | 337 | 338 | 339 | 960 | 959 | 958 | 349 | 350 | 945 | 352 | 947 | 948 | 409 | 410 | 411 | 886 | 887 | 888 | 421 | 422 | 873 | 424 | 875 | 876 | 23346 |
| 1027 | 1028 | 1024 | 273 | 269 | 270 | 1015 | 1016 | 279 | 1018 | 281 | 282 | 955 | 956 | 952 | 342 | 341 | 345 | 943 | 944 | 351 | 946 | 353 | 354 | 883 | 884 | 880 | 417 | 413 | 414 | 871 | 872 | 423 | 874 | 425 | 426 | 23346 |
| 276 | 1025 | 1029 | 268 | 272 | 1021 | 288 | 1013 | 1012 | 285 | 284 | 1009 | 348 | 953 | 957 | 949 | 344 | 340 | 360 | 941 | 940 | 357 | 356 | 937 | 420 | 881 | 885 | 412 | 416 | 877 | 432 | 869 | 868 | 429 | 428 | 865 | 23346 |
| 1026 | 275 | 274 | 1023 | 1022 | 271 | 1014 | 287 | 286 | 1011 | 1010 | 283 | 954 | 347 | 346 | 343 | 950 | 951 | 942 | 359 | 358 | 939 | 938 | 355 | 882 | 419 | 418 | 879 | 878 | 415 | 870 | 431 | 430 | 867 | 866 | 427 | 23346 |
| 433 | 860 | 435 | 436 | 863 | 864 | 552 | 542 | 543 | 745 | 755 | 754 | 505 | 788 | 507 | 508 | 791 | 792 | 480 | 470 | 471 | 826 | 827 | 817 | 577 | 716 | 579 | 580 | 719 | 720 | 624 | 614 | 615 | 682 | 683 | 673 | 23346 |
| 859 | 434 | 861 | 862 | 437 | 438 | 750 | 749 | 748 | 547 | 548 | 549 | 787 | 506 | 789 | 790 | 509 | 510 | 822 | 821 | 820 | 477 | 476 | 475 | 715 | 578 | 717 | 718 | 581 | 582 | 678 | 677 | 676 | 621 | 620 | 619 | 23346 |
| 858 | 857 | 856 | 441 | 440 | 439 | 751 | 752 | 753 | 546 | 545 | 544 | 786 | 785 | 784 | 513 | 512 | 511 | 823 | 824 | 825 | 472 | 473 | 474 | 714 | 713 | 712 | 585 | 584 | 583 | 679 | 680 | 681 | 616 | 617 | 618 | 23346 |
| 444 | 443 | 442 | 855 | 854 | 853 | 541 | 551 | 550 | 756 | 746 | 747 | 516 | 515 | 514 | 783 | 782 | 781 | 469 | 479 | 478 | 819 | 818 | 828 | 588 | 587 | 586 | 711 | 710 | 709 | 613 | 623 | 622 | 675 | 674 | 684 | 23346 |
| 457 | 458 | 459 | 838 | 839 | 840 | 445 | 446 | 447 | 850 | 851 | 852 | 529 | 530 | 531 | 766 | 767 | 768 | 517 | 518 | 519 | 778 | 779 | 780 | 601 | 602 | 603 | 694 | 695 | 696 | 589 | 590 | 591 | 706 | 707 | 708 | 23346 |
| 835 | 836 | 837 | 460 | 461 | 462 | 847 | 848 | 849 | 448 | 449 | 450 | 763 | 764 | 765 | 532 | 533 | 534 | 775 | 776 | 777 | 520 | 521 | 522 | 691 | 692 | 693 | 604 | 605 | 606 | 703 | 704 | 705 | 592 | 593 | 594 | 23346 |
| 834 | 833 | 832 | 465 | 464 | 463 | 846 | 845 | 844 | 453 | 452 | 451 | 762 | 761 | 760 | 537 | 536 | 535 | 774 | 773 | 772 | 525 | 524 | 523 | 690 | 689 | 688 | 609 | 608 | 607 | 702 | 701 | 700 | 597 | 596 | 595 | 23346 |
| 468 | 467 | 466 | 831 | 830 | 829 | 456 | 455 | 454 | 843 | 842 | 841 | 540 | 539 | 538 | 759 | 758 | 757 | 528 | 527 | 526 | 771 | 770 | 769 | 612 | 611 | 610 | 687 | 686 | 685 | 600 | 599 | 598 | 699 | 698 | 697 | 23346 |
| 481 | 482 | 483 | 814 | 815 | 816 | 493 | 494 | 801 | 496 | 803 | 804 | 553 | 554 | 555 | 742 | 743 | 744 | 565 | 566 | 729 | 568 | 731 | 732 | 625 | 626 | 627 | 670 | 671 | 672 | 637 | 638 | 657 | 640 | 659 | 660 | 23346 |
| 811 | 812 | 808 | 489 | 485 | 486 | 799 | 800 | 495 | 802 | 497 | 498 | 739 | 740 | 736 | 561 | 557 | 558 | 727 | 728 | 567 | 730 | 569 | 570 | 667 | 668 | 664 | 633 | 629 | 630 | 655 | 656 | 639 | 658 | 641 | 642 | 23346 |
| 492 | 809 | 813 | 484 | 488 | 805 | 504 | 797 | 796 | 501 | 500 | 793 | 564 | 737 | 741 | 556 | 560 | 733 | 576 | 725 | 724 | 573 | 572 | 721 | 636 | 665 | 669 | 628 | 632 | 661 | 648 | 653 | 652 | 645 | 644 | 649 | 23346 |
| 810 | 491 | 490 | 807 | 806 | 487 | 798 | 503 | 502 | 795 | 794 | 499 | 738 | 563 | 562 | 735 | 734 | 559 | 726 | 575 | 574 | 723 | 722 | 571 | 666 | 635 | 634 | 663 | 662 | 631 | 654 | 647 | 646 | 651 | 650 | 643 | 23346 |
| 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 |

*The **magic square** sum is* $S_{36\times 36} := 23346$*. The magic rectangles of order* $4\times 6$ *are of **equal magic sums** given by* $R_{4\times 6} := (3891, 2594)$.

## 2.16 Block Bordered Magic Squares of Order $38\times 38$

It is based on the Examples 2.24 and 2.25. The internal blocks are magic squares of order 36.

**Example 2.26.** *A **block bordered** magic square of order 38 with inner block as a magic square of order 36 is given by*

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 27455 |
| 1407 | 1372 | 1374 | 1376 | 1378 | 1380 | 1382 | 1384 | 1386 | 1388 | 1428 | 1430 | 1432 | 1434 | 1436 | 1438 | 1440 | 1442 | 74 | 72 | 70 | 68 | 66 | 64 | 62 | 60 | 58 | 56 | 18 | 16 | 14 | 12 | 10 | 8 | 6 | 4 | 2 | 1408 | 27455 |
| 55 | 102 | 1344 | 1345 | 99 | 79 | 1365 | 1364 | 82 | 98 | 1349 | 1348 | 95 | 174 | 1272 | 1273 | 171 | 151 | 1293 | 1292 | 154 | 170 | 1277 | 1276 | 167 | 246 | 1200 | 1201 | 243 | 223 | 1221 | 1220 | 226 | 242 | 1205 | 1204 | 239 | 1390 | 27455 |
| 53 | 1343 | 101 | 100 | 1346 | 1366 | 80 | 81 | 1363 | 1347 | 96 | 97 | 1350 | 1271 | 173 | 172 | 1274 | 1294 | 152 | 153 | 1291 | 1275 | 168 | 169 | 1278 | 1199 | 245 | 244 | 1202 | 1222 | 224 | 225 | 1219 | 1203 | 240 | 241 | 1206 | 1392 | 27455 |
| 51 | 87 | 1357 | 1356 | 90 | 83 | 1361 | 1360 | 86 | 94 | 1352 | 1353 | 91 | 159 | 1285 | 1284 | 162 | 155 | 1289 | 1288 | 158 | 166 | 1280 | 1281 | 163 | 231 | 1213 | 1212 | 234 | 227 | 1217 | 1216 | 230 | 238 | 1208 | 1209 | 235 | 1394 | 27455 |
| 49 | 1358 | 88 | 89 | 1355 | 1362 | 84 | 85 | 1359 | 1351 | 93 | 92 | 1354 | 1286 | 160 | 161 | 1283 | 1290 | 156 | 157 | 1287 | 1279 | 165 | 164 | 1282 | 1214 | 232 | 233 | 1211 | 1218 | 228 | 229 | 1215 | 1207 | 237 | 236 | 1210 | 1396 | 27455 |
| 47 | 75 | 1369 | 1368 | 78 | 103 | 1341 | 1340 | 106 | 107 | 1337 | 1336 | 110 | 147 | 1297 | 1296 | 150 | 175 | 1269 | 1268 | 178 | 179 | 1265 | 1264 | 182 | 219 | 1225 | 1224 | 222 | 247 | 1197 | 1196 | 250 | 251 | 1193 | 1192 | 254 | 1398 | 27455 |
| 45 | 1370 | 76 | 77 | 1367 | 1342 | 104 | 105 | 1339 | 1338 | 108 | 109 | 1335 | 1298 | 148 | 149 | 1295 | 1270 | 176 | 177 | 1267 | 1266 | 180 | 181 | 1263 | 1226 | 220 | 221 | 1223 | 1198 | 248 | 249 | 1195 | 1194 | 252 | 253 | 1191 | 1400 | 27455 |
| 43 | 123 | 1321 | 1320 | 126 | 115 | 1329 | 1328 | 118 | 119 | 1325 | 1324 | 122 | 195 | 1249 | 1248 | 198 | 187 | 1257 | 1256 | 190 | 191 | 1253 | 1252 | 194 | 267 | 1177 | 1176 | 270 | 259 | 1185 | 1184 | 262 | 263 | 1181 | 1180 | 266 | 1402 | 27455 |
| 41 | 1322 | 124 | 125 | 1319 | 1330 | 116 | 117 | 1327 | 1326 | 120 | 121 | 1323 | 1250 | 196 | 197 | 1247 | 1258 | 188 | 189 | 1255 | 1254 | 192 | 193 | 1251 | 1178 | 268 | 269 | 1175 | 1186 | 260 | 261 | 1183 | 1182 | 264 | 265 | 1179 | 1404 | 27455 |
| 39 | 143 | 1300 | 1301 | 146 | 111 | 1333 | 1332 | 114 | 131 | 1313 | 1312 | 134 | 215 | 1228 | 1229 | 218 | 183 | 1261 | 1260 | 186 | 203 | 1241 | 1240 | 206 | 287 | 1156 | 1157 | 290 | 255 | 1189 | 1188 | 258 | 275 | 1169 | 1168 | 278 | 1406 | 27455 |
| 35 | 1302 | 145 | 144 | 1299 | 1334 | 112 | 113 | 1331 | 1314 | 132 | 133 | 1311 | 1230 | 217 | 216 | 1227 | 1262 | 184 | 185 | 1259 | 1242 | 204 | 205 | 1239 | 1158 | 289 | 288 | 1155 | 1190 | 256 | 257 | 1187 | 1170 | 276 | 277 | 1167 | 1410 | 27455 |
| 33 | 138 | 1308 | 1309 | 135 | 139 | 1305 | 1304 | 142 | 127 | 1317 | 1316 | 130 | 210 | 1236 | 1237 | 207 | 211 | 1233 | 1232 | 214 | 199 | 1245 | 1244 | 202 | 282 | 1164 | 1165 | 279 | 283 | 1161 | 1160 | 286 | 271 | 1173 | 1172 | 274 | 1412 | 27455 |
| 31 | 1307 | 137 | 136 | 1310 | 1306 | 140 | 141 | 1303 | 1318 | 128 | 129 | 1315 | 1235 | 209 | 208 | 1238 | 1234 | 212 | 213 | 1231 | 1246 | 200 | 201 | 1243 | 1163 | 281 | 280 | 1166 | 1162 | 284 | 285 | 1159 | 1174 | 272 | 273 | 1171 | 1414 | 27455 |
| 29 | 318 | 1128 | 1129 | 315 | 295 | 1149 | 1148 | 298 | 314 | 1133 | 1132 | 311 | 390 | 1056 | 1057 | 387 | 367 | 1077 | 1076 | 370 | 386 | 1061 | 1060 | 383 | 462 | 984 | 985 | 459 | 439 | 1005 | 1004 | 442 | 458 | 989 | 988 | 455 | 1416 | 27455 |
| 27 | 1127 | 317 | 316 | 1130 | 1150 | 296 | 297 | 1147 | 1131 | 312 | 313 | 1134 | 1055 | 389 | 388 | 1058 | 1078 | 368 | 369 | 1075 | 1059 | 384 | 385 | 1062 | 983 | 461 | 460 | 986 | 1006 | 440 | 441 | 1003 | 987 | 456 | 457 | 990 | 1418 | 27455 |
| 25 | 303 | 1141 | 1140 | 306 | 299 | 1145 | 1144 | 302 | 310 | 1136 | 1137 | 307 | 375 | 1069 | 1068 | 378 | 371 | 1073 | 1072 | 374 | 382 | 1064 | 1065 | 379 | 447 | 997 | 996 | 450 | 443 | 1001 | 1000 | 446 | 454 | 992 | 993 | 451 | 1420 | 27455 |
| 23 | 1142 | 304 | 305 | 1139 | 1146 | 300 | 301 | 1143 | 1135 | 309 | 308 | 1138 | 1070 | 376 | 377 | 1067 | 1074 | 372 | 373 | 1071 | 1063 | 381 | 380 | 1066 | 998 | 448 | 449 | 995 | 1002 | 444 | 445 | 999 | 991 | 453 | 452 | 994 | 1422 | 27455 |
| 21 | 291 | 1153 | 1152 | 294 | 319 | 1125 | 1124 | 322 | 323 | 1121 | 1120 | 326 | 363 | 1081 | 1080 | 366 | 391 | 1053 | 1052 | 394 | 395 | 1049 | 1048 | 398 | 435 | 1009 | 1008 | 438 | 463 | 981 | 980 | 466 | 467 | 977 | 976 | 470 | 1424 | 27455 |
| 1 | 1154 | 292 | 293 | 1151 | 1126 | 320 | 321 | 1123 | 1122 | 324 | 325 | 1119 | 1082 | 364 | 365 | 1079 | 1054 | 392 | 393 | 1051 | 1050 | 396 | 397 | 1047 | 1010 | 436 | 437 | 1007 | 982 | 464 | 465 | 979 | 978 | 468 | 469 | 975 | 1444 | 27455 |
| 1391 | 339 | 1105 | 1104 | 342 | 331 | 1113 | 1112 | 334 | 335 | 1109 | 1108 | 338 | 411 | 1033 | 1032 | 414 | 403 | 1041 | 1040 | 406 | 407 | 1037 | 1036 | 410 | 483 | 961 | 960 | 486 | 475 | 969 | 968 | 478 | 479 | 965 | 964 | 482 | 54 | 27455 |
| 1393 | 1106 | 340 | 341 | 1103 | 1114 | 332 | 333 | 1111 | 1110 | 336 | 337 | 1107 | 1034 | 412 | 413 | 1031 | 1042 | 404 | 405 | 1039 | 1038 | 408 | 409 | 1035 | 962 | 484 | 485 | 959 | 970 | 476 | 477 | 967 | 966 | 480 | 481 | 963 | 52 | 27455 |
| 1395 | 359 | 1084 | 1085 | 362 | 327 | 1117 | 1116 | 330 | 347 | 1097 | 1096 | 350 | 431 | 1012 | 1013 | 434 | 399 | 1045 | 1044 | 402 | 419 | 1025 | 1024 | 422 | 503 | 940 | 941 | 506 | 471 | 973 | 972 | 474 | 491 | 953 | 952 | 494 | 50 | 27455 |
| 1397 | 1086 | 361 | 360 | 1083 | 1118 | 328 | 329 | 1115 | 1098 | 348 | 349 | 1095 | 1014 | 433 | 432 | 1011 | 1046 | 400 | 401 | 1043 | 1026 | 420 | 421 | 1023 | 942 | 505 | 504 | 939 | 974 | 472 | 473 | 971 | 954 | 492 | 493 | 951 | 48 | 27455 |
| 1399 | 354 | 1092 | 1093 | 351 | 355 | 1089 | 1088 | 358 | 343 | 1101 | 1100 | 346 | 426 | 1020 | 1021 | 423 | 427 | 1017 | 1016 | 430 | 415 | 1029 | 1028 | 418 | 498 | 948 | 949 | 495 | 499 | 945 | 944 | 502 | 487 | 957 | 956 | 490 | 46 | 27455 |
| 1401 | 1091 | 353 | 352 | 1094 | 1090 | 356 | 357 | 1087 | 1102 | 344 | 345 | 1099 | 1019 | 425 | 424 | 1022 | 1018 | 428 | 429 | 1015 | 1030 | 416 | 417 | 1027 | 947 | 497 | 496 | 950 | 946 | 500 | 501 | 943 | 958 | 488 | 489 | 955 | 44 | 27455 |
| 1403 | 534 | 912 | 913 | 531 | 511 | 933 | 932 | 514 | 530 | 917 | 916 | 527 | 606 | 840 | 841 | 603 | 583 | 861 | 860 | 586 | 602 | 845 | 844 | 599 | 678 | 768 | 769 | 675 | 655 | 789 | 788 | 658 | 674 | 773 | 772 | 671 | 42 | 27455 |
| 1405 | 911 | 533 | 532 | 914 | 934 | 512 | 513 | 931 | 915 | 528 | 529 | 918 | 839 | 605 | 604 | 842 | 862 | 584 | 585 | 859 | 843 | 600 | 601 | 846 | 767 | 677 | 676 | 770 | 790 | 656 | 657 | 787 | 771 | 672 | 673 | 774 | 40 | 27455 |
| 1409 | 519 | 925 | 924 | 522 | 515 | 929 | 928 | 518 | 526 | 920 | 921 | 523 | 591 | 853 | 852 | 594 | 587 | 857 | 856 | 590 | 598 | 848 | 849 | 595 | 663 | 781 | 780 | 666 | 659 | 785 | 784 | 662 | 670 | 776 | 777 | 667 | 36 | 27455 |
| 1411 | 926 | 520 | 521 | 923 | 930 | 516 | 517 | 927 | 919 | 525 | 524 | 922 | 854 | 592 | 593 | 851 | 858 | 588 | 589 | 855 | 847 | 597 | 596 | 850 | 782 | 664 | 665 | 779 | 786 | 660 | 661 | 783 | 775 | 669 | 668 | 778 | 34 | 27455 |
| 1413 | 507 | 937 | 936 | 510 | 535 | 909 | 908 | 538 | 539 | 905 | 904 | 542 | 579 | 865 | 864 | 582 | 607 | 837 | 836 | 610 | 611 | 833 | 832 | 614 | 651 | 793 | 792 | 654 | 679 | 765 | 764 | 682 | 683 | 761 | 760 | 686 | 32 | 27455 |
| 1415 | 938 | 508 | 509 | 935 | 910 | 536 | 537 | 907 | 906 | 540 | 541 | 903 | 866 | 580 | 581 | 863 | 838 | 608 | 609 | 835 | 834 | 612 | 613 | 831 | 794 | 652 | 653 | 791 | 766 | 680 | 681 | 763 | 762 | 684 | 685 | 759 | 30 | 27455 |
| 1417 | 555 | 889 | 888 | 558 | 547 | 897 | 896 | 550 | 551 | 893 | 892 | 554 | 627 | 817 | 816 | 630 | 619 | 825 | 824 | 622 | 623 | 821 | 820 | 626 | 699 | 745 | 744 | 702 | 691 | 753 | 752 | 694 | 695 | 749 | 748 | 698 | 28 | 27455 |
| 1419 | 890 | 556 | 557 | 887 | 898 | 548 | 549 | 895 | 894 | 552 | 553 | 891 | 818 | 628 | 629 | 815 | 826 | 620 | 621 | 823 | 822 | 624 | 625 | 819 | 746 | 700 | 701 | 743 | 754 | 692 | 693 | 751 | 750 | 696 | 697 | 747 | 26 | 27455 |
| 1421 | 575 | 868 | 869 | 578 | 543 | 901 | 900 | 546 | 563 | 881 | 880 | 566 | 647 | 796 | 797 | 650 | 615 | 829 | 828 | 618 | 635 | 809 | 808 | 638 | 719 | 724 | 725 | 722 | 687 | 757 | 756 | 690 | 707 | 737 | 736 | 710 | 24 | 27455 |
| 1423 | 870 | 577 | 576 | 867 | 902 | 544 | 545 | 899 | 882 | 564 | 565 | 879 | 798 | 649 | 648 | 795 | 830 | 616 | 617 | 827 | 810 | 636 | 637 | 807 | 726 | 721 | 720 | 723 | 758 | 688 | 689 | 755 | 738 | 708 | 709 | 735 | 22 | 27455 |
| 1425 | 570 | 876 | 877 | 567 | 571 | 873 | 872 | 574 | 559 | 885 | 884 | 562 | 642 | 804 | 805 | 639 | 643 | 801 | 800 | 646 | 631 | 813 | 812 | 634 | 714 | 732 | 733 | 711 | 715 | 729 | 728 | 718 | 703 | 741 | 740 | 706 | 20 | 27455 |
| 1426 | 875 | 569 | 568 | 878 | 874 | 572 | 573 | 871 | 886 | 560 | 561 | 883 | 803 | 641 | 640 | 806 | 802 | 644 | 645 | 799 | 814 | 632 | 633 | 811 | 731 | 713 | 712 | 734 | 730 | 716 | 717 | 727 | 742 | 704 | 705 | 739 | 19 | 27455 |
| 37 | 73 | 71 | 69 | 67 | 65 | 63 | 61 | 59 | 57 | 17 | 15 | 13 | 11 | 9 | 7 | 5 | 3 | 1371 | 1373 | 1375 | 1377 | 1379 | 1381 | 1383 | 1385 | 1387 | 1389 | 1427 | 1429 | 1431 | 1433 | 1435 | 1437 | 1439 | 1441 | 1443 | 38 | 27455 |
| 27455 | 27455 | 27455 | 27455 | 27455 | 27455 | 27455 | 27455 | 27455 | 27455 | 27455 | 27455 | 27455 | 27455 | 27455 | 27455 | 27455 | 27455 | 27455 | 27455 | 27455 | 27455 | 27455 | 27455 | 27455 | 27455 | 27455 | 27455 | 27455 | 27455 | 27455 | 27455 | 27455 | 27455 | 27455 | 27455 | 27455 | 27455 | 27455 |

*In this case the magic sums are* $S_{38\times38} := 27455$ *and* $S_{36\times36} := 26010$*. The magic rectangles of order* $2\times4$ *are of* ***equal magic sums*** *given by* $R_{2\times4} := (2890, 1445)$*.*

**Example 2.27.** *A* ***block bordered*** *magic square of order 38 with inner block as a magic square of order 36 is given by*

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 27455 |
| 1407 | 1372 | 1374 | 1376 | 1378 | 1380 | 1382 | 1384 | 1386 | 1388 | 1428 | 1430 | 1432 | 1434 | 1436 | 1438 | 1440 | 1442 | 74 | 72 | 70 | 68 | 66 | 64 | 62 | 60 | 58 | 56 | 18 | 16 | 14 | 12 | 10 | 8 | 6 | 4 | 2 | 1408 | 27455 |
| 55 | 75 | 1366 | 77 | 78 | 1369 | 1370 | 122 | 112 | 113 | 1332 | 1333 | 1323 | 147 | 1294 | 149 | 150 | 1297 | 1298 | 194 | 184 | 185 | 1260 | 1261 | 1251 | 219 | 1222 | 221 | 222 | 1225 | 1226 | 1179 | 256 | 257 | 1188 | 1189 | 266 | 1390 | 27455 |
| 53 | 1365 | 76 | 1367 | 1368 | 79 | 80 | 1328 | 1327 | 1326 | 119 | 118 | 117 | 1293 | 148 | 1295 | 1296 | 151 | 152 | 1256 | 1255 | 1254 | 191 | 190 | 189 | 1221 | 220 | 1223 | 1224 | 223 | 224 | 261 | 1183 | 1182 | 263 | 262 | 1184 | 1392 | 27455 |
| 51 | 1364 | 1363 | 1362 | 83 | 82 | 81 | 1329 | 1330 | 1331 | 114 | 115 | 116 | 1292 | 1291 | 1290 | 155 | 154 | 153 | 1257 | 1258 | 1259 | 186 | 187 | 188 | 1220 | 1219 | 1218 | 227 | 226 | 225 | 260 | 1186 | 1187 | 258 | 259 | 1185 | 1394 | 27455 |
| 49 | 86 | 85 | 84 | 1361 | 1360 | 1359 | 111 | 121 | 120 | 1325 | 1324 | 1334 | 158 | 157 | 156 | 1289 | 1288 | 1287 | 183 | 193 | 192 | 1253 | 1252 | 1262 | 230 | 229 | 228 | 1217 | 1216 | 1215 | 1190 | 265 | 264 | 1181 | 1180 | 255 | 1396 | 27455 |
| 47 | 99 | 100 | 101 | 1344 | 1345 | 1346 | 87 | 88 | 89 | 1356 | 1357 | 1358 | 171 | 172 | 173 | 1272 | 1273 | 1274 | 159 | 160 | 161 | 1284 | 1285 | 1286 | 243 | 244 | 245 | 1200 | 1201 | 1202 | 1214 | 232 | 233 | 1212 | 1213 | 231 | 1398 | 27455 |
| 45 | 1341 | 1342 | 1343 | 102 | 103 | 104 | 1353 | 1354 | 1355 | 90 | 91 | 92 | 1269 | 1270 | 1271 | 174 | 175 | 176 | 1281 | 1282 | 1283 | 162 | 163 | 164 | 1197 | 1198 | 1199 | 246 | 248 | 247 | 236 | 1210 | 1211 | 234 | 235 | 1209 | 1400 | 27455 |
| 43 | 1340 | 1339 | 1338 | 107 | 106 | 105 | 1352 | 1351 | 1350 | 95 | 94 | 93 | 1268 | 1267 | 1266 | 179 | 178 | 177 | 1280 | 1279 | 1278 | 167 | 166 | 165 | 1196 | 1195 | 1194 | 251 | 249 | 250 | 237 | 1207 | 1206 | 239 | 238 | 1208 | 1402 | 27455 |
| 41 | 110 | 109 | 108 | 1337 | 1336 | 1335 | 98 | 97 | 96 | 1349 | 1348 | 1347 | 182 | 181 | 180 | 1265 | 1264 | 1263 | 170 | 169 | 168 | 1277 | 1276 | 1275 | 254 | 253 | 252 | 1193 | 1192 | 1191 | 1203 | 241 | 240 | 1205 | 1204 | 242 | 1404 | 27455 |
| 39 | 123 | 124 | 125 | 1320 | 1321 | 1322 | 135 | 136 | 1307 | 138 | 1309 | 1310 | 195 | 196 | 197 | 1248 | 1249 | 1250 | 207 | 208 | 1235 | 210 | 1237 | 1238 | 267 | 268 | 269 | 1176 | 1177 | 1178 | 279 | 280 | 1163 | 282 | 1165 | 1166 | 1406 | 27455 |
| 35 | 1317 | 1318 | 1314 | 131 | 127 | 128 | 1305 | 1306 | 137 | 1308 | 139 | 140 | 1245 | 1246 | 1242 | 203 | 199 | 200 | 1233 | 1234 | 209 | 1236 | 211 | 212 | 1173 | 1174 | 1170 | 275 | 271 | 272 | 1161 | 1162 | 281 | 1164 | 283 | 284 | 1410 | 27455 |
| 33 | 134 | 1315 | 1319 | 126 | 130 | 1311 | 146 | 1303 | 1302 | 143 | 142 | 1299 | 206 | 1243 | 1247 | 198 | 202 | 1239 | 218 | 1231 | 1230 | 215 | 214 | 1227 | 278 | 1171 | 1175 | 270 | 274 | 1167 | 290 | 1159 | 1158 | 287 | 286 | 1155 | 1412 | 27455 |
| 31 | 1316 | 133 | 132 | 1313 | 1312 | 129 | 1304 | 145 | 144 | 1301 | 1300 | 141 | 1244 | 205 | 204 | 1241 | 1240 | 201 | 1232 | 217 | 216 | 1229 | 1228 | 213 | 1172 | 277 | 276 | 1169 | 1168 | 273 | 1160 | 289 | 288 | 1157 | 1156 | 285 | 1414 | 27455 |
| 29 | 291 | 1150 | 293 | 294 | 1153 | 1154 | 338 | 328 | 329 | 1116 | 1117 | 1107 | 363 | 1078 | 365 | 366 | 1081 | 1082 | 410 | 400 | 401 | 1044 | 1045 | 1035 | 435 | 1006 | 437 | 438 | 1009 | 1010 | 482 | 472 | 473 | 972 | 973 | 963 | 1416 | 27455 |
| 27 | 1149 | 292 | 1151 | 1152 | 295 | 296 | 1112 | 1111 | 1110 | 335 | 334 | 333 | 1077 | 364 | 1079 | 1080 | 367 | 368 | 1040 | 1039 | 1038 | 407 | 406 | 405 | 1005 | 436 | 1007 | 1008 | 439 | 440 | 968 | 967 | 966 | 479 | 478 | 477 | 1418 | 27455 |
| 25 | 1148 | 1147 | 1146 | 299 | 298 | 297 | 1113 | 1114 | 1115 | 330 | 331 | 332 | 1076 | 1075 | 1074 | 371 | 370 | 369 | 1041 | 1042 | 1043 | 402 | 403 | 404 | 1004 | 1003 | 1002 | 443 | 442 | 441 | 969 | 970 | 971 | 474 | 475 | 476 | 1420 | 27455 |
| 23 | 302 | 301 | 300 | 1145 | 1144 | 1143 | 327 | 337 | 336 | 1109 | 1108 | 1118 | 374 | 373 | 372 | 1073 | 1072 | 1071 | 399 | 409 | 408 | 1037 | 1036 | 1046 | 446 | 445 | 444 | 1001 | 1000 | 999 | 471 | 481 | 480 | 965 | 964 | 974 | 1422 | 27455 |
| 21 | 315 | 316 | 317 | 1128 | 1129 | 1130 | 303 | 304 | 305 | 1140 | 1141 | 1142 | 387 | 388 | 389 | 1056 | 1057 | 1058 | 375 | 376 | 377 | 1068 | 1069 | 1070 | 459 | 460 | 461 | 984 | 985 | 986 | 447 | 448 | 449 | 996 | 997 | 998 | 1424 | 27455 |
| 1 | 1125 | 1126 | 1127 | 318 | 319 | 320 | 1137 | 1138 | 1139 | 306 | 307 | 308 | 1053 | 1054 | 1055 | 390 | 391 | 392 | 1065 | 1066 | 1067 | 378 | 379 | 380 | 981 | 982 | 983 | 462 | 463 | 464 | 993 | 994 | 995 | 450 | 451 | 452 | 1444 | 27455 |
| 1391 | 1124 | 1123 | 1122 | 323 | 322 | 321 | 1136 | 1135 | 1134 | 311 | 310 | 309 | 1052 | 1051 | 1050 | 395 | 394 | 393 | 1064 | 1063 | 1062 | 383 | 382 | 381 | 980 | 979 | 978 | 467 | 466 | 465 | 992 | 991 | 990 | 455 | 454 | 453 | 54 | 27455 |
| 1393 | 326 | 325 | 324 | 1121 | 1120 | 1119 | 314 | 313 | 312 | 1133 | 1132 | 1131 | 398 | 397 | 396 | 1049 | 1048 | 1047 | 386 | 385 | 384 | 1061 | 1060 | 1059 | 470 | 469 | 468 | 977 | 976 | 975 | 458 | 457 | 456 | 989 | 988 | 987 | 52 | 27455 |
| 1395 | 339 | 340 | 341 | 1104 | 1105 | 1106 | 351 | 352 | 1091 | 354 | 1093 | 1094 | 411 | 412 | 413 | 1034 | 1033 | 1032 | 423 | 424 | 1019 | 426 | 1021 | 1022 | 483 | 484 | 485 | 960 | 961 | 962 | 495 | 496 | 947 | 498 | 949 | 950 | 50 | 27455 |
| 1397 | 1101 | 1102 | 1098 | 347 | 343 | 344 | 1089 | 1090 | 353 | 1092 | 355 | 356 | 1029 | 1030 | 1026 | 416 | 415 | 419 | 1017 | 1018 | 425 | 1020 | 427 | 428 | 957 | 958 | 954 | 491 | 487 | 488 | 945 | 946 | 497 | 948 | 499 | 500 | 48 | 27455 |
| 1399 | 350 | 1099 | 1103 | 342 | 346 | 1095 | 362 | 1087 | 1086 | 359 | 358 | 1083 | 422 | 1027 | 1031 | 1023 | 418 | 414 | 434 | 1015 | 1014 | 431 | 430 | 1011 | 494 | 955 | 959 | 486 | 490 | 951 | 506 | 943 | 942 | 503 | 502 | 939 | 46 | 27455 |
| 1401 | 1100 | 349 | 348 | 1097 | 1096 | 345 | 1088 | 361 | 360 | 1085 | 1084 | 357 | 1028 | 421 | 420 | 417 | 1024 | 1025 | 1016 | 433 | 432 | 1013 | 1012 | 429 | 956 | 493 | 492 | 953 | 952 | 489 | 944 | 505 | 504 | 941 | 940 | 501 | 44 | 27455 |
| 1403 | 507 | 934 | 509 | 510 | 937 | 938 | 626 | 616 | 617 | 819 | 829 | 828 | 579 | 862 | 581 | 582 | 865 | 866 | 554 | 544 | 545 | 900 | 901 | 891 | 651 | 790 | 653 | 654 | 793 | 794 | 698 | 688 | 689 | 756 | 757 | 747 | 42 | 27455 |
| 1405 | 933 | 508 | 935 | 936 | 511 | 512 | 824 | 823 | 822 | 621 | 622 | 623 | 861 | 580 | 863 | 864 | 583 | 584 | 896 | 895 | 894 | 551 | 550 | 549 | 789 | 652 | 791 | 792 | 655 | 656 | 752 | 751 | 750 | 695 | 694 | 693 | 40 | 27455 |
| 1409 | 932 | 931 | 930 | 515 | 514 | 513 | 825 | 826 | 827 | 620 | 619 | 618 | 860 | 859 | 858 | 587 | 586 | 585 | 897 | 898 | 899 | 546 | 547 | 548 | 788 | 787 | 786 | 659 | 658 | 657 | 753 | 754 | 755 | 690 | 691 | 692 | 36 | 27455 |
| 1411 | 518 | 517 | 516 | 929 | 928 | 927 | 615 | 625 | 624 | 830 | 820 | 821 | 590 | 589 | 588 | 857 | 856 | 855 | 543 | 553 | 552 | 893 | 892 | 902 | 662 | 661 | 660 | 785 | 784 | 783 | 687 | 697 | 696 | 749 | 748 | 758 | 34 | 27455 |
| 1413 | 531 | 532 | 533 | 912 | 913 | 914 | 519 | 520 | 521 | 924 | 925 | 926 | 603 | 604 | 605 | 840 | 841 | 842 | 591 | 592 | 593 | 852 | 853 | 854 | 675 | 676 | 677 | 768 | 769 | 770 | 663 | 664 | 665 | 780 | 781 | 782 | 32 | 27455 |
| 1415 | 909 | 910 | 911 | 534 | 535 | 536 | 921 | 922 | 923 | 522 | 523 | 524 | 837 | 838 | 839 | 606 | 607 | 608 | 849 | 850 | 851 | 594 | 595 | 596 | 765 | 766 | 767 | 678 | 679 | 680 | 777 | 778 | 779 | 666 | 667 | 668 | 30 | 27455 |
| 1417 | 908 | 907 | 906 | 539 | 538 | 537 | 920 | 919 | 918 | 527 | 526 | 525 | 836 | 835 | 834 | 611 | 610 | 609 | 848 | 847 | 846 | 599 | 598 | 597 | 764 | 763 | 762 | 683 | 682 | 681 | 776 | 775 | 774 | 671 | 670 | 669 | 28 | 27455 |
| 1419 | 542 | 541 | 540 | 905 | 904 | 903 | 530 | 529 | 528 | 917 | 916 | 915 | 614 | 613 | 612 | 833 | 832 | 831 | 602 | 601 | 600 | 845 | 844 | 843 | 686 | 685 | 684 | 761 | 760 | 759 | 674 | 673 | 672 | 773 | 772 | 771 | 26 | 27455 |
| 1421 | 555 | 556 | 557 | 888 | 889 | 890 | 567 | 568 | 875 | 570 | 877 | 878 | 627 | 628 | 629 | 816 | 817 | 818 | 639 | 640 | 803 | 642 | 805 | 806 | 699 | 700 | 701 | 744 | 745 | 746 | 711 | 712 | 731 | 714 | 733 | 734 | 24 | 27455 |
| 1423 | 885 | 886 | 882 | 563 | 559 | 560 | 873 | 874 | 569 | 876 | 571 | 572 | 813 | 814 | 810 | 635 | 631 | 632 | 801 | 802 | 641 | 804 | 643 | 644 | 741 | 742 | 738 | 707 | 703 | 704 | 729 | 730 | 713 | 732 | 715 | 716 | 22 | 27455 |
| 1425 | 566 | 883 | 887 | 558 | 562 | 879 | 578 | 871 | 870 | 575 | 574 | 867 | 638 | 811 | 815 | 630 | 634 | 807 | 650 | 799 | 798 | 647 | 646 | 795 | 710 | 739 | 743 | 702 | 706 | 735 | 722 | 727 | 726 | 719 | 718 | 723 | 20 | 27455 |
| 1426 | 884 | 565 | 564 | 881 | 880 | 561 | 872 | 577 | 576 | 869 | 868 | 573 | 812 | 637 | 636 | 809 | 808 | 633 | 800 | 649 | 648 | 797 | 796 | 645 | 740 | 709 | 708 | 737 | 736 | 705 | 728 | 721 | 720 | 725 | 724 | 717 | 19 | 27455 |
| 37 | 73 | 71 | 69 | 67 | 65 | 63 | 61 | 59 | 57 | 17 | 15 | 13 | 11 | 9 | 7 | 5 | 3 | 1371 | 1373 | 1375 | 1377 | 1379 | 1381 | 1383 | 1385 | 1387 | 1389 | 1427 | 1429 | 1431 | 1433 | 1435 | 1437 | 1439 | 1441 | 1443 | 38 | 27455 |
| 7 | 27455 | 27455 | 27455 | 27455 | 27455 | 27455 | 27455 | 27455 | 27455 | 27455 | 27455 | 27455 | 27455 | 27455 | 27455 | 27455 | 27455 | 27455 | 27455 | 27455 | 27455 | 27455 | 27455 | 27455 | 27455 | 27455 | 27455 | 27455 | 27455 | 27455 | 27455 | 27455 | 27455 | 27455 | 27455 | 27455 | 27455 | 27455 |

*In this case the magic sums are* $S_{38\times 38} := 27455$ *and* $S_{36\times 36} := 26010$*. The magic rectangles of order* $4\times 6$ *are of* ***equal magic sums*** *given by* $R_{4\times 6} := (4335, 2890)$*.*

Magic Rectangles in Construction of Magic and Block-Bordered Magic Squares, June 07, 2022, pp. 1-70, https://doi.org/10.5281/zenodo.6621071

## 2.17 Magic Square of Order $40 \times 40$

**Example 2.28.** *A magic square of order* $40 \times 40$ *constructed based on blocks of magic rectangles of order* $2 \times 4$ *is given by*

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 32020 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 12 | 1590 | 1591 | 9 | 8 | 1594 | 1595 | 5 | 44 | 1558 | 1559 | 41 | 40 | 1562 | 1563 | 37 | 76 | 1526 | 1527 | 73 | 72 | 1530 | 1531 | 69 | 108 | 1494 | 1495 | 105 | 104 | 1498 | 1499 | 101 | 140 | 1462 | 1463 | 137 | 136 | 1466 | 1467 | 133 | 32020 |
| 1589 | 11 | 10 | 1592 | 1593 | 7 | 6 | 1596 | 1557 | 43 | 42 | 1560 | 1561 | 39 | 38 | 1564 | 1525 | 75 | 74 | 1528 | 1529 | 71 | 70 | 1532 | 1493 | 107 | 106 | 1496 | 1497 | 103 | 102 | 1500 | 1461 | 139 | 138 | 1464 | 1465 | 135 | 134 | 1468 | 32020 |
| 1 | 1599 | 1598 | 4 | 13 | 1587 | 1586 | 16 | 33 | 1567 | 1566 | 36 | 45 | 1555 | 1554 | 48 | 65 | 1535 | 1534 | 68 | 77 | 1523 | 1522 | 80 | 97 | 1503 | 1502 | 100 | 109 | 1491 | 1490 | 112 | 129 | 1471 | 1470 | 132 | 141 | 1459 | 1458 | 144 | 32020 |
| 1600 | 2 | 3 | 1597 | 1588 | 14 | 15 | 1585 | 1568 | 34 | 35 | 1565 | 1556 | 46 | 47 | 1553 | 1536 | 66 | 67 | 1533 | 1524 | 78 | 79 | 1521 | 1504 | 98 | 99 | 1501 | 1492 | 110 | 111 | 1489 | 1472 | 130 | 131 | 1469 | 1460 | 142 | 143 | 1457 | 32020 |
| 25 | 1575 | 1574 | 28 | 24 | 1578 | 1579 | 21 | 57 | 1543 | 1542 | 60 | 56 | 1546 | 1547 | 53 | 89 | 1511 | 1510 | 92 | 88 | 1514 | 1515 | 85 | 121 | 1479 | 1478 | 124 | 120 | 1482 | 1483 | 117 | 153 | 1447 | 1446 | 156 | 152 | 1450 | 1451 | 149 | 32020 |
| 1576 | 26 | 27 | 1573 | 1577 | 23 | 22 | 1580 | 1544 | 58 | 59 | 1541 | 1545 | 55 | 54 | 1548 | 1512 | 90 | 91 | 1509 | 1513 | 87 | 86 | 1516 | 1480 | 122 | 123 | 1477 | 1481 | 119 | 118 | 1484 | 1448 | 154 | 155 | 1445 | 1449 | 151 | 150 | 1452 | 32020 |
| 20 | 1582 | 1583 | 17 | 29 | 1571 | 1570 | 32 | 52 | 1550 | 1551 | 49 | 61 | 1539 | 1538 | 64 | 84 | 1518 | 1519 | 81 | 93 | 1507 | 1506 | 96 | 116 | 1486 | 1487 | 113 | 125 | 1475 | 1474 | 128 | 148 | 1454 | 1455 | 145 | 157 | 1443 | 1442 | 160 | 32020 |
| 1581 | 19 | 18 | 1584 | 1572 | 30 | 31 | 1569 | 1549 | 51 | 50 | 1552 | 1540 | 62 | 63 | 1537 | 1517 | 83 | 82 | 1520 | 1508 | 94 | 95 | 1505 | 1485 | 115 | 114 | 1488 | 1476 | 126 | 127 | 1473 | 1453 | 147 | 146 | 1456 | 1444 | 158 | 159 | 1441 | 32020 |
| 12 | 1590 | 1591 | 9 | 8 | 1594 | 1595 | 5 | 44 | 1558 | 1559 | 41 | 40 | 1562 | 1563 | 37 | 76 | 1526 | 1527 | 73 | 72 | 1530 | 1531 | 69 | 108 | 1494 | 1495 | 105 | 104 | 1498 | 1499 | 101 | 140 | 1462 | 1463 | 137 | 136 | 1466 | 1467 | 133 | 32020 |
| 1589 | 11 | 10 | 1592 | 1593 | 7 | 6 | 1596 | 1557 | 43 | 42 | 1560 | 1561 | 39 | 38 | 1564 | 1525 | 75 | 74 | 1528 | 1529 | 71 | 70 | 1532 | 1493 | 107 | 106 | 1496 | 1497 | 103 | 102 | 1500 | 1461 | 139 | 138 | 1464 | 1465 | 135 | 134 | 1468 | 32020 |
| 1 | 1599 | 1598 | 4 | 13 | 1587 | 1586 | 16 | 33 | 1567 | 1566 | 36 | 45 | 1555 | 1554 | 48 | 65 | 1535 | 1534 | 68 | 77 | 1523 | 1522 | 80 | 97 | 1503 | 1502 | 100 | 109 | 1491 | 1490 | 112 | 129 | 1471 | 1470 | 132 | 141 | 1459 | 1458 | 144 | 32020 |
| 1600 | 2 | 3 | 1597 | 1588 | 14 | 15 | 1585 | 1568 | 34 | 35 | 1565 | 1556 | 46 | 47 | 1553 | 1536 | 66 | 67 | 1533 | 1524 | 78 | 79 | 1521 | 1504 | 98 | 99 | 1501 | 1492 | 110 | 111 | 1489 | 1472 | 130 | 131 | 1469 | 1460 | 142 | 143 | 1457 | 32020 |
| 25 | 1575 | 1574 | 28 | 24 | 1578 | 1579 | 21 | 57 | 1543 | 1542 | 60 | 56 | 1546 | 1547 | 53 | 89 | 1511 | 1510 | 92 | 88 | 1514 | 1515 | 85 | 121 | 1479 | 1478 | 124 | 120 | 1482 | 1483 | 117 | 153 | 1447 | 1446 | 156 | 152 | 1450 | 1451 | 149 | 32020 |
| 1576 | 26 | 27 | 1573 | 1577 | 23 | 22 | 1580 | 1544 | 58 | 59 | 1541 | 1545 | 55 | 54 | 1548 | 1512 | 90 | 91 | 1509 | 1513 | 87 | 86 | 1516 | 1480 | 122 | 123 | 1477 | 1481 | 119 | 118 | 1484 | 1448 | 154 | 155 | 1445 | 1449 | 151 | 150 | 1452 | 32020 |
| 20 | 1582 | 1583 | 17 | 29 | 1571 | 1570 | 32 | 52 | 1550 | 1551 | 49 | 61 | 1539 | 1538 | 64 | 84 | 1518 | 1519 | 81 | 93 | 1507 | 1506 | 96 | 116 | 1486 | 1487 | 113 | 125 | 1475 | 1474 | 128 | 148 | 1454 | 1455 | 145 | 157 | 1443 | 1442 | 160 | 32020 |
| 1581 | 19 | 18 | 1584 | 1572 | 30 | 31 | 1569 | 1549 | 51 | 50 | 1552 | 1540 | 62 | 63 | 1537 | 1517 | 83 | 82 | 1520 | 1508 | 94 | 95 | 1505 | 1485 | 115 | 114 | 1488 | 1476 | 126 | 127 | 1473 | 1453 | 147 | 146 | 1456 | 1444 | 158 | 159 | 1441 | 32020 |
| 172 | 1430 | 1431 | 169 | 168 | 1434 | 1435 | 165 | 204 | 1398 | 1399 | 201 | 200 | 1402 | 1403 | 197 | 236 | 1366 | 1367 | 233 | 232 | 1370 | 1371 | 229 | 268 | 1334 | 1335 | 265 | 264 | 1338 | 1339 | 261 | 300 | 1302 | 1303 | 297 | 296 | 1306 | 1307 | 293 | 32020 |
| 1429 | 171 | 170 | 1432 | 1433 | 167 | 166 | 1436 | 1397 | 203 | 202 | 1400 | 1401 | 199 | 198 | 1404 | 1365 | 235 | 234 | 1368 | 1369 | 231 | 230 | 1372 | 1333 | 267 | 266 | 1336 | 1337 | 263 | 262 | 1340 | 1301 | 299 | 298 | 1304 | 1305 | 295 | 294 | 1308 | 32020 |
| 161 | 1439 | 1438 | 164 | 173 | 1427 | 1426 | 176 | 193 | 1407 | 1406 | 196 | 205 | 1395 | 1394 | 208 | 225 | 1375 | 1374 | 228 | 237 | 1363 | 1362 | 240 | 257 | 1343 | 1342 | 260 | 269 | 1331 | 1330 | 272 | 289 | 1311 | 1310 | 292 | 301 | 1299 | 1298 | 304 | 32020 |
| 1440 | 162 | 163 | 1437 | 1428 | 174 | 175 | 1425 | 1408 | 194 | 195 | 1405 | 1396 | 206 | 207 | 1393 | 1376 | 226 | 227 | 1373 | 1364 | 238 | 239 | 1361 | 1344 | 258 | 259 | 1341 | 1332 | 270 | 271 | 1329 | 1312 | 290 | 291 | 1309 | 1300 | 302 | 303 | 1297 | 32020 |
| 185 | 1415 | 1414 | 188 | 184 | 1418 | 1419 | 181 | 217 | 1383 | 1382 | 220 | 216 | 1386 | 1387 | 213 | 249 | 1351 | 1350 | 252 | 248 | 1354 | 1355 | 245 | 281 | 1319 | 1318 | 284 | 280 | 1322 | 1323 | 277 | 313 | 1287 | 1286 | 316 | 312 | 1290 | 1291 | 309 | 32020 |
| 1416 | 186 | 187 | 1413 | 1417 | 183 | 182 | 1420 | 1384 | 218 | 219 | 1381 | 1385 | 215 | 214 | 1388 | 1352 | 250 | 251 | 1349 | 1353 | 247 | 246 | 1356 | 1320 | 282 | 283 | 1317 | 1321 | 279 | 278 | 1324 | 1288 | 314 | 315 | 1285 | 1289 | 311 | 310 | 1292 | 32020 |
| 180 | 1422 | 1423 | 177 | 189 | 1411 | 1410 | 192 | 212 | 1390 | 1391 | 209 | 221 | 1379 | 1378 | 224 | 244 | 1358 | 1359 | 241 | 253 | 1347 | 1346 | 256 | 276 | 1326 | 1327 | 273 | 285 | 1315 | 1314 | 288 | 308 | 1294 | 1295 | 305 | 317 | 1283 | 1282 | 320 | 32020 |
| 1421 | 179 | 178 | 1424 | 1412 | 190 | 191 | 1409 | 1389 | 211 | 210 | 1392 | 1380 | 222 | 223 | 1377 | 1357 | 243 | 242 | 1360 | 1348 | 254 | 255 | 1345 | 1325 | 275 | 274 | 1328 | 1316 | 286 | 287 | 1313 | 1293 | 307 | 306 | 1296 | 1284 | 318 | 319 | 1281 | 32020 |
| 332 | 1270 | 1271 | 329 | 328 | 1274 | 1275 | 325 | 364 | 1238 | 1239 | 361 | 360 | 1242 | 1243 | 357 | 396 | 1206 | 1207 | 393 | 392 | 1210 | 1211 | 389 | 428 | 1174 | 1175 | 425 | 424 | 1178 | 1179 | 421 | 460 | 1142 | 1143 | 457 | 456 | 1146 | 1147 | 453 | 32020 |
| 1269 | 331 | 330 | 1272 | 1273 | 327 | 326 | 1276 | 1237 | 363 | 362 | 1240 | 1241 | 359 | 358 | 1244 | 1205 | 395 | 394 | 1208 | 1209 | 391 | 390 | 1212 | 1173 | 427 | 426 | 1176 | 1177 | 423 | 422 | 1180 | 1141 | 459 | 458 | 1144 | 1145 | 455 | 454 | 1148 | 32020 |
| 321 | 1279 | 1278 | 324 | 333 | 1267 | 1266 | 336 | 353 | 1247 | 1246 | 356 | 365 | 1235 | 1234 | 368 | 385 | 1215 | 1214 | 388 | 397 | 1203 | 1202 | 400 | 417 | 1183 | 1182 | 420 | 429 | 1171 | 1170 | 432 | 449 | 1151 | 1150 | 452 | 461 | 1139 | 1138 | 464 | 32020 |
| 1280 | 322 | 323 | 1277 | 1268 | 334 | 335 | 1265 | 1248 | 354 | 355 | 1245 | 1236 | 366 | 367 | 1233 | 1216 | 386 | 387 | 1213 | 1204 | 398 | 399 | 1201 | 1184 | 418 | 419 | 1181 | 1172 | 430 | 431 | 1169 | 1152 | 450 | 451 | 1149 | 1140 | 462 | 463 | 1137 | 32020 |
| 345 | 1255 | 1254 | 348 | 344 | 1258 | 1259 | 341 | 377 | 1223 | 1222 | 380 | 376 | 1226 | 1227 | 373 | 409 | 1191 | 1190 | 412 | 408 | 1194 | 1195 | 405 | 441 | 1159 | 1158 | 444 | 440 | 1162 | 1163 | 437 | 473 | 1127 | 1126 | 476 | 472 | 1130 | 1131 | 469 | 32020 |
| 1256 | 346 | 347 | 1253 | 1257 | 343 | 342 | 1260 | 1224 | 378 | 379 | 1221 | 1225 | 375 | 374 | 1228 | 1192 | 410 | 411 | 1189 | 1193 | 407 | 406 | 1196 | 1160 | 442 | 443 | 1157 | 1161 | 439 | 438 | 1164 | 1128 | 474 | 475 | 1125 | 1129 | 471 | 470 | 1132 | 32020 |
| 340 | 1262 | 1263 | 337 | 349 | 1251 | 1250 | 352 | 372 | 1230 | 1231 | 369 | 381 | 1219 | 1218 | 384 | 404 | 1198 | 1199 | 401 | 413 | 1187 | 1186 | 416 | 436 | 1166 | 1167 | 433 | 445 | 1155 | 1154 | 448 | 468 | 1134 | 1135 | 465 | 477 | 1123 | 1122 | 480 | 32020 |
| 1261 | 339 | 338 | 1264 | 1252 | 350 | 351 | 1249 | 1229 | 371 | 370 | 1232 | 1220 | 382 | 383 | 1217 | 1197 | 403 | 402 | 1200 | 1188 | 414 | 415 | 1185 | 1165 | 435 | 434 | 1168 | 1156 | 446 | 447 | 1153 | 1133 | 467 | 466 | 1136 | 1124 | 478 | 479 | 1121 | 32020 |
| 492 | 1110 | 1111 | 489 | 488 | 1114 | 1115 | 485 | 524 | 1078 | 1079 | 521 | 520 | 1082 | 1083 | 517 | 556 | 1046 | 1047 | 553 | 552 | 1050 | 1051 | 549 | 588 | 1014 | 1015 | 585 | 584 | 1018 | 1019 | 581 | 620 | 982 | 983 | 617 | 616 | 986 | 987 | 613 | 32020 |
| 1109 | 491 | 490 | 1112 | 1113 | 487 | 486 | 1116 | 1077 | 523 | 522 | 1080 | 1081 | 519 | 518 | 1084 | 1045 | 555 | 554 | 1048 | 1049 | 551 | 550 | 1052 | 1013 | 587 | 586 | 1016 | 1017 | 583 | 582 | 1020 | 981 | 619 | 618 | 984 | 985 | 615 | 614 | 988 | 32020 |
| 481 | 1119 | 1118 | 484 | 493 | 1107 | 1106 | 496 | 513 | 1087 | 1086 | 516 | 525 | 1075 | 1074 | 528 | 545 | 1055 | 1054 | 548 | 557 | 1043 | 1042 | 560 | 577 | 1023 | 1022 | 580 | 589 | 1011 | 1010 | 592 | 609 | 991 | 990 | 612 | 621 | 979 | 978 | 624 | 32020 |
| 1120 | 482 | 483 | 1117 | 1108 | 494 | 495 | 1105 | 1088 | 514 | 515 | 1085 | 1076 | 526 | 527 | 1073 | 1056 | 546 | 547 | 1053 | 1044 | 558 | 559 | 1041 | 1024 | 578 | 579 | 1021 | 1012 | 590 | 591 | 1009 | 992 | 610 | 611 | 989 | 980 | 622 | 623 | 977 | 32020 |
| 505 | 1095 | 1094 | 508 | 504 | 1098 | 1099 | 501 | 537 | 1063 | 1062 | 540 | 536 | 1066 | 1067 | 533 | 569 | 1031 | 1030 | 572 | 568 | 1034 | 1035 | 565 | 601 | 999 | 998 | 604 | 600 | 1002 | 1003 | 597 | 633 | 967 | 966 | 636 | 632 | 970 | 971 | 629 | 32020 |
| 1096 | 506 | 507 | 1093 | 1097 | 503 | 502 | 1100 | 1064 | 538 | 539 | 1061 | 1065 | 535 | 534 | 1068 | 1032 | 570 | 571 | 1029 | 1033 | 567 | 566 | 1036 | 1000 | 602 | 603 | 997 | 1001 | 599 | 598 | 1004 | 968 | 634 | 635 | 965 | 969 | 631 | 630 | 972 | 32020 |
| 500 | 1102 | 1103 | 497 | 509 | 1091 | 1090 | 512 | 532 | 1070 | 1071 | 529 | 541 | 1059 | 1058 | 544 | 564 | 1038 | 1039 | 561 | 573 | 1027 | 1026 | 576 | 596 | 1006 | 1007 | 593 | 605 | 995 | 994 | 608 | 628 | 974 | 975 | 625 | 637 | 963 | 962 | 640 | 32020 |
| 1101 | 499 | 498 | 1104 | 1092 | 510 | 511 | 1089 | 1069 | 531 | 530 | 1072 | 1060 | 542 | 543 | 1057 | 1037 | 563 | 562 | 1040 | 1028 | 574 | 575 | 1025 | 1005 | 595 | 594 | 1008 | 996 | 606 | 607 | 993 | 973 | 627 | 626 | 976 | 964 | 638 | 639 | 961 | 32020 |
| 32020 | 32020 | 32020 | 32020 | 32020 | 32020 | 32020 | 32020 | 32020 | 32020 | 32020 | 32020 | 32020 | 32020 | 32020 | 32020 | 32020 | 32020 | 32020 | 32020 | 32020 | 32020 | 32020 | 32020 | 32020 | 32020 | 32020 | 32020 | 32020 | 32020 | 32020 | 32020 | 32020 | 32020 | 32020 | 32020 | 32020 | 32020 | 32020 | 32020 | 32020 |

*The **magic square** sum is* $S_{40\times 40} := 32020$. *The magic rectangles of order* $2 \times 4$ *are of **equal magic sums** given by*

$R_{2\times 4} := (3202, 1601)$. *Moreover, each block of order* $8 \times 8$ *is a magic square with magic sums* $S_{8\times 8} := 6404$.

**Remark 2.3.** *There are much more examples of magic squares based on magic triangles* $2 \times 4$. *These examples are of orders 20, 28, 44, etc. These shall be given later in another work. The examples of order 16, 32 and 40 are based on order 8 given in Example 2.1. The example of order 36 is calculated based on order 12 given in Example 2.3.*

## 2.18 Magic Square of Order $42 \times 42$

**Example 2.29.** *A magic square of order* $42 \times 42$ *constructed based on blocks of magic rectangles of order* $2 \times 14$ *is given by*

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 55293 |
| 2241 | 2239 | 2229 | 2251 | 2231 | 2249 | 2233 | 2247 | 2235 | 2245 | 2244 | 2243 | 2253 | 2227 | 1 | 27 | 3 | 25 | 5 | 23 | 7 | 21 | 9 | 19 | 18 | 17 | 13 | 15 | 1695 | 1693 | 1683 | 1705 | 1685 | 1703 | 1687 | 1701 | 1689 | 1699 | 1698 | 1697 | 1681 | 1707 | 55293 |
| 2240 | 2242 | 2252 | 2230 | 2250 | 2232 | 2248 | 2234 | 2246 | 2236 | 2237 | 2238 | 2228 | 2254 | 28 | 2 | 26 | 4 | 24 | 6 | 22 | 8 | 20 | 10 | 11 | 12 | 16 | 14 | 1694 | 1696 | 1706 | 1684 | 1704 | 1686 | 1702 | 1688 | 1700 | 1690 | 1691 | 1692 | 1708 | 1682 | 55293 |
| 43 | 69 | 57 | 55 | 47 | 65 | 49 | 63 | 51 | 61 | 60 | 59 | 67 | 45 | 1597 | 1623 | 1599 | 1621 | 1601 | 1619 | 1603 | 1617 | 1605 | 1615 | 1614 | 1613 | 1609 | 1611 | 2285 | 2286 | 2271 | 2293 | 2273 | 2291 | 2275 | 2289 | 2277 | 2287 | 2295 | 2269 | 2281 | 2283 | 55293 |
| 70 | 44 | 56 | 58 | 66 | 48 | 64 | 50 | 62 | 52 | 53 | 54 | 46 | 68 | 1624 | 1598 | 1622 | 1600 | 1620 | 1602 | 1618 | 1604 | 1616 | 1606 | 1607 | 1608 | 1612 | 1610 | 2280 | 2279 | 2294 | 2272 | 2292 | 2274 | 2290 | 2276 | 2288 | 2278 | 2270 | 2296 | 2284 | 2282 | 55293 |
| 1513 | 1539 | 1515 | 1537 | 1527 | 1525 | 1519 | 1533 | 1521 | 1531 | 1530 | 1529 | 1535 | 1517 | 2311 | 2337 | 2313 | 2335 | 2315 | 2333 | 2317 | 2331 | 2319 | 2329 | 2328 | 2327 | 2323 | 2325 | 103 | 93 | 87 | 109 | 89 | 107 | 91 | 105 | 111 | 85 | 102 | 101 | 97 | 99 | 55293 |
| 1540 | 1514 | 1538 | 1516 | 1526 | 1528 | 1534 | 1520 | 1532 | 1522 | 1523 | 1524 | 1518 | 1536 | 2338 | 2312 | 2336 | 2314 | 2334 | 2316 | 2332 | 2318 | 2330 | 2320 | 2321 | 2322 | 2326 | 2324 | 94 | 104 | 110 | 88 | 108 | 90 | 106 | 92 | 86 | 112 | 95 | 96 | 100 | 98 | 55293 |
| 1429 | 1455 | 1431 | 1453 | 1433 | 1451 | 1443 | 1441 | 1437 | 1447 | 1446 | 1445 | 1449 | 1435 | 2353 | 2379 | 2355 | 2377 | 2357 | 2375 | 2359 | 2373 | 2361 | 2371 | 2370 | 2369 | 2365 | 2367 | 127 | 153 | 129 | 151 | 131 | 149 | 141 | 139 | 135 | 145 | 144 | 143 | 147 | 133 | 55293 |
| 1456 | 1430 | 1454 | 1432 | 1452 | 1434 | 1442 | 1444 | 1448 | 1438 | 1439 | 1440 | 1436 | 1450 | 2380 | 2354 | 2378 | 2356 | 2376 | 2358 | 2374 | 2360 | 2372 | 2362 | 2363 | 2364 | 2368 | 2366 | 154 | 128 | 152 | 130 | 150 | 132 | 140 | 142 | 146 | 136 | 137 | 138 | 134 | 148 | 55293 |
| 169 | 195 | 171 | 193 | 173 | 191 | 175 | 189 | 177 | 187 | 186 | 185 | 181 | 183 | 2395 | 2421 | 2397 | 2419 | 2399 | 2417 | 2401 | 2415 | 2403 | 2413 | 2412 | 2411 | 2407 | 2409 | 1345 | 1371 | 1347 | 1369 | 1359 | 1357 | 1351 | 1365 | 1353 | 1363 | 1362 | 1361 | 1367 | 1349 | 55293 |
| 196 | 170 | 194 | 172 | 192 | 174 | 190 | 176 | 188 | 178 | 179 | 180 | 184 | 182 | 2422 | 2396 | 2420 | 2398 | 2418 | 2400 | 2416 | 2402 | 2414 | 2404 | 2405 | 2406 | 2410 | 2408 | 1372 | 1346 | 1370 | 1348 | 1358 | 1360 | 1366 | 1352 | 1364 | 1354 | 1355 | 1356 | 1350 | 1368 | 55293 |
| 421 | 447 | 423 | 445 | 425 | 443 | 427 | 441 | 429 | 439 | 437 | 438 | 433 | 435 | 1765 | 1791 | 1767 | 1789 | 1769 | 1787 | 1771 | 1785 | 1773 | 1783 | 1782 | 1781 | 1777 | 1779 | 1723 | 1749 | 1737 | 1735 | 1727 | 1745 | 1729 | 1743 | 1731 | 1741 | 1740 | 1739 | 1747 | 1725 | 55293 |
| 448 | 422 | 446 | 424 | 444 | 426 | 442 | 428 | 440 | 430 | 432 | 431 | 436 | 434 | 1792 | 1766 | 1790 | 1768 | 1788 | 1770 | 1786 | 1772 | 1784 | 1774 | 1775 | 1776 | 1780 | 1778 | 1750 | 1724 | 1736 | 1738 | 1746 | 1728 | 1744 | 1730 | 1742 | 1732 | 1733 | 1734 | 1726 | 1748 | 55293 |
| 1639 | 1665 | 1641 | 1663 | 1643 | 1661 | 1645 | 1659 | 1647 | 1657 | 1656 | 1655 | 1651 | 1653 | 1807 | 1833 | 1809 | 1831 | 1811 | 1829 | 1813 | 1827 | 1815 | 1825 | 1824 | 1823 | 1819 | 1821 | 477 | 475 | 465 | 487 | 467 | 485 | 469 | 483 | 471 | 481 | 480 | 479 | 489 | 463 | 55293 |
| 1666 | 1640 | 1664 | 1642 | 1662 | 1644 | 1660 | 1646 | 1658 | 1648 | 1649 | 1650 | 1654 | 1652 | 1834 | 1808 | 1832 | 1810 | 1830 | 1812 | 1828 | 1814 | 1826 | 1816 | 1817 | 1818 | 1822 | 1820 | 476 | 478 | 488 | 466 | 486 | 468 | 484 | 470 | 482 | 472 | 473 | 474 | 464 | 490 | 55293 |
| 1555 | 1581 | 1557 | 1579 | 1559 | 1577 | 1561 | 1575 | 1563 | 1573 | 1572 | 1571 | 1567 | 1569 | 1849 | 1875 | 1851 | 1873 | 1853 | 1871 | 1855 | 1869 | 1857 | 1867 | 1866 | 1863 | 1861 | 1865 | 505 | 531 | 507 | 529 | 509 | 527 | 511 | 525 | 513 | 523 | 522 | 521 | 517 | 519 | 55293 |
| 1582 | 1556 | 1580 | 1558 | 1578 | 1560 | 1576 | 1562 | 1574 | 1564 | 1565 | 1566 | 1570 | 1568 | 1876 | 1850 | 1874 | 1852 | 1872 | 1854 | 1870 | 1856 | 1868 | 1858 | 1859 | 1862 | 1864 | 1860 | 532 | 506 | 530 | 508 | 528 | 510 | 526 | 512 | 524 | 514 | 515 | 516 | 520 | 518 | 55293 |
| 547 | 573 | 549 | 571 | 551 | 569 | 553 | 567 | 555 | 565 | 564 | 563 | 559 | 561 | 1915 | 1893 | 1917 | 1891 | 1895 | 1913 | 1897 | 1911 | 1899 | 1909 | 1908 | 1907 | 1903 | 1905 | 1471 | 1497 | 1473 | 1495 | 1475 | 1493 | 1477 | 1491 | 1479 | 1489 | 1488 | 1487 | 1483 | 1485 | 55293 |
| 574 | 548 | 572 | 550 | 570 | 552 | 568 | 554 | 566 | 556 | 557 | 558 | 562 | 560 | 1894 | 1916 | 1892 | 1918 | 1914 | 1896 | 1912 | 1898 | 1910 | 1900 | 1901 | 1902 | 1906 | 1904 | 1498 | 1472 | 1496 | 1474 | 1494 | 1476 | 1492 | 1478 | 1490 | 1480 | 1481 | 1482 | 1486 | 1484 | 55293 |
| 1387 | 1413 | 1389 | 1411 | 1391 | 1409 | 1393 | 1407 | 1395 | 1405 | 1404 | 1403 | 1399 | 1401 | 1937 | 1959 | 1935 | 1957 | 1947 | 1955 | 1939 | 1953 | 1949 | 1933 | 1950 | 1941 | 1945 | 1951 | 589 | 615 | 591 | 613 | 593 | 611 | 595 | 609 | 597 | 607 | 606 | 605 | 601 | 603 | 55293 |
| 1414 | 1388 | 1412 | 1390 | 1410 | 1392 | 1408 | 1394 | 1406 | 1396 | 1397 | 1398 | 1402 | 1400 | 1956 | 1934 | 1958 | 1936 | 1946 | 1938 | 1954 | 1940 | 1944 | 1960 | 1943 | 1952 | 1948 | 1942 | 616 | 590 | 614 | 592 | 612 | 594 | 610 | 596 | 608 | 598 | 599 | 600 | 604 | 602 | 55293 |
| 631 | 657 | 633 | 655 | 635 | 653 | 637 | 651 | 639 | 649 | 648 | 647 | 643 | 645 | 1303 | 1329 | 1305 | 1327 | 1307 | 1325 | 1309 | 1323 | 1311 | 1321 | 1320 | 1319 | 1315 | 1317 | 1975 | 2001 | 1977 | 1999 | 1979 | 1997 | 1981 | 1995 | 1983 | 1993 | 1992 | 1991 | 1987 | 1989 | 55293 |
| 658 | 632 | 656 | 634 | 654 | 636 | 652 | 638 | 650 | 640 | 641 | 642 | 646 | 644 | 1330 | 1304 | 1328 | 1306 | 1326 | 1308 | 1324 | 1310 | 1322 | 1312 | 1313 | 1314 | 1318 | 1316 | 2002 | 1976 | 2000 | 1978 | 1998 | 1980 | 1996 | 1982 | 1994 | 1984 | 1985 | 1986 | 1990 | 1988 | 55293 |
| 2017 | 2043 | 2019 | 2041 | 2021 | 2039 | 2023 | 2037 | 2025 | 2035 | 2034 | 2033 | 2029 | 2031 | 673 | 699 | 675 | 697 | 687 | 685 | 679 | 693 | 681 | 691 | 690 | 689 | 695 | 677 | 1219 | 1245 | 1221 | 1243 | 1223 | 1241 | 1225 | 1239 | 1227 | 1237 | 1236 | 1235 | 1231 | 1233 | 55293 |
| 2044 | 2018 | 2042 | 2020 | 2040 | 2022 | 2038 | 2024 | 2036 | 2026 | 2027 | 2028 | 2032 | 2030 | 700 | 674 | 698 | 676 | 686 | 688 | 694 | 680 | 692 | 682 | 683 | 684 | 678 | 696 | 1246 | 1220 | 1244 | 1222 | 1242 | 1224 | 1240 | 1226 | 1238 | 1228 | 1229 | 1230 | 1234 | 1232 | 55293 |
| 1135 | 1161 | 1137 | 1159 | 1139 | 1157 | 1141 | 1155 | 1143 | 1153 | 1152 | 1151 | 1147 | 1149 | 715 | 741 | 717 | 739 | 719 | 737 | 721 | 735 | 723 | 733 | 732 | 731 | 727 | 729 | 2059 | 2085 | 2061 | 2083 | 2063 | 2081 | 2065 | 2079 | 2067 | 2077 | 2076 | 2075 | 2071 | 2073 | 55293 |
| 1162 | 1136 | 1160 | 1138 | 1158 | 1140 | 1156 | 1142 | 1154 | 1144 | 1145 | 1146 | 1150 | 1148 | 742 | 716 | 740 | 718 | 738 | 720 | 736 | 722 | 734 | 724 | 725 | 726 | 730 | 728 | 2086 | 2060 | 2084 | 2062 | 2082 | 2064 | 2080 | 2066 | 2078 | 2068 | 2069 | 2070 | 2074 | 2072 | 55293 |
| 2101 | 2127 | 2103 | 2125 | 2105 | 2123 | 2107 | 2121 | 2109 | 2119 | 2118 | 2117 | 2113 | 2115 | 773 | 783 | 759 | 781 | 761 | 779 | 763 | 777 | 765 | 775 | 774 | 757 | 769 | 771 | 1051 | 1077 | 1053 | 1075 | 1055 | 1073 | 1057 | 1071 | 1059 | 1069 | 1068 | 1067 | 1063 | 1065 | 55293 |
| 2128 | 2102 | 2126 | 2104 | 2124 | 2106 | 2122 | 2108 | 2120 | 2110 | 2111 | 2112 | 2116 | 2114 | 768 | 758 | 782 | 760 | 780 | 762 | 778 | 764 | 776 | 766 | 767 | 784 | 772 | 770 | 1078 | 1052 | 1076 | 1054 | 1074 | 1056 | 1072 | 1058 | 1070 | 1060 | 1061 | 1062 | 1066 | 1064 | 55293 |
| 2143 | 2169 | 2145 | 2167 | 2147 | 2165 | 2149 | 2163 | 2151 | 2161 | 2160 | 2159 | 2155 | 2157 | 799 | 825 | 801 | 823 | 803 | 821 | 805 | 819 | 807 | 817 | 816 | 815 | 811 | 813 | 967 | 993 | 969 | 991 | 971 | 989 | 973 | 987 | 975 | 985 | 984 | 983 | 979 | 981 | 55293 |
| 2170 | 2144 | 2168 | 2146 | 2166 | 2148 | 2164 | 2150 | 2162 | 2152 | 2153 | 2154 | 2158 | 2156 | 826 | 800 | 824 | 802 | 822 | 804 | 820 | 806 | 818 | 808 | 809 | 810 | 814 | 812 | 994 | 968 | 992 | 970 | 990 | 972 | 988 | 974 | 986 | 976 | 977 | 978 | 982 | 980 | 55293 |
| 883 | 909 | 885 | 907 | 887 | 905 | 889 | 903 | 891 | 901 | 900 | 899 | 895 | 897 | 841 | 867 | 843 | 865 | 845 | 863 | 847 | 861 | 849 | 859 | 858 | 857 | 853 | 855 | 2185 | 2211 | 2187 | 2209 | 2189 | 2207 | 2191 | 2205 | 2193 | 2203 | 2202 | 2201 | 2197 | 2199 | 55293 |
| 910 | 884 | 908 | 886 | 906 | 888 | 904 | 890 | 902 | 892 | 893 | 894 | 898 | 896 | 868 | 842 | 866 | 844 | 864 | 846 | 862 | 848 | 860 | 850 | 851 | 852 | 856 | 854 | 2212 | 2186 | 2210 | 2188 | 2208 | 2190 | 2206 | 2192 | 2204 | 2194 | 2195 | 2196 | 2200 | 2198 | 55293 |
| 1261 | 1287 | 1263 | 1285 | 1265 | 1283 | 1267 | 1281 | 1269 | 1279 | 1278 | 1277 | 1273 | 1275 | 211 | 237 | 213 | 235 | 215 | 233 | 217 | 231 | 219 | 229 | 228 | 227 | 223 | 225 | 2437 | 2463 | 2439 | 2461 | 2441 | 2459 | 2443 | 2457 | 2445 | 2455 | 2454 | 2453 | 2449 | 2451 | 55293 |
| 1288 | 1262 | 1286 | 1264 | 1284 | 1266 | 1282 | 1268 | 1280 | 1270 | 1271 | 1272 | 1276 | 1274 | 238 | 212 | 236 | 214 | 234 | 216 | 232 | 218 | 230 | 220 | 221 | 222 | 226 | 224 | 2464 | 2438 | 2462 | 2440 | 2460 | 2442 | 2458 | 2444 | 2456 | 2446 | 2447 | 2448 | 2452 | 2450 | 55293 |
| 2479 | 2505 | 2481 | 2503 | 2483 | 2501 | 2493 | 2491 | 2487 | 2497 | 2496 | 2495 | 2499 | 2485 | 253 | 279 | 255 | 277 | 257 | 275 | 259 | 273 | 261 | 271 | 270 | 269 | 265 | 267 | 1197 | 1183 | 1179 | 1201 | 1181 | 1199 | 1203 | 1177 | 1185 | 1195 | 1194 | 1193 | 1189 | 1191 | 55293 |
| 2506 | 2480 | 2504 | 2482 | 2502 | 2484 | 2492 | 2494 | 2498 | 2488 | 2489 | 2490 | 2486 | 2500 | 280 | 254 | 278 | 256 | 276 | 258 | 274 | 260 | 272 | 262 | 263 | 264 | 268 | 266 | 1184 | 1198 | 1202 | 1180 | 1200 | 1182 | 1178 | 1204 | 1196 | 1186 | 1187 | 1188 | 1192 | 1190 | 55293 |
| 2521 | 2547 | 2523 | 2545 | 2535 | 2533 | 2527 | 2541 | 2529 | 2539 | 2538 | 2537 | 2543 | 2525 | 295 | 321 | 297 | 319 | 299 | 317 | 301 | 315 | 303 | 313 | 312 | 311 | 307 | 309 | 1111 | 1101 | 1095 | 1117 | 1097 | 1115 | 1099 | 1113 | 1119 | 1093 | 1110 | 1109 | 1105 | 1107 | 55293 |
| 2548 | 2522 | 2546 | 2524 | 2534 | 2536 | 2542 | 2528 | 2540 | 2530 | 2531 | 2532 | 2526 | 2544 | 322 | 296 | 320 | 298 | 318 | 300 | 316 | 302 | 314 | 304 | 305 | 306 | 310 | 308 | 1102 | 1112 | 1118 | 1096 | 1116 | 1098 | 1114 | 1100 | 1094 | 1120 | 1103 | 1104 | 1108 | 1106 | 55293 |
| 337 | 363 | 351 | 349 | 341 | 359 | 343 | 357 | 345 | 355 | 354 | 353 | 361 | 339 | 1009 | 1035 | 1011 | 1033 | 1013 | 1031 | 1015 | 1029 | 1017 | 1027 | 1026 | 1025 | 1021 | 1023 | 2579 | 2580 | 2565 | 2587 | 2567 | 2585 | 2569 | 2583 | 2571 | 2581 | 2589 | 2563 | 2575 | 2577 | 55293 |
| 364 | 338 | 350 | 352 | 360 | 342 | 358 | 344 | 356 | 346 | 347 | 348 | 340 | 362 | 1036 | 1010 | 1034 | 1012 | 1032 | 1014 | 1030 | 1016 | 1028 | 1018 | 1019 | 1020 | 1024 | 1022 | 2574 | 2573 | 2588 | 2566 | 2586 | 2568 | 2584 | 2570 | 2582 | 2572 | 2564 | 2590 | 2578 | 2576 | 55293 |
| 939 | 937 | 927 | 949 | 929 | 947 | 931 | 945 | 933 | 943 | 942 | 941 | 951 | 925 | 2605 | 2631 | 2607 | 2629 | 2609 | 2627 | 2611 | 2625 | 2613 | 2623 | 2622 | 2621 | 2617 | 2619 | 393 | 391 | 381 | 403 | 383 | 401 | 385 | 399 | 387 | 397 | 396 | 395 | 405 | 379 | 55293 |
| 938 | 940 | 950 | 928 | 948 | 930 | 946 | 932 | 944 | 934 | 935 | 936 | 926 | 952 | 2632 | 2606 | 2630 | 2608 | 2628 | 2610 | 2626 | 2612 | 2624 | 2614 | 2615 | 2616 | 2620 | 2618 | 392 | 394 | 404 | 382 | 402 | 384 | 400 | 386 | 398 | 388 | 389 | 390 | 380 | 406 | 55293 |
| 55293 | 55293 | 55293 | 55293 | 55293 | 55293 | 55293 | 55293 | 55293 | 55293 | 55293 | 55293 | 55293 | 55293 | 55293 | 55293 | 55293 | 55293 | 55293 | 55293 | 55293 | 55293 | 55293 | 55293 | 55293 | 55293 | 55293 | 55293 | 55293 | 55293 | 55293 | 55293 | 55293 | 55293 | 55293 | 55293 | 55293 | 55293 | 55293 | 55293 | 55293 | 55293 | 55293 |

*The **magic square sum** is $S_{42\times 42} := 37065$. The magic rectangles of order $2\times 14$ are of **different magic sums**.*

In the above Example, the magic rectangles of order $2\times 14$ are of **different magic sums**. Below is another example with equal sums magic rectangles of order $6\times 14$

**Example 2.30.** *A magic square of order $42\times 42$ constructed based on blocks of magic rectangles of order $6\times 14$ is given by*

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 37065 |
| 169 | 1591 | 1590 | 180 | 181 | 1579 | 1578 | 192 | 193 | 1571 | 1570 | 1569 | 194 | 198 | 43 | 1717 | 1716 | 54 | 55 | 1705 | 1704 | 66 | 67 | 1697 | 1696 | 1695 | 68 | 72 | 114 | 1675 | 1674 | 110 | 97 | 1663 | 1662 | 108 | 109 | 1655 | 1654 | 1653 | 96 | 85 | 37065 |
| 170 | 1592 | 1589 | 179 | 182 | 1580 | 1577 | 191 | 1566 | 200 | 1564 | 201 | 203 | 1561 | 44 | 1718 | 1715 | 53 | 56 | 1706 | 1703 | 65 | 1692 | 74 | 1690 | 75 | 77 | 1687 | 1645 | 1676 | 1673 | 119 | 98 | 1664 | 1661 | 107 | 1650 | 116 | 1648 | 117 | 95 | 86 | 37065 |
| 171 | 1593 | 1588 | 178 | 183 | 1581 | 1576 | 190 | 1560 | 1559 | 207 | 208 | 1556 | 205 | 45 | 1719 | 1714 | 52 | 57 | 1707 | 1702 | 64 | 1686 | 1685 | 81 | 82 | 1682 | 79 | 121 | 1677 | 1672 | 1640 | 99 | 1665 | 1660 | 106 | 1644 | 1643 | 123 | 124 | 94 | 87 | 37065 |
| 1594 | 172 | 177 | 1587 | 1582 | 184 | 189 | 1575 | 210 | 206 | 1557 | 1558 | 209 | 1555 | 1720 | 46 | 51 | 1713 | 1708 | 58 | 63 | 1701 | 84 | 80 | 1683 | 1684 | 83 | 1681 | 1639 | 88 | 93 | 125 | 1666 | 100 | 105 | 1659 | 126 | 122 | 1641 | 1642 | 1671 | 1678 | 37065 |
| 1595 | 173 | 176 | 1586 | 1583 | 185 | 188 | 1574 | 199 | 1562 | 202 | 1563 | 1565 | 204 | 1721 | 47 | 50 | 1712 | 1709 | 59 | 62 | 1700 | 73 | 1688 | 76 | 1689 | 1691 | 78 | 120 | 89 | 92 | 1649 | 1667 | 101 | 104 | 1658 | 115 | 1646 | 118 | 1647 | 1670 | 1679 | 37065 |
| 1596 | 174 | 175 | 1585 | 1584 | 186 | 187 | 1573 | 1567 | 197 | 195 | 196 | 1568 | 1572 | 1722 | 48 | 49 | 1711 | 1710 | 60 | 61 | 1699 | 1693 | 71 | 69 | 70 | 1694 | 1698 | 1656 | 90 | 91 | 1652 | 1668 | 102 | 103 | 1657 | 1651 | 113 | 111 | 112 | 1669 | 1680 | 37065 |
| 757 | 1003 | 1002 | 768 | 769 | 991 | 990 | 780 | 781 | 983 | 982 | 981 | 782 | 786 | 1 | 1759 | 1758 | 12 | 13 | 1747 | 1746 | 24 | 25 | 1739 | 1738 | 1737 | 26 | 30 | 211 | 1549 | 1548 | 222 | 223 | 1537 | 1536 | 234 | 235 | 1529 | 1528 | 1527 | 236 | 240 | 37065 |
| 758 | 1004 | 1001 | 767 | 770 | 992 | 989 | 779 | 978 | 788 | 976 | 789 | 791 | 973 | 2 | 1760 | 1757 | 11 | 14 | 1748 | 1745 | 23 | 1734 | 32 | 1732 | 33 | 35 | 1729 | 212 | 1550 | 1547 | 221 | 224 | 1538 | 1535 | 233 | 1524 | 242 | 1522 | 243 | 245 | 1519 | 37065 |
| 759 | 1005 | 1000 | 766 | 771 | 993 | 988 | 778 | 972 | 971 | 795 | 796 | 968 | 793 | 3 | 1761 | 1756 | 10 | 15 | 1749 | 1744 | 22 | 1728 | 1727 | 39 | 40 | 1724 | 37 | 213 | 1551 | 1546 | 220 | 225 | 1539 | 1534 | 232 | 1518 | 1517 | 249 | 250 | 1514 | 247 | 37065 |
| 1006 | 760 | 765 | 999 | 994 | 772 | 777 | 987 | 798 | 794 | 969 | 970 | 797 | 967 | 1762 | 4 | 9 | 1755 | 1750 | 16 | 21 | 1743 | 42 | 38 | 1725 | 1726 | 41 | 1723 | 1552 | 214 | 219 | 1545 | 1540 | 226 | 231 | 1533 | 252 | 248 | 1515 | 1516 | 251 | 1513 | 37065 |
| 1007 | 761 | 764 | 998 | 995 | 773 | 776 | 986 | 787 | 974 | 790 | 975 | 977 | 792 | 1763 | 5 | 8 | 1754 | 1751 | 17 | 20 | 1742 | 31 | 1730 | 34 | 1731 | 1733 | 36 | 1553 | 215 | 218 | 1544 | 1541 | 227 | 230 | 1532 | 241 | 1520 | 244 | 1521 | 1523 | 246 | 37065 |
| 1008 | 762 | 763 | 997 | 996 | 774 | 775 | 985 | 979 | 785 | 783 | 784 | 980 | 984 | 1764 | 6 | 7 | 1753 | 1752 | 18 | 19 | 1741 | 1735 | 29 | 27 | 28 | 1736 | 1740 | 1554 | 216 | 217 | 1543 | 1542 | 228 | 229 | 1531 | 1525 | 239 | 237 | 238 | 1526 | 1530 | 37065 |
| 379 | 1381 | 1380 | 390 | 391 | 1369 | 1368 | 402 | 277 | 1487 | 1486 | 1485 | 278 | 282 | 295 | 1465 | 1464 | 306 | 307 | 1453 | 1452 | 318 | 319 | 1445 | 1444 | 1443 | 320 | 324 | 337 | 1411 | 1422 | 348 | 349 | 1423 | 1410 | 360 | 361 | 1403 | 1402 | 1401 | 362 | 366 | 37065 |
| 380 | 1382 | 1379 | 389 | 392 | 1370 | 1367 | 401 | 1482 | 284 | 1480 | 285 | 287 | 1477 | 296 | 1466 | 1463 | 305 | 308 | 1454 | 1451 | 317 | 1440 | 326 | 1438 | 327 | 329 | 1435 | 338 | 1412 | 1421 | 347 | 350 | 1424 | 1409 | 359 | 1398 | 368 | 1396 | 369 | 371 | 1393 | 37065 |
| 381 | 1383 | 1378 | 388 | 393 | 1371 | 1366 | 400 | 1476 | 1475 | 291 | 292 | 1472 | 289 | 297 | 1467 | 1462 | 304 | 309 | 1455 | 1450 | 316 | 1434 | 1433 | 333 | 334 | 1430 | 331 | 339 | 1413 | 1420 | 346 | 351 | 1425 | 1408 | 358 | 1392 | 1391 | 375 | 376 | 1388 | 373 | 37065 |
| 1384 | 382 | 387 | 1377 | 1372 | 394 | 399 | 1365 | 294 | 290 | 1473 | 1474 | 293 | 1471 | 1468 | 298 | 303 | 1461 | 1456 | 310 | 315 | 1449 | 336 | 332 | 1431 | 1432 | 335 | 1429 | 1426 | 352 | 345 | 1419 | 1414 | 340 | 357 | 1407 | 378 | 374 | 1389 | 1390 | 377 | 1387 | 37065 |
| 1385 | 383 | 386 | 1376 | 1373 | 395 | 398 | 1364 | 283 | 1478 | 286 | 1479 | 1481 | 288 | 1469 | 299 | 302 | 1460 | 1457 | 311 | 314 | 1448 | 325 | 1436 | 328 | 1437 | 1439 | 330 | 1427 | 353 | 344 | 1418 | 1415 | 341 | 356 | 1406 | 367 | 1394 | 370 | 1395 | 1397 | 372 | 37065 |
| 1386 | 384 | 385 | 1375 | 1374 | 396 | 397 | 1363 | 1483 | 281 | 279 | 280 | 1484 | 1488 | 1470 | 300 | 301 | 1459 | 1458 | 312 | 313 | 1447 | 1441 | 323 | 321 | 322 | 1442 | 1446 | 1428 | 354 | 343 | 1417 | 1416 | 342 | 355 | 1405 | 1399 | 365 | 363 | 364 | 1400 | 1404 | 37065 |
| 127 | 1633 | 1632 | 138 | 139 | 1621 | 1620 | 150 | 151 | 1613 | 1612 | 1611 | 152 | 156 | 421 | 1339 | 1338 | 432 | 433 | 1327 | 1326 | 444 | 445 | 1318 | 1319 | 1317 | 446 | 450 | 463 | 1297 | 1296 | 474 | 475 | 1285 | 1284 | 486 | 487 | 1277 | 1276 | 1275 | 488 | 492 | 37065 |
| 128 | 1634 | 1631 | 137 | 140 | 1622 | 1619 | 149 | 1608 | 158 | 1606 | 159 | 161 | 1603 | 422 | 1340 | 1337 | 431 | 434 | 1328 | 1325 | 443 | 1314 | 1312 | 452 | 453 | 455 | 1309 | 464 | 1298 | 1295 | 473 | 476 | 1286 | 1283 | 485 | 1272 | 494 | 1270 | 495 | 497 | 1267 | 37065 |
| 129 | 1635 | 1630 | 136 | 141 | 1623 | 1618 | 148 | 1602 | 1601 | 165 | 166 | 1598 | 163 | 423 | 1341 | 1336 | 430 | 435 | 1329 | 1324 | 442 | 1308 | 459 | 1307 | 460 | 1304 | 457 | 465 | 1299 | 1294 | 472 | 477 | 1287 | 1282 | 484 | 1266 | 1265 | 501 | 502 | 1262 | 499 | 37065 |
| 1636 | 130 | 135 | 1629 | 1624 | 142 | 147 | 1617 | 168 | 164 | 1599 | 1600 | 167 | 1597 | 1342 | 424 | 429 | 1335 | 1330 | 436 | 441 | 1323 | 462 | 1305 | 458 | 1306 | 461 | 1303 | 1300 | 466 | 471 | 1293 | 1288 | 478 | 483 | 1281 | 504 | 500 | 1263 | 1264 | 503 | 1261 | 37065 |
| 1637 | 131 | 134 | 1628 | 1625 | 143 | 146 | 1616 | 157 | 1604 | 160 | 1605 | 1607 | 162 | 1343 | 425 | 428 | 1334 | 1331 | 437 | 440 | 1322 | 451 | 454 | 1310 | 1311 | 1313 | 456 | 1301 | 467 | 470 | 1292 | 1289 | 479 | 482 | 1280 | 493 | 1268 | 496 | 1269 | 1271 | 498 | 37065 |
| 1638 | 132 | 133 | 1627 | 1626 | 144 | 145 | 1615 | 1609 | 155 | 153 | 154 | 1610 | 1614 | 1344 | 426 | 427 | 1333 | 1332 | 438 | 439 | 1321 | 1315 | 447 | 449 | 448 | 1316 | 1320 | 1302 | 468 | 469 | 1291 | 1290 | 480 | 481 | 1279 | 1273 | 491 | 489 | 490 | 1274 | 1278 | 37065 |
| 505 | 1255 | 1254 | 516 | 517 | 1243 | 1242 | 528 | 534 | 1235 | 1234 | 1233 | 530 | 529 | 547 | 1213 | 1212 | 558 | 559 | 1201 | 1200 | 570 | 1192 | 1193 | 571 | 1191 | 572 | 576 | 589 | 1171 | 1170 | 600 | 601 | 1159 | 1158 | 612 | 613 | 1151 | 1150 | 1149 | 614 | 618 | 37065 |
| 506 | 1256 | 1253 | 515 | 518 | 1244 | 1241 | 527 | 1225 | 536 | 1228 | 537 | 539 | 1230 | 548 | 1214 | 1211 | 557 | 560 | 1202 | 1199 | 569 | 1186 | 578 | 1188 | 579 | 581 | 1183 | 590 | 1172 | 1169 | 599 | 602 | 1160 | 1157 | 611 | 1146 | 620 | 1144 | 621 | 623 | 1141 | 37065 |
| 507 | 1257 | 1252 | 514 | 519 | 1245 | 1240 | 526 | 541 | 1223 | 543 | 544 | 1220 | 1224 | 549 | 1215 | 1210 | 556 | 561 | 1203 | 1198 | 568 | 585 | 1181 | 1182 | 586 | 1178 | 583 | 591 | 1173 | 1168 | 598 | 603 | 1161 | 1156 | 610 | 1140 | 1139 | 627 | 628 | 1136 | 625 | 37065 |
| 1258 | 508 | 513 | 1251 | 1246 | 520 | 525 | 1239 | 1219 | 542 | 1221 | 1222 | 545 | 546 | 1216 | 550 | 555 | 1209 | 1204 | 562 | 567 | 1197 | 1179 | 584 | 588 | 1180 | 587 | 1177 | 1174 | 592 | 597 | 1167 | 1162 | 604 | 609 | 1155 | 630 | 626 | 1137 | 1138 | 629 | 1135 | 37065 |
| 1259 | 509 | 512 | 1250 | 1247 | 521 | 524 | 1238 | 540 | 1226 | 538 | 1227 | 1229 | 535 | 1217 | 551 | 554 | 1208 | 1205 | 563 | 566 | 1196 | 580 | 1184 | 577 | 1185 | 1187 | 582 | 1175 | 593 | 596 | 1166 | 1163 | 605 | 608 | 1154 | 619 | 1142 | 622 | 1143 | 1145 | 624 | 37065 |
| 1260 | 510 | 511 | 1249 | 1248 | 522 | 523 | 1237 | 1236 | 533 | 531 | 532 | 1232 | 1231 | 1218 | 552 | 553 | 1207 | 1206 | 564 | 565 | 1195 | 573 | 575 | 1189 | 574 | 1190 | 1194 | 1176 | 594 | 595 | 1165 | 1164 | 606 | 607 | 1153 | 1147 | 617 | 615 | 616 | 1148 | 1152 | 37065 |
| 631 | 1129 | 1128 | 642 | 643 | 1116 | 1117 | 654 | 655 | 1109 | 1108 | 1107 | 656 | 660 | 673 | 1087 | 1086 | 684 | 685 | 1075 | 1074 | 696 | 697 | 1067 | 1066 | 1065 | 698 | 702 | 864 | 918 | 919 | 852 | 853 | 907 | 906 | 841 | 865 | 899 | 898 | 897 | 866 | 870 | 37065 |
| 632 | 1130 | 1127 | 641 | 644 | 1115 | 1118 | 653 | 1104 | 662 | 1102 | 663 | 665 | 1099 | 674 | 1088 | 1085 | 683 | 686 | 1076 | 1073 | 695 | 1062 | 704 | 1060 | 705 | 707 | 1057 | 863 | 917 | 920 | 851 | 854 | 908 | 905 | 842 | 894 | 872 | 892 | 873 | 875 | 889 | 37065 |
| 633 | 1131 | 1126 | 640 | 645 | 1114 | 1119 | 652 | 1098 | 1097 | 669 | 670 | 1094 | 667 | 675 | 1089 | 1084 | 682 | 687 | 1077 | 1072 | 694 | 1056 | 1055 | 711 | 712 | 1052 | 709 | 862 | 916 | 921 | 850 | 855 | 909 | 904 | 843 | 888 | 887 | 879 | 880 | 884 | 877 | 37065 |
| 1132 | 634 | 639 | 1125 | 1120 | 651 | 646 | 1113 | 672 | 668 | 1095 | 1096 | 671 | 1093 | 1090 | 676 | 681 | 1083 | 1078 | 688 | 693 | 1071 | 714 | 710 | 1053 | 1054 | 713 | 1051 | 903 | 849 | 844 | 915 | 910 | 856 | 861 | 922 | 882 | 878 | 885 | 886 | 881 | 883 | 37065 |
| 1133 | 635 | 638 | 1124 | 1121 | 650 | 647 | 1112 | 661 | 1100 | 664 | 1101 | 1103 | 666 | 1091 | 677 | 680 | 1082 | 1079 | 689 | 692 | 1070 | 703 | 1058 | 706 | 1059 | 1061 | 708 | 902 | 848 | 845 | 914 | 911 | 857 | 860 | 923 | 871 | 890 | 874 | 891 | 893 | 876 | 37065 |
| 1134 | 636 | 637 | 1123 | 1122 | 649 | 648 | 1111 | 1105 | 659 | 657 | 658 | 1106 | 1110 | 1092 | 678 | 679 | 1081 | 1080 | 690 | 691 | 1069 | 1063 | 701 | 699 | 700 | 1064 | 1068 | 901 | 847 | 846 | 913 | 912 | 858 | 859 | 924 | 895 | 869 | 867 | 868 | 896 | 900 | 37065 |
| 253 | 1507 | 1506 | 264 | 265 | 1494 | 1495 | 276 | 403 | 1361 | 1360 | 1359 | 404 | 408 | 799 | 961 | 960 | 810 | 811 | 949 | 948 | 822 | 823 | 941 | 940 | 939 | 824 | 828 | 715 | 1045 | 1044 | 726 | 727 | 1033 | 1032 | 738 | 739 | 1025 | 1024 | 1023 | 740 | 744 | 37065 |
| 254 | 1508 | 1505 | 263 | 266 | 1493 | 1496 | 275 | 1356 | 410 | 1354 | 411 | 413 | 1351 | 800 | 962 | 959 | 809 | 812 | 950 | 947 | 821 | 936 | 830 | 934 | 831 | 833 | 931 | 716 | 1046 | 1043 | 725 | 728 | 1034 | 1031 | 737 | 1020 | 746 | 1018 | 747 | 749 | 1015 | 37065 |
| 255 | 1509 | 1504 | 262 | 267 | 1492 | 1497 | 274 | 1350 | 1349 | 417 | 418 | 1346 | 415 | 801 | 963 | 958 | 808 | 813 | 951 | 946 | 820 | 930 | 929 | 837 | 838 | 926 | 835 | 717 | 1047 | 1042 | 724 | 729 | 1035 | 1030 | 736 | 1014 | 1013 | 753 | 754 | 1010 | 751 | 37065 |
| 1510 | 256 | 261 | 1503 | 1498 | 273 | 268 | 1491 | 420 | 416 | 1347 | 1348 | 419 | 1345 | 964 | 802 | 807 | 957 | 952 | 814 | 819 | 945 | 840 | 836 | 927 | 928 | 839 | 925 | 1048 | 718 | 723 | 1041 | 1036 | 730 | 735 | 1029 | 756 | 752 | 1011 | 1012 | 755 | 1009 | 37065 |
| 1511 | 257 | 260 | 1502 | 1499 | 272 | 269 | 1490 | 409 | 1352 | 412 | 1353 | 1355 | 414 | 965 | 803 | 806 | 956 | 953 | 815 | 818 | 944 | 829 | 932 | 832 | 933 | 935 | 834 | 1049 | 719 | 722 | 1040 | 1037 | 731 | 734 | 1028 | 745 | 1016 | 748 | 1017 | 1019 | 750 | 37065 |
| 1512 | 258 | 259 | 1501 | 1500 | 271 | 270 | 1489 | 1357 | 407 | 405 | 406 | 1358 | 1362 | 966 | 804 | 805 | 955 | 954 | 816 | 817 | 943 | 937 | 827 | 825 | 826 | 938 | 942 | 1050 | 720 | 721 | 1039 | 1038 | 732 | 733 | 1027 | 1021 | 743 | 741 | 742 | 1022 | 1026 | 37065 |
| 37065 | 37065 | 37065 | 37065 | 37065 | 37065 | 37065 | 37065 | 37065 | 37065 | 37065 | 37065 | 37065 | 37065 | 37065 | 37065 | 37065 | 37065 | 37065 | 37065 | 37065 | 37065 | 37065 | 37065 | 37065 | 37065 | 37065 | 37065 | 37065 | 37065 | 37065 | 37065 | 37065 | 37065 | 37065 | 37065 | 37065 | 37065 | 37065 | 37065 | 37065 | 37065 | 37065 |

*The **magic square sum** is $S_{42\times 42} := 37065$. The magic rectangles of order $6\times 14$ are of **equal magic sums** given by $R_{6\times 14} := (12355, 5295)$.*

Magic Rectangles in Construction of Magic and Block-Bordered Magic Squares, June 07, 2022, pp. 1-70, https://doi.org/10.5281/zenodo.6621071

## 2.19 Magic Square of Order $44 \times 44$

**Example 2.31.** *A magic square of order $44 \times 44$ constructed based on blocks of magic rectangles of order $2 \times 4$ is given by*

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 42614 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 504 | 1435 | 1434 | 501 | 5 | 1931 | 1930 | 8 | 9 | 1927 | 1926 | 12 | 13 | 1923 | 1922 | 16 | 17 | 1919 | 1918 | 20 | 21 | 1915 | 1914 | 24 | 25 | 1911 | 1910 | 28 | 29 | 1907 | 1906 | 32 | 33 | 1903 | 1902 | 36 | 37 | 1899 | 1898 | 40 | 512 | 1427 | 1426 | 509 | 42614 |
| 1433 | 502 | 503 | 1436 | 1932 | 6 | 7 | 1929 | 1928 | 10 | 11 | 1925 | 1924 | 14 | 15 | 1921 | 1920 | 18 | 19 | 1917 | 1916 | 22 | 23 | 1913 | 1912 | 26 | 27 | 1909 | 1908 | 30 | 31 | 1905 | 1904 | 34 | 35 | 1901 | 1900 | 38 | 39 | 1897 | 1425 | 510 | 511 | 1428 | 42614 |
| 48 | 1891 | 1890 | 45 | 49 | 1887 | 1886 | 52 | 53 | 1883 | 1882 | 56 | 57 | 1879 | 1878 | 60 | 61 | 1875 | 1874 | 64 | 65 | 1871 | 1870 | 68 | 69 | 1867 | 1866 | 72 | 73 | 1863 | 1862 | 76 | 77 | 1859 | 1858 | 80 | 81 | 1855 | 1854 | 84 | 88 | 1851 | 1850 | 85 | 42614 |
| 1889 | 46 | 47 | 1892 | 1888 | 50 | 51 | 1885 | 1884 | 54 | 55 | 1881 | 1880 | 58 | 59 | 1877 | 1876 | 62 | 63 | 1873 | 1872 | 66 | 67 | 1869 | 1868 | 70 | 71 | 1865 | 1864 | 74 | 75 | 1861 | 1860 | 78 | 79 | 1857 | 1856 | 82 | 83 | 1853 | 1849 | 86 | 87 | 1852 | 42614 |
| 89 | 1847 | 1846 | 92 | 96 | 1843 | 1842 | 93 | 97 | 1839 | 1838 | 100 | 101 | 1835 | 1834 | 104 | 105 | 1831 | 1830 | 108 | 109 | 1827 | 1826 | 112 | 113 | 1823 | 1822 | 116 | 117 | 1819 | 1818 | 120 | 121 | 1815 | 1814 | 124 | 128 | 1811 | 1810 | 125 | 129 | 1807 | 1806 | 132 | 42614 |
| 1848 | 90 | 91 | 1845 | 1841 | 94 | 95 | 1844 | 1840 | 98 | 99 | 1837 | 1836 | 102 | 103 | 1833 | 1832 | 106 | 107 | 1829 | 1828 | 110 | 111 | 1825 | 1824 | 114 | 115 | 1821 | 1820 | 118 | 119 | 1817 | 1816 | 122 | 123 | 1813 | 1809 | 126 | 127 | 1812 | 1808 | 130 | 131 | 1805 | 42614 |
| 133 | 1803 | 1802 | 136 | 140 | 1799 | 1798 | 137 | 141 | 1795 | 1794 | 144 | 145 | 1791 | 1790 | 148 | 149 | 1787 | 1786 | 152 | 153 | 1783 | 1782 | 156 | 157 | 1779 | 1778 | 160 | 161 | 1775 | 1774 | 164 | 165 | 1771 | 1770 | 168 | 172 | 1767 | 1766 | 169 | 173 | 1763 | 1762 | 176 | 42614 |
| 1804 | 134 | 135 | 1801 | 1797 | 138 | 139 | 1800 | 1796 | 142 | 143 | 1793 | 1792 | 146 | 147 | 1789 | 1788 | 150 | 151 | 1785 | 1784 | 154 | 155 | 1781 | 1780 | 158 | 159 | 1777 | 1776 | 162 | 163 | 1773 | 1772 | 166 | 167 | 1769 | 1765 | 170 | 171 | 1768 | 1764 | 174 | 175 | 1761 | 42614 |
| 177 | 1759 | 1758 | 180 | 181 | 1755 | 1754 | 184 | 185 | 1751 | 1750 | 188 | 189 | 1747 | 1746 | 192 | 193 | 1743 | 1742 | 196 | 197 | 1739 | 1738 | 200 | 201 | 1735 | 1734 | 204 | 205 | 1731 | 1730 | 208 | 209 | 1727 | 1726 | 212 | 213 | 1723 | 1722 | 216 | 217 | 1719 | 1718 | 220 | 42614 |
| 1760 | 178 | 179 | 1757 | 1756 | 182 | 183 | 1753 | 1752 | 186 | 187 | 1749 | 1748 | 190 | 191 | 1745 | 1744 | 194 | 195 | 1741 | 1740 | 198 | 199 | 1737 | 1736 | 202 | 203 | 1733 | 1732 | 206 | 207 | 1729 | 1728 | 210 | 211 | 1725 | 1724 | 214 | 215 | 1721 | 1720 | 218 | 219 | 1717 | 42614 |
| 221 | 1715 | 1714 | 224 | 225 | 1711 | 1710 | 228 | 229 | 1707 | 1706 | 232 | 233 | 1703 | 1702 | 236 | 237 | 1699 | 1698 | 240 | 241 | 1695 | 1694 | 244 | 245 | 1691 | 1690 | 248 | 249 | 1687 | 1686 | 252 | 253 | 1683 | 1682 | 256 | 257 | 1679 | 1678 | 260 | 261 | 1675 | 1674 | 264 | 42614 |
| 1716 | 222 | 223 | 1713 | 1712 | 226 | 227 | 1709 | 1708 | 230 | 231 | 1705 | 1704 | 234 | 235 | 1701 | 1700 | 238 | 239 | 1697 | 1696 | 242 | 243 | 1693 | 1692 | 246 | 247 | 1689 | 1688 | 250 | 251 | 1685 | 1684 | 254 | 255 | 1681 | 1680 | 258 | 259 | 1677 | 1676 | 262 | 263 | 1673 | 42614 |
| 265 | 1671 | 1670 | 268 | 269 | 1667 | 1666 | 272 | 273 | 1663 | 1662 | 276 | 277 | 1659 | 1658 | 280 | 281 | 1655 | 1654 | 284 | 285 | 1651 | 1650 | 288 | 289 | 1647 | 1646 | 292 | 293 | 1643 | 1642 | 296 | 297 | 1639 | 1638 | 300 | 301 | 1635 | 1634 | 304 | 305 | 1631 | 1630 | 308 | 42614 |
| 1672 | 266 | 267 | 1669 | 1668 | 270 | 271 | 1665 | 1664 | 274 | 275 | 1661 | 1660 | 278 | 279 | 1657 | 1656 | 282 | 283 | 1653 | 1652 | 286 | 287 | 1649 | 1648 | 290 | 291 | 1645 | 1644 | 294 | 295 | 1641 | 1640 | 298 | 299 | 1637 | 1636 | 302 | 303 | 1633 | 1632 | 306 | 307 | 1629 | 42614 |
| 309 | 1627 | 1626 | 312 | 313 | 1623 | 1622 | 316 | 317 | 1619 | 1618 | 320 | 321 | 1615 | 1614 | 324 | 325 | 1611 | 1610 | 328 | 329 | 1607 | 1606 | 332 | 333 | 1603 | 1602 | 336 | 337 | 1599 | 1598 | 340 | 341 | 1595 | 1594 | 344 | 345 | 1591 | 1590 | 348 | 349 | 1587 | 1586 | 352 | 42614 |
| 1628 | 310 | 311 | 1625 | 1624 | 314 | 315 | 1621 | 1620 | 318 | 319 | 1617 | 1616 | 322 | 323 | 1613 | 1612 | 326 | 327 | 1609 | 1608 | 330 | 331 | 1605 | 1604 | 334 | 335 | 1601 | 1600 | 338 | 339 | 1597 | 1596 | 342 | 343 | 1593 | 1592 | 346 | 347 | 1589 | 1588 | 350 | 351 | 1585 | 42614 |
| 353 | 1583 | 1582 | 356 | 357 | 1579 | 1578 | 360 | 361 | 1575 | 1574 | 364 | 365 | 1571 | 1570 | 368 | 369 | 1567 | 1566 | 372 | 373 | 1563 | 1562 | 376 | 377 | 1559 | 1558 | 380 | 381 | 1555 | 1554 | 384 | 385 | 1551 | 1550 | 388 | 389 | 1547 | 1546 | 392 | 393 | 1543 | 1542 | 396 | 42614 |
| 1584 | 354 | 355 | 1581 | 1580 | 358 | 359 | 1577 | 1576 | 362 | 363 | 1573 | 1572 | 366 | 367 | 1569 | 1568 | 370 | 371 | 1565 | 1564 | 374 | 375 | 1561 | 1560 | 378 | 379 | 1557 | 1556 | 382 | 383 | 1553 | 1552 | 386 | 387 | 1549 | 1548 | 390 | 391 | 1545 | 1544 | 394 | 395 | 1541 | 42614 |
| 397 | 1539 | 1538 | 400 | 401 | 1535 | 1534 | 404 | 405 | 1531 | 1530 | 408 | 409 | 1527 | 1526 | 412 | 413 | 1523 | 1522 | 416 | 417 | 1519 | 1518 | 420 | 421 | 1515 | 1514 | 424 | 425 | 1511 | 1510 | 428 | 429 | 1507 | 1506 | 432 | 433 | 1503 | 1502 | 436 | 437 | 1499 | 1498 | 440 | 42614 |
| 1540 | 398 | 399 | 1537 | 1536 | 402 | 403 | 1533 | 1532 | 406 | 407 | 1529 | 1528 | 410 | 411 | 1525 | 1524 | 414 | 415 | 1521 | 1520 | 418 | 419 | 1517 | 1516 | 422 | 423 | 1513 | 1512 | 426 | 427 | 1509 | 1508 | 430 | 431 | 1505 | 1504 | 434 | 435 | 1501 | 1500 | 438 | 439 | 1497 | 42614 |
| 441 | 1495 | 1494 | 444 | 445 | 1491 | 1490 | 448 | 449 | 1487 | 1486 | 452 | 453 | 1483 | 1482 | 456 | 457 | 1479 | 1478 | 460 | 461 | 1475 | 1474 | 464 | 465 | 1471 | 1470 | 468 | 469 | 1467 | 1466 | 472 | 473 | 1463 | 1462 | 476 | 477 | 1459 | 1458 | 480 | 481 | 1455 | 1454 | 484 | 42614 |
| 1496 | 442 | 443 | 1493 | 1492 | 446 | 447 | 1489 | 1488 | 450 | 451 | 1485 | 1484 | 454 | 455 | 1481 | 1480 | 458 | 459 | 1477 | 1476 | 462 | 463 | 1473 | 1472 | 466 | 467 | 1469 | 1468 | 470 | 471 | 1465 | 1464 | 474 | 475 | 1461 | 1460 | 478 | 479 | 1457 | 1456 | 482 | 483 | 1453 | 42614 |
| 485 | 1451 | 1450 | 488 | 489 | 1447 | 1446 | 492 | 493 | 1443 | 1442 | 496 | 497 | 1439 | 1438 | 500 | 1 | 1935 | 1934 | 4 | 505 | 1431 | 1430 | 508 | 41 | 1895 | 1894 | 44 | 513 | 1423 | 1422 | 516 | 517 | 1419 | 1418 | 520 | 521 | 1415 | 1414 | 524 | 525 | 1411 | 1410 | 528 | 42614 |
| 1452 | 486 | 487 | 1449 | 1448 | 490 | 491 | 1445 | 1444 | 494 | 495 | 1441 | 1440 | 498 | 499 | 1437 | 1936 | 2 | 3 | 1933 | 1432 | 506 | 507 | 1429 | 1896 | 42 | 43 | 1893 | 1424 | 514 | 515 | 1421 | 1420 | 518 | 519 | 1417 | 1416 | 522 | 523 | 1413 | 1412 | 526 | 527 | 1409 | 42614 |
| 529 | 1407 | 1406 | 532 | 533 | 1403 | 1402 | 536 | 537 | 1399 | 1398 | 540 | 541 | 1395 | 1394 | 544 | 545 | 1391 | 1390 | 548 | 549 | 1387 | 1386 | 552 | 553 | 1383 | 1382 | 556 | 557 | 1379 | 1378 | 560 | 561 | 1375 | 1374 | 564 | 565 | 1371 | 1370 | 568 | 569 | 1367 | 1366 | 572 | 42614 |
| 1408 | 530 | 531 | 1405 | 1404 | 534 | 535 | 1401 | 1400 | 538 | 539 | 1397 | 1396 | 542 | 543 | 1393 | 1392 | 546 | 547 | 1389 | 1388 | 550 | 551 | 1385 | 1384 | 554 | 555 | 1381 | 1380 | 558 | 559 | 1377 | 1376 | 562 | 563 | 1373 | 1372 | 566 | 567 | 1369 | 1368 | 570 | 571 | 1365 | 42614 |
| 573 | 1363 | 1362 | 576 | 577 | 1359 | 1358 | 580 | 581 | 1355 | 1354 | 584 | 585 | 1351 | 1350 | 588 | 589 | 1347 | 1346 | 592 | 593 | 1343 | 1342 | 596 | 597 | 1339 | 1338 | 600 | 601 | 1335 | 1334 | 604 | 605 | 1331 | 1330 | 608 | 609 | 1327 | 1326 | 612 | 613 | 1323 | 1322 | 616 | 42614 |
| 1364 | 574 | 575 | 1361 | 1360 | 578 | 579 | 1357 | 1356 | 582 | 583 | 1353 | 1352 | 586 | 587 | 1349 | 1348 | 590 | 591 | 1345 | 1344 | 594 | 595 | 1341 | 1340 | 598 | 599 | 1337 | 1336 | 602 | 603 | 1333 | 1332 | 606 | 607 | 1329 | 1328 | 610 | 611 | 1325 | 1324 | 614 | 615 | 1321 | 42614 |
| 617 | 1319 | 1318 | 620 | 621 | 1315 | 1314 | 624 | 625 | 1311 | 1310 | 628 | 629 | 1307 | 1306 | 632 | 633 | 1303 | 1302 | 636 | 637 | 1299 | 1298 | 640 | 641 | 1295 | 1294 | 644 | 645 | 1291 | 1290 | 648 | 649 | 1287 | 1286 | 652 | 653 | 1283 | 1282 | 656 | 657 | 1279 | 1278 | 660 | 42614 |
| 1320 | 618 | 619 | 1317 | 1316 | 622 | 623 | 1313 | 1312 | 626 | 627 | 1309 | 1308 | 630 | 631 | 1305 | 1304 | 634 | 635 | 1301 | 1300 | 638 | 639 | 1297 | 1296 | 642 | 643 | 1293 | 1292 | 646 | 647 | 1289 | 1288 | 650 | 651 | 1285 | 1284 | 654 | 655 | 1281 | 1280 | 658 | 659 | 1277 | 42614 |
| 661 | 1275 | 1274 | 664 | 665 | 1271 | 1270 | 668 | 669 | 1267 | 1266 | 672 | 673 | 1263 | 1262 | 676 | 677 | 1259 | 1258 | 680 | 681 | 1255 | 1254 | 684 | 685 | 1251 | 1250 | 688 | 689 | 1247 | 1246 | 692 | 693 | 1243 | 1242 | 696 | 697 | 1239 | 1238 | 700 | 701 | 1235 | 1234 | 704 | 42614 |
| 1276 | 662 | 663 | 1273 | 1272 | 666 | 667 | 1269 | 1268 | 670 | 671 | 1265 | 1264 | 674 | 675 | 1261 | 1260 | 678 | 679 | 1257 | 1256 | 682 | 683 | 1253 | 1252 | 686 | 687 | 1249 | 1248 | 690 | 691 | 1245 | 1244 | 694 | 695 | 1241 | 1240 | 698 | 699 | 1237 | 1236 | 702 | 703 | 1233 | 42614 |
| 705 | 1231 | 1230 | 708 | 709 | 1227 | 1226 | 712 | 713 | 1223 | 1222 | 716 | 717 | 1219 | 1218 | 720 | 721 | 1215 | 1214 | 724 | 725 | 1211 | 1210 | 728 | 729 | 1207 | 1206 | 732 | 733 | 1203 | 1202 | 736 | 737 | 1199 | 1198 | 740 | 741 | 1195 | 1194 | 744 | 745 | 1191 | 1190 | 748 | 42614 |
| 1232 | 706 | 707 | 1229 | 1228 | 710 | 711 | 1225 | 1224 | 714 | 715 | 1221 | 1220 | 718 | 719 | 1217 | 1216 | 722 | 723 | 1213 | 1212 | 726 | 727 | 1209 | 1208 | 730 | 731 | 1205 | 1204 | 734 | 735 | 1201 | 1200 | 738 | 739 | 1197 | 1196 | 742 | 743 | 1193 | 1192 | 746 | 747 | 1189 | 42614 |
| 749 | 1187 | 1186 | 752 | 753 | 1183 | 1182 | 756 | 757 | 1179 | 1178 | 760 | 761 | 1175 | 1174 | 764 | 765 | 1171 | 1170 | 768 | 769 | 1167 | 1166 | 772 | 773 | 1163 | 1162 | 776 | 777 | 1159 | 1158 | 780 | 781 | 1155 | 1154 | 784 | 785 | 1151 | 1150 | 788 | 789 | 1147 | 1146 | 792 | 42614 |
| 1188 | 750 | 751 | 1185 | 1184 | 754 | 755 | 1181 | 1180 | 758 | 759 | 1177 | 1176 | 762 | 763 | 1173 | 1172 | 766 | 767 | 1169 | 1168 | 770 | 771 | 1165 | 1164 | 774 | 775 | 1161 | 1160 | 778 | 779 | 1157 | 1156 | 782 | 783 | 1153 | 1152 | 786 | 787 | 1149 | 1148 | 790 | 791 | 1145 | 42614 |
| 793 | 1143 | 1142 | 796 | 797 | 1139 | 1138 | 800 | 801 | 1135 | 1134 | 804 | 805 | 1131 | 1130 | 808 | 809 | 1127 | 1126 | 812 | 813 | 1123 | 1122 | 816 | 817 | 1119 | 1118 | 820 | 821 | 1115 | 1114 | 824 | 825 | 1111 | 1110 | 828 | 829 | 1107 | 1106 | 832 | 833 | 1103 | 1102 | 836 | 42614 |
| 1144 | 794 | 795 | 1141 | 1140 | 798 | 799 | 1137 | 1136 | 802 | 803 | 1133 | 1132 | 806 | 807 | 1129 | 1128 | 810 | 811 | 1125 | 1124 | 814 | 815 | 1121 | 1120 | 818 | 819 | 1117 | 1116 | 822 | 823 | 1113 | 1112 | 826 | 827 | 1109 | 1108 | 830 | 831 | 1105 | 1104 | 834 | 835 | 1101 | 42614 |
| 837 | 1099 | 1098 | 840 | 841 | 1095 | 1094 | 844 | 845 | 1091 | 1090 | 848 | 849 | 1087 | 1086 | 852 | 853 | 1083 | 1082 | 856 | 857 | 1079 | 1078 | 860 | 861 | 1075 | 1074 | 864 | 865 | 1071 | 1070 | 868 | 869 | 1067 | 1066 | 872 | 873 | 1063 | 1062 | 876 | 877 | 1059 | 1058 | 880 | 42614 |
| 1100 | 838 | 839 | 1097 | 1096 | 842 | 843 | 1093 | 1092 | 846 | 847 | 1089 | 1088 | 850 | 851 | 1085 | 1084 | 854 | 855 | 1081 | 1080 | 858 | 859 | 1077 | 1076 | 862 | 863 | 1073 | 1072 | 866 | 867 | 1069 | 1068 | 870 | 871 | 1065 | 1064 | 874 | 875 | 1061 | 1060 | 878 | 879 | 1057 | 42614 |
| 881 | 1055 | 1054 | 884 | 885 | 1051 | 1050 | 888 | 889 | 1047 | 1046 | 892 | 893 | 1043 | 1042 | 896 | 897 | 1039 | 1038 | 900 | 901 | 1035 | 1034 | 904 | 905 | 1031 | 1030 | 908 | 909 | 1027 | 1026 | 912 | 913 | 1023 | 1022 | 916 | 917 | 1019 | 1018 | 920 | 921 | 1015 | 1014 | 924 | 42614 |
| 1056 | 882 | 883 | 1053 | 1052 | 886 | 887 | 1049 | 1048 | 890 | 891 | 1045 | 1044 | 894 | 895 | 1041 | 1040 | 898 | 899 | 1037 | 1036 | 902 | 903 | 1033 | 1032 | 906 | 907 | 1029 | 1028 | 910 | 911 | 1025 | 1024 | 914 | 915 | 1021 | 1020 | 918 | 919 | 1017 | 1016 | 922 | 923 | 1013 | 42614 |
| 925 | 1011 | 1010 | 928 | 929 | 1007 | 1006 | 932 | 933 | 1003 | 1002 | 936 | 937 | 999 | 998 | 940 | 941 | 995 | 994 | 944 | 945 | 991 | 990 | 948 | 949 | 987 | 986 | 952 | 953 | 983 | 982 | 956 | 957 | 979 | 978 | 960 | 961 | 975 | 974 | 964 | 965 | 971 | 970 | 968 | 42614 |
| 1012 | 926 | 927 | 1009 | 1008 | 930 | 931 | 1005 | 1004 | 934 | 935 | 1001 | 1000 | 938 | 939 | 997 | 996 | 942 | 943 | 993 | 992 | 946 | 947 | 989 | 988 | 950 | 951 | 985 | 984 | 954 | 955 | 981 | 980 | 958 | 959 | 977 | 976 | 962 | 963 | 973 | 972 | 966 | 967 | 969 | 42614 |
| 42614 | 42614 | 42614 | 42614 | 42614 | 42614 | 42614 | 42614 | 42614 | 42614 | 42614 | 42614 | 42614 | 42614 | 42614 | 42614 | 42614 | 42614 | 42614 | 42614 | 42614 | 42614 | 42614 | 42614 | 42614 | 42614 | 42614 | 42614 | 42614 | 42614 | 42614 | 42614 | 42614 | 42614 | 42614 | 42614 | 42614 | 42614 | 42614 | 42614 | 42614 | 42614 | 42614 | 42614 | 42614 |

*The **magic square sum** is $S_{44\times 44} := 42614$. The magic rectangles of order $2 \times 4$ are of **equal magic sums** given by $R_{2\times 4} := (3874, 1937)$.*

## 2.20 Block Bordered Magic Square of Order $46 \times 46$

**Example 2.32.** *A* ***block bordered*** *magic square of order 46 with inner block as a magic square of order 44 is given by*

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 48691 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2071 | 2028 | 2030 | 2032 | 2034 | 2036 | 2038 | 2040 | 2042 | 2044 | 2046 | 2048 | 2096 | 2098 | 2100 | 2102 | 2104 | 2106 | 2108 | 2110 | 2112 | 2114 | 90 | 88 | 86 | 84 | 82 | 80 | 78 | 76 | 74 | 72 | 70 | 68 | 22 | 20 | 18 | 16 | 14 | 12 | 10 | 8 | 6 | 4 | 2 | 2072 | 48691 |
| 67 | 594 | 1525 | 1524 | 591 | 95 | 2021 | 2020 | 98 | 99 | 2017 | 2016 | 102 | 103 | 2013 | 2012 | 106 | 107 | 2009 | 2008 | 110 | 111 | 2005 | 2004 | 114 | 115 | 2001 | 2000 | 118 | 119 | 1997 | 1996 | 122 | 123 | 1993 | 1992 | 126 | 127 | 1989 | 1988 | 130 | 602 | 1517 | 1516 | 599 | 2050 | 48691 |
| 65 | 1523 | 592 | 593 | 1526 | 2022 | 96 | 97 | 2019 | 2018 | 100 | 101 | 2015 | 2014 | 104 | 105 | 2011 | 2010 | 108 | 109 | 2007 | 2006 | 112 | 113 | 2003 | 2002 | 116 | 117 | 1999 | 1998 | 120 | 121 | 1995 | 1994 | 124 | 125 | 1991 | 1990 | 128 | 129 | 1987 | 1515 | 600 | 601 | 1518 | 2052 | 48691 |
| 63 | 138 | 1981 | 1980 | 135 | 139 | 1977 | 1976 | 142 | 143 | 1973 | 1972 | 146 | 147 | 1969 | 1968 | 150 | 151 | 1965 | 1964 | 154 | 155 | 1961 | 1960 | 158 | 159 | 1957 | 1956 | 162 | 163 | 1953 | 1952 | 166 | 167 | 1949 | 1948 | 170 | 171 | 1945 | 1944 | 174 | 178 | 1941 | 1940 | 175 | 2054 | 48691 |
| 61 | 1979 | 136 | 137 | 1982 | 1978 | 140 | 141 | 1975 | 1974 | 144 | 145 | 1971 | 1970 | 148 | 149 | 1967 | 1966 | 152 | 153 | 1963 | 1962 | 156 | 157 | 1959 | 1958 | 160 | 161 | 1955 | 1954 | 164 | 165 | 1951 | 1950 | 168 | 169 | 1947 | 1946 | 172 | 173 | 1943 | 1939 | 176 | 177 | 1942 | 2056 | 48691 |
| 59 | 179 | 1937 | 1936 | 182 | 186 | 1933 | 1932 | 183 | 187 | 1929 | 1928 | 190 | 191 | 1925 | 1924 | 194 | 195 | 1921 | 1920 | 198 | 199 | 1917 | 1916 | 202 | 203 | 1913 | 1912 | 206 | 207 | 1909 | 1908 | 210 | 211 | 1905 | 1904 | 214 | 218 | 1901 | 1900 | 215 | 219 | 1897 | 1896 | 222 | 2058 | 48691 |
| 57 | 1938 | 180 | 181 | 1935 | 1931 | 184 | 185 | 1934 | 1930 | 188 | 189 | 1927 | 1926 | 192 | 193 | 1923 | 1922 | 196 | 197 | 1919 | 1918 | 200 | 201 | 1915 | 1914 | 204 | 205 | 1911 | 1910 | 208 | 209 | 1907 | 1906 | 212 | 213 | 1903 | 1899 | 216 | 217 | 1902 | 1898 | 220 | 221 | 1895 | 2060 | 48691 |
| 55 | 223 | 1893 | 1892 | 226 | 230 | 1889 | 1888 | 227 | 231 | 1885 | 1884 | 234 | 235 | 1881 | 1880 | 238 | 239 | 1877 | 1876 | 242 | 243 | 1873 | 1872 | 246 | 247 | 1869 | 1868 | 250 | 251 | 1865 | 1864 | 254 | 255 | 1861 | 1860 | 258 | 262 | 1857 | 1856 | 259 | 263 | 1853 | 1852 | 266 | 2062 | 48691 |
| 53 | 1894 | 224 | 225 | 1891 | 1887 | 228 | 229 | 1890 | 1886 | 232 | 233 | 1883 | 1882 | 236 | 237 | 1879 | 1878 | 240 | 241 | 1875 | 1874 | 244 | 245 | 1871 | 1870 | 248 | 249 | 1867 | 1866 | 252 | 253 | 1863 | 1862 | 256 | 257 | 1859 | 1855 | 260 | 261 | 1858 | 1854 | 264 | 265 | 1851 | 2064 | 48691 |
| 51 | 267 | 1849 | 1848 | 270 | 271 | 1845 | 1844 | 274 | 275 | 1841 | 1840 | 278 | 279 | 1837 | 1836 | 282 | 283 | 1833 | 1832 | 286 | 287 | 1829 | 1828 | 290 | 291 | 1825 | 1824 | 294 | 295 | 1821 | 1820 | 298 | 299 | 1817 | 1816 | 302 | 303 | 1813 | 1812 | 306 | 307 | 1809 | 1808 | 310 | 2066 | 48691 |
| 49 | 1850 | 268 | 269 | 1847 | 1846 | 272 | 273 | 1843 | 1842 | 276 | 277 | 1839 | 1838 | 280 | 281 | 1835 | 1834 | 284 | 285 | 1831 | 1830 | 288 | 289 | 1827 | 1826 | 292 | 293 | 1823 | 1822 | 296 | 297 | 1819 | 1818 | 300 | 301 | 1815 | 1814 | 304 | 305 | 1811 | 1810 | 308 | 309 | 1807 | 2068 | 48691 |
| 47 | 311 | 1805 | 1804 | 314 | 315 | 1801 | 1800 | 318 | 319 | 1797 | 1796 | 322 | 323 | 1793 | 1792 | 326 | 327 | 1789 | 1788 | 330 | 331 | 1785 | 1784 | 334 | 335 | 1781 | 1780 | 338 | 339 | 1777 | 1776 | 342 | 343 | 1773 | 1772 | 346 | 347 | 1769 | 1768 | 350 | 351 | 1765 | 1764 | 354 | 2070 | 48691 |
| 43 | 1806 | 312 | 313 | 1803 | 1802 | 316 | 317 | 1799 | 1798 | 320 | 321 | 1795 | 1794 | 324 | 325 | 1791 | 1790 | 328 | 329 | 1787 | 1786 | 332 | 333 | 1783 | 1782 | 336 | 337 | 1779 | 1778 | 340 | 341 | 1775 | 1774 | 344 | 345 | 1771 | 1770 | 348 | 349 | 1767 | 1766 | 352 | 353 | 1763 | 2074 | 48691 |
| 41 | 355 | 1761 | 1760 | 358 | 359 | 1757 | 1756 | 362 | 363 | 1753 | 1752 | 366 | 367 | 1749 | 1748 | 370 | 371 | 1745 | 1744 | 374 | 375 | 1741 | 1740 | 378 | 379 | 1737 | 1736 | 382 | 383 | 1733 | 1732 | 386 | 387 | 1729 | 1728 | 390 | 391 | 1725 | 1724 | 394 | 395 | 1721 | 1720 | 398 | 2076 | 48691 |
| 39 | 1762 | 356 | 357 | 1759 | 1758 | 360 | 361 | 1755 | 1754 | 364 | 365 | 1751 | 1750 | 368 | 369 | 1747 | 1746 | 372 | 373 | 1743 | 1742 | 376 | 377 | 1739 | 1738 | 380 | 381 | 1735 | 1734 | 384 | 385 | 1731 | 1730 | 388 | 389 | 1727 | 1726 | 392 | 393 | 1723 | 1722 | 396 | 397 | 1719 | 2078 | 48691 |
| 37 | 399 | 1717 | 1716 | 402 | 403 | 1713 | 1712 | 406 | 407 | 1709 | 1708 | 410 | 411 | 1705 | 1704 | 414 | 415 | 1701 | 1700 | 418 | 419 | 1697 | 1696 | 422 | 423 | 1693 | 1692 | 426 | 427 | 1689 | 1688 | 430 | 431 | 1685 | 1684 | 434 | 435 | 1681 | 1680 | 438 | 439 | 1677 | 1676 | 442 | 2080 | 48691 |
| 35 | 1718 | 400 | 401 | 1715 | 1714 | 404 | 405 | 1711 | 1710 | 408 | 409 | 1707 | 1706 | 412 | 413 | 1703 | 1702 | 416 | 417 | 1699 | 1698 | 420 | 421 | 1695 | 1694 | 424 | 425 | 1691 | 1690 | 428 | 429 | 1687 | 1686 | 432 | 433 | 1683 | 1682 | 436 | 437 | 1679 | 1678 | 440 | 441 | 1675 | 2082 | 48691 |
| 33 | 443 | 1673 | 1672 | 446 | 447 | 1669 | 1668 | 450 | 451 | 1665 | 1664 | 454 | 455 | 1661 | 1660 | 458 | 459 | 1657 | 1656 | 462 | 463 | 1653 | 1652 | 466 | 467 | 1649 | 1648 | 470 | 471 | 1645 | 1644 | 474 | 475 | 1641 | 1640 | 478 | 479 | 1637 | 1636 | 482 | 483 | 1633 | 1632 | 486 | 2084 | 48691 |
| 31 | 1674 | 444 | 445 | 1671 | 1670 | 448 | 449 | 1667 | 1666 | 452 | 453 | 1663 | 1662 | 456 | 457 | 1659 | 1658 | 460 | 461 | 1655 | 1654 | 464 | 465 | 1651 | 1650 | 468 | 469 | 1647 | 1646 | 472 | 473 | 1643 | 1642 | 476 | 477 | 1639 | 1638 | 480 | 481 | 1635 | 1634 | 484 | 485 | 1631 | 2086 | 48691 |
| 29 | 487 | 1629 | 1628 | 490 | 491 | 1625 | 1624 | 494 | 495 | 1621 | 1620 | 498 | 499 | 1617 | 1616 | 502 | 503 | 1613 | 1612 | 506 | 507 | 1609 | 1608 | 510 | 511 | 1605 | 1604 | 514 | 515 | 1601 | 1600 | 518 | 519 | 1597 | 1596 | 522 | 523 | 1593 | 1592 | 526 | 527 | 1589 | 1588 | 530 | 2088 | 48691 |
| 27 | 1630 | 488 | 489 | 1627 | 1626 | 492 | 493 | 1623 | 1622 | 496 | 497 | 1619 | 1618 | 500 | 501 | 1615 | 1614 | 504 | 505 | 1611 | 1610 | 508 | 509 | 1607 | 1606 | 512 | 513 | 1603 | 1602 | 516 | 517 | 1599 | 1598 | 520 | 521 | 1595 | 1594 | 524 | 525 | 1591 | 1590 | 528 | 529 | 1587 | 2090 | 48691 |
| 25 | 531 | 1585 | 1584 | 534 | 535 | 1581 | 1580 | 538 | 539 | 1577 | 1576 | 542 | 543 | 1573 | 1572 | 546 | 547 | 1569 | 1568 | 550 | 551 | 1565 | 1564 | 554 | 555 | 1561 | 1560 | 558 | 559 | 1557 | 1556 | 562 | 563 | 1553 | 1552 | 566 | 567 | 1549 | 1548 | 570 | 571 | 1545 | 1544 | 574 | 2092 | 48691 |
| 1 | 1586 | 532 | 533 | 1583 | 1582 | 536 | 537 | 1579 | 1578 | 540 | 541 | 1575 | 1574 | 544 | 545 | 1571 | 1570 | 548 | 549 | 1567 | 1566 | 552 | 553 | 1563 | 1562 | 556 | 557 | 1559 | 1558 | 560 | 561 | 1555 | 1554 | 564 | 565 | 1551 | 1550 | 568 | 569 | 1547 | 1546 | 572 | 573 | 1543 | 2116 | 48691 |
| 2051 | 575 | 1541 | 1540 | 578 | 579 | 1537 | 1536 | 582 | 583 | 1533 | 1532 | 586 | 587 | 1529 | 1528 | 590 | 91 | 2025 | 2024 | 94 | 595 | 1521 | 1520 | 598 | 131 | 1985 | 1984 | 134 | 603 | 1513 | 1512 | 606 | 607 | 1509 | 1508 | 610 | 611 | 1505 | 1504 | 614 | 615 | 1501 | 1500 | 618 | 66 | 48691 |
| 2053 | 1542 | 576 | 577 | 1539 | 1538 | 580 | 581 | 1535 | 1534 | 584 | 585 | 1531 | 1530 | 588 | 589 | 1527 | 2026 | 92 | 93 | 2023 | 1522 | 596 | 597 | 1519 | 1986 | 132 | 133 | 1983 | 1514 | 604 | 605 | 1511 | 1510 | 608 | 609 | 1507 | 1506 | 612 | 613 | 1503 | 1502 | 616 | 617 | 1499 | 64 | 48691 |
| 2055 | 619 | 1497 | 1496 | 622 | 623 | 1493 | 1492 | 626 | 627 | 1489 | 1488 | 630 | 631 | 1485 | 1484 | 634 | 635 | 1481 | 1480 | 638 | 639 | 1477 | 1476 | 642 | 643 | 1473 | 1472 | 646 | 647 | 1469 | 1468 | 650 | 651 | 1465 | 1464 | 654 | 655 | 1461 | 1460 | 658 | 659 | 1457 | 1456 | 662 | 62 | 48691 |
| 2057 | 1498 | 620 | 621 | 1495 | 1494 | 624 | 625 | 1491 | 1490 | 628 | 629 | 1487 | 1486 | 632 | 633 | 1483 | 1482 | 636 | 637 | 1479 | 1478 | 640 | 641 | 1475 | 1474 | 644 | 645 | 1471 | 1470 | 648 | 649 | 1467 | 1466 | 652 | 653 | 1463 | 1462 | 656 | 657 | 1459 | 1458 | 660 | 661 | 1455 | 60 | 48691 |
| 2059 | 663 | 1453 | 1452 | 666 | 667 | 1449 | 1448 | 670 | 671 | 1445 | 1444 | 674 | 675 | 1441 | 1440 | 678 | 679 | 1437 | 1436 | 682 | 683 | 1433 | 1432 | 686 | 687 | 1429 | 1428 | 690 | 691 | 1425 | 1424 | 694 | 695 | 1421 | 1420 | 698 | 699 | 1417 | 1416 | 702 | 703 | 1413 | 1412 | 706 | 58 | 48691 |
| 2061 | 1454 | 664 | 665 | 1451 | 1450 | 668 | 669 | 1447 | 1446 | 672 | 673 | 1443 | 1442 | 676 | 677 | 1439 | 1438 | 680 | 681 | 1435 | 1434 | 684 | 685 | 1431 | 1430 | 688 | 689 | 1427 | 1426 | 692 | 693 | 1423 | 1422 | 696 | 697 | 1419 | 1418 | 700 | 701 | 1415 | 1414 | 704 | 705 | 1411 | 56 | 48691 |
| 2063 | 707 | 1409 | 1408 | 710 | 711 | 1405 | 1404 | 714 | 715 | 1401 | 1400 | 718 | 719 | 1397 | 1396 | 722 | 723 | 1393 | 1392 | 726 | 727 | 1389 | 1388 | 730 | 731 | 1385 | 1384 | 734 | 735 | 1381 | 1380 | 738 | 739 | 1377 | 1376 | 742 | 743 | 1373 | 1372 | 746 | 747 | 1369 | 1368 | 750 | 54 | 48691 |
| 2065 | 1410 | 708 | 709 | 1407 | 1406 | 712 | 713 | 1403 | 1402 | 716 | 717 | 1399 | 1398 | 720 | 721 | 1395 | 1394 | 724 | 725 | 1391 | 1390 | 728 | 729 | 1387 | 1386 | 732 | 733 | 1383 | 1382 | 736 | 737 | 1379 | 1378 | 740 | 741 | 1375 | 1374 | 744 | 745 | 1371 | 1370 | 748 | 749 | 1367 | 52 | 48691 |
| 2067 | 751 | 1365 | 1364 | 754 | 755 | 1361 | 1360 | 758 | 759 | 1357 | 1356 | 762 | 763 | 1353 | 1352 | 766 | 767 | 1349 | 1348 | 770 | 771 | 1345 | 1344 | 774 | 775 | 1341 | 1340 | 778 | 779 | 1337 | 1336 | 782 | 783 | 1333 | 1332 | 786 | 787 | 1329 | 1328 | 790 | 791 | 1325 | 1324 | 794 | 50 | 48691 |
| 2069 | 1366 | 752 | 753 | 1363 | 1362 | 756 | 757 | 1359 | 1358 | 760 | 761 | 1355 | 1354 | 764 | 765 | 1351 | 1350 | 768 | 769 | 1347 | 1346 | 772 | 773 | 1343 | 1342 | 776 | 777 | 1339 | 1338 | 780 | 781 | 1335 | 1334 | 784 | 785 | 1331 | 1330 | 788 | 789 | 1327 | 1326 | 792 | 793 | 1323 | 48 | 48691 |
| 2073 | 795 | 1321 | 1320 | 798 | 799 | 1317 | 1316 | 802 | 803 | 1313 | 1312 | 806 | 807 | 1309 | 1308 | 810 | 811 | 1305 | 1304 | 814 | 815 | 1301 | 1300 | 818 | 819 | 1297 | 1296 | 822 | 823 | 1293 | 1292 | 826 | 827 | 1289 | 1288 | 830 | 831 | 1285 | 1284 | 834 | 835 | 1281 | 1280 | 838 | 44 | 48691 |
| 2075 | 1322 | 796 | 797 | 1319 | 1318 | 800 | 801 | 1315 | 1314 | 804 | 805 | 1311 | 1310 | 808 | 809 | 1307 | 1306 | 812 | 813 | 1303 | 1302 | 816 | 817 | 1299 | 1298 | 820 | 821 | 1295 | 1294 | 824 | 825 | 1291 | 1290 | 828 | 829 | 1287 | 1286 | 832 | 833 | 1283 | 1282 | 836 | 837 | 1279 | 42 | 48691 |
| 2077 | 839 | 1277 | 1276 | 842 | 843 | 1273 | 1272 | 846 | 847 | 1269 | 1268 | 850 | 851 | 1265 | 1264 | 854 | 855 | 1261 | 1260 | 858 | 859 | 1257 | 1256 | 862 | 863 | 1253 | 1252 | 866 | 867 | 1249 | 1248 | 870 | 871 | 1245 | 1244 | 874 | 875 | 1241 | 1240 | 878 | 879 | 1237 | 1236 | 882 | 40 | 48691 |
| 2079 | 1278 | 840 | 841 | 1275 | 1274 | 844 | 845 | 1271 | 1270 | 848 | 849 | 1267 | 1266 | 852 | 853 | 1263 | 1262 | 856 | 857 | 1259 | 1258 | 860 | 861 | 1255 | 1254 | 864 | 865 | 1251 | 1250 | 868 | 869 | 1247 | 1246 | 872 | 873 | 1243 | 1242 | 876 | 877 | 1239 | 1238 | 880 | 881 | 1235 | 38 | 48691 |
| 2081 | 883 | 1233 | 1232 | 886 | 887 | 1229 | 1228 | 890 | 891 | 1225 | 1224 | 894 | 895 | 1221 | 1220 | 898 | 899 | 1217 | 1216 | 902 | 903 | 1213 | 1212 | 906 | 907 | 1209 | 1208 | 910 | 911 | 1205 | 1204 | 914 | 915 | 1201 | 1200 | 918 | 919 | 1197 | 1196 | 922 | 923 | 1193 | 1192 | 926 | 36 | 48691 |
| 2083 | 1234 | 884 | 885 | 1231 | 1230 | 888 | 889 | 1227 | 1226 | 892 | 893 | 1223 | 1222 | 896 | 897 | 1219 | 1218 | 900 | 901 | 1215 | 1214 | 904 | 905 | 1211 | 1210 | 908 | 909 | 1207 | 1206 | 912 | 913 | 1203 | 1202 | 916 | 917 | 1199 | 1198 | 920 | 921 | 1195 | 1194 | 924 | 925 | 1191 | 34 | 48691 |
| 2085 | 927 | 1189 | 1188 | 930 | 931 | 1185 | 1184 | 934 | 935 | 1181 | 1180 | 938 | 939 | 1177 | 1176 | 942 | 943 | 1173 | 1172 | 946 | 947 | 1169 | 1168 | 950 | 951 | 1165 | 1164 | 954 | 955 | 1161 | 1160 | 958 | 959 | 1157 | 1156 | 962 | 963 | 1153 | 1152 | 966 | 967 | 1149 | 1148 | 970 | 32 | 48691 |
| 2087 | 1190 | 928 | 929 | 1187 | 1186 | 932 | 933 | 1183 | 1182 | 936 | 937 | 1179 | 1178 | 940 | 941 | 1175 | 1174 | 944 | 945 | 1171 | 1170 | 948 | 949 | 1167 | 1166 | 952 | 953 | 1163 | 1162 | 956 | 957 | 1159 | 1158 | 960 | 961 | 1155 | 1154 | 964 | 965 | 1151 | 1150 | 968 | 969 | 1147 | 30 | 48691 |
| 2089 | 971 | 1145 | 1144 | 974 | 975 | 1141 | 1140 | 978 | 979 | 1137 | 1136 | 982 | 983 | 1133 | 1132 | 986 | 987 | 1129 | 1128 | 990 | 991 | 1125 | 1124 | 994 | 995 | 1121 | 1120 | 998 | 999 | 1117 | 1116 | 1002 | 1003 | 1113 | 1112 | 1006 | 1007 | 1109 | 1108 | 1010 | 1011 | 1105 | 1104 | 1014 | 28 | 48691 |
| 2091 | 1146 | 972 | 973 | 1143 | 1142 | 976 | 977 | 1139 | 1138 | 980 | 981 | 1135 | 1134 | 984 | 985 | 1131 | 1130 | 988 | 989 | 1127 | 1126 | 992 | 993 | 1123 | 1122 | 996 | 997 | 1119 | 1118 | 1000 | 1001 | 1115 | 1114 | 1004 | 1005 | 1111 | 1110 | 1008 | 1009 | 1107 | 1106 | 1012 | 1013 | 1103 | 26 | 48691 |
| 2093 | 1015 | 1101 | 1100 | 1018 | 1019 | 1097 | 1096 | 1022 | 1023 | 1093 | 1092 | 1026 | 1027 | 1089 | 1088 | 1030 | 1031 | 1085 | 1084 | 1034 | 1035 | 1081 | 1080 | 1038 | 1039 | 1077 | 1076 | 1042 | 1043 | 1073 | 1072 | 1046 | 1047 | 1069 | 1068 | 1050 | 1051 | 1065 | 1064 | 1054 | 1055 | 1061 | 1060 | 1058 | 24 | 48691 |
| 2094 | 1102 | 1016 | 1017 | 1099 | 1098 | 1020 | 1021 | 1095 | 1094 | 1024 | 1025 | 1091 | 1090 | 1028 | 1029 | 1087 | 1086 | 1032 | 1033 | 1083 | 1082 | 1036 | 1037 | 1079 | 1078 | 1040 | 1041 | 1075 | 1074 | 1044 | 1045 | 1071 | 1070 | 1048 | 1049 | 1067 | 1066 | 1052 | 1053 | 1063 | 1062 | 1056 | 1057 | 1059 | 23 | 48691 |
| 45 | 89 | 87 | 85 | 83 | 81 | 79 | 77 | 75 | 73 | 71 | 69 | 21 | 19 | 17 | 15 | 13 | 11 | 9 | 7 | 5 | 3 | 2027 | 2029 | 2031 | 2033 | 2035 | 2037 | 2039 | 2041 | 2043 | 2045 | 2047 | 2049 | 2095 | 2097 | 2099 | 2101 | 2103 | 2105 | 2107 | 2109 | 2111 | 2113 | 2115 | 46 | 48691 |
| 48691 | 48691 | 48691 | 48691 | 48691 | 48691 | 48691 | 48691 | 48691 | 48691 | 48691 | 48691 | 48691 | 48691 | 48691 | 48691 | 48691 | 48691 | 48691 | 48691 | 48691 | 48691 | 48691 | 48691 | 48691 | 48691 | 48691 | 48691 | 48691 | 48691 | 48691 | 48691 | 48691 | 48691 | 48691 | 48691 | 48691 | 48691 | 48691 | 48691 | 48691 | 48691 | 48691 | 48691 | 48691 | 48691 | 48691 |

*In this case the magic sums are* $S_{46\times 46} := 48691$ *and* $S_{44\times 44} := 46574$. *The magic rectangles of order* $2\times 4$ *are of* ***equal magic sums*** *given by* $R_{2\times 4} := (4234, 2117)$.

## 2.21 Magic Square of Order $48 \times 48$

**Example 2.33.** *A magic square of order* $48 \times 48$ *constructed based on blocks of magic rectangles of order* $2 \times 4$ *is given by*

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 55320 |
| 28 | 2278 | 2279 | 25 | 5 | 2299 | 2298 | 8 | 24 | 2283 | 2282 | 21 | 100 | 2206 | 2207 | 97 | 77 | 2227 | 2226 | 80 | 96 | 2211 | 2210 | 93 | 172 | 2134 | 2135 | 169 | 149 | 2155 | 2154 | 152 | 168 | 2139 | 2138 | 165 | 244 | 2062 | 2063 | 241 | 221 | 2083 | 2082 | 224 | 240 | 2067 | 2066 | 237 | 55320 |
| 2277 | 27 | 26 | 2280 | 2300 | 6 | 7 | 2297 | 2281 | 22 | 23 | 2284 | 2205 | 99 | 98 | 2208 | 2228 | 78 | 79 | 2225 | 2209 | 94 | 95 | 2212 | 2133 | 171 | 170 | 2136 | 2156 | 150 | 151 | 2153 | 2137 | 166 | 167 | 2140 | 2061 | 243 | 242 | 2064 | 2084 | 222 | 223 | 2081 | 2065 | 238 | 239 | 2068 | 55320 |
| 13 | 2291 | 2290 | 16 | 9 | 2295 | 2294 | 12 | 20 | 2286 | 2287 | 17 | 85 | 2219 | 2218 | 88 | 81 | 2223 | 2222 | 84 | 92 | 2214 | 2215 | 89 | 157 | 2147 | 2146 | 160 | 153 | 2151 | 2150 | 156 | 164 | 2142 | 2143 | 161 | 229 | 2075 | 2074 | 232 | 225 | 2079 | 2078 | 228 | 236 | 2070 | 2071 | 233 | 55320 |
| 2292 | 14 | 15 | 2289 | 2296 | 10 | 11 | 2293 | 2285 | 19 | 18 | 2288 | 2220 | 86 | 87 | 2217 | 2224 | 82 | 83 | 2221 | 2213 | 91 | 90 | 2216 | 2148 | 158 | 159 | 2145 | 2152 | 154 | 155 | 2149 | 2141 | 163 | 162 | 2144 | 2076 | 230 | 231 | 2073 | 2080 | 226 | 227 | 2077 | 2069 | 235 | 234 | 2072 | 55320 |
| 1 | 2303 | 2302 | 4 | 29 | 2275 | 2274 | 32 | 33 | 2271 | 2270 | 36 | 73 | 2231 | 2230 | 76 | 101 | 2203 | 2202 | 104 | 105 | 2199 | 2198 | 108 | 145 | 2159 | 2158 | 148 | 173 | 2131 | 2130 | 176 | 177 | 2127 | 2126 | 180 | 217 | 2087 | 2086 | 220 | 245 | 2059 | 2058 | 248 | 249 | 2055 | 2054 | 252 | 55320 |
| 2304 | 2 | 3 | 2301 | 2276 | 30 | 31 | 2273 | 2272 | 34 | 35 | 2269 | 2232 | 74 | 75 | 2229 | 2204 | 102 | 103 | 2201 | 2200 | 106 | 107 | 2197 | 2160 | 146 | 147 | 2157 | 2132 | 174 | 175 | 2129 | 2128 | 178 | 179 | 2125 | 2088 | 218 | 219 | 2085 | 2060 | 246 | 247 | 2057 | 2056 | 250 | 251 | 2053 | 55320 |
| 49 | 2255 | 2254 | 52 | 41 | 2263 | 2262 | 44 | 45 | 2259 | 2258 | 48 | 121 | 2183 | 2182 | 124 | 113 | 2191 | 2190 | 116 | 117 | 2187 | 2186 | 120 | 193 | 2111 | 2110 | 196 | 185 | 2119 | 2118 | 188 | 189 | 2115 | 2114 | 192 | 265 | 2039 | 2038 | 268 | 257 | 2047 | 2046 | 260 | 261 | 2043 | 2042 | 264 | 55320 |
| 2256 | 50 | 51 | 2253 | 2264 | 42 | 43 | 2261 | 2260 | 46 | 47 | 2257 | 2184 | 122 | 123 | 2181 | 2192 | 114 | 115 | 2189 | 2188 | 118 | 119 | 2185 | 2112 | 194 | 195 | 2109 | 2120 | 186 | 187 | 2117 | 2116 | 190 | 191 | 2113 | 2040 | 266 | 267 | 2037 | 2048 | 258 | 259 | 2045 | 2044 | 262 | 263 | 2041 | 55320 |
| 69 | 2234 | 2235 | 72 | 37 | 2267 | 2266 | 40 | 57 | 2247 | 2246 | 60 | 141 | 2162 | 2163 | 144 | 109 | 2195 | 2194 | 112 | 129 | 2175 | 2174 | 132 | 213 | 2090 | 2091 | 216 | 181 | 2123 | 2122 | 184 | 201 | 2103 | 2102 | 204 | 285 | 2018 | 2019 | 288 | 253 | 2051 | 2050 | 256 | 273 | 2031 | 2030 | 276 | 55320 |
| 2236 | 71 | 70 | 2233 | 2268 | 38 | 39 | 2265 | 2248 | 58 | 59 | 2245 | 2164 | 143 | 142 | 2161 | 2196 | 110 | 111 | 2193 | 2176 | 130 | 131 | 2173 | 2092 | 215 | 214 | 2089 | 2124 | 182 | 183 | 2121 | 2104 | 202 | 203 | 2101 | 2020 | 287 | 286 | 2017 | 2052 | 254 | 255 | 2049 | 2032 | 274 | 275 | 2029 | 55320 |
| 64 | 2242 | 2243 | 61 | 65 | 2239 | 2238 | 68 | 53 | 2251 | 2250 | 56 | 136 | 2170 | 2171 | 133 | 137 | 2167 | 2166 | 140 | 125 | 2179 | 2178 | 128 | 208 | 2098 | 2099 | 205 | 209 | 2095 | 2094 | 212 | 197 | 2107 | 2106 | 200 | 280 | 2026 | 2027 | 277 | 281 | 2023 | 2022 | 284 | 269 | 2035 | 2034 | 272 | 55320 |
| 2241 | 63 | 62 | 2244 | 2240 | 66 | 67 | 2237 | 2252 | 54 | 55 | 2249 | 2169 | 135 | 134 | 2172 | 2168 | 138 | 139 | 2165 | 2180 | 126 | 127 | 2177 | 2097 | 207 | 206 | 2100 | 2096 | 210 | 211 | 2093 | 2108 | 198 | 199 | 2105 | 2025 | 279 | 278 | 2028 | 2024 | 282 | 283 | 2021 | 2036 | 270 | 271 | 2033 | 55320 |
| 316 | 1990 | 1991 | 313 | 293 | 2011 | 2010 | 296 | 312 | 1995 | 1994 | 309 | 388 | 1918 | 1919 | 385 | 365 | 1939 | 1938 | 368 | 384 | 1923 | 1922 | 381 | 460 | 1846 | 1847 | 457 | 437 | 1867 | 1866 | 440 | 456 | 1851 | 1850 | 453 | 532 | 1774 | 1775 | 529 | 509 | 1795 | 1794 | 512 | 528 | 1779 | 1778 | 525 | 55320 |
| 1989 | 315 | 314 | 1992 | 2012 | 294 | 295 | 2009 | 1993 | 310 | 311 | 1996 | 1917 | 387 | 386 | 1920 | 1940 | 366 | 367 | 1937 | 1921 | 382 | 383 | 1924 | 1845 | 459 | 458 | 1848 | 1868 | 438 | 439 | 1865 | 1849 | 454 | 455 | 1852 | 1773 | 531 | 530 | 1776 | 1796 | 510 | 511 | 1793 | 1777 | 526 | 527 | 1780 | 55320 |
| 301 | 2003 | 2002 | 304 | 297 | 2007 | 2006 | 300 | 308 | 1998 | 1999 | 305 | 373 | 1931 | 1930 | 376 | 369 | 1935 | 1934 | 372 | 380 | 1926 | 1927 | 377 | 445 | 1859 | 1858 | 448 | 441 | 1863 | 1862 | 444 | 452 | 1854 | 1855 | 449 | 517 | 1787 | 1786 | 520 | 513 | 1791 | 1790 | 516 | 524 | 1782 | 1783 | 521 | 55320 |
| 2004 | 302 | 303 | 2001 | 2008 | 298 | 299 | 2005 | 1997 | 307 | 306 | 2000 | 1932 | 374 | 375 | 1929 | 1936 | 370 | 371 | 1933 | 1925 | 379 | 378 | 1928 | 1860 | 446 | 447 | 1857 | 1864 | 442 | 443 | 1861 | 1853 | 451 | 450 | 1856 | 1788 | 518 | 519 | 1785 | 1792 | 514 | 515 | 1789 | 1781 | 523 | 522 | 1784 | 55320 |
| 289 | 2015 | 2014 | 292 | 317 | 1987 | 1986 | 320 | 321 | 1983 | 1982 | 324 | 361 | 1943 | 1942 | 364 | 389 | 1915 | 1914 | 392 | 393 | 1911 | 1910 | 396 | 433 | 1871 | 1870 | 436 | 461 | 1843 | 1842 | 464 | 465 | 1839 | 1838 | 468 | 505 | 1799 | 1798 | 508 | 533 | 1771 | 1770 | 536 | 537 | 1767 | 1766 | 540 | 55320 |
| 2016 | 290 | 291 | 2013 | 1988 | 318 | 319 | 1985 | 1984 | 322 | 323 | 1981 | 1944 | 362 | 363 | 1941 | 1916 | 390 | 391 | 1913 | 1912 | 394 | 395 | 1909 | 1872 | 434 | 435 | 1869 | 1844 | 462 | 463 | 1841 | 1840 | 466 | 467 | 1837 | 1800 | 506 | 507 | 1797 | 1772 | 534 | 535 | 1769 | 1768 | 538 | 539 | 1765 | 55320 |
| 337 | 1967 | 1966 | 340 | 329 | 1975 | 1974 | 332 | 333 | 1971 | 1970 | 336 | 409 | 1895 | 1894 | 412 | 401 | 1903 | 1902 | 404 | 405 | 1899 | 1898 | 408 | 481 | 1823 | 1822 | 484 | 473 | 1831 | 1830 | 476 | 477 | 1827 | 1826 | 480 | 553 | 1751 | 1750 | 556 | 545 | 1759 | 1758 | 548 | 549 | 1755 | 1754 | 552 | 55320 |
| 1968 | 338 | 339 | 1965 | 1976 | 330 | 331 | 1973 | 1972 | 334 | 335 | 1969 | 1896 | 410 | 411 | 1893 | 1904 | 402 | 403 | 1901 | 1900 | 406 | 407 | 1897 | 1824 | 482 | 483 | 1821 | 1832 | 474 | 475 | 1829 | 1828 | 478 | 479 | 1825 | 1752 | 554 | 555 | 1749 | 1760 | 546 | 547 | 1757 | 1756 | 550 | 551 | 1753 | 55320 |
| 357 | 1946 | 1947 | 360 | 325 | 1979 | 1978 | 328 | 345 | 1959 | 1958 | 348 | 429 | 1874 | 1875 | 432 | 397 | 1907 | 1906 | 400 | 417 | 1887 | 1886 | 420 | 501 | 1802 | 1803 | 504 | 469 | 1835 | 1834 | 472 | 489 | 1815 | 1814 | 492 | 573 | 1730 | 1731 | 576 | 541 | 1763 | 1762 | 544 | 561 | 1743 | 1742 | 564 | 55320 |
| 1948 | 359 | 358 | 1945 | 1980 | 326 | 327 | 1977 | 1960 | 346 | 347 | 1957 | 1876 | 431 | 430 | 1873 | 1908 | 398 | 399 | 1905 | 1888 | 418 | 419 | 1885 | 1804 | 503 | 502 | 1801 | 1836 | 470 | 471 | 1833 | 1816 | 490 | 491 | 1813 | 1732 | 575 | 574 | 1729 | 1764 | 542 | 543 | 1761 | 1744 | 562 | 563 | 1741 | 55320 |
| 352 | 1954 | 1955 | 349 | 353 | 1951 | 1950 | 356 | 341 | 1963 | 1962 | 344 | 424 | 1882 | 1883 | 421 | 425 | 1879 | 1878 | 428 | 413 | 1891 | 1890 | 416 | 496 | 1810 | 1811 | 493 | 497 | 1807 | 1806 | 500 | 485 | 1819 | 1818 | 488 | 568 | 1738 | 1739 | 565 | 569 | 1735 | 1734 | 572 | 557 | 1747 | 1746 | 560 | 55320 |
| 1953 | 351 | 350 | 1956 | 1952 | 354 | 355 | 1949 | 1964 | 342 | 343 | 1961 | 1881 | 423 | 422 | 1884 | 1880 | 426 | 427 | 1877 | 1892 | 414 | 415 | 1889 | 1809 | 495 | 494 | 1812 | 1808 | 498 | 499 | 1805 | 1820 | 486 | 487 | 1817 | 1737 | 567 | 566 | 1740 | 1736 | 570 | 571 | 1733 | 1748 | 558 | 559 | 1745 | 55320 |
| 604 | 1702 | 1703 | 601 | 581 | 1723 | 1722 | 584 | 600 | 1707 | 1706 | 597 | 676 | 1630 | 1631 | 673 | 653 | 1651 | 1650 | 656 | 672 | 1635 | 1634 | 669 | 748 | 1558 | 1559 | 745 | 725 | 1579 | 1578 | 728 | 744 | 1563 | 1562 | 741 | 820 | 1486 | 1487 | 817 | 797 | 1507 | 1506 | 800 | 816 | 1491 | 1490 | 813 | 55320 |
| 1701 | 603 | 602 | 1704 | 1724 | 582 | 583 | 1721 | 1705 | 598 | 599 | 1708 | 1629 | 675 | 674 | 1632 | 1652 | 654 | 655 | 1649 | 1633 | 670 | 671 | 1636 | 1557 | 747 | 746 | 1560 | 1580 | 726 | 727 | 1577 | 1561 | 742 | 743 | 1564 | 1485 | 819 | 818 | 1488 | 1508 | 798 | 799 | 1505 | 1489 | 814 | 815 | 1492 | 55320 |
| 589 | 1715 | 1714 | 592 | 585 | 1719 | 1718 | 588 | 596 | 1710 | 1711 | 593 | 661 | 1643 | 1642 | 664 | 657 | 1647 | 1646 | 660 | 668 | 1638 | 1639 | 665 | 733 | 1571 | 1570 | 736 | 729 | 1575 | 1574 | 732 | 740 | 1566 | 1567 | 737 | 805 | 1499 | 1498 | 808 | 801 | 1503 | 1502 | 804 | 812 | 1494 | 1495 | 809 | 55320 |
| 1716 | 590 | 591 | 1713 | 1720 | 586 | 587 | 1717 | 1709 | 595 | 594 | 1712 | 1644 | 662 | 663 | 1641 | 1648 | 658 | 659 | 1645 | 1637 | 667 | 666 | 1640 | 1572 | 734 | 735 | 1569 | 1576 | 730 | 731 | 1573 | 1565 | 739 | 738 | 1568 | 1500 | 806 | 807 | 1497 | 1504 | 802 | 803 | 1501 | 1493 | 811 | 810 | 1496 | 55320 |
| 577 | 1727 | 1726 | 580 | 605 | 1699 | 1698 | 608 | 609 | 1695 | 1694 | 612 | 649 | 1655 | 1654 | 652 | 677 | 1627 | 1626 | 680 | 681 | 1623 | 1622 | 684 | 721 | 1583 | 1582 | 724 | 749 | 1555 | 1554 | 752 | 753 | 1551 | 1550 | 756 | 793 | 1511 | 1510 | 796 | 821 | 1483 | 1482 | 824 | 825 | 1479 | 1478 | 828 | 55320 |
| 1728 | 578 | 579 | 1725 | 1700 | 606 | 607 | 1697 | 1696 | 610 | 611 | 1693 | 1656 | 650 | 651 | 1653 | 1628 | 678 | 679 | 1625 | 1624 | 682 | 683 | 1621 | 1584 | 722 | 723 | 1581 | 1556 | 750 | 751 | 1553 | 1552 | 754 | 755 | 1549 | 1512 | 794 | 795 | 1509 | 1484 | 822 | 823 | 1481 | 1480 | 826 | 827 | 1477 | 55320 |
| 625 | 1679 | 1678 | 628 | 617 | 1687 | 1686 | 620 | 621 | 1683 | 1682 | 624 | 697 | 1607 | 1606 | 700 | 689 | 1615 | 1614 | 692 | 693 | 1611 | 1610 | 696 | 769 | 1535 | 1534 | 772 | 761 | 1543 | 1542 | 764 | 765 | 1539 | 1538 | 768 | 841 | 1463 | 1462 | 844 | 833 | 1471 | 1470 | 836 | 837 | 1467 | 1466 | 840 | 55320 |
| 1680 | 626 | 627 | 1677 | 1688 | 618 | 619 | 1685 | 1684 | 622 | 623 | 1681 | 1608 | 698 | 699 | 1605 | 1616 | 690 | 691 | 1613 | 1612 | 694 | 695 | 1609 | 1536 | 770 | 771 | 1533 | 1544 | 762 | 763 | 1541 | 1540 | 766 | 767 | 1537 | 1464 | 842 | 843 | 1461 | 1472 | 834 | 835 | 1469 | 1468 | 838 | 839 | 1465 | 55320 |
| 645 | 1658 | 1659 | 648 | 613 | 1691 | 1690 | 616 | 633 | 1671 | 1670 | 636 | 717 | 1586 | 1587 | 720 | 685 | 1619 | 1618 | 688 | 705 | 1599 | 1598 | 708 | 789 | 1514 | 1515 | 792 | 757 | 1547 | 1546 | 760 | 777 | 1527 | 1526 | 780 | 861 | 1442 | 1443 | 864 | 829 | 1475 | 1474 | 832 | 849 | 1455 | 1454 | 852 | 55320 |
| 1660 | 647 | 646 | 1657 | 1692 | 614 | 615 | 1689 | 1672 | 634 | 635 | 1669 | 1588 | 719 | 718 | 1585 | 1620 | 686 | 687 | 1617 | 1600 | 706 | 707 | 1597 | 1516 | 791 | 790 | 1513 | 1548 | 758 | 759 | 1545 | 1528 | 778 | 779 | 1525 | 1444 | 863 | 862 | 1441 | 1476 | 830 | 831 | 1473 | 1456 | 850 | 851 | 1453 | 55320 |
| 640 | 1666 | 1667 | 637 | 641 | 1663 | 1662 | 644 | 629 | 1675 | 1674 | 632 | 712 | 1594 | 1595 | 709 | 713 | 1591 | 1590 | 716 | 701 | 1603 | 1602 | 704 | 784 | 1522 | 1523 | 781 | 785 | 1519 | 1518 | 788 | 773 | 1531 | 1530 | 776 | 856 | 1450 | 1451 | 853 | 857 | 1447 | 1446 | 860 | 845 | 1459 | 1458 | 848 | 55320 |
| 1665 | 639 | 638 | 1668 | 1664 | 642 | 643 | 1661 | 1676 | 630 | 631 | 1673 | 1593 | 711 | 710 | 1596 | 1592 | 714 | 715 | 1589 | 1604 | 702 | 703 | 1601 | 1521 | 783 | 782 | 1524 | 1520 | 786 | 787 | 1517 | 1532 | 774 | 775 | 1529 | 1449 | 855 | 854 | 1452 | 1448 | 858 | 859 | 1445 | 1460 | 846 | 847 | 1457 | 55320 |
| 892 | 1414 | 1415 | 889 | 869 | 1435 | 1434 | 872 | 888 | 1419 | 1418 | 885 | 964 | 1342 | 1343 | 961 | 941 | 1363 | 1362 | 944 | 960 | 1347 | 1346 | 957 | 1036 | 1270 | 1271 | 1033 | 1013 | 1291 | 1290 | 1016 | 1032 | 1275 | 1274 | 1029 | 1108 | 1198 | 1199 | 1105 | 1085 | 1219 | 1218 | 1088 | 1104 | 1203 | 1202 | 1101 | 55320 |
| 1413 | 891 | 890 | 1416 | 1436 | 870 | 871 | 1433 | 1417 | 886 | 887 | 1420 | 1341 | 963 | 962 | 1344 | 1364 | 942 | 943 | 1361 | 1345 | 958 | 959 | 1348 | 1269 | 1035 | 1034 | 1272 | 1292 | 1014 | 1015 | 1289 | 1273 | 1030 | 1031 | 1276 | 1197 | 1107 | 1106 | 1200 | 1220 | 1086 | 1087 | 1217 | 1201 | 1102 | 1103 | 1204 | 55320 |
| 877 | 1427 | 1426 | 880 | 873 | 1431 | 1430 | 876 | 884 | 1422 | 1423 | 881 | 949 | 1355 | 1354 | 952 | 945 | 1359 | 1358 | 948 | 956 | 1350 | 1351 | 953 | 1021 | 1283 | 1282 | 1024 | 1017 | 1287 | 1286 | 1020 | 1028 | 1278 | 1279 | 1025 | 1093 | 1211 | 1210 | 1096 | 1089 | 1215 | 1214 | 1092 | 1100 | 1206 | 1207 | 1097 | 55320 |
| 1428 | 878 | 879 | 1425 | 1432 | 874 | 875 | 1429 | 1421 | 883 | 882 | 1424 | 1356 | 950 | 951 | 1353 | 1360 | 946 | 947 | 1357 | 1349 | 955 | 954 | 1352 | 1284 | 1022 | 1023 | 1281 | 1288 | 1018 | 1019 | 1285 | 1277 | 1027 | 1026 | 1280 | 1212 | 1094 | 1095 | 1209 | 1216 | 1090 | 1091 | 1213 | 1205 | 1099 | 1098 | 1208 | 55320 |
| 865 | 1439 | 1438 | 868 | 893 | 1411 | 1410 | 896 | 897 | 1407 | 1406 | 900 | 937 | 1367 | 1366 | 940 | 965 | 1339 | 1338 | 968 | 969 | 1335 | 1334 | 972 | 1009 | 1295 | 1294 | 1012 | 1037 | 1267 | 1266 | 1040 | 1041 | 1263 | 1262 | 1044 | 1081 | 1223 | 1222 | 1084 | 1109 | 1195 | 1194 | 1112 | 1113 | 1191 | 1190 | 1116 | 55320 |
| 1440 | 866 | 867 | 1437 | 1412 | 894 | 895 | 1409 | 1408 | 898 | 899 | 1405 | 1368 | 938 | 939 | 1365 | 1340 | 966 | 967 | 1337 | 1336 | 970 | 971 | 1333 | 1296 | 1010 | 1011 | 1293 | 1268 | 1038 | 1039 | 1265 | 1264 | 1042 | 1043 | 1261 | 1224 | 1082 | 1083 | 1221 | 1196 | 1110 | 1111 | 1193 | 1192 | 1114 | 1115 | 1189 | 55320 |
| 913 | 1391 | 1390 | 916 | 905 | 1399 | 1398 | 908 | 909 | 1395 | 1394 | 912 | 985 | 1319 | 1318 | 988 | 977 | 1327 | 1326 | 980 | 981 | 1323 | 1322 | 984 | 1057 | 1247 | 1246 | 1060 | 1049 | 1255 | 1254 | 1052 | 1053 | 1251 | 1250 | 1056 | 1129 | 1175 | 1174 | 1132 | 1121 | 1183 | 1182 | 1124 | 1125 | 1179 | 1178 | 1128 | 55320 |
| 1392 | 914 | 915 | 1389 | 1400 | 906 | 907 | 1397 | 1396 | 910 | 911 | 1393 | 1320 | 986 | 987 | 1317 | 1328 | 978 | 979 | 1325 | 1324 | 982 | 983 | 1321 | 1248 | 1058 | 1059 | 1245 | 1256 | 1050 | 1051 | 1253 | 1252 | 1054 | 1055 | 1249 | 1176 | 1130 | 1131 | 1173 | 1184 | 1122 | 1123 | 1181 | 1180 | 1126 | 1127 | 1177 | 55320 |
| 933 | 1370 | 1371 | 936 | 901 | 1403 | 1402 | 904 | 921 | 1383 | 1382 | 924 | 1005 | 1298 | 1299 | 1008 | 973 | 1331 | 1330 | 976 | 993 | 1311 | 1310 | 996 | 1077 | 1226 | 1227 | 1080 | 1045 | 1259 | 1258 | 1048 | 1065 | 1239 | 1238 | 1068 | 1149 | 1154 | 1155 | 1152 | 1117 | 1187 | 1186 | 1120 | 1137 | 1167 | 1166 | 1140 | 55320 |
| 1372 | 935 | 934 | 1369 | 1404 | 902 | 903 | 1401 | 1384 | 922 | 923 | 1381 | 1300 | 1007 | 1006 | 1297 | 1332 | 974 | 975 | 1329 | 1312 | 994 | 995 | 1309 | 1228 | 1079 | 1078 | 1225 | 1260 | 1046 | 1047 | 1257 | 1240 | 1066 | 1067 | 1237 | 1156 | 1151 | 1150 | 1153 | 1188 | 1118 | 1119 | 1185 | 1168 | 1138 | 1139 | 1165 | 55320 |
| 928 | 1378 | 1379 | 925 | 929 | 1375 | 1374 | 932 | 917 | 1387 | 1386 | 920 | 1000 | 1306 | 1307 | 997 | 1001 | 1303 | 1302 | 1004 | 989 | 1315 | 1314 | 992 | 1072 | 1234 | 1235 | 1069 | 1073 | 1231 | 1230 | 1076 | 1061 | 1243 | 1242 | 1064 | 1144 | 1162 | 1163 | 1141 | 1145 | 1159 | 1158 | 1148 | 1133 | 1171 | 1170 | 1136 | 55320 |
| 1377 | 927 | 926 | 1380 | 1376 | 930 | 931 | 1373 | 1388 | 918 | 919 | 1385 | 1305 | 999 | 998 | 1308 | 1304 | 1002 | 1003 | 1301 | 1316 | 990 | 991 | 1313 | 1233 | 1071 | 1070 | 1236 | 1232 | 1074 | 1075 | 1229 | 1244 | 1062 | 1063 | 1241 | 1161 | 1143 | 1142 | 1164 | 1160 | 1146 | 1147 | 1157 | 1172 | 1134 | 1135 | 1169 | 55320 |
| 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 |

*The* **magic square sum** *is* $S_{48\times48} := 55320$. *The magic rectangles of order* $2 \times 4$ *are of* **equal magic sums** *given by* $R_{2\times4} := (4610, 2305)$. *In this case, the blocks of order* $12 \times 12$ *are magic squares of equal magic sums.*

**Example 2.34.** *A magic square of order $48 \times 48$ constructed based on blocks of magic rectangles of order $4 \times 6$ is given by*

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 55320 |
| 1 | 2300 | 3 | 4 | 2303 | 2304 | 48 | 38 | 39 | 2266 | 2267 | 2257 | 73 | 2228 | 75 | 76 | 2231 | 2232 | 120 | 110 | 111 | 2194 | 2195 | 2185 | 145 | 2156 | 147 | 148 | 2159 | 2160 | 192 | 182 | 183 | 2122 | 2123 | 2113 | 217 | 2084 | 219 | 220 | 2087 | 2088 | 2041 | 254 | 255 | 2050 | 2051 | 264 | 55320 |
| 2299 | 2 | 2301 | 2302 | 5 | 6 | 2262 | 2261 | 2260 | 45 | 44 | 43 | 2227 | 74 | 2229 | 2230 | 77 | 78 | 2190 | 2189 | 2188 | 117 | 116 | 115 | 2155 | 146 | 2157 | 2158 | 149 | 150 | 2118 | 2117 | 2116 | 189 | 188 | 187 | 2083 | 218 | 2085 | 2086 | 221 | 222 | 259 | 2045 | 2044 | 261 | 260 | 2046 | 55320 |
| 2298 | 2297 | 2296 | 9 | 8 | 7 | 2263 | 2264 | 2265 | 40 | 41 | 42 | 2226 | 2225 | 2224 | 81 | 80 | 79 | 2191 | 2192 | 2193 | 112 | 113 | 114 | 2154 | 2153 | 2152 | 153 | 152 | 151 | 2119 | 2120 | 2121 | 184 | 185 | 186 | 2082 | 2081 | 2080 | 225 | 224 | 223 | 258 | 2048 | 2049 | 256 | 257 | 2047 | 55320 |
| 12 | 11 | 10 | 2295 | 2294 | 2293 | 37 | 47 | 46 | 2259 | 2258 | 2268 | 84 | 83 | 82 | 2223 | 2222 | 2221 | 109 | 119 | 118 | 2187 | 2186 | 2196 | 156 | 155 | 154 | 2151 | 2150 | 2149 | 181 | 191 | 190 | 2115 | 2114 | 2124 | 228 | 227 | 226 | 2079 | 2078 | 2077 | 2052 | 263 | 262 | 2043 | 2042 | 253 | 55320 |
| 25 | 26 | 27 | 2278 | 2279 | 2280 | 13 | 14 | 15 | 2290 | 2291 | 2292 | 97 | 98 | 99 | 2206 | 2207 | 2208 | 85 | 86 | 87 | 2218 | 2219 | 2220 | 169 | 170 | 171 | 2134 | 2135 | 2136 | 157 | 158 | 159 | 2146 | 2147 | 2148 | 241 | 242 | 243 | 2062 | 2063 | 2064 | 2076 | 230 | 231 | 2074 | 2075 | 229 | 55320 |
| 2275 | 2276 | 2277 | 28 | 29 | 30 | 2287 | 2288 | 2289 | 16 | 17 | 18 | 2203 | 2204 | 2205 | 100 | 101 | 102 | 2215 | 2216 | 2217 | 88 | 89 | 90 | 2131 | 2132 | 2133 | 172 | 173 | 174 | 2143 | 2144 | 2145 | 160 | 161 | 162 | 2059 | 2060 | 2061 | 244 | 245 | 246 | 234 | 2072 | 2073 | 232 | 233 | 2071 | 55320 |
| 2274 | 2273 | 2272 | 33 | 32 | 31 | 2286 | 2285 | 2284 | 21 | 20 | 19 | 2202 | 2201 | 2200 | 105 | 104 | 103 | 2214 | 2213 | 2212 | 93 | 92 | 91 | 2130 | 2129 | 2128 | 177 | 176 | 175 | 2142 | 2141 | 2140 | 165 | 164 | 163 | 2058 | 2057 | 2056 | 249 | 248 | 247 | 235 | 2069 | 2068 | 237 | 236 | 2070 | 55320 |
| 36 | 35 | 34 | 2271 | 2270 | 2269 | 24 | 23 | 22 | 2283 | 2282 | 2281 | 108 | 107 | 106 | 2199 | 2198 | 2197 | 96 | 95 | 94 | 2211 | 2210 | 2209 | 180 | 179 | 178 | 2127 | 2126 | 2125 | 168 | 167 | 166 | 2139 | 2138 | 2137 | 252 | 251 | 250 | 2055 | 2054 | 2053 | 2065 | 239 | 238 | 2067 | 2066 | 240 | 55320 |
| 49 | 50 | 51 | 2254 | 2255 | 2256 | 61 | 62 | 2241 | 64 | 2243 | 2244 | 121 | 122 | 123 | 2182 | 2183 | 2184 | 133 | 134 | 2169 | 136 | 2171 | 2172 | 193 | 194 | 195 | 2110 | 2111 | 2112 | 205 | 206 | 2097 | 208 | 2099 | 2100 | 265 | 266 | 267 | 2038 | 2039 | 2040 | 277 | 278 | 2025 | 280 | 2027 | 2028 | 55320 |
| 2251 | 2252 | 2248 | 57 | 53 | 54 | 2239 | 2240 | 63 | 2242 | 65 | 66 | 2179 | 2180 | 2176 | 129 | 125 | 126 | 2167 | 2168 | 135 | 2170 | 137 | 138 | 2107 | 2108 | 2104 | 201 | 197 | 198 | 2095 | 2096 | 207 | 2098 | 209 | 210 | 2035 | 2036 | 2032 | 273 | 269 | 270 | 2023 | 2024 | 279 | 2026 | 281 | 282 | 55320 |
| 60 | 2249 | 2253 | 52 | 56 | 2245 | 72 | 2237 | 2236 | 69 | 68 | 2233 | 132 | 2177 | 2181 | 124 | 128 | 2173 | 144 | 2165 | 2164 | 141 | 140 | 2161 | 204 | 2105 | 2109 | 196 | 200 | 2101 | 216 | 2093 | 2092 | 213 | 212 | 2089 | 276 | 2033 | 2037 | 268 | 272 | 2029 | 288 | 2021 | 2020 | 285 | 284 | 2017 | 55320 |
| 2250 | 59 | 58 | 2247 | 2246 | 55 | 2238 | 71 | 70 | 2235 | 2234 | 67 | 2178 | 131 | 130 | 2175 | 2174 | 127 | 2166 | 143 | 142 | 2163 | 2162 | 139 | 2106 | 203 | 202 | 2103 | 2102 | 199 | 2094 | 215 | 214 | 2091 | 2090 | 211 | 2034 | 275 | 274 | 2031 | 2030 | 271 | 2022 | 287 | 286 | 2019 | 2018 | 283 | 55320 |
| 289 | 2012 | 291 | 292 | 2015 | 2016 | 336 | 326 | 327 | 1978 | 1979 | 1969 | 361 | 1940 | 363 | 364 | 1943 | 1944 | 408 | 398 | 399 | 1906 | 1907 | 1897 | 433 | 1868 | 435 | 436 | 1871 | 1872 | 409 | 410 | 411 | 1894 | 1895 | 1896 | 505 | 1796 | 507 | 508 | 1799 | 1800 | 552 | 542 | 543 | 1762 | 1763 | 1753 | 55320 |
| 2011 | 290 | 2013 | 2014 | 293 | 294 | 1974 | 1973 | 1972 | 333 | 332 | 331 | 1939 | 362 | 1941 | 1942 | 365 | 366 | 1902 | 1901 | 1900 | 405 | 404 | 403 | 1867 | 434 | 1869 | 1870 | 437 | 438 | 1891 | 1892 | 1888 | 417 | 413 | 414 | 1795 | 506 | 1797 | 1798 | 509 | 510 | 1758 | 1757 | 1756 | 549 | 548 | 547 | 55320 |
| 2010 | 2009 | 2008 | 297 | 296 | 295 | 1975 | 1976 | 1977 | 328 | 329 | 330 | 1938 | 1937 | 1936 | 369 | 368 | 367 | 1903 | 1904 | 1905 | 400 | 401 | 402 | 1866 | 1865 | 1864 | 441 | 440 | 439 | 420 | 1889 | 1893 | 412 | 416 | 1885 | 1794 | 1793 | 1792 | 513 | 512 | 511 | 1759 | 1760 | 1761 | 544 | 545 | 546 | 55320 |
| 300 | 299 | 298 | 2007 | 2006 | 2005 | 325 | 335 | 334 | 1971 | 1970 | 1980 | 372 | 371 | 370 | 1935 | 1934 | 1933 | 397 | 407 | 406 | 1899 | 1898 | 1908 | 444 | 443 | 442 | 1863 | 1862 | 1861 | 1890 | 419 | 418 | 1887 | 1886 | 415 | 516 | 515 | 514 | 1791 | 1790 | 1789 | 541 | 551 | 550 | 1755 | 1754 | 1764 | 55320 |
| 313 | 314 | 315 | 1990 | 1991 | 1992 | 301 | 302 | 303 | 2002 | 2003 | 2004 | 385 | 386 | 387 | 1918 | 1919 | 1920 | 373 | 374 | 375 | 1930 | 1931 | 1932 | 457 | 458 | 459 | 1846 | 1847 | 1848 | 445 | 446 | 447 | 1858 | 1859 | 1860 | 529 | 530 | 531 | 1774 | 1775 | 1776 | 517 | 518 | 519 | 1786 | 1787 | 1788 | 55320 |
| 1987 | 1988 | 1989 | 316 | 317 | 318 | 1999 | 2000 | 2001 | 304 | 305 | 306 | 1915 | 1916 | 1917 | 388 | 389 | 390 | 1927 | 1928 | 1929 | 376 | 377 | 378 | 1843 | 1844 | 1845 | 460 | 461 | 462 | 1855 | 1856 | 1857 | 448 | 449 | 450 | 1771 | 1772 | 1773 | 532 | 533 | 534 | 1783 | 1784 | 1785 | 520 | 521 | 522 | 55320 |
| 1986 | 1985 | 1984 | 321 | 320 | 319 | 1998 | 1997 | 1996 | 309 | 308 | 307 | 1914 | 1913 | 1912 | 393 | 392 | 391 | 1926 | 1925 | 1924 | 381 | 380 | 379 | 1842 | 1841 | 1840 | 465 | 464 | 463 | 1854 | 1853 | 1852 | 453 | 452 | 451 | 1770 | 1769 | 1768 | 537 | 536 | 535 | 1782 | 1781 | 1780 | 525 | 524 | 523 | 55320 |
| 324 | 323 | 322 | 1983 | 1982 | 1981 | 312 | 311 | 310 | 1995 | 1994 | 1993 | 396 | 395 | 394 | 1911 | 1910 | 1909 | 384 | 383 | 382 | 1923 | 1922 | 1921 | 468 | 467 | 466 | 1839 | 1838 | 1837 | 456 | 455 | 454 | 1851 | 1850 | 1849 | 540 | 539 | 538 | 1767 | 1766 | 1765 | 528 | 527 | 526 | 1779 | 1778 | 1777 | 55320 |
| 337 | 338 | 339 | 1966 | 1967 | 1968 | 349 | 350 | 1953 | 352 | 1955 | 1956 | 480 | 470 | 471 | 1834 | 1835 | 1825 | 421 | 422 | 1881 | 424 | 1883 | 1884 | 481 | 482 | 483 | 1822 | 1823 | 1824 | 493 | 494 | 1809 | 496 | 1811 | 1812 | 553 | 554 | 555 | 1750 | 1751 | 1752 | 565 | 566 | 1737 | 568 | 1739 | 1740 | 55320 |
| 1963 | 1964 | 1960 | 345 | 341 | 342 | 1951 | 1952 | 351 | 1954 | 353 | 354 | 1830 | 1829 | 1828 | 477 | 476 | 475 | 1879 | 1880 | 423 | 1882 | 425 | 426 | 1819 | 1820 | 1816 | 489 | 485 | 486 | 1807 | 1808 | 495 | 1810 | 497 | 498 | 1747 | 1748 | 1744 | 561 | 557 | 558 | 1735 | 1736 | 567 | 1738 | 569 | 570 | 55320 |
| 348 | 1961 | 1965 | 340 | 344 | 1957 | 360 | 1949 | 1948 | 357 | 356 | 1945 | 1831 | 1832 | 1833 | 472 | 473 | 474 | 432 | 1877 | 1876 | 429 | 428 | 1873 | 492 | 1817 | 1821 | 484 | 488 | 1813 | 504 | 1805 | 1804 | 501 | 500 | 1801 | 564 | 1745 | 1749 | 556 | 560 | 1741 | 576 | 1733 | 1732 | 573 | 572 | 1729 | 55320 |
| 1962 | 347 | 346 | 1959 | 1958 | 343 | 1950 | 359 | 358 | 1947 | 1946 | 355 | 469 | 479 | 478 | 1827 | 1826 | 1836 | 1878 | 431 | 430 | 1875 | 1874 | 427 | 1818 | 491 | 490 | 1815 | 1814 | 487 | 1806 | 503 | 502 | 1803 | 1802 | 499 | 1746 | 563 | 562 | 1743 | 1742 | 559 | 1734 | 575 | 574 | 1731 | 1730 | 571 | 55320 |
| 577 | 1724 | 579 | 580 | 1727 | 1728 | 624 | 614 | 615 | 1690 | 1691 | 1681 | 649 | 1652 | 651 | 652 | 1655 | 1656 | 625 | 626 | 627 | 1678 | 1679 | 1680 | 721 | 1580 | 723 | 724 | 1583 | 1584 | 768 | 758 | 759 | 1546 | 1547 | 1537 | 793 | 1508 | 795 | 796 | 1511 | 1512 | 840 | 830 | 831 | 1474 | 1475 | 1465 | 55320 |
| 1723 | 578 | 1725 | 1726 | 581 | 582 | 1686 | 1685 | 1684 | 621 | 620 | 619 | 1651 | 650 | 1653 | 1654 | 653 | 654 | 1675 | 1676 | 1672 | 633 | 629 | 630 | 1579 | 722 | 1581 | 1582 | 725 | 726 | 1542 | 1541 | 1540 | 765 | 764 | 763 | 1507 | 794 | 1509 | 1510 | 797 | 798 | 1470 | 1469 | 1468 | 837 | 836 | 835 | 55320 |
| 1722 | 1721 | 1720 | 585 | 584 | 583 | 1687 | 1688 | 1689 | 616 | 617 | 618 | 1650 | 1649 | 1648 | 657 | 656 | 655 | 636 | 1673 | 1677 | 628 | 632 | 1669 | 1578 | 1577 | 1576 | 729 | 728 | 727 | 1543 | 1544 | 1545 | 760 | 761 | 762 | 1506 | 1505 | 1504 | 801 | 800 | 799 | 1471 | 1472 | 1473 | 832 | 833 | 834 | 55320 |
| 588 | 587 | 586 | 1719 | 1718 | 1717 | 613 | 623 | 622 | 1683 | 1682 | 1692 | 660 | 659 | 658 | 1647 | 1646 | 1645 | 1674 | 635 | 634 | 1671 | 1670 | 631 | 732 | 731 | 730 | 1575 | 1574 | 1573 | 757 | 767 | 766 | 1539 | 1538 | 1548 | 804 | 803 | 802 | 1503 | 1502 | 1501 | 829 | 839 | 838 | 1467 | 1466 | 1476 | 55320 |
| 601 | 602 | 603 | 1702 | 1703 | 1704 | 589 | 590 | 591 | 1714 | 1715 | 1716 | 673 | 674 | 675 | 1630 | 1631 | 1632 | 661 | 662 | 663 | 1642 | 1643 | 1644 | 745 | 746 | 747 | 1558 | 1559 | 1560 | 733 | 734 | 735 | 1570 | 1571 | 1572 | 817 | 818 | 819 | 1486 | 1487 | 1488 | 805 | 806 | 807 | 1498 | 1499 | 1500 | 55320 |
| 1699 | 1700 | 1701 | 604 | 605 | 606 | 1711 | 1712 | 1713 | 592 | 593 | 594 | 1627 | 1628 | 1629 | 676 | 677 | 678 | 1639 | 1640 | 1641 | 664 | 665 | 666 | 1555 | 1556 | 1557 | 748 | 749 | 750 | 1567 | 1568 | 1569 | 736 | 737 | 738 | 1483 | 1484 | 1485 | 820 | 821 | 822 | 1495 | 1496 | 1497 | 808 | 809 | 810 | 55320 |
| 1698 | 1697 | 1696 | 609 | 608 | 607 | 1710 | 1709 | 1708 | 597 | 596 | 595 | 1626 | 1625 | 1624 | 681 | 680 | 679 | 1638 | 1637 | 1636 | 669 | 668 | 667 | 1554 | 1553 | 1552 | 753 | 752 | 751 | 1566 | 1565 | 1564 | 741 | 740 | 739 | 1482 | 1481 | 1480 | 825 | 824 | 823 | 1494 | 1493 | 1492 | 813 | 812 | 811 | 55320 |
| 612 | 611 | 610 | 1695 | 1694 | 1693 | 600 | 599 | 598 | 1707 | 1706 | 1705 | 684 | 683 | 682 | 1623 | 1622 | 1621 | 672 | 671 | 670 | 1635 | 1634 | 1633 | 756 | 755 | 754 | 1551 | 1550 | 1549 | 744 | 743 | 742 | 1563 | 1562 | 1561 | 828 | 827 | 826 | 1479 | 1478 | 1477 | 816 | 815 | 814 | 1491 | 1490 | 1489 | 55320 |
| 696 | 686 | 687 | 1618 | 1619 | 1609 | 637 | 638 | 1665 | 640 | 1667 | 1668 | 697 | 698 | 699 | 1606 | 1607 | 1608 | 709 | 710 | 1593 | 712 | 1595 | 1596 | 769 | 770 | 771 | 1534 | 1535 | 1536 | 781 | 782 | 1521 | 784 | 1523 | 1524 | 841 | 842 | 843 | 1462 | 1463 | 1464 | 853 | 854 | 1449 | 856 | 1451 | 1452 | 55320 |
| 1614 | 1613 | 1612 | 693 | 692 | 691 | 1663 | 1664 | 639 | 1666 | 641 | 642 | 1603 | 1604 | 1600 | 705 | 701 | 702 | 1591 | 1592 | 711 | 1594 | 713 | 714 | 1531 | 1532 | 1528 | 777 | 773 | 774 | 1519 | 1520 | 783 | 1522 | 785 | 786 | 1459 | 1460 | 1456 | 849 | 845 | 846 | 1447 | 1448 | 855 | 1450 | 857 | 858 | 55320 |
| 1615 | 1616 | 1617 | 688 | 689 | 690 | 648 | 1661 | 1660 | 645 | 644 | 1657 | 708 | 1601 | 1605 | 700 | 704 | 1597 | 720 | 1589 | 1588 | 717 | 716 | 1585 | 780 | 1529 | 1533 | 772 | 776 | 1525 | 792 | 1517 | 1516 | 789 | 788 | 1513 | 852 | 1457 | 1461 | 844 | 848 | 1453 | 864 | 1445 | 1444 | 861 | 860 | 1441 | 55320 |
| 685 | 695 | 694 | 1611 | 1610 | 1620 | 1662 | 647 | 646 | 1659 | 1658 | 643 | 1602 | 707 | 706 | 1599 | 1598 | 703 | 1590 | 719 | 718 | 1587 | 1586 | 715 | 1530 | 779 | 778 | 1527 | 1526 | 775 | 1518 | 791 | 790 | 1515 | 1514 | 787 | 1458 | 851 | 850 | 1455 | 1454 | 847 | 1446 | 863 | 862 | 1443 | 1442 | 859 | 55320 |
| 865 | 1436 | 867 | 868 | 1439 | 1440 | 1393 | 902 | 903 | 1402 | 1403 | 912 | 937 | 1364 | 939 | 940 | 1367 | 1368 | 984 | 974 | 975 | 1330 | 1331 | 1321 | 1009 | 1292 | 1011 | 1012 | 1295 | 1296 | 1056 | 1046 | 1047 | 1258 | 1259 | 1249 | 1081 | 1220 | 1083 | 1084 | 1223 | 1224 | 1128 | 1118 | 1119 | 1186 | 1187 | 1177 | 55320 |
| 1435 | 866 | 1437 | 1438 | 869 | 870 | 907 | 1397 | 1396 | 909 | 908 | 1398 | 1363 | 938 | 1365 | 1366 | 941 | 942 | 1326 | 1325 | 1324 | 981 | 980 | 979 | 1291 | 1010 | 1293 | 1294 | 1013 | 1014 | 1254 | 1253 | 1252 | 1053 | 1052 | 1051 | 1219 | 1082 | 1221 | 1222 | 1085 | 1086 | 1182 | 1181 | 1180 | 1125 | 1124 | 1123 | 55320 |
| 1434 | 1433 | 1432 | 873 | 872 | 871 | 906 | 1400 | 1401 | 904 | 905 | 1399 | 1362 | 1361 | 1360 | 945 | 944 | 943 | 1327 | 1328 | 1329 | 976 | 977 | 978 | 1290 | 1289 | 1288 | 1017 | 1016 | 1015 | 1255 | 1256 | 1257 | 1048 | 1049 | 1050 | 1218 | 1217 | 1216 | 1089 | 1088 | 1087 | 1183 | 1184 | 1185 | 1120 | 1121 | 1122 | 55320 |
| 876 | 875 | 874 | 1431 | 1430 | 1429 | 1404 | 911 | 910 | 1395 | 1394 | 901 | 948 | 947 | 946 | 1359 | 1358 | 1357 | 973 | 983 | 982 | 1323 | 1322 | 1332 | 1020 | 1019 | 1018 | 1287 | 1286 | 1285 | 1045 | 1055 | 1054 | 1251 | 1250 | 1260 | 1092 | 1091 | 1090 | 1215 | 1214 | 1213 | 1117 | 1127 | 1126 | 1179 | 1178 | 1188 | 55320 |
| 889 | 890 | 891 | 1414 | 1415 | 1416 | 877 | 878 | 879 | 1426 | 1427 | 1428 | 961 | 962 | 963 | 1342 | 1343 | 1344 | 949 | 950 | 951 | 1354 | 1355 | 1356 | 1033 | 1034 | 1035 | 1270 | 1271 | 1272 | 1021 | 1022 | 1023 | 1282 | 1283 | 1284 | 1105 | 1106 | 1107 | 1198 | 1199 | 1200 | 1093 | 1094 | 1095 | 1210 | 1211 | 1212 | 55320 |
| 1411 | 1412 | 1413 | 892 | 894 | 893 | 1423 | 1424 | 1425 | 880 | 881 | 882 | 1339 | 1340 | 1341 | 964 | 965 | 966 | 1351 | 1352 | 1353 | 952 | 953 | 954 | 1267 | 1268 | 1269 | 1036 | 1037 | 1038 | 1279 | 1280 | 1281 | 1024 | 1025 | 1026 | 1195 | 1196 | 1197 | 1108 | 1109 | 1110 | 1207 | 1208 | 1209 | 1096 | 1097 | 1098 | 55320 |
| 1410 | 1409 | 1408 | 897 | 896 | 895 | 1422 | 1421 | 1420 | 885 | 884 | 883 | 1338 | 1337 | 1336 | 969 | 968 | 967 | 1350 | 1349 | 1348 | 957 | 956 | 955 | 1266 | 1265 | 1264 | 1041 | 1040 | 1039 | 1278 | 1277 | 1276 | 1029 | 1028 | 1027 | 1194 | 1193 | 1192 | 1113 | 1112 | 1111 | 1206 | 1205 | 1204 | 1101 | 1100 | 1099 | 55320 |
| 900 | 899 | 898 | 1407 | 1405 | 1406 | 888 | 887 | 886 | 1419 | 1418 | 1417 | 972 | 971 | 970 | 1335 | 1334 | 1333 | 960 | 959 | 958 | 1347 | 1346 | 1345 | 1044 | 1043 | 1042 | 1263 | 1262 | 1261 | 1032 | 1031 | 1030 | 1275 | 1274 | 1273 | 1116 | 1115 | 1114 | 1191 | 1190 | 1189 | 1104 | 1103 | 1102 | 1203 | 1202 | 1201 | 55320 |
| 913 | 914 | 915 | 1390 | 1391 | 1392 | 925 | 926 | 1377 | 928 | 1379 | 1380 | 985 | 986 | 987 | 1318 | 1319 | 1320 | 997 | 998 | 1305 | 1000 | 1307 | 1308 | 1057 | 1058 | 1059 | 1246 | 1247 | 1248 | 1069 | 1070 | 1233 | 1072 | 1235 | 1236 | 1129 | 1130 | 1131 | 1174 | 1175 | 1176 | 1141 | 1142 | 1161 | 1144 | 1163 | 1164 | 55320 |
| 1387 | 1388 | 1384 | 921 | 917 | 918 | 1375 | 1376 | 927 | 1378 | 929 | 930 | 1315 | 1316 | 1312 | 993 | 989 | 990 | 1303 | 1304 | 999 | 1306 | 1001 | 1002 | 1243 | 1244 | 1240 | 1065 | 1061 | 1062 | 1231 | 1232 | 1071 | 1234 | 1073 | 1074 | 1171 | 1172 | 1168 | 1137 | 1133 | 1134 | 1159 | 1160 | 1143 | 1162 | 1145 | 1146 | 55320 |
| 924 | 1385 | 1389 | 916 | 920 | 1381 | 936 | 1373 | 1372 | 933 | 932 | 1369 | 996 | 1313 | 1317 | 988 | 992 | 1309 | 1008 | 1301 | 1300 | 1005 | 1004 | 1297 | 1068 | 1241 | 1245 | 1060 | 1064 | 1237 | 1080 | 1229 | 1228 | 1077 | 1076 | 1225 | 1140 | 1169 | 1173 | 1132 | 1136 | 1165 | 1152 | 1157 | 1156 | 1149 | 1148 | 1153 | 55320 |
| 1386 | 923 | 922 | 1383 | 1382 | 919 | 1374 | 935 | 934 | 1371 | 1370 | 931 | 1314 | 995 | 994 | 1311 | 1310 | 991 | 1302 | 1007 | 1006 | 1299 | 1298 | 1003 | 1242 | 1067 | 1066 | 1239 | 1238 | 1063 | 1230 | 1079 | 1078 | 1227 | 1226 | 1075 | 1170 | 1139 | 1138 | 1167 | 1166 | 1135 | 1158 | 1151 | 1150 | 1155 | 1154 | 1147 | 55320 |
| 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 | 55320 |

*The **magic square sum** is $S_{48\times 48} := 55320$. The magic rectangles of order $4 \times 6$ are of **equal magic sums** given by $R_{4\times 6} := (6915, 4610)$. In this case, the blocks of order $12 \times 12$ are magic squares of equal magic sums.*

Magic Rectangles in Construction of Magic and Block-Bordered Magic Squares, June 07, 2022, pp. 1-70, https://doi.org/10.5281/zenodo.6621071

# 3 Odd Order Magic and Block-Bordered Magic Squares

Below are few odd magic squares constructed with blocks of unequal sums of **magic rectangles**.

## 3.1 Magic Square of Order $15 \times 15$

**Example 3.1.** *A magic square of order* $15 \times 15$ *constructed based on blocks of* ***magic rectangles*** *of order* $3 \times 5$ *given in Example 4.14 is given by*

| | | | | | | | | | | | | | | | 1695 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 209 | 205 | 199 | 200 | 202 | 14 | 10 | 4 | 5 | 7 | 134 | 130 | 124 | 125 | 127 | 1695 |
| 196 | 198 | 203 | 208 | 210 | 1 | 3 | 8 | 13 | 15 | 121 | 123 | 128 | 133 | 135 | 1695 |
| 204 | 206 | 207 | 201 | 197 | 9 | 11 | 12 | 6 | 2 | 129 | 131 | 132 | 126 | 122 | 1695 |
| 149 | 145 | 139 | 140 | 142 | 44 | 40 | 34 | 35 | 37 | 164 | 160 | 154 | 155 | 157 | 1695 |
| 136 | 138 | 143 | 148 | 150 | 31 | 33 | 38 | 43 | 45 | 151 | 153 | 158 | 163 | 165 | 1695 |
| 144 | 146 | 147 | 141 | 137 | 39 | 41 | 42 | 36 | 32 | 159 | 161 | 162 | 156 | 152 | 1695 |
| 59 | 55 | 49 | 50 | 52 | 119 | 115 | 109 | 110 | 112 | 179 | 175 | 169 | 170 | 172 | 1695 |
| 46 | 48 | 53 | 58 | 60 | 106 | 108 | 113 | 118 | 120 | 166 | 168 | 173 | 178 | 180 | 1695 |
| 54 | 56 | 57 | 51 | 47 | 114 | 116 | 117 | 111 | 107 | 174 | 176 | 177 | 171 | 167 | 1695 |
| 74 | 70 | 64 | 65 | 67 | 194 | 190 | 184 | 185 | 187 | 89 | 85 | 79 | 80 | 82 | 1695 |
| 61 | 63 | 68 | 73 | 75 | 181 | 183 | 188 | 193 | 195 | 76 | 78 | 83 | 88 | 90 | 1695 |
| 69 | 71 | 72 | 66 | 62 | 189 | 191 | 192 | 186 | 182 | 84 | 86 | 87 | 81 | 77 | 1695 |
| 104 | 100 | 94 | 95 | 97 | 224 | 220 | 214 | 215 | 217 | 29 | 25 | 19 | 20 | 22 | 1695 |
| 91 | 93 | 98 | 103 | 105 | 211 | 213 | 218 | 223 | 225 | 16 | 18 | 23 | 28 | 30 | 1695 |
| 99 | 101 | 102 | 96 | 92 | 219 | 221 | 222 | 216 | 212 | 24 | 26 | 27 | 21 | 17 | 1695 |
| 1695 | 1695 | 1695 | 1695 | 1695 | 1695 | 1695 | 1695 | 1695 | 1695 | 1695 | 1695 | 1695 | 1695 | 1695 | 1695 |

*In this case the magic sum is* $S_{15\times 15} := 1695$. *The magic rectangles of order* $3 \times 5$ *are of* ***different magic sums***.

## 3.2 Block Bordered Magic Square of Order $17 \times 17$

It is based on the Example 3.1 with single external border, and internal block is magic square of order 15 given in Example 3.1.

**Example 3.2.** *A magic square of order* $17 \times 17$ *is constructed based on magic square of order* $15 \times 15$ *given in Example 3.1. It is given by*

| | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | | | | | 2465 |
| 16 | 288 | 286 | 284 | 282 | 280 | 278 | 276 | 275 | 20 | 22 | 24 | 26 | 28 | 30 | 32 | 18 | 2465 |
| 1 | 241 | 237 | 231 | 232 | 234 | 46 | 42 | 36 | 37 | 39 | 166 | 162 | 156 | 157 | 159 | 289 | 2465 |
| 3 | 228 | 230 | 235 | 240 | 242 | 33 | 35 | 40 | 45 | 47 | 153 | 155 | 160 | 165 | 167 | 287 | 2465 |
| 5 | 236 | 238 | 239 | 233 | 229 | 41 | 43 | 44 | 38 | 34 | 161 | 163 | 164 | 158 | 154 | 285 | 2465 |
| 7 | 181 | 177 | 171 | 172 | 174 | 76 | 72 | 66 | 67 | 69 | 196 | 192 | 186 | 187 | 189 | 283 | 2465 |
| 9 | 168 | 170 | 175 | 180 | 182 | 63 | 65 | 70 | 75 | 77 | 183 | 185 | 190 | 195 | 197 | 281 | 2465 |
| 11 | 176 | 178 | 179 | 173 | 169 | 71 | 73 | 74 | 68 | 64 | 191 | 193 | 194 | 188 | 184 | 279 | 2465 |
| 13 | 91 | 87 | 81 | 82 | 84 | 151 | 147 | 141 | 142 | 144 | 211 | 207 | 201 | 202 | 204 | 277 | 2465 |
| 273 | 78 | 80 | 85 | 90 | 92 | 138 | 140 | 145 | 150 | 152 | 198 | 200 | 205 | 210 | 212 | 17 | 2465 |
| 271 | 86 | 88 | 89 | 83 | 79 | 146 | 148 | 149 | 143 | 139 | 206 | 208 | 209 | 203 | 199 | 19 | 2465 |
| 269 | 106 | 102 | 96 | 97 | 99 | 226 | 222 | 216 | 217 | 219 | 121 | 117 | 111 | 112 | 114 | 21 | 2465 |
| 267 | 93 | 95 | 100 | 105 | 107 | 213 | 215 | 220 | 225 | 227 | 108 | 110 | 115 | 120 | 122 | 23 | 2465 |
| 265 | 101 | 103 | 104 | 98 | 94 | 221 | 223 | 224 | 218 | 214 | 116 | 118 | 119 | 113 | 109 | 25 | 2465 |
| 263 | 136 | 132 | 126 | 127 | 129 | 256 | 252 | 246 | 247 | 249 | 61 | 57 | 51 | 52 | 54 | 27 | 2465 |
| 261 | 123 | 125 | 130 | 135 | 137 | 243 | 245 | 250 | 255 | 257 | 48 | 50 | 55 | 60 | 62 | 29 | 2465 |
| 259 | 131 | 133 | 134 | 128 | 124 | 251 | 253 | 254 | 248 | 244 | 56 | 58 | 59 | 53 | 49 | 31 | 2465 |
| 272 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 15 | 270 | 268 | 266 | 264 | 262 | 260 | 258 | 274 | 2465 |
| 2465 | 2465 | 2465 | 2465 | 2465 | 2465 | 2465 | 2465 | 2465 | 2465 | 2465 | 2465 | 2465 | 2465 | 2465 | 2465 | 2465 | 2465 |

*In this case the magic sum is $S_{17\times17} := 2465$ and $S_{15\times15} := 2175$. The magic rectangles of order $3\times5$ are of* ***different magic sums****.*

## 3.3 Block Bordered Magic Square of Order $19\times19$

It is based on the Example 3.1 with single double external border, and internal block is magic square of order 15 given in Example 3.1.

**Example 3.3.** *A magic square of order $19\times19$ is constructed based on magic square of order $15\times15$ given in Example 3.1. It is given by*

| | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | | | | | | | 3439 |
| 20 | 36 | 34 | 32 | 30 | 28 | 26 | 24 | 22 | 345 | 346 | 348 | 350 | 352 | 354 | 356 | 358 | 360 | 18 | 3439 |
| 361 | 52 | 324 | 322 | 320 | 318 | 316 | 314 | 312 | 311 | 56 | 58 | 60 | 62 | 64 | 66 | 68 | 54 | 1 | 3439 |
| 359 | 37 | 277 | 273 | 267 | 268 | 270 | 82 | 78 | 72 | 73 | 75 | 202 | 198 | 192 | 193 | 195 | 325 | 3 | 3439 |
| 357 | 39 | 264 | 266 | 271 | 276 | 278 | 69 | 71 | 76 | 81 | 83 | 189 | 191 | 196 | 201 | 203 | 323 | 5 | 3439 |
| 355 | 41 | 272 | 274 | 275 | 269 | 265 | 77 | 79 | 80 | 74 | 70 | 197 | 199 | 200 | 194 | 190 | 321 | 7 | 3439 |
| 353 | 43 | 217 | 213 | 207 | 208 | 210 | 112 | 108 | 102 | 103 | 105 | 232 | 228 | 222 | 223 | 225 | 319 | 9 | 3439 |
| 351 | 45 | 204 | 206 | 211 | 216 | 218 | 99 | 101 | 106 | 111 | 113 | 219 | 221 | 226 | 231 | 233 | 317 | 11 | 3439 |
| 349 | 47 | 212 | 214 | 215 | 209 | 205 | 107 | 109 | 110 | 104 | 100 | 227 | 229 | 230 | 224 | 220 | 315 | 13 | 3439 |
| 347 | 49 | 127 | 123 | 117 | 118 | 120 | 187 | 183 | 177 | 178 | 180 | 247 | 243 | 237 | 238 | 240 | 313 | 15 | 3439 |
| 19 | 309 | 114 | 116 | 121 | 126 | 128 | 174 | 176 | 181 | 186 | 188 | 234 | 236 | 241 | 246 | 248 | 53 | 343 | 3439 |
| 21 | 307 | 122 | 124 | 125 | 119 | 115 | 182 | 184 | 185 | 179 | 175 | 242 | 244 | 245 | 239 | 235 | 55 | 341 | 3439 |
| 23 | 305 | 142 | 138 | 132 | 133 | 135 | 262 | 258 | 252 | 253 | 255 | 157 | 153 | 147 | 148 | 150 | 57 | 339 | 3439 |
| 25 | 303 | 129 | 131 | 136 | 141 | 143 | 249 | 251 | 256 | 261 | 263 | 144 | 146 | 151 | 156 | 158 | 59 | 337 | 3439 |
| 27 | 301 | 137 | 139 | 140 | 134 | 130 | 257 | 259 | 260 | 254 | 250 | 152 | 154 | 155 | 149 | 145 | 61 | 335 | 3439 |
| 29 | 299 | 172 | 168 | 162 | 163 | 165 | 292 | 288 | 282 | 283 | 285 | 97 | 93 | 87 | 88 | 90 | 63 | 333 | 3439 |
| 31 | 297 | 159 | 161 | 166 | 171 | 173 | 279 | 281 | 286 | 291 | 293 | 84 | 86 | 91 | 96 | 98 | 65 | 331 | 3439 |
| 33 | 295 | 167 | 169 | 170 | 164 | 160 | 287 | 289 | 290 | 284 | 280 | 92 | 94 | 95 | 89 | 85 | 67 | 329 | 3439 |
| 35 | 308 | 38 | 40 | 42 | 44 | 46 | 48 | 50 | 51 | 306 | 304 | 302 | 300 | 298 | 296 | 294 | 310 | 327 | 3439 |
| 344 | 326 | 328 | 330 | 332 | 334 | 336 | 338 | 340 | 17 | 16 | 14 | 12 | 10 | 8 | 6 | 4 | 2 | 342 | 3439 |
| 3439 | 3439 | 3439 | 3439 | 3439 | 3439 | 3439 | 3439 | 3439 | 3439 | 3439 | 3439 | 3439 | 3439 | 3439 | 3439 | 3439 | 3439 | 3439 | 3439 |

*In this case the magic sum is* $S_{19\times 19} := 3439$, $S_{17\times 17} := 3077$ *and* $S_{15\times 15} := 2715$. *The magic rectangles of order* $3\times 5$ *are of* ***different magic sums***.

## 3.4 Magic Square of Order $21\times 21$

**Example 3.4.** *A magic square of order* $21\times 21$ *constructed based on blocks of magic rectangles of order* $3\times 7$ *is given by*

| | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | | | | | | | | | 4641 |
| 1 | 12 | 13 | 6 | 17 | 20 | 8 | 358 | 369 | 370 | 363 | 374 | 377 | 365 | 274 | 285 | 286 | 279 | 290 | 293 | 281 | 4641 |
| 18 | 19 | 15 | 11 | 7 | 3 | 4 | 375 | 376 | 372 | 368 | 364 | 360 | 361 | 291 | 292 | 288 | 284 | 280 | 276 | 277 | 4641 |
| 14 | 2 | 5 | 16 | 9 | 10 | 21 | 371 | 359 | 362 | 373 | 366 | 367 | 378 | 287 | 275 | 278 | 289 | 282 | 283 | 294 | 4641 |
| 232 | 243 | 244 | 237 | 248 | 251 | 239 | 379 | 390 | 391 | 384 | 395 | 398 | 386 | 22 | 33 | 34 | 27 | 38 | 41 | 29 | 4641 |
| 249 | 250 | 246 | 242 | 238 | 234 | 235 | 396 | 397 | 393 | 389 | 385 | 381 | 382 | 39 | 40 | 36 | 32 | 28 | 24 | 25 | 4641 |
| 245 | 233 | 236 | 247 | 240 | 241 | 252 | 392 | 380 | 383 | 394 | 387 | 388 | 399 | 35 | 23 | 26 | 37 | 30 | 31 | 42 | 4641 |
| 253 | 264 | 265 | 258 | 269 | 272 | 260 | 295 | 306 | 307 | 300 | 311 | 314 | 302 | 85 | 96 | 97 | 90 | 101 | 104 | 92 | 4641 |
| 270 | 271 | 267 | 263 | 259 | 255 | 256 | 312 | 313 | 309 | 305 | 301 | 297 | 298 | 102 | 103 | 99 | 95 | 91 | 87 | 88 | 4641 |
| 266 | 254 | 257 | 268 | 261 | 262 | 273 | 308 | 296 | 299 | 310 | 303 | 304 | 315 | 98 | 86 | 89 | 100 | 93 | 94 | 105 | 4641 |
| 106 | 117 | 118 | 111 | 122 | 125 | 113 | 211 | 222 | 223 | 216 | 227 | 230 | 218 | 316 | 327 | 328 | 321 | 332 | 335 | 323 | 4641 |
| 123 | 124 | 120 | 116 | 112 | 108 | 109 | 228 | 229 | 225 | 221 | 217 | 213 | 214 | 333 | 334 | 330 | 326 | 322 | 318 | 319 | 4641 |
| 119 | 107 | 110 | 121 | 114 | 115 | 126 | 224 | 212 | 215 | 226 | 219 | 220 | 231 | 329 | 317 | 320 | 331 | 324 | 325 | 336 | 4641 |
| 337 | 348 | 349 | 342 | 353 | 356 | 344 | 127 | 138 | 139 | 132 | 143 | 146 | 134 | 169 | 180 | 181 | 174 | 185 | 188 | 176 | 4641 |
| 354 | 355 | 351 | 347 | 343 | 339 | 340 | 144 | 145 | 141 | 137 | 133 | 129 | 130 | 186 | 187 | 183 | 179 | 175 | 171 | 172 | 4641 |
| 350 | 338 | 341 | 352 | 345 | 346 | 357 | 140 | 128 | 131 | 142 | 135 | 136 | 147 | 182 | 170 | 173 | 184 | 177 | 178 | 189 | 4641 |
| 400 | 411 | 412 | 405 | 416 | 419 | 407 | 43 | 54 | 55 | 48 | 59 | 62 | 50 | 190 | 201 | 202 | 195 | 206 | 209 | 197 | 4641 |
| 417 | 418 | 414 | 410 | 406 | 402 | 403 | 60 | 61 | 57 | 53 | 49 | 45 | 46 | 207 | 208 | 204 | 200 | 196 | 192 | 193 | 4641 |
| 413 | 401 | 404 | 415 | 408 | 409 | 420 | 56 | 44 | 47 | 58 | 51 | 52 | 63 | 203 | 191 | 194 | 205 | 198 | 199 | 210 | 4641 |
| 148 | 159 | 160 | 153 | 164 | 167 | 155 | 64 | 75 | 76 | 69 | 80 | 83 | 71 | 421 | 432 | 433 | 426 | 437 | 440 | 428 | 4641 |
| 165 | 166 | 162 | 158 | 154 | 150 | 151 | 81 | 82 | 78 | 74 | 70 | 66 | 67 | 438 | 439 | 435 | 431 | 427 | 423 | 424 | 4641 |
| 161 | 149 | 152 | 163 | 156 | 157 | 168 | 77 | 65 | 68 | 79 | 72 | 73 | 84 | 434 | 422 | 425 | 436 | 429 | 430 | 441 | 4641 |
| 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 |

*In this case the magic sum is $S_{21\times21} := 4641$. The magic rectangles of order $3\times7$ are of* ***different magic sums****.*

## 3.5 Block Bordered Magic Square of Order $23\times23$

It is based on the Example 3.4 with single external border, and internal block is magic square of order 21 given in Example 3.4.

**Example 3.5.** *A magic square of order $23\times23$ is constructed based on magic square of order $21\times21$ given in Example 3.4. It is given by*

| | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | | | | | | | | | | | 6095 |
| 508 | 43 | 41 | 39 | 37 | 35 | 33 | 31 | 29 | 27 | 25 | 23 | 511 | 513 | 515 | 517 | 519 | 521 | 523 | 525 | 527 | 529 | 24 | 6095 |
| 486 | 45 | 56 | 57 | 50 | 61 | 64 | 52 | 402 | 413 | 414 | 407 | 418 | 421 | 409 | 318 | 329 | 330 | 323 | 334 | 337 | 325 | 44 | 6095 |
| 488 | 62 | 63 | 59 | 55 | 51 | 47 | 48 | 419 | 420 | 416 | 412 | 408 | 404 | 405 | 335 | 336 | 332 | 328 | 324 | 320 | 321 | 42 | 6095 |
| 490 | 58 | 46 | 49 | 60 | 53 | 54 | 65 | 415 | 403 | 406 | 417 | 410 | 411 | 422 | 331 | 319 | 322 | 333 | 326 | 327 | 338 | 40 | 6095 |
| 492 | 276 | 287 | 288 | 281 | 292 | 295 | 283 | 423 | 434 | 435 | 428 | 439 | 442 | 430 | 66 | 77 | 78 | 71 | 82 | 85 | 73 | 38 | 6095 |
| 494 | 293 | 294 | 290 | 286 | 282 | 278 | 279 | 440 | 441 | 437 | 433 | 429 | 425 | 426 | 83 | 84 | 80 | 76 | 72 | 68 | 69 | 36 | 6095 |
| 496 | 289 | 277 | 280 | 291 | 284 | 285 | 296 | 436 | 424 | 427 | 438 | 431 | 432 | 443 | 79 | 67 | 70 | 81 | 74 | 75 | 86 | 34 | 6095 |
| 498 | 297 | 308 | 309 | 302 | 313 | 316 | 304 | 339 | 350 | 351 | 344 | 355 | 358 | 346 | 129 | 140 | 141 | 134 | 145 | 148 | 136 | 32 | 6095 |
| 500 | 314 | 315 | 311 | 307 | 303 | 299 | 300 | 356 | 357 | 353 | 349 | 345 | 341 | 342 | 146 | 147 | 143 | 139 | 135 | 131 | 132 | 30 | 6095 |
| 502 | 310 | 298 | 301 | 312 | 305 | 306 | 317 | 352 | 340 | 343 | 354 | 347 | 348 | 359 | 142 | 130 | 133 | 144 | 137 | 138 | 149 | 28 | 6095 |
| 504 | 150 | 161 | 162 | 155 | 166 | 169 | 157 | 255 | 266 | 267 | 260 | 271 | 274 | 262 | 360 | 371 | 372 | 365 | 376 | 379 | 367 | 26 | 6095 |
| 21 | 167 | 168 | 164 | 160 | 156 | 152 | 153 | 272 | 273 | 269 | 265 | 261 | 257 | 258 | 377 | 378 | 374 | 370 | 366 | 362 | 363 | 509 | 6095 |
| 20 | 163 | 151 | 154 | 165 | 158 | 159 | 170 | 268 | 256 | 259 | 270 | 263 | 264 | 275 | 373 | 361 | 364 | 375 | 368 | 369 | 380 | 510 | 6095 |
| 18 | 381 | 392 | 393 | 386 | 397 | 400 | 388 | 171 | 182 | 183 | 176 | 187 | 190 | 178 | 213 | 224 | 225 | 218 | 229 | 232 | 220 | 512 | 6095 |
| 16 | 398 | 399 | 395 | 391 | 387 | 383 | 384 | 188 | 189 | 185 | 181 | 177 | 173 | 174 | 230 | 231 | 227 | 223 | 219 | 215 | 216 | 514 | 6095 |
| 14 | 394 | 382 | 385 | 396 | 389 | 390 | 401 | 184 | 172 | 175 | 186 | 179 | 180 | 191 | 226 | 214 | 217 | 228 | 221 | 222 | 233 | 516 | 6095 |
| 12 | 444 | 455 | 456 | 449 | 460 | 463 | 451 | 87 | 98 | 99 | 92 | 103 | 106 | 94 | 234 | 245 | 246 | 239 | 250 | 253 | 241 | 518 | 6095 |
| 10 | 461 | 462 | 458 | 454 | 450 | 446 | 447 | 104 | 105 | 101 | 97 | 93 | 89 | 90 | 251 | 252 | 248 | 244 | 240 | 236 | 237 | 520 | 6095 |
| 8 | 457 | 445 | 448 | 459 | 452 | 453 | 464 | 100 | 88 | 91 | 102 | 95 | 96 | 107 | 247 | 235 | 238 | 249 | 242 | 243 | 254 | 522 | 6095 |
| 6 | 192 | 203 | 204 | 197 | 208 | 211 | 199 | 108 | 119 | 120 | 113 | 124 | 127 | 115 | 465 | 476 | 477 | 470 | 481 | 484 | 472 | 524 | 6095 |
| 4 | 209 | 210 | 206 | 202 | 198 | 194 | 195 | 125 | 126 | 122 | 118 | 114 | 110 | 111 | 482 | 483 | 479 | 475 | 471 | 467 | 468 | 526 | 6095 |
| 2 | 205 | 193 | 196 | 207 | 200 | 201 | 212 | 121 | 109 | 112 | 123 | 116 | 117 | 128 | 478 | 466 | 469 | 480 | 473 | 474 | 485 | 528 | 6095 |
| 506 | 487 | 489 | 491 | 493 | 495 | 497 | 499 | 501 | 503 | 505 | 507 | 19 | 17 | 15 | 13 | 11 | 9 | 7 | 5 | 3 | 1 | 22 | 6095 |
| 6095 | 6095 | 6095 | 6095 | 6095 | 6095 | 6095 | 6095 | 6095 | 6095 | 6095 | 6095 | 6095 | 6095 | 6095 | 6095 | 6095 | 6095 | 6095 | 6095 | 6095 | 6095 | 6095 | 6095 |

*In this case the magic sum is $S_{23\times 23} := 6095$ and $S_{21\times 21} := 5595$. The magic rectangles of order $3\times 7$ are of* ***different magic sums****.*

## 3.6 Magic Square of Order $27\times 27$

**Example 3.6.** *A magic square of order $27\times 27$ constructed based on blocks of magic rectangles of order $3\times 9$ is given by*

| | | | | | | | | | | | | | | | | | | | | | | | | | | | 9855 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 15 | 5 | 16 | 21 | 22 | 9 | 26 | 11 | 622 | 636 | 626 | 637 | 642 | 643 | 630 | 647 | 632 | 433 | 447 | 437 | 448 | 453 | 454 | 441 | 458 | 443 | 9855 |
| 24 | 25 | 18 | 20 | 14 | 8 | 10 | 3 | 4 | 645 | 646 | 639 | 641 | 635 | 629 | 631 | 624 | 625 | 456 | 457 | 450 | 452 | 446 | 440 | 442 | 435 | 436 | 9855 |
| 17 | 2 | 19 | 6 | 7 | 12 | 23 | 13 | 27 | 638 | 623 | 640 | 627 | 628 | 633 | 644 | 634 | 648 | 449 | 434 | 451 | 438 | 439 | 444 | 455 | 445 | 459 | 9855 |
| 379 | 393 | 383 | 394 | 399 | 400 | 387 | 404 | 389 | 649 | 663 | 653 | 664 | 669 | 670 | 657 | 674 | 659 | 28 | 42 | 32 | 43 | 48 | 49 | 36 | 53 | 38 | 9855 |
| 402 | 403 | 396 | 398 | 392 | 386 | 388 | 381 | 382 | 672 | 673 | 666 | 668 | 662 | 656 | 658 | 651 | 652 | 51 | 52 | 45 | 47 | 41 | 35 | 37 | 30 | 31 | 9855 |
| 395 | 380 | 397 | 384 | 385 | 390 | 401 | 391 | 405 | 665 | 650 | 667 | 654 | 655 | 660 | 671 | 661 | 675 | 44 | 29 | 46 | 33 | 34 | 39 | 50 | 40 | 54 | 9855 |
| 109 | 123 | 113 | 124 | 129 | 130 | 117 | 134 | 119 | 460 | 474 | 464 | 475 | 480 | 481 | 468 | 485 | 470 | 487 | 501 | 491 | 502 | 507 | 508 | 495 | 512 | 497 | 9855 |
| 132 | 133 | 126 | 128 | 122 | 116 | 118 | 111 | 112 | 483 | 484 | 477 | 479 | 473 | 467 | 469 | 462 | 463 | 510 | 511 | 504 | 506 | 500 | 494 | 496 | 489 | 490 | 9855 |
| 125 | 110 | 127 | 114 | 115 | 120 | 131 | 121 | 135 | 476 | 461 | 478 | 465 | 466 | 471 | 482 | 472 | 486 | 503 | 488 | 505 | 492 | 493 | 498 | 509 | 499 | 513 | 9855 |
| 406 | 420 | 410 | 421 | 426 | 427 | 414 | 431 | 416 | 514 | 528 | 518 | 529 | 534 | 535 | 522 | 539 | 524 | 136 | 150 | 140 | 151 | 156 | 157 | 144 | 161 | 146 | 9855 |
| 429 | 430 | 423 | 425 | 419 | 413 | 415 | 408 | 409 | 537 | 538 | 531 | 533 | 527 | 521 | 523 | 516 | 517 | 159 | 160 | 153 | 155 | 149 | 143 | 145 | 138 | 139 | 9855 |
| 422 | 407 | 424 | 411 | 412 | 417 | 428 | 418 | 432 | 530 | 515 | 532 | 519 | 520 | 525 | 536 | 526 | 540 | 152 | 137 | 154 | 141 | 142 | 147 | 158 | 148 | 162 | 9855 |
| 541 | 555 | 545 | 556 | 561 | 562 | 549 | 566 | 551 | 352 | 366 | 356 | 367 | 372 | 373 | 360 | 377 | 362 | 163 | 177 | 167 | 178 | 183 | 184 | 171 | 188 | 173 | 9855 |
| 564 | 565 | 558 | 560 | 554 | 548 | 550 | 543 | 544 | 375 | 376 | 369 | 371 | 365 | 359 | 361 | 354 | 355 | 186 | 187 | 180 | 182 | 176 | 170 | 172 | 165 | 166 | 9855 |
| 557 | 542 | 559 | 546 | 547 | 552 | 563 | 553 | 567 | 368 | 353 | 370 | 357 | 358 | 363 | 374 | 364 | 378 | 179 | 164 | 181 | 168 | 169 | 174 | 185 | 175 | 189 | 9855 |
| 568 | 582 | 572 | 583 | 588 | 589 | 576 | 593 | 578 | 190 | 204 | 194 | 205 | 210 | 211 | 198 | 215 | 200 | 298 | 312 | 302 | 313 | 318 | 319 | 306 | 323 | 308 | 9855 |
| 591 | 592 | 585 | 587 | 581 | 575 | 577 | 570 | 571 | 213 | 214 | 207 | 209 | 203 | 197 | 199 | 192 | 193 | 321 | 322 | 315 | 317 | 311 | 305 | 307 | 300 | 301 | 9855 |
| 584 | 569 | 586 | 573 | 574 | 579 | 590 | 580 | 594 | 206 | 191 | 208 | 195 | 196 | 201 | 212 | 202 | 216 | 314 | 299 | 316 | 303 | 304 | 309 | 320 | 310 | 324 | 9855 |
| 217 | 231 | 221 | 232 | 237 | 238 | 225 | 242 | 227 | 244 | 258 | 248 | 259 | 264 | 265 | 252 | 269 | 254 | 595 | 609 | 599 | 610 | 615 | 616 | 603 | 620 | 605 | 9855 |
| 240 | 241 | 234 | 236 | 230 | 224 | 226 | 219 | 220 | 267 | 268 | 261 | 263 | 257 | 251 | 253 | 246 | 247 | 618 | 619 | 612 | 614 | 608 | 602 | 604 | 597 | 598 | 9855 |
| 233 | 218 | 235 | 222 | 223 | 228 | 239 | 229 | 243 | 260 | 245 | 262 | 249 | 250 | 255 | 266 | 256 | 270 | 611 | 596 | 613 | 600 | 601 | 606 | 617 | 607 | 621 | 9855 |
| 676 | 690 | 680 | 691 | 696 | 697 | 684 | 701 | 686 | 55 | 69 | 59 | 70 | 75 | 76 | 63 | 80 | 65 | 325 | 339 | 329 | 340 | 345 | 346 | 333 | 350 | 335 | 9855 |
| 699 | 700 | 693 | 695 | 689 | 683 | 685 | 678 | 679 | 78 | 79 | 72 | 74 | 68 | 62 | 64 | 57 | 58 | 348 | 349 | 342 | 344 | 338 | 332 | 334 | 327 | 328 | 9855 |
| 692 | 677 | 694 | 681 | 682 | 687 | 698 | 688 | 702 | 71 | 56 | 73 | 60 | 61 | 66 | 77 | 67 | 81 | 341 | 326 | 343 | 330 | 331 | 336 | 347 | 337 | 351 | 9855 |
| 271 | 285 | 275 | 286 | 291 | 292 | 279 | 296 | 281 | 82 | 96 | 86 | 97 | 102 | 103 | 90 | 107 | 92 | 703 | 717 | 707 | 718 | 723 | 724 | 711 | 728 | 713 | 9855 |
| 294 | 295 | 288 | 290 | 284 | 278 | 280 | 273 | 274 | 105 | 106 | 99 | 101 | 95 | 89 | 91 | 84 | 85 | 726 | 727 | 720 | 722 | 716 | 710 | 712 | 705 | 706 | 9855 |
| 287 | 272 | 289 | 276 | 277 | 282 | 293 | 283 | 297 | 98 | 83 | 100 | 87 | 88 | 93 | 104 | 94 | 108 | 719 | 704 | 721 | 708 | 709 | 714 | 725 | 715 | 729 | 9855 |
| 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 |

In this case the magic sum is $S_{27\times 27} := 9855$. The magic rectangles of order $3\times 9$ are of **different magic sums**.

## 3.7 Block Bordered Magic Square of Order $29\times 29$

It is based on the Example 3.6 with single external border, and internal block is magic square of order 27, where magic rectangles are of order $3\times 9$.

**Example 3.7.** *A magic square of order $29\times 29$ is constructed based on magic square of order $27\times 27$ given in Example 3.6. It is given by*

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 12209 |
| 814 | 786 | 788 | 790 | 792 | 794 | 796 | 798 | 800 | 802 | 804 | 806 | 808 | 810 | 27 | 26 | 24 | 22 | 20 | 18 | 16 | 14 | 12 | 10 | 8 | 6 | 4 | 2 | 812 | 12209 |
| 55 | 57 | 71 | 61 | 72 | 77 | 78 | 65 | 82 | 67 | 678 | 692 | 682 | 693 | 698 | 699 | 686 | 703 | 688 | 489 | 503 | 493 | 504 | 509 | 510 | 497 | 514 | 499 | 787 | 12209 |
| 53 | 80 | 81 | 74 | 76 | 70 | 64 | 66 | 59 | 60 | 701 | 702 | 695 | 697 | 691 | 685 | 687 | 680 | 681 | 512 | 513 | 506 | 508 | 502 | 496 | 498 | 491 | 492 | 789 | 12209 |
| 51 | 73 | 58 | 75 | 62 | 63 | 68 | 79 | 69 | 83 | 694 | 679 | 696 | 683 | 684 | 689 | 700 | 690 | 704 | 505 | 490 | 507 | 494 | 495 | 500 | 511 | 501 | 515 | 791 | 12209 |
| 49 | 435 | 449 | 439 | 450 | 455 | 456 | 443 | 460 | 445 | 705 | 719 | 709 | 720 | 725 | 726 | 713 | 730 | 715 | 84 | 98 | 88 | 99 | 104 | 105 | 92 | 109 | 94 | 793 | 12209 |
| 47 | 458 | 459 | 452 | 454 | 448 | 442 | 444 | 437 | 438 | 728 | 729 | 722 | 724 | 718 | 712 | 714 | 707 | 708 | 107 | 108 | 101 | 103 | 97 | 91 | 93 | 86 | 87 | 795 | 12209 |
| 45 | 451 | 436 | 453 | 440 | 441 | 446 | 457 | 447 | 461 | 721 | 706 | 723 | 710 | 711 | 716 | 727 | 717 | 731 | 100 | 85 | 102 | 89 | 90 | 95 | 106 | 96 | 110 | 797 | 12209 |
| 43 | 165 | 179 | 169 | 180 | 185 | 186 | 173 | 190 | 175 | 516 | 530 | 520 | 531 | 536 | 537 | 524 | 541 | 526 | 543 | 557 | 547 | 558 | 563 | 564 | 551 | 568 | 553 | 799 | 12209 |
| 41 | 188 | 189 | 182 | 184 | 178 | 172 | 174 | 167 | 168 | 539 | 540 | 533 | 535 | 529 | 523 | 525 | 518 | 519 | 566 | 567 | 560 | 562 | 556 | 550 | 552 | 545 | 546 | 801 | 12209 |
| 39 | 181 | 166 | 183 | 170 | 171 | 176 | 187 | 177 | 191 | 532 | 517 | 534 | 521 | 522 | 527 | 538 | 528 | 542 | 559 | 544 | 561 | 548 | 549 | 554 | 565 | 555 | 569 | 803 | 12209 |
| 37 | 462 | 476 | 466 | 477 | 482 | 483 | 470 | 487 | 472 | 570 | 584 | 574 | 585 | 590 | 591 | 578 | 595 | 580 | 192 | 206 | 196 | 207 | 212 | 213 | 200 | 217 | 202 | 805 | 12209 |
| 35 | 485 | 486 | 479 | 481 | 475 | 469 | 471 | 464 | 465 | 593 | 594 | 587 | 589 | 583 | 577 | 579 | 572 | 573 | 215 | 216 | 209 | 211 | 205 | 199 | 201 | 194 | 195 | 807 | 12209 |
| 33 | 478 | 463 | 480 | 467 | 468 | 473 | 484 | 474 | 488 | 586 | 571 | 588 | 575 | 576 | 581 | 592 | 582 | 596 | 208 | 193 | 210 | 197 | 198 | 203 | 214 | 204 | 218 | 809 | 12209 |
| 31 | 597 | 611 | 601 | 612 | 617 | 618 | 605 | 622 | 607 | 408 | 422 | 412 | 423 | 428 | 429 | 416 | 433 | 418 | 219 | 233 | 223 | 234 | 239 | 240 | 227 | 244 | 229 | 811 | 12209 |
| 29 | 620 | 621 | 614 | 616 | 610 | 604 | 606 | 599 | 600 | 431 | 432 | 425 | 427 | 421 | 415 | 417 | 410 | 411 | 242 | 243 | 236 | 238 | 232 | 226 | 228 | 221 | 222 | 813 | 12209 |
| 817 | 613 | 598 | 615 | 602 | 603 | 608 | 619 | 609 | 623 | 424 | 409 | 426 | 413 | 414 | 419 | 430 | 420 | 434 | 235 | 220 | 237 | 224 | 225 | 230 | 241 | 231 | 245 | 25 | 12209 |
| 819 | 624 | 638 | 628 | 639 | 644 | 645 | 632 | 649 | 634 | 246 | 260 | 250 | 261 | 266 | 267 | 254 | 271 | 256 | 354 | 368 | 358 | 369 | 374 | 375 | 362 | 379 | 364 | 23 | 12209 |
| 821 | 647 | 648 | 641 | 643 | 637 | 631 | 633 | 626 | 627 | 269 | 270 | 263 | 265 | 259 | 253 | 255 | 248 | 249 | 377 | 378 | 371 | 373 | 367 | 361 | 363 | 356 | 357 | 21 | 12209 |
| 823 | 640 | 625 | 642 | 629 | 630 | 635 | 646 | 636 | 650 | 262 | 247 | 264 | 251 | 252 | 257 | 268 | 258 | 272 | 370 | 355 | 372 | 359 | 360 | 365 | 376 | 366 | 380 | 19 | 12209 |
| 825 | 273 | 287 | 277 | 288 | 293 | 294 | 281 | 298 | 283 | 300 | 314 | 304 | 315 | 320 | 321 | 308 | 325 | 310 | 651 | 665 | 655 | 666 | 671 | 672 | 659 | 676 | 661 | 17 | 12209 |
| 827 | 296 | 297 | 290 | 292 | 286 | 280 | 282 | 275 | 276 | 323 | 324 | 317 | 319 | 313 | 307 | 309 | 302 | 303 | 674 | 675 | 668 | 670 | 664 | 658 | 660 | 653 | 654 | 15 | 12209 |
| 829 | 289 | 274 | 291 | 278 | 279 | 284 | 295 | 285 | 299 | 316 | 301 | 318 | 305 | 306 | 311 | 322 | 312 | 326 | 667 | 652 | 669 | 656 | 657 | 662 | 673 | 663 | 677 | 13 | 12209 |
| 831 | 732 | 746 | 736 | 747 | 752 | 753 | 740 | 757 | 742 | 111 | 125 | 115 | 126 | 131 | 132 | 119 | 136 | 121 | 381 | 395 | 385 | 396 | 401 | 402 | 389 | 406 | 391 | 11 | 12209 |
| 833 | 755 | 756 | 749 | 751 | 745 | 739 | 741 | 734 | 735 | 134 | 135 | 128 | 130 | 124 | 118 | 120 | 113 | 114 | 404 | 405 | 398 | 400 | 394 | 388 | 390 | 383 | 384 | 9 | 12209 |
| 835 | 748 | 733 | 750 | 737 | 738 | 743 | 754 | 744 | 758 | 127 | 112 | 129 | 116 | 117 | 122 | 133 | 123 | 137 | 397 | 382 | 399 | 386 | 387 | 392 | 403 | 393 | 407 | 7 | 12209 |
| 837 | 327 | 341 | 331 | 342 | 347 | 348 | 335 | 352 | 337 | 138 | 152 | 142 | 153 | 158 | 159 | 146 | 163 | 148 | 759 | 773 | 763 | 774 | 779 | 780 | 767 | 784 | 769 | 5 | 12209 |
| 839 | 350 | 351 | 344 | 346 | 340 | 334 | 336 | 329 | 330 | 161 | 162 | 155 | 157 | 151 | 145 | 147 | 140 | 141 | 782 | 783 | 776 | 778 | 772 | 766 | 768 | 761 | 762 | 3 | 12209 |
| 841 | 343 | 328 | 345 | 332 | 333 | 338 | 349 | 339 | 353 | 154 | 139 | 156 | 143 | 144 | 149 | 160 | 150 | 164 | 775 | 760 | 777 | 764 | 765 | 770 | 781 | 771 | 785 | 1 | 12209 |
| 30 | 56 | 54 | 52 | 50 | 48 | 46 | 44 | 42 | 40 | 38 | 36 | 34 | 32 | 815 | 816 | 818 | 820 | 822 | 824 | 826 | 828 | 830 | 832 | 834 | 836 | 838 | 840 | 28 | 12209 |
| 12209 | 12209 | 12209 | 12209 | 12209 | 12209 | 12209 | 12209 | 12209 | 12209 | 12209 | 12209 | 12209 | 12209 | 12209 | 12209 | 12209 | 12209 | 12209 | 12209 | 12209 | 12209 | 12209 | 12209 | 12209 | 12209 | 12209 | 12209 | 12209 | 12209 |

*In this case the magic sum is $S_{29\times 29}:=12209$ and $S_{27\times 27}:=11367$. The magic rectangles of order $3\times 9$ are of* ***different magic sums****.*

## 3.8 Block Bordered Magic Square of Order $31\times 31$

It is based on the Example 3.6 with single external border, and internal block is magic square of order 27, where magic rectangles are of order $3\times 9$

**Example 3.8.** *A magic square of order $31\times 31$ is constructed based on magic square of order $27\times 27$ given in Example 3.6. It is given by*

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 14911 |
| 932 | 902 | 904 | 906 | 908 | 910 | 912 | 914 | 916 | 918 | 920 | 922 | 924 | 926 | 928 | 29 | 28 | 26 | 24 | 22 | 20 | 18 | 16 | 14 | 12 | 10 | 8 | 6 | 4 | 2 | 930 | 14911 |
| 59 | 874 | 846 | 848 | 850 | 852 | 854 | 856 | 858 | 860 | 862 | 864 | 866 | 868 | 870 | 87 | 86 | 84 | 82 | 80 | 78 | 76 | 74 | 72 | 70 | 68 | 66 | 64 | 62 | 872 | 903 | 14911 |
| 57 | 115 | 117 | 131 | 121 | 132 | 137 | 138 | 125 | 142 | 127 | 738 | 752 | 742 | 753 | 758 | 759 | 746 | 763 | 748 | 549 | 563 | 553 | 564 | 569 | 570 | 557 | 574 | 559 | 847 | 905 | 14911 |
| 55 | 113 | 140 | 141 | 134 | 136 | 130 | 124 | 126 | 119 | 120 | 761 | 762 | 755 | 757 | 751 | 745 | 747 | 740 | 741 | 572 | 573 | 566 | 568 | 562 | 556 | 558 | 551 | 552 | 849 | 907 | 14911 |
| 53 | 111 | 133 | 118 | 135 | 122 | 123 | 128 | 139 | 129 | 143 | 754 | 739 | 756 | 743 | 744 | 749 | 760 | 750 | 764 | 565 | 550 | 567 | 554 | 555 | 560 | 571 | 561 | 575 | 851 | 909 | 14911 |
| 51 | 109 | 495 | 509 | 499 | 510 | 515 | 516 | 503 | 520 | 505 | 765 | 779 | 769 | 780 | 785 | 786 | 773 | 790 | 775 | 144 | 158 | 148 | 159 | 164 | 165 | 152 | 169 | 154 | 853 | 911 | 14911 |
| 49 | 107 | 518 | 519 | 512 | 514 | 508 | 502 | 504 | 497 | 498 | 788 | 789 | 782 | 784 | 778 | 772 | 774 | 767 | 768 | 167 | 168 | 161 | 163 | 157 | 151 | 153 | 146 | 147 | 855 | 913 | 14911 |
| 47 | 105 | 511 | 496 | 513 | 500 | 501 | 506 | 517 | 507 | 521 | 781 | 766 | 783 | 770 | 771 | 776 | 787 | 777 | 791 | 160 | 145 | 162 | 149 | 150 | 155 | 166 | 156 | 170 | 857 | 915 | 14911 |
| 45 | 103 | 225 | 239 | 229 | 240 | 245 | 246 | 233 | 250 | 235 | 576 | 590 | 580 | 591 | 596 | 597 | 584 | 601 | 586 | 603 | 617 | 607 | 618 | 623 | 624 | 611 | 628 | 613 | 859 | 917 | 14911 |
| 43 | 101 | 248 | 249 | 242 | 244 | 238 | 232 | 234 | 227 | 228 | 599 | 600 | 593 | 595 | 589 | 583 | 585 | 578 | 579 | 626 | 627 | 620 | 622 | 616 | 610 | 612 | 605 | 606 | 861 | 919 | 14911 |
| 41 | 99 | 241 | 226 | 243 | 230 | 231 | 236 | 247 | 237 | 251 | 592 | 577 | 594 | 581 | 582 | 587 | 598 | 588 | 602 | 619 | 604 | 621 | 608 | 609 | 614 | 625 | 615 | 629 | 863 | 921 | 14911 |
| 39 | 97 | 522 | 536 | 526 | 537 | 542 | 543 | 530 | 547 | 532 | 630 | 644 | 634 | 645 | 650 | 651 | 638 | 655 | 640 | 252 | 266 | 256 | 267 | 272 | 273 | 260 | 277 | 262 | 865 | 923 | 14911 |
| 37 | 95 | 545 | 546 | 539 | 541 | 535 | 529 | 531 | 524 | 525 | 653 | 654 | 647 | 649 | 643 | 637 | 639 | 632 | 633 | 275 | 276 | 269 | 271 | 265 | 259 | 261 | 254 | 255 | 867 | 925 | 14911 |
| 35 | 93 | 538 | 523 | 540 | 527 | 528 | 533 | 544 | 534 | 548 | 646 | 631 | 648 | 635 | 636 | 641 | 652 | 642 | 656 | 268 | 253 | 270 | 257 | 258 | 263 | 274 | 264 | 278 | 869 | 927 | 14911 |
| 33 | 91 | 657 | 671 | 661 | 672 | 677 | 678 | 665 | 682 | 667 | 468 | 482 | 472 | 483 | 488 | 489 | 476 | 493 | 478 | 279 | 293 | 283 | 294 | 299 | 300 | 287 | 304 | 289 | 871 | 929 | 14911 |
| 31 | 89 | 680 | 681 | 674 | 676 | 670 | 664 | 666 | 659 | 660 | 491 | 492 | 485 | 487 | 481 | 475 | 477 | 470 | 471 | 302 | 303 | 296 | 298 | 292 | 286 | 288 | 281 | 282 | 873 | 931 | 14911 |
| 935 | 877 | 673 | 658 | 675 | 662 | 663 | 668 | 679 | 669 | 683 | 484 | 469 | 486 | 473 | 474 | 479 | 490 | 480 | 494 | 295 | 280 | 297 | 284 | 285 | 290 | 301 | 291 | 305 | 85 | 27 | 14911 |
| 937 | 879 | 684 | 698 | 688 | 699 | 704 | 705 | 692 | 709 | 694 | 306 | 320 | 310 | 321 | 326 | 327 | 314 | 331 | 316 | 414 | 428 | 418 | 429 | 434 | 435 | 422 | 439 | 424 | 83 | 25 | 14911 |
| 939 | 881 | 707 | 708 | 701 | 703 | 697 | 691 | 693 | 686 | 687 | 329 | 330 | 323 | 325 | 319 | 313 | 315 | 308 | 309 | 437 | 438 | 431 | 433 | 427 | 421 | 423 | 416 | 417 | 81 | 23 | 14911 |
| 941 | 883 | 700 | 685 | 702 | 689 | 690 | 695 | 706 | 696 | 710 | 322 | 307 | 324 | 311 | 312 | 317 | 328 | 318 | 332 | 430 | 415 | 432 | 419 | 420 | 425 | 436 | 426 | 440 | 79 | 21 | 14911 |
| 943 | 885 | 333 | 347 | 337 | 348 | 353 | 354 | 341 | 358 | 343 | 360 | 374 | 364 | 375 | 380 | 381 | 368 | 385 | 370 | 711 | 725 | 715 | 726 | 731 | 732 | 719 | 736 | 721 | 77 | 19 | 14911 |
| 945 | 887 | 356 | 357 | 350 | 352 | 346 | 340 | 342 | 335 | 336 | 383 | 384 | 377 | 379 | 373 | 367 | 369 | 362 | 363 | 734 | 735 | 728 | 730 | 724 | 718 | 720 | 713 | 714 | 75 | 17 | 14911 |
| 947 | 889 | 349 | 334 | 351 | 338 | 339 | 344 | 355 | 345 | 359 | 376 | 361 | 378 | 365 | 366 | 371 | 382 | 372 | 386 | 727 | 712 | 729 | 716 | 717 | 722 | 733 | 723 | 737 | 73 | 15 | 14911 |
| 949 | 891 | 792 | 806 | 796 | 807 | 812 | 813 | 800 | 817 | 802 | 171 | 185 | 175 | 186 | 191 | 192 | 179 | 196 | 181 | 441 | 455 | 445 | 456 | 461 | 462 | 449 | 466 | 451 | 71 | 13 | 14911 |
| 951 | 893 | 815 | 816 | 809 | 811 | 805 | 799 | 801 | 794 | 795 | 194 | 195 | 188 | 190 | 184 | 178 | 180 | 173 | 174 | 464 | 465 | 458 | 460 | 454 | 448 | 450 | 443 | 444 | 69 | 11 | 14911 |
| 953 | 895 | 808 | 793 | 810 | 797 | 798 | 803 | 814 | 804 | 818 | 187 | 172 | 189 | 176 | 177 | 182 | 193 | 183 | 197 | 457 | 442 | 459 | 446 | 447 | 452 | 463 | 453 | 467 | 67 | 9 | 14911 |
| 955 | 897 | 387 | 401 | 391 | 402 | 407 | 408 | 395 | 412 | 397 | 198 | 212 | 202 | 213 | 218 | 219 | 206 | 223 | 208 | 819 | 833 | 823 | 834 | 839 | 840 | 827 | 844 | 829 | 65 | 7 | 14911 |
| 957 | 899 | 410 | 411 | 404 | 406 | 400 | 394 | 396 | 389 | 390 | 221 | 222 | 215 | 217 | 211 | 205 | 207 | 200 | 201 | 842 | 843 | 836 | 838 | 832 | 826 | 828 | 821 | 822 | 63 | 5 | 14911 |
| 959 | 901 | 403 | 388 | 405 | 392 | 393 | 398 | 409 | 399 | 413 | 214 | 199 | 216 | 203 | 204 | 209 | 220 | 210 | 224 | 835 | 820 | 837 | 824 | 825 | 830 | 841 | 831 | 845 | 61 | 3 | 14911 |
| 961 | 90 | 116 | 114 | 112 | 110 | 108 | 106 | 104 | 102 | 100 | 98 | 96 | 94 | 92 | 875 | 876 | 878 | 880 | 882 | 884 | 886 | 888 | 890 | 892 | 894 | 896 | 898 | 900 | 88 | 1 | 14911 |
| 32 | 60 | 58 | 56 | 54 | 52 | 50 | 48 | 46 | 44 | 42 | 40 | 38 | 36 | 34 | 933 | 934 | 936 | 938 | 940 | 942 | 944 | 946 | 948 | 950 | 952 | 954 | 956 | 958 | 960 | 30 | 14911 |
| 14911 | 14911 | 14911 | 14911 | 14911 | 14911 | 14911 | 14911 | 14911 | 14911 | 14911 | 14911 | 14911 | 14911 | 14911 | 14911 | 14911 | 14911 | 14911 | 14911 | 14911 | 14911 | 14911 | 14911 | 14911 | 14911 | 14911 | 14911 | 14911 | 14911 | 14911 | 14911 |

*In this case the magic sums are* $S_{31\times31} := 14911$, $S_{29\times29} := 13949$ *and* $S_{27\times27} := 12987$*. The magic rectangles of order* $3\times9$ *are of* ***different magic sums****.*

## 3.9 Magic Square of Order $33 \times 33$

**Example 3.9.** *A magic square of order* $33\times33$ *constructed based on blocks of magic rectangles of order* $3\times11$ *is given by*

 Magic Rectangles in Construction of Magic and Block-Bordered Magic Squares, June 07, 2022, pp. 1-70, *https://doi.org/10.5281/zenodo.6621071*

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 17985 |
| 715 | 722 | 696 | 700 | 717 | 702 | 719 | 706 | 709 | 725 | 699 | 22 | 29 | 3 | 7 | 24 | 9 | 26 | 13 | 16 | 32 | 6 | 913 | 920 | 894 | 898 | 915 | 900 | 917 | 904 | 907 | 923 | 897 | 17985 |
| 694 | 713 | 723 | 716 | 712 | 710 | 708 | 704 | 697 | 707 | 726 | 1 | 20 | 30 | 23 | 19 | 17 | 15 | 11 | 4 | 14 | 33 | 892 | 911 | 921 | 914 | 910 | 908 | 906 | 902 | 895 | 905 | 924 | 17985 |
| 721 | 695 | 711 | 714 | 701 | 718 | 703 | 720 | 724 | 698 | 705 | 28 | 2 | 18 | 21 | 8 | 25 | 10 | 27 | 31 | 5 | 12 | 919 | 893 | 909 | 912 | 899 | 916 | 901 | 918 | 922 | 896 | 903 | 17985 |
| 946 | 953 | 927 | 931 | 948 | 933 | 950 | 937 | 940 | 956 | 930 | 649 | 656 | 630 | 634 | 651 | 636 | 653 | 640 | 643 | 659 | 633 | 55 | 62 | 36 | 40 | 57 | 42 | 59 | 46 | 49 | 65 | 39 | 17985 |
| 925 | 944 | 954 | 947 | 943 | 941 | 939 | 935 | 928 | 938 | 957 | 628 | 647 | 657 | 650 | 646 | 644 | 642 | 638 | 631 | 641 | 660 | 34 | 53 | 63 | 56 | 52 | 50 | 48 | 44 | 37 | 47 | 66 | 17985 |
| 952 | 926 | 942 | 945 | 932 | 949 | 934 | 951 | 955 | 929 | 936 | 655 | 629 | 645 | 648 | 635 | 652 | 637 | 654 | 658 | 632 | 639 | 61 | 35 | 51 | 54 | 41 | 58 | 43 | 60 | 64 | 38 | 45 | 17985 |
| 88 | 95 | 69 | 73 | 90 | 75 | 92 | 79 | 82 | 98 | 72 | 979 | 986 | 960 | 964 | 981 | 966 | 983 | 970 | 973 | 989 | 963 | 583 | 590 | 564 | 568 | 585 | 570 | 587 | 574 | 577 | 593 | 567 | 17985 |
| 67 | 86 | 96 | 89 | 85 | 83 | 81 | 77 | 70 | 80 | 99 | 958 | 977 | 987 | 980 | 976 | 974 | 972 | 968 | 961 | 971 | 990 | 562 | 581 | 591 | 584 | 580 | 578 | 576 | 572 | 565 | 575 | 594 | 17985 |
| 94 | 68 | 84 | 87 | 74 | 91 | 76 | 93 | 97 | 71 | 78 | 985 | 959 | 975 | 978 | 965 | 982 | 967 | 984 | 988 | 962 | 969 | 589 | 563 | 579 | 582 | 569 | 586 | 571 | 588 | 592 | 566 | 573 | 17985 |
| 220 | 227 | 201 | 205 | 222 | 207 | 224 | 211 | 214 | 230 | 204 | 748 | 755 | 729 | 733 | 750 | 735 | 752 | 739 | 742 | 758 | 732 | 682 | 689 | 663 | 667 | 684 | 669 | 686 | 673 | 676 | 692 | 666 | 17985 |
| 199 | 218 | 228 | 221 | 217 | 215 | 213 | 209 | 202 | 212 | 231 | 727 | 746 | 756 | 749 | 745 | 743 | 741 | 737 | 730 | 740 | 759 | 661 | 680 | 690 | 683 | 679 | 677 | 675 | 671 | 664 | 674 | 693 | 17985 |
| 226 | 200 | 216 | 219 | 206 | 223 | 208 | 225 | 229 | 203 | 210 | 754 | 728 | 744 | 747 | 734 | 751 | 736 | 753 | 757 | 731 | 738 | 688 | 662 | 678 | 681 | 668 | 685 | 670 | 687 | 691 | 665 | 672 | 17985 |
| 781 | 788 | 762 | 766 | 783 | 768 | 785 | 772 | 775 | 791 | 765 | 616 | 623 | 597 | 601 | 618 | 603 | 620 | 607 | 610 | 626 | 600 | 253 | 260 | 234 | 238 | 255 | 240 | 257 | 244 | 247 | 263 | 237 | 17985 |
| 760 | 779 | 789 | 782 | 778 | 776 | 774 | 770 | 763 | 773 | 792 | 595 | 614 | 624 | 617 | 613 | 611 | 609 | 605 | 598 | 608 | 627 | 232 | 251 | 261 | 254 | 250 | 248 | 246 | 242 | 235 | 245 | 264 | 17985 |
| 787 | 761 | 777 | 780 | 767 | 784 | 769 | 786 | 790 | 764 | 771 | 622 | 596 | 612 | 615 | 602 | 619 | 604 | 621 | 625 | 599 | 606 | 259 | 233 | 249 | 252 | 239 | 256 | 241 | 258 | 262 | 236 | 243 | 17985 |
| 286 | 293 | 267 | 271 | 288 | 273 | 290 | 277 | 280 | 296 | 270 | 550 | 557 | 531 | 535 | 552 | 537 | 554 | 541 | 544 | 560 | 534 | 814 | 821 | 795 | 799 | 816 | 801 | 818 | 805 | 808 | 824 | 798 | 17985 |
| 265 | 284 | 294 | 287 | 283 | 281 | 279 | 275 | 268 | 278 | 297 | 529 | 548 | 558 | 551 | 547 | 545 | 543 | 539 | 532 | 542 | 561 | 793 | 812 | 822 | 815 | 811 | 809 | 807 | 803 | 796 | 806 | 825 | 17985 |
| 292 | 266 | 282 | 285 | 272 | 289 | 274 | 291 | 295 | 269 | 276 | 556 | 530 | 546 | 549 | 536 | 553 | 538 | 555 | 559 | 533 | 540 | 820 | 794 | 810 | 813 | 800 | 817 | 802 | 819 | 823 | 797 | 804 | 17985 |
| 847 | 854 | 828 | 832 | 849 | 834 | 851 | 838 | 841 | 857 | 831 | 484 | 491 | 465 | 469 | 486 | 471 | 488 | 475 | 478 | 494 | 468 | 319 | 326 | 300 | 304 | 321 | 306 | 323 | 310 | 313 | 329 | 303 | 17985 |
| 826 | 845 | 855 | 848 | 844 | 842 | 840 | 836 | 829 | 839 | 858 | 463 | 482 | 492 | 485 | 481 | 479 | 477 | 473 | 466 | 476 | 495 | 298 | 317 | 327 | 320 | 316 | 314 | 312 | 308 | 301 | 311 | 330 | 17985 |
| 853 | 827 | 843 | 846 | 833 | 850 | 835 | 852 | 856 | 830 | 837 | 490 | 464 | 480 | 483 | 470 | 487 | 472 | 489 | 493 | 467 | 474 | 325 | 299 | 315 | 318 | 305 | 322 | 307 | 324 | 328 | 302 | 309 | 17985 |
| 418 | 425 | 399 | 403 | 420 | 405 | 422 | 409 | 412 | 428 | 402 | 352 | 359 | 333 | 337 | 354 | 339 | 356 | 343 | 346 | 362 | 336 | 880 | 887 | 861 | 865 | 882 | 867 | 884 | 871 | 874 | 890 | 864 | 17985 |
| 397 | 416 | 426 | 419 | 415 | 413 | 411 | 407 | 400 | 410 | 429 | 331 | 350 | 360 | 353 | 349 | 347 | 345 | 341 | 334 | 344 | 363 | 859 | 878 | 888 | 881 | 877 | 875 | 873 | 869 | 862 | 872 | 891 | 17985 |
| 424 | 398 | 414 | 417 | 404 | 421 | 406 | 423 | 427 | 401 | 408 | 358 | 332 | 348 | 351 | 338 | 355 | 340 | 357 | 361 | 335 | 342 | 886 | 860 | 876 | 879 | 866 | 883 | 868 | 885 | 889 | 863 | 870 | 17985 |
| 517 | 524 | 498 | 502 | 519 | 504 | 521 | 508 | 511 | 527 | 501 | 121 | 128 | 102 | 106 | 123 | 108 | 125 | 112 | 115 | 131 | 105 | 1012 | 1019 | 993 | 997 | 1014 | 999 | 1016 | 1003 | 1006 | 1022 | 996 | 17985 |
| 496 | 515 | 525 | 518 | 514 | 512 | 510 | 506 | 499 | 509 | 528 | 100 | 119 | 129 | 122 | 118 | 116 | 114 | 110 | 103 | 113 | 132 | 991 | 1010 | 1020 | 1013 | 1009 | 1007 | 1005 | 1001 | 994 | 1004 | 1023 | 17985 |
| 523 | 497 | 513 | 516 | 503 | 520 | 505 | 522 | 526 | 500 | 507 | 127 | 101 | 117 | 120 | 107 | 124 | 109 | 126 | 130 | 104 | 111 | 1018 | 992 | 1008 | 1011 | 998 | 1015 | 1000 | 1017 | 1021 | 995 | 1002 | 17985 |
| 1045 | 1052 | 1026 | 1030 | 1047 | 1032 | 1049 | 1036 | 1039 | 1055 | 1029 | 451 | 458 | 432 | 436 | 453 | 438 | 455 | 442 | 445 | 461 | 435 | 154 | 161 | 135 | 139 | 156 | 141 | 158 | 145 | 148 | 164 | 138 | 17985 |
| 1024 | 1043 | 1053 | 1046 | 1042 | 1040 | 1038 | 1034 | 1027 | 1037 | 1056 | 430 | 449 | 459 | 452 | 448 | 446 | 444 | 440 | 433 | 443 | 462 | 133 | 152 | 162 | 155 | 151 | 149 | 147 | 143 | 136 | 146 | 165 | 17985 |
| 1051 | 1025 | 1041 | 1044 | 1031 | 1048 | 1033 | 1050 | 1054 | 1028 | 1035 | 457 | 431 | 447 | 450 | 437 | 454 | 439 | 456 | 460 | 434 | 441 | 160 | 134 | 150 | 153 | 140 | 157 | 142 | 159 | 163 | 137 | 144 | 17985 |
| 187 | 194 | 168 | 172 | 189 | 174 | 191 | 178 | 181 | 197 | 171 | 1078 | 1085 | 1059 | 1063 | 1080 | 1065 | 1082 | 1069 | 1072 | 1088 | 1062 | 385 | 392 | 366 | 370 | 387 | 372 | 389 | 376 | 379 | 395 | 369 | 17985 |
| 166 | 185 | 195 | 188 | 184 | 182 | 180 | 176 | 169 | 179 | 198 | 1057 | 1076 | 1086 | 1079 | 1075 | 1073 | 1071 | 1067 | 1060 | 1070 | 1089 | 364 | 383 | 393 | 386 | 382 | 380 | 378 | 374 | 367 | 377 | 396 | 17985 |
| 193 | 167 | 183 | 186 | 173 | 190 | 175 | 192 | 196 | 170 | 177 | 1084 | 1058 | 1074 | 1077 | 1064 | 1081 | 1066 | 1083 | 1087 | 1061 | 1068 | 391 | 365 | 381 | 384 | 371 | 388 | 373 | 390 | 394 | 368 | 375 | 17985 |
| 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 |

*In this case the magic sum is $S_{33\times 33} := 17985$. The magic rectangles of order $3\times 11$ are of **different magic sums**.*

## 3.10 Magic Square of Order $35\times 35$

**Example 3.10.** *A magic square of order $35\times 35$ constructed based on blocks of magic rectangles of order $5\times 7$ is given by*

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 21455 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 505 | 516 | 503 | 496 | 510 | 514 | 512 | 15 | 26 | 13 | 6 | 20 | 24 | 22 | 960 | 971 | 958 | 951 | 965 | 969 | 967 | 1100 | 1111 | 1098 | 1091 | 1105 | 1109 | 1107 | 470 | 481 | 468 | 461 | 475 | 479 | 477 | 21455 |
| 491 | 523 | 517 | 501 | 521 | 509 | 494 | 1 | 33 | 27 | 11 | 31 | 19 | 4 | 946 | 978 | 972 | 956 | 976 | 964 | 949 | 1086 | 1118 | 1112 | 1096 | 1116 | 1104 | 1089 | 456 | 488 | 482 | 466 | 486 | 474 | 459 | 21455 |
| 518 | 492 | 519 | 508 | 497 | 524 | 498 | 28 | 2 | 29 | 18 | 7 | 34 | 8 | 973 | 947 | 974 | 963 | 952 | 979 | 953 | 1113 | 1087 | 1114 | 1103 | 1092 | 1119 | 1093 | 483 | 457 | 484 | 473 | 462 | 489 | 463 | 21455 |
| 522 | 507 | 495 | 515 | 499 | 493 | 525 | 32 | 17 | 5 | 25 | 9 | 3 | 35 | 977 | 962 | 950 | 970 | 954 | 948 | 980 | 1117 | 1102 | 1090 | 1110 | 1094 | 1088 | 1120 | 487 | 472 | 460 | 480 | 464 | 458 | 490 | 21455 |
| 504 | 502 | 506 | 520 | 513 | 500 | 511 | 14 | 12 | 16 | 30 | 23 | 10 | 21 | 959 | 957 | 961 | 975 | 968 | 955 | 966 | 1099 | 1097 | 1101 | 1115 | 1108 | 1095 | 1106 | 469 | 467 | 471 | 485 | 478 | 465 | 476 | 21455 |
| 890 | 901 | 888 | 881 | 895 | 899 | 897 | 1135 | 1146 | 1133 | 1126 | 1140 | 1144 | 1142 | 50 | 61 | 48 | 41 | 55 | 59 | 57 | 575 | 586 | 573 | 566 | 580 | 584 | 582 | 400 | 411 | 398 | 391 | 405 | 409 | 407 | 21455 |
| 876 | 908 | 902 | 886 | 906 | 894 | 879 | 1121 | 1153 | 1147 | 1131 | 1151 | 1139 | 1124 | 36 | 68 | 62 | 46 | 66 | 54 | 39 | 561 | 593 | 587 | 571 | 591 | 579 | 564 | 386 | 418 | 412 | 396 | 416 | 404 | 389 | 21455 |
| 903 | 877 | 904 | 893 | 882 | 909 | 883 | 1148 | 1122 | 1149 | 1138 | 1127 | 1154 | 1128 | 63 | 37 | 64 | 53 | 42 | 69 | 43 | 588 | 562 | 589 | 578 | 567 | 594 | 568 | 413 | 387 | 414 | 403 | 392 | 419 | 393 | 21455 |
| 907 | 892 | 880 | 900 | 884 | 878 | 910 | 1152 | 1137 | 1125 | 1145 | 1129 | 1123 | 1155 | 67 | 52 | 40 | 60 | 44 | 38 | 70 | 592 | 577 | 565 | 585 | 569 | 563 | 595 | 417 | 402 | 390 | 410 | 394 | 388 | 420 | 21455 |
| 889 | 887 | 891 | 905 | 898 | 885 | 896 | 1134 | 1132 | 1136 | 1150 | 1143 | 1130 | 1141 | 49 | 47 | 51 | 65 | 58 | 45 | 56 | 574 | 572 | 576 | 590 | 583 | 570 | 581 | 399 | 397 | 401 | 415 | 408 | 395 | 406 | 21455 |
| 435 | 446 | 433 | 426 | 440 | 444 | 442 | 925 | 936 | 923 | 916 | 930 | 934 | 932 | 995 | 1006 | 993 | 986 | 1000 | 1004 | 1002 | 155 | 166 | 153 | 146 | 160 | 164 | 162 | 540 | 551 | 538 | 531 | 545 | 549 | 547 | 21455 |
| 421 | 453 | 447 | 431 | 451 | 439 | 424 | 911 | 943 | 937 | 921 | 941 | 929 | 914 | 981 | 1013 | 1007 | 991 | 1011 | 999 | 984 | 141 | 173 | 167 | 151 | 171 | 159 | 144 | 526 | 558 | 552 | 536 | 556 | 544 | 529 | 21455 |
| 448 | 422 | 449 | 438 | 427 | 454 | 428 | 938 | 912 | 939 | 928 | 917 | 944 | 918 | 1008 | 982 | 1009 | 998 | 987 | 1014 | 988 | 168 | 142 | 169 | 158 | 147 | 174 | 148 | 553 | 527 | 554 | 543 | 532 | 559 | 533 | 21455 |
| 452 | 437 | 425 | 445 | 429 | 423 | 455 | 942 | 927 | 915 | 935 | 919 | 913 | 945 | 1012 | 997 | 985 | 1005 | 989 | 983 | 1015 | 172 | 157 | 145 | 165 | 149 | 143 | 175 | 557 | 542 | 530 | 550 | 534 | 528 | 560 | 21455 |
| 434 | 432 | 436 | 450 | 443 | 430 | 441 | 924 | 922 | 926 | 940 | 933 | 920 | 931 | 994 | 992 | 996 | 1010 | 1003 | 990 | 1001 | 154 | 152 | 156 | 170 | 163 | 150 | 161 | 539 | 537 | 541 | 555 | 548 | 535 | 546 | 21455 |
| 190 | 201 | 188 | 181 | 195 | 199 | 197 | 365 | 376 | 363 | 356 | 370 | 374 | 372 | 610 | 621 | 608 | 601 | 615 | 619 | 617 | 855 | 866 | 853 | 846 | 860 | 864 | 862 | 1030 | 1041 | 1028 | 1021 | 1035 | 1039 | 1037 | 21455 |
| 176 | 208 | 202 | 186 | 206 | 194 | 179 | 351 | 383 | 377 | 361 | 381 | 369 | 354 | 596 | 628 | 622 | 606 | 626 | 614 | 599 | 841 | 873 | 867 | 851 | 871 | 859 | 844 | 1016 | 1048 | 1042 | 1026 | 1046 | 1034 | 1019 | 21455 |
| 203 | 177 | 204 | 193 | 182 | 209 | 183 | 378 | 352 | 379 | 368 | 357 | 384 | 358 | 623 | 597 | 624 | 613 | 602 | 629 | 603 | 868 | 842 | 869 | 858 | 847 | 874 | 848 | 1043 | 1017 | 1044 | 1033 | 1022 | 1049 | 1023 | 21455 |
| 207 | 192 | 180 | 200 | 184 | 178 | 210 | 382 | 367 | 355 | 375 | 359 | 353 | 385 | 627 | 612 | 600 | 620 | 604 | 598 | 630 | 872 | 857 | 845 | 865 | 849 | 843 | 875 | 1047 | 1032 | 1020 | 1040 | 1024 | 1018 | 1050 | 21455 |
| 189 | 187 | 191 | 205 | 198 | 185 | 196 | 364 | 362 | 366 | 380 | 373 | 360 | 371 | 609 | 607 | 611 | 625 | 618 | 605 | 616 | 854 | 852 | 856 | 870 | 863 | 850 | 861 | 1029 | 1027 | 1031 | 1045 | 1038 | 1025 | 1036 | 21455 |
| 680 | 691 | 678 | 671 | 685 | 689 | 687 | 1065 | 1076 | 1063 | 1056 | 1070 | 1074 | 1072 | 225 | 236 | 223 | 216 | 230 | 234 | 232 | 295 | 306 | 293 | 286 | 300 | 304 | 302 | 785 | 796 | 783 | 776 | 790 | 794 | 792 | 21455 |
| 666 | 698 | 692 | 676 | 696 | 684 | 669 | 1051 | 1083 | 1077 | 1061 | 1081 | 1069 | 1054 | 211 | 243 | 237 | 221 | 241 | 229 | 214 | 281 | 313 | 307 | 291 | 311 | 299 | 284 | 771 | 803 | 797 | 781 | 801 | 789 | 774 | 21455 |
| 693 | 667 | 694 | 683 | 672 | 699 | 673 | 1078 | 1052 | 1079 | 1068 | 1057 | 1084 | 1058 | 238 | 212 | 239 | 228 | 217 | 244 | 218 | 308 | 282 | 309 | 298 | 287 | 314 | 288 | 798 | 772 | 799 | 788 | 777 | 804 | 778 | 21455 |
| 697 | 682 | 670 | 690 | 674 | 668 | 700 | 1082 | 1067 | 1055 | 1075 | 1059 | 1053 | 1085 | 242 | 227 | 215 | 235 | 219 | 213 | 245 | 312 | 297 | 285 | 305 | 289 | 283 | 315 | 802 | 787 | 775 | 795 | 779 | 773 | 805 | 21455 |
| 679 | 677 | 681 | 695 | 688 | 675 | 686 | 1064 | 1062 | 1066 | 1080 | 1073 | 1060 | 1071 | 224 | 222 | 226 | 240 | 233 | 220 | 231 | 294 | 292 | 296 | 310 | 303 | 290 | 301 | 784 | 782 | 786 | 800 | 793 | 780 | 791 | 21455 |
| 820 | 831 | 818 | 811 | 825 | 829 | 827 | 645 | 656 | 643 | 636 | 650 | 654 | 652 | 1170 | 1181 | 1168 | 1161 | 1175 | 1179 | 1177 | 85 | 96 | 83 | 76 | 90 | 94 | 92 | 330 | 341 | 328 | 321 | 335 | 339 | 337 | 21455 |
| 806 | 838 | 832 | 816 | 836 | 824 | 809 | 631 | 663 | 657 | 641 | 661 | 649 | 634 | 1156 | 1188 | 1182 | 1166 | 1186 | 1174 | 1159 | 71 | 103 | 97 | 81 | 101 | 89 | 74 | 316 | 348 | 342 | 326 | 346 | 334 | 319 | 21455 |
| 833 | 807 | 834 | 823 | 812 | 839 | 813 | 658 | 632 | 659 | 648 | 637 | 664 | 638 | 1183 | 1157 | 1184 | 1173 | 1162 | 1189 | 1163 | 98 | 72 | 99 | 88 | 77 | 104 | 78 | 343 | 317 | 344 | 333 | 322 | 349 | 323 | 21455 |
| 837 | 822 | 810 | 830 | 814 | 808 | 840 | 662 | 647 | 635 | 655 | 639 | 633 | 665 | 1187 | 1172 | 1160 | 1180 | 1164 | 1158 | 1190 | 102 | 87 | 75 | 95 | 79 | 73 | 105 | 347 | 332 | 320 | 340 | 324 | 318 | 350 | 21455 |
| 819 | 817 | 821 | 835 | 828 | 815 | 826 | 644 | 642 | 646 | 660 | 653 | 640 | 651 | 1169 | 1167 | 1171 | 1185 | 1178 | 1165 | 1176 | 84 | 82 | 86 | 100 | 93 | 80 | 91 | 329 | 327 | 331 | 345 | 338 | 325 | 336 | 21455 |
| 750 | 761 | 748 | 741 | 755 | 759 | 757 | 120 | 131 | 118 | 111 | 125 | 129 | 127 | 260 | 271 | 258 | 251 | 265 | 269 | 267 | 1205 | 1216 | 1203 | 1196 | 1210 | 1214 | 1212 | 715 | 726 | 713 | 706 | 720 | 724 | 722 | 21455 |
| 736 | 768 | 762 | 746 | 766 | 754 | 739 | 106 | 138 | 132 | 116 | 136 | 124 | 109 | 246 | 278 | 272 | 256 | 276 | 264 | 249 | 1191 | 1223 | 1217 | 1201 | 1221 | 1209 | 1194 | 701 | 733 | 727 | 711 | 731 | 719 | 704 | 21455 |
| 763 | 737 | 764 | 753 | 742 | 769 | 743 | 133 | 107 | 134 | 123 | 112 | 139 | 113 | 273 | 247 | 274 | 263 | 252 | 279 | 253 | 1218 | 1192 | 1219 | 1208 | 1197 | 1224 | 1198 | 728 | 702 | 729 | 718 | 707 | 734 | 708 | 21455 |
| 767 | 752 | 740 | 760 | 744 | 738 | 770 | 137 | 122 | 110 | 130 | 114 | 108 | 140 | 277 | 262 | 250 | 270 | 254 | 248 | 280 | 1222 | 1207 | 1195 | 1215 | 1199 | 1193 | 1225 | 732 | 717 | 705 | 725 | 709 | 703 | 735 | 21455 |
| 749 | 747 | 751 | 765 | 758 | 745 | 756 | 119 | 117 | 121 | 135 | 128 | 115 | 126 | 259 | 257 | 261 | 275 | 268 | 255 | 266 | 1204 | 1202 | 1206 | 1220 | 1213 | 1200 | 1211 | 714 | 712 | 716 | 730 | 723 | 710 | 721 | 21455 |
| 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 |

*In this case the magic sum is $S_{35\times 35} := 21455$. The magic rectangles of order $5\times 7$ are of **different magic sums**.*

## 3.11 Block Bordered Magic Square of Order $37\times 37$

It is based on the Examples 3.9 and 3.10 with double external borders, and internal blocks are magic squares of orders 33 and 35, where the magic rectangles are of orders $3\times 11$ and $5\times 7$.

**Example 3.11.** *A magic square of order $37\times 37$ is constructed based on magic square of order $33\times 33$ given in Example 3.9. It is given by*

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 25345 |
| 1334 | 1298 | 1300 | 1302 | 1304 | 1306 | 1308 | 1310 | 1312 | 1314 | 1316 | 1318 | 1320 | 1322 | 1324 | 1326 | 1328 | 1330 | 35 | 34 | 32 | 30 | 28 | 26 | 24 | 22 | 20 | 18 | 16 | 14 | 12 | 10 | 8 | 6 | 4 | 2 | 1332 | 25345 |
| 71 | 1264 | 1230 | 1232 | 1234 | 1236 | 1238 | 1240 | 1242 | 1244 | 1246 | 1248 | 1250 | 1252 | 1254 | 1256 | 1258 | 1260 | 105 | 104 | 102 | 100 | 98 | 96 | 94 | 92 | 90 | 88 | 86 | 84 | 82 | 80 | 78 | 76 | 74 | 1262 | 1299 | 25345 |
| 69 | 139 | 855 | 862 | 836 | 840 | 857 | 842 | 859 | 846 | 849 | 865 | 839 | 162 | 169 | 143 | 147 | 164 | 149 | 166 | 153 | 156 | 172 | 146 | 1053 | 1060 | 1034 | 1038 | 1055 | 1040 | 1057 | 1044 | 1047 | 1063 | 1037 | 1231 | 1301 | 25345 |
| 67 | 137 | 834 | 853 | 863 | 856 | 852 | 850 | 848 | 844 | 837 | 847 | 866 | 141 | 160 | 170 | 163 | 159 | 157 | 155 | 151 | 144 | 154 | 173 | 1032 | 1051 | 1061 | 1054 | 1050 | 1048 | 1046 | 1042 | 1035 | 1045 | 1064 | 1233 | 1303 | 25345 |
| 65 | 135 | 861 | 835 | 851 | 854 | 841 | 858 | 843 | 860 | 864 | 838 | 845 | 168 | 142 | 158 | 161 | 148 | 165 | 150 | 167 | 171 | 145 | 152 | 1059 | 1033 | 1049 | 1052 | 1039 | 1056 | 1041 | 1058 | 1062 | 1036 | 1043 | 1235 | 1305 | 25345 |
| 63 | 133 | 1086 | 1093 | 1067 | 1071 | 1088 | 1073 | 1090 | 1077 | 1080 | 1096 | 1070 | 789 | 796 | 770 | 774 | 791 | 776 | 793 | 780 | 783 | 799 | 773 | 195 | 202 | 176 | 180 | 197 | 182 | 199 | 186 | 189 | 205 | 179 | 1237 | 1307 | 25345 |
| 61 | 131 | 1065 | 1084 | 1094 | 1087 | 1083 | 1081 | 1079 | 1075 | 1068 | 1078 | 1097 | 768 | 787 | 797 | 790 | 786 | 784 | 782 | 778 | 771 | 781 | 800 | 174 | 193 | 203 | 196 | 192 | 190 | 188 | 184 | 177 | 187 | 206 | 1239 | 1309 | 25345 |
| 59 | 129 | 1092 | 1066 | 1082 | 1085 | 1072 | 1089 | 1074 | 1091 | 1095 | 1069 | 1076 | 795 | 769 | 785 | 788 | 775 | 792 | 777 | 794 | 798 | 772 | 779 | 201 | 175 | 191 | 194 | 181 | 198 | 183 | 200 | 204 | 178 | 185 | 1241 | 1311 | 25345 |
| 57 | 127 | 228 | 235 | 209 | 213 | 230 | 215 | 232 | 219 | 222 | 238 | 212 | 1119 | 1126 | 1100 | 1104 | 1121 | 1106 | 1123 | 1110 | 1113 | 1129 | 1103 | 723 | 730 | 704 | 708 | 725 | 710 | 727 | 714 | 717 | 733 | 707 | 1243 | 1313 | 25345 |
| 55 | 125 | 207 | 226 | 236 | 229 | 225 | 223 | 221 | 217 | 210 | 220 | 239 | 1098 | 1117 | 1127 | 1120 | 1116 | 1114 | 1112 | 1108 | 1101 | 1111 | 1130 | 702 | 721 | 731 | 724 | 720 | 718 | 716 | 712 | 705 | 715 | 734 | 1245 | 1315 | 25345 |
| 53 | 123 | 234 | 208 | 224 | 227 | 214 | 231 | 216 | 233 | 237 | 211 | 218 | 1125 | 1099 | 1115 | 1118 | 1105 | 1122 | 1107 | 1124 | 1128 | 1102 | 1109 | 729 | 703 | 719 | 722 | 709 | 726 | 711 | 728 | 732 | 706 | 713 | 1247 | 1317 | 25345 |
| 51 | 121 | 360 | 367 | 341 | 345 | 362 | 347 | 364 | 351 | 354 | 370 | 344 | 888 | 895 | 869 | 873 | 890 | 875 | 892 | 879 | 882 | 898 | 872 | 822 | 829 | 803 | 807 | 824 | 809 | 826 | 813 | 816 | 832 | 806 | 1249 | 1319 | 25345 |
| 49 | 119 | 339 | 358 | 368 | 361 | 357 | 355 | 353 | 349 | 342 | 352 | 371 | 867 | 886 | 896 | 889 | 885 | 883 | 881 | 877 | 870 | 880 | 899 | 801 | 820 | 830 | 823 | 819 | 817 | 815 | 811 | 804 | 814 | 833 | 1251 | 1321 | 25345 |
| 47 | 117 | 366 | 340 | 356 | 359 | 346 | 363 | 348 | 365 | 369 | 343 | 350 | 894 | 868 | 884 | 887 | 874 | 891 | 876 | 893 | 897 | 871 | 878 | 828 | 802 | 818 | 821 | 808 | 825 | 810 | 827 | 831 | 805 | 812 | 1253 | 1323 | 25345 |
| 45 | 115 | 921 | 928 | 902 | 906 | 923 | 908 | 925 | 912 | 915 | 931 | 905 | 756 | 763 | 737 | 741 | 758 | 743 | 760 | 747 | 750 | 766 | 740 | 393 | 400 | 374 | 378 | 395 | 380 | 397 | 384 | 387 | 403 | 377 | 1255 | 1325 | 25345 |
| 43 | 113 | 900 | 919 | 929 | 922 | 918 | 916 | 914 | 910 | 903 | 913 | 932 | 735 | 754 | 764 | 757 | 753 | 751 | 749 | 745 | 738 | 748 | 767 | 372 | 391 | 401 | 394 | 390 | 388 | 386 | 382 | 375 | 385 | 404 | 1257 | 1327 | 25345 |
| 41 | 111 | 927 | 901 | 917 | 920 | 907 | 924 | 909 | 926 | 930 | 904 | 911 | 762 | 736 | 752 | 755 | 742 | 759 | 744 | 761 | 765 | 739 | 746 | 399 | 373 | 389 | 392 | 379 | 396 | 381 | 398 | 402 | 376 | 383 | 1259 | 1329 | 25345 |
| 39 | 109 | 426 | 433 | 407 | 411 | 428 | 413 | 430 | 417 | 420 | 436 | 410 | 690 | 697 | 671 | 675 | 692 | 677 | 694 | 681 | 684 | 700 | 674 | 954 | 961 | 935 | 939 | 956 | 941 | 958 | 945 | 948 | 964 | 938 | 1261 | 1331 | 25345 |
| 37 | 107 | 405 | 424 | 434 | 427 | 423 | 421 | 419 | 415 | 408 | 418 | 437 | 669 | 688 | 698 | 691 | 687 | 685 | 683 | 679 | 672 | 682 | 701 | 933 | 952 | 962 | 955 | 951 | 949 | 947 | 943 | 936 | 946 | 965 | 1263 | 1333 | 25345 |
| 1337 | 1267 | 432 | 406 | 422 | 425 | 412 | 429 | 414 | 431 | 435 | 409 | 416 | 696 | 670 | 686 | 689 | 676 | 693 | 678 | 695 | 699 | 673 | 680 | 960 | 934 | 950 | 953 | 940 | 957 | 942 | 959 | 963 | 937 | 944 | 103 | 33 | 25345 |
| 1339 | 1269 | 987 | 994 | 968 | 972 | 989 | 974 | 991 | 978 | 981 | 997 | 971 | 624 | 631 | 605 | 609 | 626 | 611 | 628 | 615 | 618 | 634 | 608 | 459 | 466 | 440 | 444 | 461 | 446 | 463 | 450 | 453 | 469 | 443 | 101 | 31 | 25345 |
| 1341 | 1271 | 966 | 985 | 995 | 988 | 984 | 982 | 980 | 976 | 969 | 979 | 998 | 603 | 622 | 632 | 625 | 621 | 619 | 617 | 613 | 606 | 616 | 635 | 438 | 457 | 467 | 460 | 456 | 454 | 452 | 448 | 441 | 451 | 470 | 99 | 29 | 25345 |
| 1343 | 1273 | 993 | 967 | 983 | 986 | 973 | 990 | 975 | 992 | 996 | 970 | 977 | 630 | 604 | 620 | 623 | 610 | 627 | 612 | 629 | 633 | 607 | 614 | 465 | 439 | 455 | 458 | 445 | 462 | 447 | 464 | 468 | 442 | 449 | 97 | 27 | 25345 |
| 1345 | 1275 | 558 | 565 | 539 | 543 | 560 | 545 | 562 | 549 | 552 | 568 | 542 | 492 | 499 | 473 | 477 | 494 | 479 | 496 | 483 | 486 | 502 | 476 | 1020 | 1027 | 1001 | 1005 | 1022 | 1007 | 1024 | 1011 | 1014 | 1030 | 1004 | 95 | 25 | 25345 |
| 1347 | 1277 | 537 | 556 | 566 | 559 | 555 | 553 | 551 | 547 | 540 | 550 | 569 | 471 | 490 | 500 | 493 | 489 | 487 | 485 | 481 | 474 | 484 | 503 | 999 | 1018 | 1028 | 1021 | 1017 | 1015 | 1013 | 1009 | 1002 | 1012 | 1031 | 93 | 23 | 25345 |
| 1349 | 1279 | 564 | 538 | 554 | 557 | 544 | 561 | 546 | 563 | 567 | 541 | 548 | 498 | 472 | 488 | 491 | 478 | 495 | 480 | 497 | 501 | 475 | 482 | 1026 | 1000 | 1016 | 1019 | 1006 | 1023 | 1008 | 1025 | 1029 | 1003 | 1010 | 91 | 21 | 25345 |
| 1351 | 1281 | 657 | 664 | 638 | 642 | 659 | 644 | 661 | 648 | 651 | 667 | 641 | 261 | 268 | 242 | 246 | 263 | 248 | 265 | 252 | 255 | 271 | 245 | 1152 | 1159 | 1133 | 1137 | 1154 | 1139 | 1156 | 1143 | 1146 | 1162 | 1136 | 89 | 19 | 25345 |
| 1353 | 1283 | 636 | 655 | 665 | 658 | 654 | 652 | 650 | 646 | 639 | 649 | 668 | 240 | 259 | 269 | 262 | 258 | 256 | 254 | 250 | 243 | 253 | 272 | 1131 | 1150 | 1160 | 1153 | 1149 | 1147 | 1145 | 1141 | 1134 | 1144 | 1163 | 87 | 17 | 25345 |
| 1355 | 1285 | 663 | 637 | 653 | 656 | 643 | 660 | 645 | 662 | 666 | 640 | 647 | 267 | 241 | 257 | 260 | 247 | 264 | 249 | 266 | 270 | 244 | 251 | 1158 | 1132 | 1148 | 1151 | 1138 | 1155 | 1140 | 1157 | 1161 | 1135 | 1142 | 85 | 15 | 25345 |
| 1357 | 1287 | 1185 | 1192 | 1166 | 1170 | 1187 | 1172 | 1189 | 1176 | 1179 | 1195 | 1169 | 591 | 598 | 572 | 576 | 593 | 578 | 595 | 582 | 585 | 601 | 575 | 294 | 301 | 275 | 279 | 296 | 281 | 298 | 285 | 288 | 304 | 278 | 83 | 13 | 25345 |
| 1359 | 1289 | 1164 | 1183 | 1193 | 1186 | 1182 | 1180 | 1178 | 1174 | 1167 | 1177 | 1196 | 570 | 589 | 599 | 592 | 588 | 586 | 584 | 580 | 573 | 583 | 602 | 273 | 292 | 302 | 295 | 291 | 289 | 287 | 283 | 276 | 286 | 305 | 81 | 11 | 25345 |
| 1361 | 1291 | 1191 | 1165 | 1181 | 1184 | 1171 | 1188 | 1173 | 1190 | 1194 | 1168 | 1175 | 597 | 571 | 587 | 590 | 577 | 594 | 579 | 596 | 600 | 574 | 581 | 300 | 274 | 290 | 293 | 280 | 297 | 282 | 299 | 303 | 277 | 284 | 79 | 9 | 25345 |
| 1363 | 1293 | 327 | 334 | 308 | 312 | 329 | 314 | 331 | 318 | 321 | 337 | 311 | 1218 | 1225 | 1199 | 1203 | 1220 | 1205 | 1222 | 1209 | 1212 | 1228 | 1202 | 525 | 532 | 506 | 510 | 527 | 512 | 529 | 516 | 519 | 535 | 509 | 77 | 7 | 25345 |
| 1365 | 1295 | 306 | 325 | 335 | 328 | 324 | 322 | 320 | 316 | 309 | 319 | 338 | 1197 | 1216 | 1226 | 1219 | 1215 | 1213 | 1211 | 1207 | 1200 | 1210 | 1229 | 504 | 523 | 533 | 526 | 522 | 520 | 518 | 514 | 507 | 517 | 536 | 75 | 5 | 25345 |
| 1367 | 1297 | 333 | 307 | 323 | 326 | 313 | 330 | 315 | 332 | 336 | 310 | 317 | 1224 | 1198 | 1214 | 1217 | 1204 | 1221 | 1206 | 1223 | 1227 | 1201 | 1208 | 531 | 505 | 521 | 524 | 511 | 528 | 513 | 530 | 534 | 508 | 515 | 73 | 3 | 25345 |
| 1369 | 108 | 140 | 138 | 136 | 134 | 132 | 130 | 128 | 126 | 124 | 122 | 120 | 118 | 116 | 114 | 112 | 110 | 1265 | 1266 | 1268 | 1270 | 1272 | 1274 | 1276 | 1278 | 1280 | 1282 | 1284 | 1286 | 1288 | 1290 | 1292 | 1294 | 1296 | 106 | 1 | 25345 |
| 38 | 72 | 70 | 68 | 66 | 64 | 62 | 60 | 58 | 56 | 54 | 52 | 50 | 48 | 46 | 44 | 42 | 40 | 1335 | 1336 | 1338 | 1340 | 1342 | 1344 | 1346 | 1348 | 1350 | 1352 | 1354 | 1356 | 1358 | 1360 | 1362 | 1364 | 1366 | 1368 | 36 | 25345 |
| 25345 | 25345 | 25345 | 25345 | 25345 | 25345 | 25345 | 25345 | 25345 | 25345 | 25345 | 25345 | 25345 | 25345 | 25345 | 25345 | 25345 | 25345 | 25345 | 25345 | 25345 | 25345 | 25345 | 25345 | 25345 | 25345 | 25345 | 25345 | 25345 | 25345 | 25345 | 25345 | 25345 | 25345 | 25345 | 25345 | 25345 | 25345 |

*In this case the magic sums are $S_{37\times 37} := 25345$, $S_{35\times 35} := 23975$ and $S_{33\times 33} := 22605$. The magic rectangles of order $3\times 9$ are of **different magic sums**.*

**Example 3.12.** *A magic square of order $37\times 37$ is constructed based on magic square of order $35\times 35$ given in Example 3.10. It is given by*

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 25345 |
| 1334 | 1298 | 1300 | 1302 | 1304 | 1306 | 1308 | 1310 | 1312 | 1314 | 1316 | 1318 | 1320 | 1322 | 1324 | 1326 | 1328 | 1330 | 35 | 34 | 32 | 30 | 28 | 26 | 24 | 22 | 20 | 18 | 16 | 14 | 12 | 10 | 8 | 6 | 4 | 2 | 1332 | 25345 |
| 71 | 577 | 588 | 575 | 568 | 582 | 586 | 584 | 87 | 98 | 85 | 78 | 92 | 96 | 94 | 1032 | 1043 | 1030 | 1023 | 1037 | 1041 | 1039 | 1172 | 1183 | 1170 | 1163 | 1177 | 1181 | 1179 | 542 | 553 | 540 | 533 | 547 | 551 | 549 | 1299 | 25345 |
| 69 | 563 | 595 | 589 | 573 | 593 | 581 | 566 | 73 | 105 | 99 | 83 | 103 | 91 | 76 | 1018 | 1050 | 1044 | 1028 | 1048 | 1036 | 1021 | 1158 | 1190 | 1184 | 1168 | 1188 | 1176 | 1161 | 528 | 560 | 554 | 538 | 558 | 546 | 531 | 1301 | 25345 |
| 67 | 590 | 564 | 591 | 580 | 569 | 596 | 570 | 100 | 74 | 101 | 90 | 79 | 106 | 80 | 1045 | 1019 | 1046 | 1035 | 1024 | 1051 | 1025 | 1185 | 1159 | 1186 | 1175 | 1164 | 1191 | 1165 | 555 | 529 | 556 | 545 | 534 | 561 | 535 | 1303 | 25345 |
| 65 | 594 | 579 | 567 | 587 | 571 | 565 | 597 | 104 | 89 | 77 | 97 | 81 | 75 | 107 | 1049 | 1034 | 1022 | 1042 | 1026 | 1020 | 1052 | 1189 | 1174 | 1162 | 1182 | 1166 | 1160 | 1192 | 559 | 544 | 532 | 552 | 536 | 530 | 562 | 1305 | 25345 |
| 63 | 576 | 574 | 578 | 592 | 585 | 572 | 583 | 86 | 84 | 88 | 102 | 95 | 82 | 93 | 1031 | 1029 | 1033 | 1047 | 1040 | 1027 | 1038 | 1171 | 1169 | 1173 | 1187 | 1180 | 1167 | 1178 | 541 | 539 | 543 | 557 | 550 | 537 | 548 | 1307 | 25345 |
| 61 | 962 | 973 | 960 | 953 | 967 | 971 | 969 | 1207 | 1218 | 1205 | 1198 | 1212 | 1216 | 1214 | 122 | 133 | 120 | 113 | 127 | 131 | 129 | 647 | 658 | 645 | 638 | 652 | 656 | 654 | 472 | 483 | 470 | 463 | 477 | 481 | 479 | 1309 | 25345 |
| 59 | 948 | 980 | 974 | 958 | 978 | 966 | 951 | 1193 | 1225 | 1219 | 1203 | 1223 | 1211 | 1196 | 108 | 140 | 134 | 118 | 138 | 126 | 111 | 633 | 665 | 659 | 643 | 663 | 651 | 636 | 458 | 490 | 484 | 468 | 488 | 476 | 461 | 1311 | 25345 |
| 57 | 975 | 949 | 976 | 965 | 954 | 981 | 955 | 1220 | 1194 | 1221 | 1210 | 1199 | 1226 | 1200 | 135 | 109 | 136 | 125 | 114 | 141 | 115 | 660 | 634 | 661 | 650 | 639 | 666 | 640 | 485 | 459 | 486 | 475 | 464 | 491 | 465 | 1313 | 25345 |
| 55 | 979 | 964 | 952 | 972 | 956 | 950 | 982 | 1224 | 1209 | 1197 | 1217 | 1201 | 1195 | 1227 | 139 | 124 | 112 | 132 | 116 | 110 | 142 | 664 | 649 | 637 | 657 | 641 | 635 | 667 | 489 | 474 | 462 | 482 | 466 | 460 | 492 | 1315 | 25345 |
| 53 | 961 | 959 | 963 | 977 | 970 | 957 | 968 | 1206 | 1204 | 1208 | 1222 | 1215 | 1202 | 1213 | 121 | 119 | 123 | 137 | 130 | 117 | 128 | 646 | 644 | 648 | 662 | 655 | 642 | 653 | 471 | 469 | 473 | 487 | 480 | 467 | 478 | 1317 | 25345 |
| 51 | 507 | 518 | 505 | 498 | 512 | 516 | 514 | 997 | 1008 | 995 | 988 | 1002 | 1006 | 1004 | 1067 | 1078 | 1065 | 1058 | 1072 | 1076 | 1074 | 227 | 238 | 225 | 218 | 232 | 236 | 234 | 612 | 623 | 610 | 603 | 617 | 621 | 619 | 1319 | 25345 |
| 49 | 493 | 525 | 519 | 503 | 523 | 511 | 496 | 983 | 1015 | 1009 | 993 | 1013 | 1001 | 986 | 1053 | 1085 | 1079 | 1063 | 1083 | 1071 | 1056 | 213 | 245 | 239 | 223 | 243 | 231 | 216 | 598 | 630 | 624 | 608 | 628 | 616 | 601 | 1321 | 25345 |
| 47 | 520 | 494 | 521 | 510 | 499 | 526 | 500 | 1010 | 984 | 1011 | 1000 | 989 | 1016 | 990 | 1080 | 1054 | 1081 | 1070 | 1059 | 1086 | 1060 | 240 | 214 | 241 | 230 | 219 | 246 | 220 | 625 | 599 | 626 | 615 | 604 | 631 | 605 | 1323 | 25345 |
| 45 | 524 | 509 | 497 | 517 | 501 | 495 | 527 | 1014 | 999 | 987 | 1007 | 991 | 985 | 1017 | 1084 | 1069 | 1057 | 1077 | 1061 | 1055 | 1087 | 244 | 229 | 217 | 237 | 221 | 215 | 247 | 629 | 614 | 602 | 622 | 606 | 600 | 632 | 1325 | 25345 |
| 43 | 506 | 504 | 508 | 522 | 515 | 502 | 513 | 996 | 994 | 998 | 1012 | 1005 | 992 | 1003 | 1066 | 1064 | 1068 | 1082 | 1075 | 1062 | 1073 | 226 | 224 | 228 | 242 | 235 | 222 | 233 | 611 | 609 | 613 | 627 | 620 | 607 | 618 | 1327 | 25345 |
| 41 | 262 | 273 | 260 | 253 | 267 | 271 | 269 | 437 | 448 | 435 | 428 | 442 | 446 | 444 | 682 | 693 | 680 | 673 | 687 | 691 | 689 | 927 | 938 | 925 | 918 | 932 | 936 | 934 | 1102 | 1113 | 1100 | 1093 | 1107 | 1111 | 1109 | 1329 | 25345 |
| 39 | 248 | 280 | 274 | 258 | 278 | 266 | 251 | 423 | 455 | 449 | 433 | 453 | 441 | 426 | 668 | 700 | 694 | 678 | 698 | 686 | 671 | 913 | 945 | 939 | 923 | 943 | 931 | 916 | 1088 | 1120 | 1114 | 1098 | 1118 | 1106 | 1091 | 1331 | 25345 |
| 37 | 275 | 249 | 276 | 265 | 254 | 281 | 255 | 450 | 424 | 451 | 440 | 429 | 456 | 430 | 695 | 669 | 696 | 685 | 674 | 701 | 675 | 940 | 914 | 941 | 930 | 919 | 946 | 920 | 1115 | 1089 | 1116 | 1105 | 1094 | 1121 | 1095 | 1333 | 25345 |
| 1337 | 279 | 264 | 252 | 272 | 256 | 250 | 282 | 454 | 439 | 427 | 447 | 431 | 425 | 457 | 699 | 684 | 672 | 692 | 676 | 670 | 702 | 944 | 929 | 917 | 937 | 921 | 915 | 947 | 1119 | 1104 | 1092 | 1112 | 1096 | 1090 | 1122 | 33 | 25345 |
| 1339 | 261 | 259 | 263 | 277 | 270 | 257 | 268 | 436 | 434 | 438 | 452 | 445 | 432 | 443 | 681 | 679 | 683 | 697 | 690 | 677 | 688 | 926 | 924 | 928 | 942 | 935 | 922 | 933 | 1101 | 1099 | 1103 | 1117 | 1110 | 1097 | 1108 | 31 | 25345 |
| 1341 | 752 | 763 | 750 | 743 | 757 | 761 | 759 | 1137 | 1148 | 1135 | 1128 | 1142 | 1146 | 1144 | 297 | 308 | 295 | 288 | 302 | 306 | 304 | 367 | 378 | 365 | 358 | 372 | 376 | 374 | 857 | 868 | 855 | 848 | 862 | 866 | 864 | 29 | 25345 |
| 1343 | 738 | 770 | 764 | 748 | 768 | 756 | 741 | 1123 | 1155 | 1149 | 1133 | 1153 | 1141 | 1126 | 283 | 315 | 309 | 293 | 313 | 301 | 286 | 353 | 385 | 379 | 363 | 383 | 371 | 356 | 843 | 875 | 869 | 853 | 873 | 861 | 846 | 27 | 25345 |
| 1345 | 765 | 739 | 766 | 755 | 744 | 771 | 745 | 1150 | 1124 | 1151 | 1140 | 1129 | 1156 | 1130 | 310 | 284 | 311 | 300 | 289 | 316 | 290 | 380 | 354 | 381 | 370 | 359 | 386 | 360 | 870 | 844 | 871 | 860 | 849 | 876 | 850 | 25 | 25345 |
| 1347 | 769 | 754 | 742 | 762 | 746 | 740 | 772 | 1154 | 1139 | 1127 | 1147 | 1131 | 1125 | 1157 | 314 | 299 | 287 | 307 | 291 | 285 | 317 | 384 | 369 | 357 | 377 | 361 | 355 | 387 | 874 | 859 | 847 | 867 | 851 | 845 | 877 | 23 | 25345 |
| 1349 | 751 | 749 | 753 | 767 | 760 | 747 | 758 | 1136 | 1134 | 1138 | 1152 | 1145 | 1132 | 1143 | 296 | 294 | 298 | 312 | 305 | 292 | 303 | 366 | 364 | 368 | 382 | 375 | 362 | 373 | 856 | 854 | 858 | 872 | 865 | 852 | 863 | 21 | 25345 |
| 1351 | 892 | 903 | 890 | 883 | 897 | 901 | 899 | 717 | 728 | 715 | 708 | 722 | 726 | 724 | 1242 | 1253 | 1240 | 1233 | 1247 | 1251 | 1249 | 157 | 168 | 155 | 148 | 162 | 166 | 164 | 402 | 413 | 400 | 393 | 407 | 411 | 409 | 19 | 25345 |
| 1353 | 878 | 910 | 904 | 888 | 908 | 896 | 881 | 703 | 735 | 729 | 713 | 733 | 721 | 706 | 1228 | 1260 | 1254 | 1238 | 1258 | 1246 | 1231 | 143 | 175 | 169 | 153 | 173 | 161 | 146 | 388 | 420 | 414 | 398 | 418 | 406 | 391 | 17 | 25345 |
| 1355 | 905 | 879 | 906 | 895 | 884 | 911 | 885 | 730 | 704 | 731 | 720 | 709 | 736 | 710 | 1255 | 1229 | 1256 | 1245 | 1234 | 1261 | 1235 | 170 | 144 | 171 | 160 | 149 | 176 | 150 | 415 | 389 | 416 | 405 | 394 | 421 | 395 | 15 | 25345 |
| 1357 | 909 | 894 | 882 | 902 | 886 | 880 | 912 | 734 | 719 | 707 | 727 | 711 | 705 | 737 | 1259 | 1244 | 1232 | 1252 | 1236 | 1230 | 1262 | 174 | 159 | 147 | 167 | 151 | 145 | 177 | 419 | 404 | 392 | 412 | 396 | 390 | 422 | 13 | 25345 |
| 1359 | 891 | 889 | 893 | 907 | 900 | 887 | 898 | 716 | 714 | 718 | 732 | 725 | 712 | 723 | 1241 | 1239 | 1243 | 1257 | 1250 | 1237 | 1248 | 156 | 154 | 158 | 172 | 165 | 152 | 163 | 401 | 399 | 403 | 417 | 410 | 397 | 408 | 11 | 25345 |
| 1361 | 822 | 833 | 820 | 813 | 827 | 831 | 829 | 192 | 203 | 190 | 183 | 197 | 201 | 199 | 332 | 343 | 330 | 323 | 337 | 341 | 339 | 1277 | 1288 | 1275 | 1268 | 1282 | 1286 | 1284 | 787 | 798 | 785 | 778 | 792 | 796 | 794 | 9 | 25345 |
| 1363 | 808 | 840 | 834 | 818 | 838 | 826 | 811 | 178 | 210 | 204 | 188 | 208 | 196 | 181 | 318 | 350 | 344 | 328 | 348 | 336 | 321 | 1263 | 1295 | 1289 | 1273 | 1293 | 1281 | 1266 | 773 | 805 | 799 | 783 | 803 | 791 | 776 | 7 | 25345 |
| 1365 | 835 | 809 | 836 | 825 | 814 | 841 | 815 | 205 | 179 | 206 | 195 | 184 | 211 | 185 | 345 | 319 | 346 | 335 | 324 | 351 | 325 | 1290 | 1264 | 1291 | 1280 | 1269 | 1296 | 1270 | 800 | 774 | 801 | 790 | 779 | 806 | 780 | 5 | 25345 |
| 1367 | 839 | 824 | 812 | 832 | 816 | 810 | 842 | 209 | 194 | 182 | 202 | 186 | 180 | 212 | 349 | 334 | 322 | 342 | 326 | 320 | 352 | 1294 | 1279 | 1267 | 1287 | 1271 | 1265 | 1297 | 804 | 789 | 777 | 797 | 781 | 775 | 807 | 3 | 25345 |
| 1369 | 821 | 819 | 823 | 837 | 830 | 817 | 828 | 191 | 189 | 193 | 207 | 200 | 187 | 198 | 331 | 329 | 333 | 347 | 340 | 327 | 338 | 1276 | 1274 | 1278 | 1292 | 1285 | 1272 | 1283 | 786 | 784 | 788 | 802 | 795 | 782 | 793 | 1 | 25345 |
| 38 | 72 | 70 | 68 | 66 | 64 | 62 | 60 | 58 | 56 | 54 | 52 | 50 | 48 | 46 | 44 | 42 | 40 | 1335 | 1336 | 1338 | 1340 | 1342 | 1344 | 1346 | 1348 | 1350 | 1352 | 1354 | 1356 | 1358 | 1360 | 1362 | 1364 | 1366 | 1368 | 36 | 25345 |
| 25345 | 25345 | 25345 | 25345 | 25345 | 25345 | 25345 | 25345 | 25345 | 25345 | 25345 | 25345 | 25345 | 25345 | 25345 | 25345 | 25345 | 25345 | 25345 | 25345 | 25345 | 25345 | 25345 | 25345 | 25345 | 25345 | 25345 | 25345 | 25345 | 25345 | 25345 | 25345 | 25345 | 25345 | 25345 | 25345 | 25345 | 25345 |

*In this case the magic sums are $S_{37\times 37} := 25345$, $S_{35\times 35} := 23975$ and $S_{33\times 33} := 22605$. The magic rectangles of order $5\times 7$ are of **different magic sums**.*

## 3.12 Magic Square of Order $39\times 39$

**Example 3.13.** *A magic square of order $39\times 39$ constructed based on blocks of magic rectangles of order $3\times 13$ is given by*

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 29679 |
| 1321 | 1289 | 1308 | 1313 | 1295 | 1309 | 1297 | 1318 | 1303 | 1320 | 1324 | 1292 | 1302 | 34 | 2 | 21 | 26 | 8 | 22 | 10 | 31 | 16 | 33 | 37 | 5 | 15 | 970 | 938 | 957 | 962 | 944 | 958 | 946 | 967 | 952 | 969 | 973 | 941 | 951 | 29679 |
| 1288 | 1310 | 1323 | 1314 | 1315 | 1316 | 1307 | 1298 | 1299 | 1300 | 1291 | 1304 | 1326 | 1 | 23 | 36 | 27 | 28 | 29 | 20 | 11 | 12 | 13 | 4 | 17 | 39 | 937 | 959 | 972 | 963 | 964 | 965 | 956 | 947 | 948 | 949 | 940 | 953 | 975 | 29679 |
| 1312 | 1322 | 1290 | 1294 | 1311 | 1296 | 1317 | 1305 | 1319 | 1301 | 1306 | 1325 | 1293 | 25 | 35 | 3 | 7 | 24 | 9 | 30 | 18 | 32 | 14 | 19 | 38 | 6 | 961 | 971 | 939 | 943 | 960 | 945 | 966 | 954 | 968 | 950 | 955 | 974 | 942 | 29679 |
| 73 | 41 | 60 | 65 | 47 | 61 | 49 | 70 | 55 | 72 | 76 | 44 | 54 | 892 | 860 | 879 | 884 | 866 | 880 | 868 | 889 | 874 | 891 | 895 | 863 | 873 | 1360 | 1328 | 1347 | 1352 | 1334 | 1348 | 1336 | 1357 | 1342 | 1359 | 1363 | 1331 | 1341 | 29679 |
| 40 | 62 | 75 | 66 | 67 | 68 | 59 | 50 | 51 | 52 | 43 | 56 | 78 | 859 | 881 | 894 | 885 | 886 | 887 | 878 | 869 | 870 | 871 | 862 | 875 | 897 | 1327 | 1349 | 1362 | 1353 | 1354 | 1355 | 1346 | 1337 | 1338 | 1339 | 1330 | 1343 | 1365 | 29679 |
| 64 | 74 | 42 | 46 | 63 | 48 | 69 | 57 | 71 | 53 | 58 | 77 | 45 | 883 | 893 | 861 | 865 | 882 | 867 | 888 | 876 | 890 | 872 | 877 | 896 | 864 | 1351 | 1361 | 1329 | 1333 | 1350 | 1335 | 1356 | 1344 | 1358 | 1340 | 1345 | 1364 | 1332 | 29679 |
| 814 | 782 | 801 | 806 | 788 | 802 | 790 | 811 | 796 | 813 | 817 | 785 | 795 | 1399 | 1367 | 1386 | 1391 | 1373 | 1387 | 1375 | 1396 | 1381 | 1398 | 1402 | 1370 | 1380 | 112 | 80 | 99 | 104 | 86 | 100 | 88 | 109 | 94 | 111 | 115 | 83 | 93 | 29679 |
| 781 | 803 | 816 | 807 | 808 | 809 | 800 | 791 | 792 | 793 | 784 | 797 | 819 | 1366 | 1388 | 1401 | 1392 | 1393 | 1394 | 1385 | 1376 | 1377 | 1378 | 1369 | 1382 | 1404 | 79 | 101 | 114 | 105 | 106 | 107 | 98 | 89 | 90 | 91 | 82 | 95 | 117 | 29679 |
| 805 | 815 | 783 | 787 | 804 | 789 | 810 | 798 | 812 | 794 | 799 | 818 | 786 | 1390 | 1400 | 1368 | 1372 | 1389 | 1374 | 1395 | 1383 | 1397 | 1379 | 1384 | 1403 | 1371 | 103 | 113 | 81 | 85 | 102 | 87 | 108 | 96 | 110 | 92 | 97 | 116 | 84 | 29679 |
| 1009 | 977 | 996 | 1001 | 983 | 997 | 985 | 1006 | 991 | 1008 | 1012 | 980 | 990 | 1048 | 1016 | 1035 | 1040 | 1022 | 1036 | 1024 | 1045 | 1030 | 1047 | 1051 | 1019 | 1029 | 268 | 236 | 255 | 260 | 242 | 256 | 244 | 265 | 250 | 267 | 271 | 239 | 249 | 29679 |
| 976 | 998 | 1011 | 1002 | 1003 | 1004 | 995 | 986 | 987 | 988 | 979 | 992 | 1014 | 1015 | 1037 | 1050 | 1041 | 1042 | 1043 | 1034 | 1025 | 1026 | 1027 | 1018 | 1031 | 1053 | 235 | 257 | 270 | 261 | 262 | 263 | 254 | 245 | 246 | 247 | 238 | 251 | 273 | 29679 |
| 1000 | 1010 | 978 | 982 | 999 | 984 | 1005 | 993 | 1007 | 989 | 994 | 1013 | 981 | 1039 | 1049 | 1017 | 1021 | 1038 | 1023 | 1044 | 1032 | 1046 | 1028 | 1033 | 1052 | 1020 | 259 | 269 | 237 | 241 | 258 | 243 | 264 | 252 | 266 | 248 | 253 | 272 | 240 | 29679 |
| 307 | 275 | 294 | 299 | 281 | 295 | 283 | 304 | 289 | 306 | 310 | 278 | 288 | 1087 | 1055 | 1074 | 1079 | 1061 | 1075 | 1063 | 1084 | 1069 | 1086 | 1090 | 1058 | 1068 | 931 | 899 | 918 | 923 | 905 | 919 | 907 | 928 | 913 | 930 | 934 | 902 | 912 | 29679 |
| 274 | 296 | 309 | 300 | 301 | 302 | 293 | 284 | 285 | 286 | 277 | 290 | 312 | 1054 | 1076 | 1089 | 1080 | 1081 | 1082 | 1073 | 1064 | 1065 | 1066 | 1057 | 1070 | 1092 | 898 | 920 | 933 | 924 | 925 | 926 | 917 | 908 | 909 | 910 | 901 | 914 | 936 | 29679 |
| 298 | 308 | 276 | 280 | 297 | 282 | 303 | 291 | 305 | 287 | 292 | 311 | 279 | 1078 | 1088 | 1056 | 1060 | 1077 | 1062 | 1083 | 1071 | 1085 | 1067 | 1072 | 1091 | 1059 | 922 | 932 | 900 | 904 | 921 | 906 | 927 | 915 | 929 | 911 | 916 | 935 | 903 | 29679 |
| 853 | 821 | 840 | 845 | 827 | 841 | 829 | 850 | 835 | 852 | 856 | 824 | 834 | 1126 | 1094 | 1113 | 1118 | 1100 | 1114 | 1102 | 1123 | 1108 | 1125 | 1129 | 1097 | 1107 | 346 | 314 | 333 | 338 | 320 | 334 | 322 | 343 | 328 | 345 | 349 | 317 | 327 | 29679 |
| 820 | 842 | 855 | 846 | 847 | 848 | 839 | 830 | 831 | 832 | 823 | 836 | 858 | 1093 | 1115 | 1128 | 1119 | 1120 | 1121 | 1112 | 1103 | 1104 | 1105 | 1096 | 1109 | 1131 | 313 | 335 | 348 | 339 | 340 | 341 | 332 | 323 | 324 | 325 | 316 | 329 | 351 | 29679 |
| 844 | 854 | 822 | 826 | 843 | 828 | 849 | 837 | 851 | 833 | 838 | 857 | 825 | 1117 | 1127 | 1095 | 1099 | 1116 | 1101 | 1122 | 1110 | 1124 | 1106 | 1111 | 1130 | 1098 | 337 | 347 | 315 | 319 | 336 | 321 | 342 | 330 | 344 | 326 | 331 | 350 | 318 | 29679 |
| 385 | 353 | 372 | 377 | 359 | 373 | 361 | 382 | 367 | 384 | 388 | 356 | 366 | 775 | 743 | 762 | 767 | 749 | 763 | 751 | 772 | 757 | 774 | 778 | 746 | 756 | 1165 | 1133 | 1152 | 1157 | 1139 | 1153 | 1141 | 1162 | 1147 | 1164 | 1168 | 1136 | 1146 | 29679 |
| 352 | 374 | 387 | 378 | 379 | 380 | 371 | 362 | 363 | 364 | 355 | 368 | 390 | 742 | 764 | 777 | 768 | 769 | 770 | 761 | 752 | 753 | 754 | 745 | 758 | 780 | 1132 | 1154 | 1167 | 1158 | 1159 | 1160 | 1151 | 1142 | 1143 | 1144 | 1135 | 1148 | 1170 | 29679 |
| 376 | 386 | 354 | 358 | 375 | 360 | 381 | 369 | 383 | 365 | 370 | 389 | 357 | 766 | 776 | 744 | 748 | 765 | 750 | 771 | 759 | 773 | 755 | 760 | 779 | 747 | 1156 | 1166 | 1134 | 1138 | 1155 | 1140 | 1161 | 1149 | 1163 | 1145 | 1150 | 1169 | 1137 | 29679 |
| 1204 | 1172 | 1191 | 1196 | 1178 | 1192 | 1180 | 1201 | 1186 | 1203 | 1207 | 1175 | 1185 | 424 | 392 | 411 | 416 | 398 | 412 | 400 | 421 | 406 | 423 | 427 | 395 | 405 | 697 | 665 | 684 | 689 | 671 | 685 | 673 | 694 | 679 | 696 | 700 | 668 | 678 | 29679 |
| 1171 | 1193 | 1206 | 1197 | 1198 | 1199 | 1190 | 1181 | 1182 | 1183 | 1174 | 1187 | 1209 | 391 | 413 | 426 | 417 | 418 | 419 | 410 | 401 | 402 | 403 | 394 | 407 | 429 | 664 | 686 | 699 | 690 | 691 | 692 | 683 | 674 | 675 | 676 | 667 | 680 | 702 | 29679 |
| 1195 | 1205 | 1173 | 1177 | 1194 | 1179 | 1200 | 1188 | 1202 | 1184 | 1189 | 1208 | 1176 | 415 | 425 | 393 | 397 | 414 | 399 | 420 | 408 | 422 | 404 | 409 | 428 | 396 | 688 | 698 | 666 | 670 | 687 | 672 | 693 | 681 | 695 | 677 | 682 | 701 | 669 | 29679 |
| 619 | 587 | 606 | 611 | 593 | 607 | 595 | 616 | 601 | 618 | 622 | 590 | 600 | 463 | 431 | 450 | 455 | 437 | 451 | 439 | 460 | 445 | 462 | 466 | 434 | 444 | 1243 | 1211 | 1230 | 1235 | 1217 | 1231 | 1219 | 1240 | 1225 | 1242 | 1246 | 1214 | 1224 | 29679 |
| 586 | 608 | 621 | 612 | 613 | 614 | 605 | 596 | 597 | 598 | 589 | 602 | 624 | 430 | 452 | 465 | 456 | 457 | 458 | 449 | 440 | 441 | 442 | 433 | 446 | 468 | 1210 | 1232 | 1245 | 1236 | 1237 | 1238 | 1229 | 1220 | 1221 | 1222 | 1213 | 1226 | 1248 | 29679 |
| 610 | 620 | 588 | 592 | 609 | 594 | 615 | 603 | 617 | 599 | 604 | 623 | 591 | 454 | 464 | 432 | 436 | 453 | 438 | 459 | 447 | 461 | 443 | 448 | 467 | 435 | 1234 | 1244 | 1212 | 1216 | 1233 | 1218 | 1239 | 1227 | 1241 | 1223 | 1228 | 1247 | 1215 | 29679 |
| 1282 | 1250 | 1269 | 1274 | 1256 | 1270 | 1258 | 1279 | 1264 | 1281 | 1285 | 1253 | 1263 | 502 | 470 | 489 | 494 | 476 | 490 | 478 | 499 | 484 | 501 | 505 | 473 | 483 | 541 | 509 | 528 | 533 | 515 | 529 | 517 | 538 | 523 | 540 | 544 | 512 | 522 | 29679 |
| 1249 | 1271 | 1284 | 1275 | 1276 | 1277 | 1268 | 1259 | 1260 | 1261 | 1252 | 1265 | 1287 | 469 | 491 | 504 | 495 | 496 | 497 | 488 | 479 | 480 | 481 | 472 | 485 | 507 | 508 | 530 | 543 | 534 | 535 | 536 | 527 | 518 | 519 | 520 | 511 | 524 | 546 | 29679 |
| 1273 | 1283 | 1251 | 1255 | 1272 | 1257 | 1278 | 1266 | 1280 | 1262 | 1267 | 1286 | 1254 | 493 | 503 | 471 | 475 | 492 | 477 | 498 | 486 | 500 | 482 | 487 | 506 | 474 | 532 | 542 | 510 | 514 | 531 | 516 | 537 | 525 | 539 | 521 | 526 | 545 | 513 | 29679 |
| 1438 | 1406 | 1425 | 1430 | 1412 | 1426 | 1414 | 1435 | 1420 | 1437 | 1441 | 1409 | 1419 | 151 | 119 | 138 | 143 | 125 | 139 | 127 | 148 | 133 | 150 | 154 | 122 | 132 | 736 | 704 | 723 | 728 | 710 | 724 | 712 | 733 | 718 | 735 | 739 | 707 | 717 | 29679 |
| 1405 | 1427 | 1440 | 1431 | 1432 | 1433 | 1424 | 1415 | 1416 | 1417 | 1408 | 1421 | 1443 | 118 | 140 | 153 | 144 | 145 | 146 | 137 | 128 | 129 | 130 | 121 | 134 | 156 | 703 | 725 | 738 | 729 | 730 | 731 | 722 | 713 | 714 | 715 | 706 | 719 | 741 | 29679 |
| 1429 | 1439 | 1407 | 1411 | 1428 | 1413 | 1434 | 1422 | 1436 | 1418 | 1423 | 1442 | 1410 | 142 | 152 | 120 | 124 | 141 | 126 | 147 | 135 | 149 | 131 | 136 | 155 | 123 | 727 | 737 | 705 | 709 | 726 | 711 | 732 | 720 | 734 | 716 | 721 | 740 | 708 | 29679 |
| 190 | 158 | 177 | 182 | 164 | 178 | 166 | 187 | 172 | 189 | 193 | 161 | 171 | 658 | 626 | 645 | 650 | 632 | 646 | 634 | 655 | 640 | 657 | 661 | 629 | 639 | 1477 | 1445 | 1464 | 1469 | 1451 | 1465 | 1453 | 1474 | 1459 | 1476 | 1480 | 1448 | 1458 | 29679 |
| 157 | 179 | 192 | 183 | 184 | 185 | 176 | 167 | 168 | 169 | 160 | 173 | 195 | 625 | 647 | 660 | 651 | 652 | 653 | 644 | 635 | 636 | 637 | 628 | 641 | 663 | 1444 | 1466 | 1479 | 1470 | 1471 | 1472 | 1463 | 1454 | 1455 | 1456 | 1447 | 1460 | 1482 | 29679 |
| 181 | 191 | 159 | 163 | 180 | 165 | 186 | 174 | 188 | 170 | 175 | 194 | 162 | 649 | 659 | 627 | 631 | 648 | 633 | 654 | 642 | 656 | 638 | 643 | 662 | 630 | 1468 | 1478 | 1446 | 1450 | 1467 | 1452 | 1473 | 1461 | 1475 | 1457 | 1462 | 1481 | 1449 | 29679 |
| 580 | 548 | 567 | 572 | 554 | 568 | 556 | 577 | 562 | 579 | 583 | 551 | 561 | 1516 | 1484 | 1503 | 1508 | 1490 | 1504 | 1492 | 1513 | 1498 | 1515 | 1519 | 1487 | 1497 | 229 | 197 | 216 | 221 | 203 | 217 | 205 | 226 | 211 | 228 | 232 | 200 | 210 | 29679 |
| 547 | 569 | 582 | 573 | 574 | 575 | 566 | 557 | 558 | 559 | 550 | 563 | 585 | 1483 | 1505 | 1518 | 1509 | 1510 | 1511 | 1502 | 1493 | 1494 | 1495 | 1486 | 1499 | 1521 | 196 | 218 | 231 | 222 | 223 | 224 | 215 | 206 | 207 | 208 | 199 | 212 | 234 | 29679 |
| 571 | 581 | 549 | 553 | 570 | 555 | 576 | 564 | 578 | 560 | 565 | 584 | 552 | 1507 | 1517 | 1485 | 1489 | 1506 | 1491 | 1512 | 1500 | 1514 | 1496 | 1501 | 1520 | 1488 | 220 | 230 | 198 | 202 | 219 | 204 | 225 | 213 | 227 | 209 | 214 | 233 | 201 | 29679 |
| 29679 | 29679 | 29679 | 29679 | 29679 | 29679 | 29679 | 29679 | 29679 | 29679 | 29679 | 29679 | 29679 | 29679 | 29679 | 29679 | 29679 | 29679 | 29679 | 29679 | 29679 | 29679 | 29679 | 29679 | 29679 | 29679 | 29679 | 29679 | 29679 | 29679 | 29679 | 29679 | 29679 | 29679 | 29679 | 29679 | 29679 | 29679 | 29679 | 29679 |

*In this case the magic sum is $S_{39\times 39} := 29679$. The magic rectangles of order $3\times 13$ are of* ***different magic sums****.*

## 3.13 Block Bordered Magic Square of Order $41\times 41$

It is based on the Example 3.13 with single external border, and the internal block is a magic square of order 39, where the magic rectangles are of orders $3\times 13$.

Magic Rectangles in Construction of Magic and Block-Bordered Magic Squares, June 07, 2022, pp. 1-70, https://doi.org/10.5281/zenodo.6621071

**Example 3.14.** *A magic square of order* $41 \times 41$ *is constructed based on magic square of order* $39 \times 39$ *given in Example 3.13. It is given by*

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 34481 |
| 1642 | 1602 | 1604 | 1606 | 1608 | 1610 | 1612 | 1614 | 1616 | 1618 | 1620 | 1622 | 1624 | 1626 | 1628 | 1630 | 1632 | 1634 | 1636 | 1638 | 39 | 38 | 36 | 34 | 32 | 30 | 28 | 26 | 24 | 22 | 20 | 18 | 16 | 14 | 12 | 10 | 8 | 6 | 4 | 2 | 1640 | 34481 |
| 79 | 1401 | 1369 | 1388 | 1393 | 1375 | 1389 | 1377 | 1398 | 1383 | 1400 | 1404 | 1372 | 1382 | 114 | 82 | 101 | 106 | 88 | 102 | 90 | 111 | 96 | 113 | 117 | 85 | 95 | 1050 | 1018 | 1037 | 1042 | 1024 | 1038 | 1026 | 1047 | 1032 | 1049 | 1053 | 1021 | 1031 | 1603 | 34481 |
| 77 | 1368 | 1390 | 1403 | 1394 | 1395 | 1396 | 1387 | 1378 | 1379 | 1380 | 1371 | 1384 | 1406 | 81 | 103 | 116 | 107 | 108 | 109 | 100 | 91 | 92 | 93 | 84 | 97 | 119 | 1017 | 1039 | 1052 | 1043 | 1044 | 1045 | 1036 | 1027 | 1028 | 1029 | 1020 | 1033 | 1055 | 1605 | 34481 |
| 75 | 1392 | 1402 | 1370 | 1374 | 1391 | 1376 | 1397 | 1385 | 1399 | 1381 | 1386 | 1405 | 1373 | 105 | 115 | 83 | 87 | 104 | 89 | 110 | 98 | 112 | 94 | 99 | 118 | 86 | 1041 | 1051 | 1019 | 1023 | 1040 | 1025 | 1046 | 1034 | 1048 | 1030 | 1035 | 1054 | 1022 | 1607 | 34481 |
| 73 | 153 | 121 | 140 | 145 | 127 | 141 | 129 | 150 | 135 | 152 | 156 | 124 | 134 | 972 | 940 | 959 | 964 | 946 | 960 | 948 | 969 | 954 | 971 | 975 | 943 | 953 | 1440 | 1408 | 1427 | 1432 | 1414 | 1428 | 1416 | 1437 | 1422 | 1439 | 1443 | 1411 | 1421 | 1609 | 34481 |
| 71 | 120 | 142 | 155 | 146 | 147 | 148 | 139 | 130 | 131 | 132 | 123 | 136 | 158 | 939 | 961 | 974 | 965 | 966 | 967 | 958 | 949 | 950 | 951 | 942 | 955 | 977 | 1407 | 1429 | 1442 | 1433 | 1434 | 1435 | 1426 | 1417 | 1418 | 1419 | 1410 | 1423 | 1445 | 1611 | 34481 |
| 69 | 144 | 154 | 122 | 126 | 143 | 128 | 149 | 137 | 151 | 133 | 138 | 157 | 125 | 963 | 973 | 941 | 945 | 962 | 947 | 968 | 956 | 970 | 952 | 957 | 976 | 944 | 1431 | 1441 | 1409 | 1413 | 1430 | 1415 | 1436 | 1424 | 1438 | 1420 | 1425 | 1444 | 1412 | 1613 | 34481 |
| 67 | 894 | 862 | 881 | 886 | 868 | 882 | 870 | 891 | 876 | 893 | 897 | 865 | 875 | 1479 | 1447 | 1466 | 1471 | 1453 | 1467 | 1455 | 1476 | 1461 | 1478 | 1482 | 1450 | 1460 | 192 | 160 | 179 | 184 | 166 | 180 | 168 | 189 | 174 | 191 | 195 | 163 | 173 | 1615 | 34481 |
| 65 | 861 | 883 | 896 | 887 | 888 | 889 | 880 | 871 | 872 | 873 | 864 | 877 | 899 | 1446 | 1468 | 1481 | 1472 | 1473 | 1474 | 1465 | 1456 | 1457 | 1458 | 1449 | 1462 | 1484 | 159 | 181 | 194 | 185 | 186 | 187 | 178 | 169 | 170 | 171 | 162 | 175 | 197 | 1617 | 34481 |
| 63 | 885 | 895 | 863 | 867 | 884 | 869 | 890 | 878 | 892 | 874 | 879 | 898 | 866 | 1470 | 1480 | 1448 | 1452 | 1469 | 1454 | 1475 | 1463 | 1477 | 1459 | 1464 | 1483 | 1451 | 183 | 193 | 161 | 165 | 182 | 167 | 188 | 176 | 190 | 172 | 177 | 196 | 164 | 1619 | 34481 |
| 61 | 1089 | 1057 | 1076 | 1081 | 1063 | 1077 | 1065 | 1086 | 1071 | 1088 | 1092 | 1060 | 1070 | 1128 | 1096 | 1115 | 1120 | 1102 | 1116 | 1104 | 1125 | 1110 | 1127 | 1131 | 1099 | 1109 | 348 | 316 | 335 | 340 | 322 | 336 | 324 | 345 | 330 | 347 | 351 | 319 | 329 | 1621 | 34481 |
| 59 | 1056 | 1078 | 1091 | 1082 | 1083 | 1084 | 1075 | 1066 | 1067 | 1068 | 1059 | 1072 | 1094 | 1095 | 1117 | 1130 | 1121 | 1122 | 1123 | 1114 | 1105 | 1106 | 1107 | 1098 | 1111 | 1133 | 315 | 337 | 350 | 341 | 342 | 343 | 334 | 325 | 326 | 327 | 318 | 331 | 353 | 1623 | 34481 |
| 57 | 1080 | 1090 | 1058 | 1062 | 1079 | 1064 | 1085 | 1073 | 1087 | 1069 | 1074 | 1093 | 1061 | 1119 | 1129 | 1097 | 1101 | 1118 | 1103 | 1124 | 1112 | 1126 | 1108 | 1113 | 1132 | 1100 | 339 | 349 | 317 | 321 | 338 | 323 | 344 | 332 | 346 | 328 | 333 | 352 | 320 | 1625 | 34481 |
| 55 | 387 | 355 | 374 | 379 | 361 | 375 | 363 | 384 | 369 | 386 | 390 | 358 | 368 | 1167 | 1135 | 1154 | 1159 | 1141 | 1155 | 1143 | 1164 | 1149 | 1166 | 1170 | 1138 | 1148 | 1011 | 979 | 998 | 1003 | 985 | 999 | 987 | 1008 | 993 | 1010 | 1014 | 982 | 992 | 1627 | 34481 |
| 53 | 354 | 376 | 389 | 380 | 381 | 382 | 373 | 364 | 365 | 366 | 357 | 370 | 392 | 1134 | 1156 | 1169 | 1160 | 1161 | 1162 | 1153 | 1144 | 1145 | 1146 | 1137 | 1150 | 1172 | 978 | 1000 | 1013 | 1004 | 1005 | 1006 | 997 | 988 | 989 | 990 | 981 | 994 | 1016 | 1629 | 34481 |
| 51 | 378 | 388 | 356 | 360 | 377 | 362 | 383 | 371 | 385 | 367 | 372 | 391 | 359 | 1158 | 1168 | 1136 | 1140 | 1157 | 1142 | 1163 | 1151 | 1165 | 1147 | 1152 | 1171 | 1139 | 1002 | 1012 | 980 | 984 | 1001 | 986 | 1007 | 995 | 1009 | 991 | 996 | 1015 | 983 | 1631 | 34481 |
| 49 | 933 | 901 | 920 | 925 | 907 | 921 | 909 | 930 | 915 | 932 | 936 | 904 | 914 | 1206 | 1174 | 1193 | 1198 | 1180 | 1194 | 1182 | 1203 | 1188 | 1205 | 1209 | 1177 | 1187 | 426 | 394 | 413 | 418 | 400 | 414 | 402 | 423 | 408 | 425 | 429 | 397 | 407 | 1633 | 34481 |
| 47 | 900 | 922 | 935 | 926 | 927 | 928 | 919 | 910 | 911 | 912 | 903 | 916 | 938 | 1173 | 1195 | 1208 | 1199 | 1200 | 1201 | 1192 | 1183 | 1184 | 1185 | 1176 | 1189 | 1211 | 393 | 415 | 428 | 419 | 420 | 421 | 412 | 403 | 404 | 405 | 396 | 409 | 431 | 1635 | 34481 |
| 45 | 924 | 934 | 902 | 906 | 923 | 908 | 929 | 917 | 931 | 913 | 918 | 937 | 905 | 1197 | 1207 | 1175 | 1179 | 1196 | 1181 | 1202 | 1190 | 1204 | 1186 | 1191 | 1210 | 1178 | 417 | 427 | 395 | 399 | 416 | 401 | 422 | 410 | 424 | 406 | 411 | 430 | 398 | 1637 | 34481 |
| 43 | 465 | 433 | 452 | 457 | 439 | 453 | 441 | 462 | 447 | 464 | 468 | 436 | 446 | 855 | 823 | 842 | 847 | 829 | 843 | 831 | 852 | 837 | 854 | 858 | 826 | 836 | 1245 | 1213 | 1232 | 1237 | 1219 | 1233 | 1221 | 1242 | 1227 | 1244 | 1248 | 1216 | 1226 | 1639 | 34481 |
| 41 | 432 | 454 | 467 | 458 | 459 | 460 | 451 | 442 | 443 | 444 | 435 | 448 | 470 | 822 | 844 | 857 | 848 | 849 | 850 | 841 | 832 | 833 | 834 | 825 | 838 | 860 | 1212 | 1234 | 1247 | 1238 | 1239 | 1240 | 1231 | 1222 | 1223 | 1224 | 1215 | 1228 | 1250 | 1641 | 34481 |
| 1645 | 456 | 466 | 434 | 438 | 455 | 440 | 461 | 449 | 463 | 445 | 450 | 469 | 437 | 846 | 856 | 824 | 828 | 845 | 830 | 851 | 839 | 853 | 835 | 840 | 859 | 827 | 1236 | 1246 | 1214 | 1218 | 1235 | 1220 | 1241 | 1229 | 1243 | 1225 | 1230 | 1249 | 1217 | 37 | 34481 |
| 1647 | 1284 | 1252 | 1271 | 1276 | 1258 | 1272 | 1260 | 1281 | 1266 | 1283 | 1287 | 1255 | 1265 | 504 | 472 | 491 | 496 | 478 | 492 | 480 | 501 | 486 | 503 | 507 | 475 | 485 | 777 | 745 | 764 | 769 | 751 | 765 | 753 | 774 | 759 | 776 | 780 | 748 | 758 | 35 | 34481 |
| 1649 | 1251 | 1273 | 1286 | 1277 | 1278 | 1279 | 1270 | 1261 | 1262 | 1263 | 1254 | 1267 | 1289 | 471 | 493 | 506 | 497 | 498 | 499 | 490 | 481 | 482 | 483 | 474 | 487 | 509 | 744 | 766 | 779 | 770 | 771 | 772 | 763 | 754 | 755 | 756 | 747 | 760 | 782 | 33 | 34481 |
| 1651 | 1275 | 1285 | 1253 | 1257 | 1274 | 1259 | 1280 | 1268 | 1282 | 1264 | 1269 | 1288 | 1256 | 495 | 505 | 473 | 477 | 494 | 479 | 500 | 488 | 502 | 484 | 489 | 508 | 476 | 768 | 778 | 746 | 750 | 767 | 752 | 773 | 761 | 775 | 757 | 762 | 781 | 749 | 31 | 34481 |
| 1653 | 699 | 667 | 686 | 691 | 673 | 687 | 675 | 696 | 681 | 698 | 702 | 670 | 680 | 543 | 511 | 530 | 535 | 517 | 531 | 519 | 540 | 525 | 542 | 546 | 514 | 524 | 1323 | 1291 | 1310 | 1315 | 1297 | 1311 | 1299 | 1320 | 1305 | 1322 | 1326 | 1294 | 1304 | 29 | 34481 |
| 1655 | 666 | 688 | 701 | 692 | 693 | 694 | 685 | 676 | 677 | 678 | 669 | 682 | 704 | 510 | 532 | 545 | 536 | 537 | 538 | 529 | 520 | 521 | 522 | 513 | 526 | 548 | 1290 | 1312 | 1325 | 1316 | 1317 | 1318 | 1309 | 1300 | 1301 | 1302 | 1293 | 1306 | 1328 | 27 | 34481 |
| 1657 | 690 | 700 | 668 | 672 | 689 | 674 | 695 | 683 | 697 | 679 | 684 | 703 | 671 | 534 | 544 | 512 | 516 | 533 | 518 | 539 | 527 | 541 | 523 | 528 | 547 | 515 | 1314 | 1324 | 1292 | 1296 | 1313 | 1298 | 1319 | 1307 | 1321 | 1303 | 1308 | 1327 | 1295 | 25 | 34481 |
| 1659 | 1362 | 1330 | 1349 | 1354 | 1336 | 1350 | 1338 | 1359 | 1344 | 1361 | 1365 | 1333 | 1343 | 582 | 550 | 569 | 574 | 556 | 570 | 558 | 579 | 564 | 581 | 585 | 553 | 563 | 621 | 589 | 608 | 613 | 595 | 609 | 597 | 618 | 603 | 620 | 624 | 592 | 602 | 23 | 34481 |
| 1661 | 1329 | 1351 | 1364 | 1355 | 1356 | 1357 | 1348 | 1339 | 1340 | 1341 | 1332 | 1345 | 1367 | 549 | 571 | 584 | 575 | 576 | 577 | 568 | 559 | 560 | 561 | 552 | 565 | 587 | 588 | 610 | 623 | 614 | 615 | 616 | 607 | 598 | 599 | 600 | 591 | 604 | 626 | 21 | 34481 |
| 1663 | 1353 | 1363 | 1331 | 1335 | 1352 | 1337 | 1358 | 1346 | 1360 | 1342 | 1347 | 1366 | 1334 | 573 | 583 | 551 | 555 | 572 | 557 | 578 | 566 | 580 | 562 | 567 | 586 | 554 | 612 | 622 | 590 | 594 | 611 | 596 | 617 | 605 | 619 | 601 | 606 | 625 | 593 | 19 | 34481 |
| 1665 | 1518 | 1486 | 1505 | 1510 | 1492 | 1506 | 1494 | 1515 | 1500 | 1517 | 1521 | 1489 | 1499 | 231 | 199 | 218 | 223 | 205 | 219 | 207 | 228 | 213 | 230 | 234 | 202 | 212 | 816 | 784 | 803 | 808 | 790 | 804 | 792 | 813 | 798 | 815 | 819 | 787 | 797 | 17 | 34481 |
| 1667 | 1485 | 1507 | 1520 | 1511 | 1512 | 1513 | 1504 | 1495 | 1496 | 1497 | 1488 | 1501 | 1523 | 198 | 220 | 233 | 224 | 225 | 226 | 217 | 208 | 209 | 210 | 201 | 214 | 236 | 783 | 805 | 818 | 809 | 810 | 811 | 802 | 793 | 794 | 795 | 786 | 799 | 821 | 15 | 34481 |
| 1669 | 1509 | 1519 | 1487 | 1491 | 1508 | 1493 | 1514 | 1502 | 1516 | 1498 | 1503 | 1522 | 1490 | 222 | 232 | 200 | 204 | 221 | 206 | 227 | 215 | 229 | 211 | 216 | 235 | 203 | 807 | 817 | 785 | 789 | 806 | 791 | 812 | 800 | 814 | 796 | 801 | 820 | 788 | 13 | 34481 |
| 1671 | 270 | 238 | 257 | 262 | 244 | 258 | 246 | 267 | 252 | 269 | 273 | 241 | 251 | 738 | 706 | 725 | 730 | 712 | 726 | 714 | 735 | 720 | 737 | 741 | 709 | 719 | 1557 | 1525 | 1544 | 1549 | 1531 | 1545 | 1533 | 1554 | 1539 | 1556 | 1560 | 1528 | 1538 | 11 | 34481 |
| 1673 | 237 | 259 | 272 | 263 | 264 | 265 | 256 | 247 | 248 | 249 | 240 | 253 | 275 | 705 | 727 | 740 | 731 | 732 | 733 | 724 | 715 | 716 | 717 | 708 | 721 | 743 | 1524 | 1546 | 1559 | 1550 | 1551 | 1552 | 1543 | 1534 | 1535 | 1536 | 1527 | 1540 | 1562 | 9 | 34481 |
| 1675 | 261 | 271 | 239 | 243 | 260 | 245 | 266 | 254 | 268 | 250 | 255 | 274 | 242 | 729 | 739 | 707 | 711 | 728 | 713 | 734 | 722 | 736 | 718 | 723 | 742 | 710 | 1548 | 1558 | 1526 | 1530 | 1547 | 1532 | 1553 | 1541 | 1555 | 1537 | 1542 | 1561 | 1529 | 7 | 34481 |
| 1677 | 660 | 628 | 647 | 652 | 634 | 648 | 636 | 657 | 642 | 659 | 663 | 631 | 641 | 1596 | 1564 | 1583 | 1588 | 1570 | 1584 | 1572 | 1593 | 1578 | 1595 | 1599 | 1567 | 1577 | 309 | 277 | 296 | 301 | 283 | 297 | 285 | 306 | 291 | 308 | 312 | 280 | 290 | 5 | 34481 |
| 1679 | 627 | 649 | 662 | 653 | 654 | 655 | 646 | 637 | 638 | 639 | 630 | 643 | 665 | 1563 | 1585 | 1598 | 1589 | 1590 | 1591 | 1582 | 1573 | 1574 | 1575 | 1566 | 1579 | 1601 | 276 | 298 | 311 | 302 | 303 | 304 | 295 | 286 | 287 | 288 | 279 | 292 | 314 | 3 | 34481 |
| 1681 | 651 | 661 | 629 | 633 | 650 | 635 | 656 | 644 | 658 | 640 | 645 | 664 | 632 | 1587 | 1597 | 1565 | 1569 | 1586 | 1571 | 1592 | 1580 | 1594 | 1576 | 1581 | 1600 | 1568 | 300 | 310 | 278 | 282 | 299 | 284 | 305 | 293 | 307 | 289 | 294 | 313 | 281 | 1 | 34481 |
| 42 | 80 | 78 | 76 | 74 | 72 | 70 | 68 | 66 | 64 | 62 | 60 | 58 | 56 | 54 | 52 | 50 | 48 | 46 | 44 | 1643 | 1644 | 1646 | 1648 | 1650 | 1652 | 1654 | 1656 | 1658 | 1660 | 1662 | 1664 | 1666 | 1668 | 1670 | 1672 | 1674 | 1676 | 1678 | 1680 | 40 | 34481 |
| 34481 | 34481 | 34481 | 34481 | 34481 | 34481 | 34481 | 34481 | 34481 | 34481 | 34481 | 34481 | 34481 | 34481 | 34481 | 34481 | 34481 | 34481 | 34481 | 34481 | 34481 | 34481 | 34481 | 34481 | 34481 | 34481 | 34481 | 34481 | 34481 | 34481 | 34481 | 34481 | 34481 | 34481 | 34481 | 34481 | 34481 | 34481 | 34481 | 34481 | 34481 | 34481 |

*In this case the magic sums are* $S_{41\times 41} := 25345$ *and* $S_{39\times 39} := 23975$. *The magic rectangles of order* $3\times 13$ *are of* ***different magic sums****.*

Magic Rectangles in Construction of Magic and Block-Bordered Magic Squares, June 07, 2022, pp. 1-70, https://doi.org/10.5281/zenodo.6621071

## 3.14 Magic Square of Order $45 \times 45$

**Example 3.15.** *A magic square of order* $45 \times 45$ *constructed based on blocks of magic rectangles of order* $5 \times 9$ *is given by*

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 45585 |
| 875 | 898 | 874 | 876 | 862 | 867 | 864 | 886 | 900 | 740 | 763 | 739 | 741 | 727 | 732 | 729 | 751 | 765 | 1550 | 1573 | 1549 | 1551 | 1537 | 1542 | 1539 | 1561 | 1575 | 1865 | 1888 | 1864 | 1866 | 1852 | 1857 | 1854 | 1876 | 1890 | 20 | 43 | 19 | 21 | 7 | 12 | 9 | 31 | 45 | 45585 |
| 872 | 877 | 873 | 893 | 869 | 895 | 865 | 899 | 859 | 737 | 742 | 738 | 758 | 734 | 760 | 730 | 764 | 724 | 1547 | 1552 | 1548 | 1568 | 1544 | 1570 | 1540 | 1574 | 1534 | 1862 | 1867 | 1863 | 1883 | 1859 | 1885 | 1855 | 1889 | 1849 | 17 | 22 | 18 | 38 | 14 | 40 | 10 | 44 | 4 | 45585 |
| 890 | 888 | 860 | 871 | 878 | 885 | 896 | 868 | 866 | 755 | 753 | 725 | 736 | 743 | 750 | 761 | 733 | 731 | 1565 | 1563 | 1535 | 1546 | 1553 | 1560 | 1571 | 1543 | 1541 | 1880 | 1878 | 1850 | 1861 | 1868 | 1875 | 1886 | 1858 | 1856 | 35 | 33 | 5 | 16 | 23 | 30 | 41 | 13 | 11 | 45585 |
| 897 | 857 | 891 | 861 | 887 | 863 | 883 | 879 | 884 | 762 | 722 | 756 | 726 | 752 | 728 | 748 | 744 | 749 | 1572 | 1532 | 1566 | 1536 | 1562 | 1538 | 1558 | 1554 | 1559 | 1887 | 1847 | 1881 | 1851 | 1877 | 1853 | 1873 | 1869 | 1874 | 42 | 2 | 36 | 6 | 32 | 8 | 28 | 24 | 29 | 45585 |
| 856 | 870 | 892 | 889 | 894 | 880 | 882 | 858 | 881 | 721 | 735 | 757 | 754 | 759 | 745 | 747 | 723 | 746 | 1531 | 1545 | 1567 | 1564 | 1569 | 1555 | 1557 | 1533 | 1556 | 1846 | 1860 | 1882 | 1879 | 1884 | 1870 | 1872 | 1848 | 1871 | 1 | 15 | 37 | 34 | 39 | 25 | 27 | 3 | 26 | 45585 |
| 1910 | 1933 | 1909 | 1911 | 1897 | 1902 | 1899 | 1921 | 1935 | 965 | 988 | 964 | 966 | 952 | 957 | 954 | 976 | 990 | 1460 | 1483 | 1459 | 1461 | 1447 | 1452 | 1449 | 1471 | 1485 | 65 | 88 | 64 | 66 | 52 | 57 | 54 | 76 | 90 | 650 | 673 | 649 | 651 | 637 | 642 | 639 | 661 | 675 | 45585 |
| 1907 | 1912 | 1908 | 1928 | 1904 | 1930 | 1900 | 1934 | 1894 | 962 | 967 | 963 | 983 | 959 | 985 | 955 | 989 | 949 | 1457 | 1462 | 1458 | 1478 | 1454 | 1480 | 1450 | 1484 | 1444 | 62 | 67 | 63 | 83 | 59 | 85 | 55 | 89 | 49 | 647 | 652 | 648 | 668 | 644 | 670 | 640 | 674 | 634 | 45585 |
| 1925 | 1923 | 1895 | 1906 | 1913 | 1920 | 1931 | 1903 | 1901 | 980 | 978 | 950 | 961 | 968 | 975 | 986 | 958 | 956 | 1475 | 1473 | 1445 | 1456 | 1463 | 1470 | 1481 | 1453 | 1451 | 80 | 78 | 50 | 61 | 68 | 75 | 86 | 58 | 56 | 665 | 663 | 635 | 646 | 653 | 660 | 671 | 643 | 641 | 45585 |
| 1932 | 1892 | 1926 | 1896 | 1922 | 1898 | 1918 | 1914 | 1919 | 987 | 947 | 981 | 951 | 977 | 953 | 973 | 969 | 974 | 1482 | 1442 | 1476 | 1446 | 1472 | 1448 | 1468 | 1464 | 1469 | 87 | 47 | 81 | 51 | 77 | 53 | 73 | 69 | 74 | 672 | 632 | 666 | 636 | 662 | 638 | 658 | 654 | 659 | 45585 |
| 1891 | 1905 | 1927 | 1924 | 1929 | 1915 | 1917 | 1893 | 1916 | 946 | 960 | 982 | 979 | 984 | 970 | 972 | 948 | 971 | 1441 | 1455 | 1477 | 1474 | 1479 | 1465 | 1467 | 1443 | 1466 | 46 | 60 | 82 | 79 | 84 | 70 | 72 | 48 | 71 | 631 | 645 | 667 | 664 | 669 | 655 | 657 | 633 | 656 | 45585 |
| 830 | 853 | 829 | 831 | 817 | 822 | 819 | 841 | 855 | 785 | 808 | 784 | 786 | 772 | 777 | 774 | 796 | 810 | 200 | 223 | 199 | 201 | 187 | 192 | 189 | 211 | 225 | 1595 | 1618 | 1594 | 1596 | 1582 | 1587 | 1584 | 1606 | 1620 | 1640 | 1663 | 1639 | 1641 | 1627 | 1632 | 1629 | 1651 | 1665 | 45585 |
| 827 | 832 | 828 | 848 | 824 | 850 | 820 | 854 | 814 | 782 | 787 | 783 | 803 | 779 | 805 | 775 | 809 | 769 | 197 | 202 | 198 | 218 | 194 | 220 | 190 | 224 | 184 | 1592 | 1597 | 1593 | 1613 | 1589 | 1615 | 1585 | 1619 | 1579 | 1637 | 1642 | 1638 | 1658 | 1634 | 1660 | 1630 | 1664 | 1624 | 45585 |
| 845 | 843 | 815 | 826 | 833 | 840 | 851 | 823 | 821 | 800 | 798 | 770 | 781 | 788 | 795 | 806 | 778 | 776 | 215 | 213 | 185 | 196 | 203 | 210 | 221 | 193 | 191 | 1610 | 1608 | 1580 | 1591 | 1598 | 1605 | 1616 | 1588 | 1586 | 1655 | 1653 | 1625 | 1636 | 1643 | 1650 | 1661 | 1633 | 1631 | 45585 |
| 852 | 812 | 846 | 816 | 842 | 818 | 838 | 834 | 839 | 807 | 767 | 801 | 771 | 797 | 773 | 793 | 789 | 794 | 222 | 182 | 216 | 186 | 212 | 188 | 208 | 204 | 209 | 1617 | 1577 | 1611 | 1581 | 1607 | 1583 | 1603 | 1599 | 1604 | 1662 | 1622 | 1656 | 1626 | 1652 | 1628 | 1648 | 1644 | 1649 | 45585 |
| 811 | 825 | 847 | 844 | 849 | 835 | 837 | 813 | 836 | 766 | 780 | 802 | 799 | 804 | 790 | 792 | 768 | 791 | 181 | 195 | 217 | 214 | 219 | 205 | 207 | 183 | 206 | 1576 | 1590 | 1612 | 1609 | 1614 | 1600 | 1602 | 1578 | 1601 | 1621 | 1635 | 1657 | 1654 | 1659 | 1645 | 1647 | 1623 | 1646 | 45585 |
| 920 | 943 | 919 | 921 | 907 | 912 | 909 | 931 | 945 | 1685 | 1708 | 1684 | 1686 | 1672 | 1677 | 1674 | 1696 | 1710 | 695 | 718 | 694 | 696 | 682 | 687 | 684 | 706 | 720 | 245 | 268 | 244 | 246 | 232 | 237 | 234 | 256 | 270 | 1505 | 1528 | 1504 | 1506 | 1492 | 1497 | 1494 | 1516 | 1530 | 45585 |
| 917 | 922 | 918 | 938 | 914 | 940 | 910 | 944 | 904 | 1682 | 1687 | 1683 | 1703 | 1679 | 1705 | 1675 | 1709 | 1669 | 692 | 697 | 693 | 713 | 689 | 715 | 685 | 719 | 679 | 242 | 247 | 243 | 263 | 239 | 265 | 235 | 269 | 229 | 1502 | 1507 | 1503 | 1523 | 1499 | 1525 | 1495 | 1529 | 1489 | 45585 |
| 935 | 933 | 905 | 916 | 923 | 930 | 941 | 913 | 911 | 1700 | 1698 | 1670 | 1681 | 1688 | 1695 | 1706 | 1678 | 1676 | 710 | 708 | 680 | 691 | 698 | 705 | 716 | 688 | 686 | 260 | 258 | 230 | 241 | 248 | 255 | 266 | 238 | 236 | 1520 | 1518 | 1490 | 1501 | 1508 | 1515 | 1526 | 1498 | 1496 | 45585 |
| 942 | 902 | 936 | 906 | 932 | 908 | 928 | 924 | 929 | 1707 | 1667 | 1701 | 1671 | 1697 | 1673 | 1693 | 1689 | 1694 | 717 | 677 | 711 | 681 | 707 | 683 | 703 | 699 | 704 | 267 | 227 | 261 | 231 | 257 | 233 | 253 | 249 | 254 | 1527 | 1487 | 1521 | 1491 | 1517 | 1493 | 1513 | 1509 | 1514 | 45585 |
| 901 | 915 | 937 | 934 | 939 | 925 | 927 | 903 | 926 | 1666 | 1680 | 1702 | 1699 | 1704 | 1690 | 1692 | 1668 | 1691 | 676 | 690 | 712 | 709 | 714 | 700 | 702 | 678 | 701 | 226 | 240 | 262 | 259 | 264 | 250 | 252 | 228 | 251 | 1486 | 1500 | 1522 | 1519 | 1524 | 1510 | 1512 | 1488 | 1511 | 45585 |
| 290 | 313 | 289 | 291 | 277 | 282 | 279 | 301 | 315 | 605 | 628 | 604 | 606 | 592 | 597 | 594 | 616 | 630 | 1010 | 1033 | 1009 | 1011 | 997 | 1002 | 999 | 1021 | 1035 | 1415 | 1438 | 1414 | 1416 | 1402 | 1407 | 1404 | 1426 | 1440 | 1730 | 1753 | 1729 | 1731 | 1717 | 1722 | 1719 | 1741 | 1755 | 45585 |
| 287 | 292 | 288 | 308 | 284 | 310 | 280 | 314 | 274 | 602 | 607 | 603 | 623 | 599 | 625 | 595 | 629 | 589 | 1007 | 1012 | 1008 | 1028 | 1004 | 1030 | 1000 | 1034 | 994 | 1412 | 1417 | 1413 | 1433 | 1409 | 1435 | 1405 | 1439 | 1399 | 1727 | 1732 | 1728 | 1748 | 1724 | 1750 | 1720 | 1754 | 1714 | 45585 |
| 305 | 303 | 275 | 286 | 293 | 300 | 311 | 283 | 281 | 620 | 618 | 590 | 601 | 608 | 615 | 626 | 598 | 596 | 1025 | 1023 | 995 | 1006 | 1013 | 1020 | 1031 | 1003 | 1001 | 1430 | 1428 | 1400 | 1411 | 1418 | 1425 | 1436 | 1408 | 1406 | 1745 | 1743 | 1715 | 1726 | 1733 | 1740 | 1751 | 1723 | 1721 | 45585 |
| 312 | 272 | 306 | 276 | 302 | 278 | 298 | 294 | 299 | 627 | 587 | 621 | 591 | 617 | 593 | 613 | 609 | 614 | 1032 | 992 | 1026 | 996 | 1022 | 998 | 1018 | 1014 | 1019 | 1437 | 1397 | 1431 | 1401 | 1427 | 1403 | 1423 | 1419 | 1424 | 1752 | 1712 | 1746 | 1716 | 1742 | 1718 | 1738 | 1734 | 1739 | 45585 |
| 271 | 285 | 307 | 304 | 309 | 295 | 297 | 273 | 296 | 586 | 600 | 622 | 619 | 624 | 610 | 612 | 588 | 611 | 991 | 1005 | 1027 | 1024 | 1029 | 1015 | 1017 | 993 | 1016 | 1396 | 1410 | 1432 | 1429 | 1434 | 1420 | 1422 | 1398 | 1421 | 1711 | 1725 | 1747 | 1744 | 1749 | 1735 | 1737 | 1713 | 1736 | 45585 |
| 515 | 538 | 514 | 516 | 502 | 507 | 504 | 526 | 540 | 1775 | 1798 | 1774 | 1776 | 1762 | 1767 | 1764 | 1786 | 1800 | 1325 | 1348 | 1324 | 1326 | 1312 | 1317 | 1314 | 1336 | 1350 | 335 | 358 | 334 | 336 | 322 | 327 | 324 | 346 | 360 | 1100 | 1123 | 1099 | 1101 | 1087 | 1092 | 1089 | 1111 | 1125 | 45585 |
| 512 | 517 | 513 | 533 | 509 | 535 | 505 | 539 | 499 | 1772 | 1777 | 1773 | 1793 | 1769 | 1795 | 1765 | 1799 | 1759 | 1322 | 1327 | 1323 | 1343 | 1319 | 1345 | 1315 | 1349 | 1309 | 332 | 337 | 333 | 353 | 329 | 355 | 325 | 359 | 319 | 1097 | 1102 | 1098 | 1118 | 1094 | 1120 | 1090 | 1124 | 1084 | 45585 |
| 530 | 528 | 500 | 511 | 518 | 525 | 536 | 508 | 506 | 1790 | 1788 | 1760 | 1771 | 1778 | 1785 | 1796 | 1768 | 1766 | 1340 | 1338 | 1310 | 1321 | 1328 | 1335 | 1346 | 1318 | 1316 | 350 | 348 | 320 | 331 | 338 | 345 | 356 | 328 | 326 | 1115 | 1113 | 1085 | 1096 | 1103 | 1110 | 1121 | 1093 | 1091 | 45585 |
| 537 | 497 | 531 | 501 | 527 | 503 | 523 | 519 | 524 | 1797 | 1757 | 1791 | 1761 | 1787 | 1763 | 1783 | 1779 | 1784 | 1347 | 1307 | 1341 | 1311 | 1337 | 1313 | 1333 | 1329 | 1334 | 357 | 317 | 351 | 321 | 347 | 323 | 343 | 339 | 344 | 1122 | 1082 | 1116 | 1086 | 1112 | 1088 | 1108 | 1104 | 1109 | 45585 |
| 496 | 510 | 532 | 529 | 534 | 520 | 522 | 498 | 521 | 1756 | 1770 | 1792 | 1789 | 1794 | 1780 | 1782 | 1758 | 1781 | 1306 | 1320 | 1342 | 1339 | 1344 | 1330 | 1332 | 1308 | 1331 | 316 | 330 | 352 | 349 | 354 | 340 | 342 | 318 | 341 | 1081 | 1095 | 1117 | 1114 | 1119 | 1105 | 1107 | 1083 | 1106 | 45585 |
| 380 | 403 | 379 | 381 | 367 | 372 | 369 | 391 | 405 | 425 | 448 | 424 | 426 | 412 | 417 | 414 | 436 | 450 | 1820 | 1843 | 1819 | 1821 | 1807 | 1812 | 1809 | 1831 | 1845 | 1235 | 1258 | 1234 | 1236 | 1222 | 1227 | 1224 | 1246 | 1260 | 1190 | 1213 | 1189 | 1191 | 1177 | 1182 | 1179 | 1201 | 1215 | 45585 |
| 377 | 382 | 378 | 398 | 374 | 400 | 370 | 404 | 364 | 422 | 427 | 423 | 443 | 419 | 445 | 415 | 449 | 409 | 1817 | 1822 | 1818 | 1838 | 1814 | 1840 | 1810 | 1844 | 1804 | 1232 | 1237 | 1233 | 1253 | 1229 | 1255 | 1225 | 1259 | 1219 | 1187 | 1192 | 1188 | 1208 | 1184 | 1210 | 1180 | 1214 | 1174 | 45585 |
| 395 | 393 | 365 | 376 | 383 | 390 | 401 | 373 | 371 | 440 | 438 | 410 | 421 | 428 | 435 | 446 | 418 | 416 | 1835 | 1833 | 1805 | 1816 | 1823 | 1830 | 1841 | 1813 | 1811 | 1250 | 1248 | 1220 | 1231 | 1238 | 1245 | 1256 | 1228 | 1226 | 1205 | 1203 | 1175 | 1186 | 1193 | 1200 | 1211 | 1183 | 1181 | 45585 |
| 402 | 362 | 396 | 366 | 392 | 368 | 388 | 384 | 389 | 447 | 407 | 441 | 411 | 437 | 413 | 433 | 429 | 434 | 1842 | 1802 | 1836 | 1806 | 1832 | 1808 | 1828 | 1824 | 1829 | 1257 | 1217 | 1251 | 1221 | 1247 | 1223 | 1243 | 1239 | 1244 | 1212 | 1172 | 1206 | 1176 | 1202 | 1178 | 1198 | 1194 | 1199 | 45585 |
| 361 | 375 | 397 | 394 | 399 | 385 | 387 | 363 | 386 | 406 | 420 | 442 | 439 | 444 | 430 | 432 | 408 | 431 | 1801 | 1815 | 1837 | 1834 | 1839 | 1825 | 1827 | 1803 | 1826 | 1216 | 1230 | 1252 | 1249 | 1254 | 1240 | 1242 | 1218 | 1241 | 1171 | 1185 | 1207 | 1204 | 1209 | 1195 | 1197 | 1173 | 1196 | 45585 |
| 1370 | 1393 | 1369 | 1371 | 1357 | 1362 | 1359 | 1381 | 1395 | 1955 | 1978 | 1954 | 1956 | 1942 | 1947 | 1944 | 1966 | 1980 | 560 | 583 | 559 | 561 | 547 | 552 | 549 | 571 | 585 | 1055 | 1078 | 1054 | 1056 | 1042 | 1047 | 1044 | 1066 | 1080 | 110 | 133 | 109 | 111 | 97 | 102 | 99 | 121 | 135 | 45585 |
| 1367 | 1372 | 1368 | 1388 | 1364 | 1390 | 1360 | 1394 | 1354 | 1952 | 1957 | 1953 | 1973 | 1949 | 1975 | 1945 | 1979 | 1939 | 557 | 562 | 558 | 578 | 554 | 580 | 550 | 584 | 544 | 1052 | 1057 | 1053 | 1073 | 1049 | 1075 | 1045 | 1079 | 1039 | 107 | 112 | 108 | 128 | 104 | 130 | 100 | 134 | 94 | 45585 |
| 1385 | 1383 | 1355 | 1366 | 1373 | 1380 | 1391 | 1363 | 1361 | 1970 | 1968 | 1940 | 1951 | 1958 | 1965 | 1976 | 1948 | 1946 | 575 | 573 | 545 | 556 | 563 | 570 | 581 | 553 | 551 | 1070 | 1068 | 1040 | 1051 | 1058 | 1065 | 1076 | 1048 | 1046 | 125 | 123 | 95 | 106 | 113 | 120 | 131 | 103 | 101 | 45585 |
| 1392 | 1352 | 1386 | 1356 | 1382 | 1358 | 1378 | 1374 | 1379 | 1977 | 1937 | 1971 | 1941 | 1967 | 1943 | 1963 | 1959 | 1964 | 582 | 542 | 576 | 546 | 572 | 548 | 568 | 564 | 569 | 1077 | 1037 | 1071 | 1041 | 1067 | 1043 | 1063 | 1059 | 1064 | 132 | 92 | 126 | 96 | 122 | 98 | 118 | 114 | 119 | 45585 |
| 1351 | 1365 | 1387 | 1384 | 1389 | 1375 | 1377 | 1353 | 1376 | 1936 | 1950 | 1972 | 1969 | 1974 | 1960 | 1962 | 1938 | 1961 | 541 | 555 | 577 | 574 | 579 | 565 | 567 | 543 | 566 | 1036 | 1050 | 1072 | 1069 | 1074 | 1060 | 1062 | 1038 | 1061 | 91 | 105 | 127 | 124 | 129 | 115 | 117 | 93 | 116 | 45585 |
| 2000 | 2023 | 1999 | 2001 | 1987 | 1992 | 1989 | 2011 | 2025 | 155 | 178 | 154 | 156 | 142 | 147 | 144 | 166 | 180 | 470 | 493 | 469 | 471 | 457 | 462 | 459 | 481 | 495 | 1280 | 1303 | 1279 | 1281 | 1267 | 1272 | 1269 | 1291 | 1305 | 1145 | 1168 | 1144 | 1146 | 1132 | 1137 | 1134 | 1156 | 1170 | 45585 |
| 1997 | 2002 | 1998 | 2018 | 1994 | 2020 | 1990 | 2024 | 1984 | 152 | 157 | 153 | 173 | 149 | 175 | 145 | 179 | 139 | 467 | 472 | 468 | 488 | 464 | 490 | 460 | 494 | 454 | 1277 | 1282 | 1278 | 1298 | 1274 | 1300 | 1270 | 1304 | 1264 | 1142 | 1147 | 1143 | 1163 | 1139 | 1165 | 1135 | 1169 | 1129 | 45585 |
| 2015 | 2013 | 1985 | 1996 | 2003 | 2010 | 2021 | 1993 | 1991 | 170 | 168 | 140 | 151 | 158 | 165 | 176 | 148 | 146 | 485 | 483 | 455 | 466 | 473 | 480 | 491 | 463 | 461 | 1295 | 1293 | 1265 | 1276 | 1283 | 1290 | 1301 | 1273 | 1271 | 1160 | 1158 | 1130 | 1141 | 1148 | 1155 | 1166 | 1138 | 1136 | 45585 |
| 2022 | 1982 | 2016 | 1986 | 2012 | 1988 | 2008 | 2004 | 2009 | 177 | 137 | 171 | 141 | 167 | 143 | 163 | 159 | 164 | 492 | 452 | 486 | 456 | 482 | 458 | 478 | 474 | 479 | 1302 | 1262 | 1296 | 1266 | 1292 | 1268 | 1288 | 1284 | 1289 | 1167 | 1127 | 1161 | 1131 | 1157 | 1133 | 1153 | 1149 | 1154 | 45585 |
| 1981 | 1995 | 2017 | 2014 | 2019 | 2005 | 2007 | 1983 | 2006 | 136 | 150 | 172 | 169 | 174 | 160 | 162 | 138 | 161 | 451 | 465 | 487 | 484 | 489 | 475 | 477 | 453 | 476 | 1261 | 1275 | 1297 | 1294 | 1299 | 1285 | 1287 | 1263 | 1286 | 1126 | 1140 | 1162 | 1159 | 1164 | 1150 | 1152 | 1128 | 1151 | 45585 |
| 45585 | 45585 | 45585 | 45585 | 45585 | 45585 | 45585 | 45585 | 45585 | 45585 | 45585 | 45585 | 45585 | 45585 | 45585 | 45585 | 45585 | 45585 | 45585 | 45585 | 45585 | 45585 | 45585 | 45585 | 45585 | 45585 | 45585 | 45585 | 45585 | 45585 | 45585 | 45585 | 45585 | 45585 | 45585 | 45585 | 45585 | 45585 | 45585 | 45585 | 45585 | 45585 | 45585 | 45585 | 45585 | 45585 |

*In this case the magic sum is* $S_{45\times 45} := 45585$. *The magic rectangles of order* $3 \times 15$ *are of* ***different magic sums***.

**Example 3.16.** *A magic square of order $45 \times 45$ constructed based on blocks of magic rectangles of order $3 \times 15$ is given by*

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 45585 |
| 1 | 28 | 26 | 4 | 9 | 27 | 25 | 12 | 35 | 36 | 17 | 22 | 43 | 44 | 16 | 1666 | 1693 | 1691 | 1669 | 1674 | 1692 | 1690 | 1677 | 1700 | 1701 | 1682 | 1687 | 1708 | 1709 | 1681 | 1306 | 1333 | 1331 | 1309 | 1314 | 1332 | 1330 | 1317 | 1340 | 1341 | 1322 | 1327 | 1348 | 1349 | 1321 | 45585 |
| 38 | 39 | 40 | 41 | 31 | 32 | 33 | 23 | 13 | 14 | 15 | 5 | 6 | 7 | 8 | 1703 | 1704 | 1705 | 1706 | 1696 | 1697 | 1698 | 1688 | 1678 | 1679 | 1680 | 1670 | 1671 | 1672 | 1673 | 1343 | 1344 | 1345 | 1346 | 1336 | 1337 | 1338 | 1328 | 1318 | 1319 | 1320 | 1310 | 1311 | 1312 | 1313 | 45585 |
| 30 | 2 | 3 | 24 | 29 | 10 | 11 | 34 | 21 | 19 | 37 | 42 | 20 | 18 | 45 | 1695 | 1667 | 1668 | 1689 | 1694 | 1675 | 1676 | 1699 | 1686 | 1684 | 1702 | 1707 | 1685 | 1683 | 1710 | 1335 | 1307 | 1308 | 1329 | 1334 | 1315 | 1316 | 1339 | 1326 | 1324 | 1342 | 1347 | 1325 | 1323 | 1350 | 45585 |
| 1216 | 1243 | 1241 | 1219 | 1224 | 1242 | 1240 | 1227 | 1250 | 1251 | 1232 | 1237 | 1258 | 1259 | 1231 | 1711 | 1738 | 1736 | 1714 | 1719 | 1737 | 1735 | 1722 | 1745 | 1746 | 1727 | 1732 | 1753 | 1754 | 1726 | 46 | 73 | 71 | 49 | 54 | 72 | 70 | 57 | 80 | 81 | 62 | 67 | 88 | 89 | 61 | 45585 |
| 1253 | 1254 | 1255 | 1256 | 1246 | 1247 | 1248 | 1238 | 1228 | 1229 | 1230 | 1220 | 1221 | 1222 | 1223 | 1748 | 1749 | 1750 | 1751 | 1741 | 1742 | 1743 | 1733 | 1723 | 1724 | 1725 | 1715 | 1716 | 1717 | 1718 | 83 | 84 | 85 | 86 | 76 | 77 | 78 | 68 | 58 | 59 | 60 | 50 | 51 | 52 | 53 | 45585 |
| 1245 | 1217 | 1218 | 1239 | 1244 | 1225 | 1226 | 1249 | 1236 | 1234 | 1252 | 1257 | 1235 | 1233 | 1260 | 1740 | 1712 | 1713 | 1734 | 1739 | 1720 | 1721 | 1744 | 1731 | 1729 | 1747 | 1752 | 1730 | 1728 | 1755 | 75 | 47 | 48 | 69 | 74 | 55 | 56 | 79 | 66 | 64 | 82 | 87 | 65 | 63 | 90 | 45585 |
| 1126 | 1153 | 1151 | 1129 | 1134 | 1152 | 1150 | 1137 | 1160 | 1161 | 1142 | 1147 | 1168 | 1169 | 1141 | 1756 | 1783 | 1781 | 1759 | 1764 | 1782 | 1780 | 1767 | 1790 | 1791 | 1772 | 1777 | 1798 | 1799 | 1771 | 91 | 118 | 116 | 94 | 99 | 117 | 115 | 102 | 125 | 126 | 107 | 112 | 133 | 134 | 106 | 45585 |
| 1163 | 1164 | 1165 | 1166 | 1156 | 1157 | 1158 | 1148 | 1138 | 1139 | 1140 | 1130 | 1131 | 1132 | 1133 | 1793 | 1794 | 1795 | 1796 | 1786 | 1787 | 1788 | 1778 | 1768 | 1769 | 1770 | 1760 | 1761 | 1762 | 1763 | 128 | 129 | 130 | 131 | 121 | 122 | 123 | 113 | 103 | 104 | 105 | 95 | 96 | 97 | 98 | 45585 |
| 1155 | 1127 | 1128 | 1149 | 1154 | 1135 | 1136 | 1159 | 1146 | 1144 | 1162 | 1167 | 1145 | 1143 | 1170 | 1785 | 1757 | 1758 | 1779 | 1784 | 1765 | 1766 | 1789 | 1776 | 1774 | 1792 | 1797 | 1775 | 1773 | 1800 | 120 | 92 | 93 | 114 | 119 | 100 | 101 | 124 | 111 | 109 | 127 | 132 | 110 | 108 | 135 | 45585 |
| 136 | 163 | 161 | 139 | 144 | 162 | 160 | 147 | 170 | 171 | 152 | 157 | 178 | 179 | 151 | 1801 | 1828 | 1826 | 1804 | 1809 | 1827 | 1825 | 1812 | 1835 | 1836 | 1817 | 1822 | 1843 | 1844 | 1816 | 1036 | 1063 | 1061 | 1039 | 1044 | 1062 | 1060 | 1047 | 1070 | 1071 | 1052 | 1057 | 1078 | 1079 | 1051 | 45585 |
| 173 | 174 | 175 | 176 | 166 | 167 | 168 | 158 | 148 | 149 | 150 | 140 | 141 | 142 | 143 | 1838 | 1839 | 1840 | 1841 | 1831 | 1832 | 1833 | 1823 | 1813 | 1814 | 1815 | 1805 | 1806 | 1807 | 1808 | 1073 | 1074 | 1075 | 1076 | 1066 | 1067 | 1068 | 1058 | 1048 | 1049 | 1050 | 1040 | 1041 | 1042 | 1043 | 45585 |
| 165 | 137 | 138 | 159 | 164 | 145 | 146 | 169 | 156 | 154 | 172 | 177 | 155 | 153 | 180 | 1830 | 1802 | 1803 | 1824 | 1829 | 1810 | 1811 | 1834 | 1821 | 1819 | 1837 | 1842 | 1820 | 1818 | 1845 | 1065 | 1037 | 1038 | 1059 | 1064 | 1045 | 1046 | 1069 | 1056 | 1054 | 1072 | 1077 | 1055 | 1053 | 1080 | 45585 |
| 361 | 388 | 386 | 364 | 369 | 387 | 385 | 372 | 395 | 396 | 377 | 382 | 403 | 404 | 376 | 1351 | 1378 | 1376 | 1354 | 1359 | 1377 | 1375 | 1362 | 1385 | 1386 | 1367 | 1372 | 1393 | 1394 | 1366 | 1261 | 1288 | 1286 | 1264 | 1269 | 1287 | 1285 | 1272 | 1295 | 1296 | 1277 | 1282 | 1303 | 1304 | 1276 | 45585 |
| 398 | 399 | 400 | 401 | 391 | 392 | 393 | 383 | 373 | 374 | 375 | 365 | 366 | 367 | 368 | 1388 | 1389 | 1390 | 1391 | 1381 | 1382 | 1383 | 1373 | 1363 | 1364 | 1365 | 1355 | 1356 | 1357 | 1358 | 1298 | 1299 | 1300 | 1301 | 1291 | 1292 | 1293 | 1283 | 1273 | 1274 | 1275 | 1265 | 1266 | 1267 | 1268 | 45585 |
| 390 | 362 | 363 | 384 | 389 | 370 | 371 | 394 | 381 | 379 | 397 | 402 | 380 | 378 | 405 | 1380 | 1352 | 1353 | 1374 | 1379 | 1360 | 1361 | 1384 | 1371 | 1369 | 1387 | 1392 | 1370 | 1368 | 1395 | 1290 | 1262 | 1263 | 1284 | 1289 | 1270 | 1271 | 1294 | 1281 | 1279 | 1297 | 1302 | 1280 | 1278 | 1305 | 45585 |
| 1171 | 1198 | 1196 | 1174 | 1179 | 1197 | 1195 | 1182 | 1205 | 1206 | 1187 | 1192 | 1213 | 1214 | 1186 | 1396 | 1423 | 1421 | 1399 | 1404 | 1422 | 1420 | 1407 | 1430 | 1431 | 1412 | 1417 | 1438 | 1439 | 1411 | 406 | 433 | 431 | 409 | 414 | 432 | 430 | 417 | 440 | 441 | 422 | 427 | 448 | 449 | 421 | 45585 |
| 1208 | 1209 | 1210 | 1211 | 1201 | 1202 | 1203 | 1193 | 1183 | 1184 | 1185 | 1175 | 1176 | 1177 | 1178 | 1433 | 1434 | 1435 | 1436 | 1426 | 1427 | 1428 | 1418 | 1408 | 1409 | 1410 | 1400 | 1401 | 1402 | 1403 | 443 | 444 | 445 | 446 | 436 | 437 | 438 | 428 | 418 | 419 | 420 | 410 | 411 | 412 | 413 | 45585 |
| 1200 | 1172 | 1173 | 1194 | 1199 | 1180 | 1181 | 1204 | 1191 | 1189 | 1207 | 1212 | 1190 | 1188 | 1215 | 1425 | 1397 | 1398 | 1419 | 1424 | 1405 | 1406 | 1429 | 1416 | 1414 | 1432 | 1437 | 1415 | 1413 | 1440 | 435 | 407 | 408 | 429 | 434 | 415 | 416 | 439 | 426 | 424 | 442 | 447 | 425 | 423 | 450 | 45585 |
| 1081 | 1108 | 1106 | 1084 | 1089 | 1107 | 1105 | 1092 | 1115 | 1116 | 1097 | 1102 | 1123 | 1124 | 1096 | 1441 | 1468 | 1466 | 1444 | 1449 | 1467 | 1465 | 1452 | 1475 | 1476 | 1457 | 1462 | 1483 | 1484 | 1456 | 451 | 478 | 476 | 454 | 459 | 477 | 475 | 462 | 485 | 486 | 467 | 472 | 493 | 494 | 466 | 45585 |
| 1118 | 1119 | 1120 | 1121 | 1111 | 1112 | 1113 | 1103 | 1093 | 1094 | 1095 | 1085 | 1086 | 1087 | 1088 | 1478 | 1479 | 1480 | 1481 | 1471 | 1472 | 1473 | 1463 | 1453 | 1454 | 1455 | 1445 | 1446 | 1447 | 1448 | 488 | 489 | 490 | 491 | 481 | 482 | 483 | 473 | 463 | 464 | 465 | 455 | 456 | 457 | 458 | 45585 |
| 1110 | 1082 | 1083 | 1104 | 1109 | 1090 | 1091 | 1114 | 1101 | 1099 | 1117 | 1122 | 1100 | 1098 | 1125 | 1470 | 1442 | 1443 | 1464 | 1469 | 1450 | 1451 | 1474 | 1461 | 1459 | 1477 | 1482 | 1460 | 1458 | 1485 | 480 | 452 | 453 | 474 | 479 | 460 | 461 | 484 | 471 | 469 | 487 | 492 | 470 | 468 | 495 | 45585 |
| 496 | 523 | 521 | 499 | 504 | 522 | 520 | 507 | 530 | 531 | 512 | 517 | 538 | 539 | 511 | 991 | 1018 | 1016 | 994 | 999 | 1017 | 1015 | 1002 | 1025 | 1026 | 1007 | 1012 | 1033 | 1034 | 1006 | 1486 | 1513 | 1511 | 1489 | 1494 | 1512 | 1510 | 1497 | 1520 | 1521 | 1502 | 1507 | 1528 | 1529 | 1501 | 45585 |
| 533 | 534 | 535 | 536 | 526 | 527 | 528 | 518 | 508 | 509 | 510 | 500 | 501 | 502 | 503 | 1028 | 1029 | 1030 | 1031 | 1021 | 1022 | 1023 | 1013 | 1003 | 1004 | 1005 | 995 | 996 | 997 | 998 | 1523 | 1524 | 1525 | 1526 | 1516 | 1517 | 1518 | 1508 | 1498 | 1499 | 1500 | 1490 | 1491 | 1492 | 1493 | 45585 |
| 525 | 497 | 498 | 519 | 524 | 505 | 506 | 529 | 516 | 514 | 532 | 537 | 515 | 513 | 540 | 1020 | 992 | 993 | 1014 | 1019 | 1000 | 1001 | 1024 | 1011 | 1009 | 1027 | 1032 | 1010 | 1008 | 1035 | 1515 | 1487 | 1488 | 1509 | 1514 | 1495 | 1496 | 1519 | 1506 | 1504 | 1522 | 1527 | 1505 | 1503 | 1530 | 45585 |
| 1531 | 1558 | 1556 | 1534 | 1539 | 1557 | 1555 | 1542 | 1565 | 1566 | 1547 | 1552 | 1573 | 1574 | 1546 | 541 | 568 | 566 | 544 | 549 | 567 | 565 | 552 | 575 | 576 | 557 | 562 | 583 | 584 | 556 | 901 | 928 | 926 | 904 | 909 | 927 | 925 | 912 | 935 | 936 | 917 | 922 | 943 | 944 | 916 | 45585 |
| 1568 | 1569 | 1570 | 1571 | 1561 | 1562 | 1563 | 1553 | 1543 | 1544 | 1545 | 1535 | 1536 | 1537 | 1538 | 578 | 579 | 580 | 581 | 571 | 572 | 573 | 563 | 553 | 554 | 555 | 545 | 546 | 547 | 548 | 938 | 939 | 940 | 941 | 931 | 932 | 933 | 923 | 913 | 914 | 915 | 905 | 906 | 907 | 908 | 45585 |
| 1560 | 1532 | 1533 | 1554 | 1559 | 1540 | 1541 | 1564 | 1551 | 1549 | 1567 | 1572 | 1550 | 1548 | 1575 | 570 | 542 | 543 | 564 | 569 | 550 | 551 | 574 | 561 | 559 | 577 | 582 | 560 | 558 | 585 | 930 | 902 | 903 | 924 | 929 | 910 | 911 | 934 | 921 | 919 | 937 | 942 | 920 | 918 | 945 | 45585 |
| 1576 | 1603 | 1601 | 1579 | 1584 | 1602 | 1600 | 1587 | 1610 | 1611 | 1592 | 1597 | 1618 | 1619 | 1591 | 586 | 613 | 611 | 589 | 594 | 612 | 610 | 597 | 620 | 621 | 602 | 607 | 628 | 629 | 601 | 811 | 838 | 836 | 814 | 819 | 837 | 835 | 822 | 845 | 846 | 827 | 832 | 853 | 854 | 826 | 45585 |
| 1613 | 1614 | 1615 | 1616 | 1606 | 1607 | 1608 | 1598 | 1588 | 1589 | 1590 | 1580 | 1581 | 1582 | 1583 | 623 | 624 | 625 | 626 | 616 | 617 | 618 | 608 | 598 | 599 | 600 | 590 | 591 | 592 | 593 | 848 | 849 | 850 | 851 | 841 | 842 | 843 | 833 | 823 | 824 | 825 | 815 | 816 | 817 | 818 | 45585 |
| 1605 | 1577 | 1578 | 1599 | 1604 | 1585 | 1586 | 1609 | 1596 | 1594 | 1612 | 1617 | 1595 | 1593 | 1620 | 615 | 587 | 588 | 609 | 614 | 595 | 596 | 619 | 606 | 604 | 622 | 627 | 605 | 603 | 630 | 840 | 812 | 813 | 834 | 839 | 820 | 821 | 844 | 831 | 829 | 847 | 852 | 830 | 828 | 855 | 45585 |
| 721 | 748 | 746 | 724 | 729 | 747 | 745 | 732 | 755 | 756 | 737 | 742 | 763 | 764 | 736 | 631 | 658 | 656 | 634 | 639 | 657 | 655 | 642 | 665 | 666 | 647 | 652 | 673 | 674 | 646 | 1621 | 1648 | 1646 | 1624 | 1629 | 1647 | 1645 | 1632 | 1655 | 1656 | 1637 | 1642 | 1663 | 1664 | 1636 | 45585 |
| 758 | 759 | 760 | 761 | 751 | 752 | 753 | 743 | 733 | 734 | 735 | 725 | 726 | 727 | 728 | 668 | 669 | 670 | 671 | 661 | 662 | 663 | 653 | 643 | 644 | 645 | 635 | 636 | 637 | 638 | 1658 | 1659 | 1660 | 1661 | 1651 | 1652 | 1653 | 1643 | 1633 | 1634 | 1635 | 1625 | 1626 | 1627 | 1628 | 45585 |
| 750 | 722 | 723 | 744 | 749 | 730 | 731 | 754 | 741 | 739 | 757 | 762 | 740 | 738 | 765 | 660 | 632 | 633 | 654 | 659 | 640 | 641 | 664 | 651 | 649 | 667 | 672 | 650 | 648 | 675 | 1650 | 1622 | 1623 | 1644 | 1649 | 1630 | 1631 | 1654 | 1641 | 1639 | 1657 | 1662 | 1640 | 1638 | 1665 | 45585 |
| 946 | 973 | 971 | 949 | 954 | 972 | 970 | 957 | 980 | 981 | 962 | 967 | 988 | 989 | 961 | 181 | 208 | 206 | 184 | 189 | 207 | 205 | 192 | 215 | 216 | 197 | 202 | 223 | 224 | 196 | 1846 | 1873 | 1871 | 1849 | 1854 | 1872 | 1870 | 1857 | 1880 | 1881 | 1862 | 1867 | 1888 | 1889 | 1861 | 45585 |
| 983 | 984 | 985 | 986 | 976 | 977 | 978 | 968 | 958 | 959 | 960 | 950 | 951 | 952 | 953 | 218 | 219 | 220 | 221 | 211 | 212 | 213 | 203 | 193 | 194 | 195 | 185 | 186 | 187 | 188 | 1883 | 1884 | 1885 | 1886 | 1876 | 1877 | 1878 | 1868 | 1858 | 1859 | 1860 | 1850 | 1851 | 1852 | 1853 | 45585 |
| 975 | 947 | 948 | 969 | 974 | 955 | 956 | 979 | 966 | 964 | 982 | 987 | 965 | 963 | 990 | 210 | 182 | 183 | 204 | 209 | 190 | 191 | 214 | 201 | 199 | 217 | 222 | 200 | 198 | 225 | 1875 | 1847 | 1848 | 1869 | 1874 | 1855 | 1856 | 1879 | 1866 | 1864 | 1882 | 1887 | 1865 | 1863 | 1890 | 45585 |
| 1891 | 1918 | 1916 | 1894 | 1899 | 1917 | 1915 | 1902 | 1925 | 1926 | 1907 | 1912 | 1933 | 1934 | 1906 | 226 | 253 | 251 | 229 | 234 | 252 | 250 | 237 | 260 | 261 | 242 | 247 | 268 | 269 | 241 | 856 | 883 | 881 | 859 | 864 | 882 | 880 | 867 | 890 | 891 | 872 | 877 | 898 | 899 | 871 | 45585 |
| 1928 | 1929 | 1930 | 1931 | 1921 | 1922 | 1923 | 1913 | 1903 | 1904 | 1905 | 1895 | 1896 | 1897 | 1898 | 263 | 264 | 265 | 266 | 256 | 257 | 258 | 248 | 238 | 239 | 240 | 230 | 231 | 232 | 233 | 893 | 894 | 895 | 896 | 886 | 887 | 888 | 878 | 868 | 869 | 870 | 860 | 861 | 862 | 863 | 45585 |
| 1920 | 1892 | 1893 | 1914 | 1919 | 1900 | 1901 | 1924 | 1911 | 1909 | 1927 | 1932 | 1910 | 1908 | 1935 | 255 | 227 | 228 | 249 | 254 | 235 | 236 | 259 | 246 | 244 | 262 | 267 | 245 | 243 | 270 | 885 | 857 | 858 | 879 | 884 | 865 | 866 | 889 | 876 | 874 | 892 | 897 | 875 | 873 | 900 | 45585 |
| 1936 | 1963 | 1961 | 1939 | 1944 | 1962 | 1960 | 1947 | 1970 | 1971 | 1952 | 1957 | 1978 | 1979 | 1951 | 271 | 298 | 296 | 274 | 279 | 297 | 295 | 282 | 305 | 306 | 287 | 292 | 313 | 314 | 286 | 766 | 793 | 791 | 769 | 774 | 792 | 790 | 777 | 800 | 801 | 782 | 787 | 808 | 809 | 781 | 45585 |
| 1973 | 1974 | 1975 | 1976 | 1966 | 1967 | 1968 | 1958 | 1948 | 1949 | 1950 | 1940 | 1941 | 1942 | 1943 | 308 | 309 | 310 | 311 | 301 | 302 | 303 | 293 | 283 | 284 | 285 | 275 | 276 | 277 | 278 | 803 | 804 | 805 | 806 | 796 | 797 | 798 | 788 | 778 | 779 | 780 | 770 | 771 | 772 | 773 | 45585 |
| 1965 | 1937 | 1938 | 1959 | 1964 | 1945 | 1946 | 1969 | 1956 | 1954 | 1972 | 1977 | 1955 | 1953 | 1980 | 300 | 272 | 273 | 294 | 299 | 280 | 281 | 304 | 291 | 289 | 307 | 312 | 290 | 288 | 315 | 795 | 767 | 768 | 789 | 794 | 775 | 776 | 799 | 786 | 784 | 802 | 807 | 785 | 783 | 810 | 45585 |
| 676 | 703 | 701 | 679 | 684 | 702 | 700 | 687 | 710 | 711 | 692 | 697 | 718 | 719 | 691 | 316 | 343 | 341 | 319 | 324 | 342 | 340 | 327 | 350 | 351 | 332 | 337 | 358 | 359 | 331 | 1981 | 2008 | 2006 | 1984 | 1989 | 2007 | 2005 | 1992 | 2015 | 2016 | 1997 | 2002 | 2023 | 2024 | 1996 | 45585 |
| 713 | 714 | 715 | 716 | 706 | 707 | 708 | 698 | 688 | 689 | 690 | 680 | 681 | 682 | 683 | 353 | 354 | 355 | 356 | 346 | 347 | 348 | 338 | 328 | 329 | 330 | 320 | 321 | 322 | 323 | 2018 | 2019 | 2020 | 2021 | 2011 | 2012 | 2013 | 2003 | 1993 | 1994 | 1995 | 1985 | 1986 | 1987 | 1988 | 45585 |
| 705 | 677 | 678 | 699 | 704 | 685 | 686 | 709 | 696 | 694 | 712 | 717 | 695 | 693 | 720 | 345 | 317 | 318 | 339 | 344 | 325 | 326 | 349 | 336 | 334 | 352 | 357 | 335 | 333 | 360 | 2010 | 1982 | 1983 | 2004 | 2009 | 1990 | 1991 | 2014 | 2001 | 1999 | 2017 | 2022 | 2000 | 1998 | 2025 | 45585 |
| 45585 | 45585 | 45585 | 45585 | 45585 | 45585 | 45585 | 45585 | 45585 | 45585 | 45585 | 45585 | 45585 | 45585 | 45585 | 45585 | 45585 | 45585 | 45585 | 45585 | 45585 | 45585 | 45585 | 45585 | 45585 | 45585 | 45585 | 45585 | 45585 | 45585 | 45585 | 45585 | 45585 | 45585 | 45585 | 45585 | 45585 | 45585 | 45585 | 45585 | 45585 | 45585 | 45585 | 45585 | 45585 | 45585 |

*In this case the magic sum is $S_{45\times45} := 45585$. The magic rectangles of order $3 \times 15$ are of* ***different magic sums****.*

Magic Rectangles in Construction of Magic and Block-Bordered Magic Squares, June 07, 2022, pp. 1-70, https://doi.org/10.5281/zenodo.6621071

## 3.15 Block Bordered Magic Square of Order $47 \times 47$

It is based on Examples 3.15 and 3.16 with single external border, and internal blocks are magic squares of order 45, where the magic rectangles are of orders $5 \times 9$ and $3 \times 15$

**Example 3.17.** *A magic square of order $47 \times 47$ is constructed based on magic squares of order $45 \times 45$ given in Examples 3.15. It is given by*

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 51935 |
| 2164 | 2118 | 2120 | 2122 | 2124 | 2126 | 2128 | 2130 | 2132 | 2134 | 2136 | 2138 | 2140 | 2142 | 2144 | 2146 | 2148 | 2150 | 2152 | 2154 | 2156 | 2158 | 2160 | 45 | 44 | 42 | 40 | 38 | 36 | 34 | 32 | 30 | 28 | 26 | 24 | 22 | 20 | 18 | 16 | 14 | 12 | 10 | 8 | 6 | 4 | 2 | 2162 | 51935 |
| 91 | 967 | 990 | 966 | 968 | 954 | 959 | 956 | 978 | 992 | 832 | 855 | 831 | 833 | 819 | 824 | 821 | 843 | 857 | 1642 | 1665 | 1641 | 1643 | 1629 | 1634 | 1631 | 1653 | 1667 | 1957 | 1980 | 1956 | 1958 | 1944 | 1949 | 1946 | 1968 | 1982 | 112 | 135 | 111 | 113 | 99 | 104 | 101 | 123 | 137 | 2119 | 51935 |
| 89 | 964 | 969 | 965 | 985 | 961 | 987 | 957 | 991 | 951 | 829 | 834 | 830 | 850 | 826 | 852 | 822 | 856 | 816 | 1639 | 1644 | 1640 | 1660 | 1636 | 1662 | 1632 | 1666 | 1626 | 1954 | 1959 | 1955 | 1975 | 1951 | 1977 | 1947 | 1981 | 1941 | 109 | 114 | 110 | 130 | 106 | 132 | 102 | 136 | 96 | 2121 | 51935 |
| 87 | 982 | 980 | 952 | 963 | 970 | 977 | 988 | 960 | 958 | 847 | 845 | 817 | 828 | 835 | 842 | 853 | 825 | 823 | 1657 | 1655 | 1627 | 1638 | 1645 | 1652 | 1663 | 1635 | 1633 | 1972 | 1970 | 1942 | 1953 | 1960 | 1967 | 1978 | 1950 | 1948 | 127 | 125 | 97 | 108 | 115 | 122 | 133 | 105 | 103 | 2123 | 51935 |
| 85 | 989 | 949 | 983 | 953 | 979 | 955 | 975 | 971 | 976 | 854 | 814 | 848 | 818 | 844 | 820 | 840 | 836 | 841 | 1664 | 1624 | 1658 | 1628 | 1654 | 1630 | 1650 | 1646 | 1651 | 1979 | 1939 | 1973 | 1943 | 1969 | 1945 | 1965 | 1961 | 1966 | 134 | 94 | 128 | 98 | 124 | 100 | 120 | 116 | 121 | 2125 | 51935 |
| 83 | 948 | 962 | 984 | 981 | 986 | 972 | 974 | 950 | 973 | 813 | 827 | 849 | 846 | 851 | 837 | 839 | 815 | 838 | 1623 | 1637 | 1659 | 1656 | 1661 | 1647 | 1649 | 1625 | 1648 | 1938 | 1952 | 1974 | 1971 | 1976 | 1962 | 1964 | 1940 | 1963 | 93 | 107 | 129 | 126 | 131 | 117 | 119 | 95 | 118 | 2127 | 51935 |
| 81 | 2002 | 2025 | 2001 | 2003 | 1989 | 1994 | 1991 | 2013 | 2027 | 1057 | 1080 | 1056 | 1058 | 1044 | 1049 | 1046 | 1068 | 1082 | 1552 | 1575 | 1551 | 1553 | 1539 | 1544 | 1541 | 1563 | 1577 | 157 | 180 | 156 | 158 | 144 | 149 | 146 | 168 | 182 | 742 | 765 | 741 | 743 | 729 | 734 | 731 | 753 | 767 | 2129 | 51935 |
| 79 | 1999 | 2004 | 2000 | 2020 | 1996 | 2022 | 1992 | 2026 | 1986 | 1054 | 1059 | 1055 | 1075 | 1051 | 1077 | 1047 | 1081 | 1041 | 1549 | 1554 | 1550 | 1570 | 1546 | 1572 | 1542 | 1576 | 1536 | 154 | 159 | 155 | 175 | 151 | 177 | 147 | 181 | 141 | 739 | 744 | 740 | 760 | 736 | 762 | 732 | 766 | 726 | 2131 | 51935 |
| 77 | 2017 | 2015 | 1987 | 1998 | 2005 | 2012 | 2023 | 1995 | 1993 | 1072 | 1070 | 1042 | 1053 | 1060 | 1067 | 1078 | 1050 | 1048 | 1567 | 1565 | 1537 | 1548 | 1555 | 1562 | 1573 | 1545 | 1543 | 172 | 170 | 142 | 153 | 160 | 167 | 178 | 150 | 148 | 757 | 755 | 727 | 738 | 745 | 752 | 763 | 735 | 733 | 2133 | 51935 |
| 75 | 2024 | 1984 | 2018 | 1988 | 2014 | 1990 | 2010 | 2006 | 2011 | 1079 | 1039 | 1073 | 1043 | 1069 | 1045 | 1065 | 1061 | 1066 | 1574 | 1534 | 1568 | 1538 | 1564 | 1540 | 1560 | 1556 | 1561 | 179 | 139 | 173 | 143 | 169 | 145 | 165 | 161 | 166 | 764 | 724 | 758 | 728 | 754 | 730 | 750 | 746 | 751 | 2135 | 51935 |
| 73 | 1983 | 1997 | 2019 | 2016 | 2021 | 2007 | 2009 | 1985 | 2008 | 1038 | 1052 | 1074 | 1071 | 1076 | 1062 | 1064 | 1040 | 1063 | 1533 | 1547 | 1569 | 1566 | 1571 | 1557 | 1559 | 1535 | 1558 | 138 | 152 | 174 | 171 | 176 | 162 | 164 | 140 | 163 | 723 | 737 | 759 | 756 | 761 | 747 | 749 | 725 | 748 | 2137 | 51935 |
| 71 | 922 | 945 | 921 | 923 | 909 | 914 | 911 | 933 | 947 | 877 | 900 | 876 | 878 | 864 | 869 | 866 | 888 | 902 | 292 | 315 | 291 | 293 | 279 | 284 | 281 | 303 | 317 | 1687 | 1710 | 1686 | 1688 | 1674 | 1679 | 1676 | 1698 | 1712 | 1732 | 1755 | 1731 | 1733 | 1719 | 1724 | 1721 | 1743 | 1757 | 2139 | 51935 |
| 69 | 919 | 924 | 920 | 940 | 916 | 942 | 912 | 946 | 906 | 874 | 879 | 875 | 895 | 871 | 897 | 867 | 901 | 861 | 289 | 294 | 290 | 310 | 286 | 312 | 282 | 316 | 276 | 1684 | 1689 | 1685 | 1705 | 1681 | 1707 | 1677 | 1711 | 1671 | 1729 | 1734 | 1730 | 1750 | 1726 | 1752 | 1722 | 1756 | 1716 | 2141 | 51935 |
| 67 | 937 | 935 | 907 | 918 | 925 | 932 | 943 | 915 | 913 | 892 | 890 | 862 | 873 | 880 | 887 | 898 | 870 | 868 | 307 | 305 | 277 | 288 | 295 | 302 | 313 | 285 | 283 | 1702 | 1700 | 1672 | 1683 | 1690 | 1697 | 1708 | 1680 | 1678 | 1747 | 1745 | 1717 | 1728 | 1735 | 1742 | 1753 | 1725 | 1723 | 2143 | 51935 |
| 65 | 944 | 904 | 938 | 908 | 934 | 910 | 930 | 926 | 931 | 899 | 859 | 893 | 863 | 889 | 865 | 885 | 881 | 886 | 314 | 274 | 308 | 278 | 304 | 280 | 300 | 296 | 301 | 1709 | 1669 | 1703 | 1673 | 1699 | 1675 | 1695 | 1691 | 1696 | 1754 | 1714 | 1748 | 1718 | 1744 | 1720 | 1740 | 1736 | 1741 | 2145 | 51935 |
| 63 | 903 | 917 | 939 | 936 | 941 | 927 | 929 | 905 | 928 | 858 | 872 | 894 | 891 | 896 | 882 | 884 | 860 | 883 | 273 | 287 | 309 | 306 | 311 | 297 | 299 | 275 | 298 | 1668 | 1682 | 1704 | 1701 | 1706 | 1692 | 1694 | 1670 | 1693 | 1713 | 1727 | 1749 | 1746 | 1751 | 1737 | 1739 | 1715 | 1738 | 2147 | 51935 |
| 61 | 1012 | 1035 | 1011 | 1013 | 999 | 1004 | 1001 | 1023 | 1037 | 1777 | 1800 | 1776 | 1778 | 1764 | 1769 | 1766 | 1788 | 1802 | 787 | 810 | 786 | 788 | 774 | 779 | 776 | 798 | 812 | 337 | 360 | 336 | 338 | 324 | 329 | 326 | 348 | 362 | 1597 | 1620 | 1596 | 1598 | 1584 | 1589 | 1586 | 1608 | 1622 | 2149 | 51935 |
| 59 | 1009 | 1014 | 1010 | 1030 | 1006 | 1032 | 1002 | 1036 | 996 | 1774 | 1779 | 1775 | 1795 | 1771 | 1797 | 1767 | 1801 | 1761 | 784 | 789 | 785 | 805 | 781 | 807 | 777 | 811 | 771 | 334 | 339 | 335 | 355 | 331 | 357 | 327 | 361 | 321 | 1594 | 1599 | 1595 | 1615 | 1591 | 1617 | 1587 | 1621 | 1581 | 2151 | 51935 |
| 57 | 1027 | 1025 | 997 | 1008 | 1015 | 1022 | 1033 | 1005 | 1003 | 1792 | 1790 | 1762 | 1773 | 1780 | 1787 | 1798 | 1770 | 1768 | 802 | 800 | 772 | 783 | 790 | 797 | 808 | 780 | 778 | 352 | 350 | 322 | 333 | 340 | 347 | 358 | 330 | 328 | 1612 | 1610 | 1582 | 1593 | 1600 | 1607 | 1618 | 1590 | 1588 | 2153 | 51935 |
| 55 | 1034 | 994 | 1028 | 998 | 1024 | 1000 | 1020 | 1016 | 1021 | 1799 | 1759 | 1793 | 1763 | 1789 | 1765 | 1785 | 1781 | 1786 | 809 | 769 | 803 | 773 | 799 | 775 | 795 | 791 | 796 | 359 | 319 | 353 | 323 | 349 | 325 | 345 | 341 | 346 | 1619 | 1579 | 1613 | 1583 | 1609 | 1585 | 1605 | 1601 | 1606 | 2155 | 51935 |
| 53 | 993 | 1007 | 1029 | 1026 | 1031 | 1017 | 1019 | 995 | 1018 | 1758 | 1772 | 1794 | 1791 | 1796 | 1782 | 1784 | 1760 | 1783 | 768 | 782 | 804 | 801 | 806 | 792 | 794 | 770 | 793 | 318 | 332 | 354 | 351 | 356 | 342 | 344 | 320 | 343 | 1578 | 1592 | 1614 | 1611 | 1616 | 1602 | 1604 | 1580 | 1603 | 2157 | 51935 |
| 51 | 382 | 405 | 381 | 383 | 369 | 374 | 371 | 393 | 407 | 697 | 720 | 696 | 698 | 684 | 689 | 686 | 708 | 722 | 1102 | 1125 | 1101 | 1103 | 1089 | 1094 | 1091 | 1113 | 1127 | 1507 | 1530 | 1506 | 1508 | 1494 | 1499 | 1496 | 1518 | 1532 | 1822 | 1845 | 1821 | 1823 | 1809 | 1814 | 1811 | 1833 | 1847 | 2159 | 51935 |
| 49 | 379 | 384 | 380 | 400 | 376 | 402 | 372 | 406 | 366 | 694 | 699 | 695 | 715 | 691 | 717 | 687 | 721 | 681 | 1099 | 1104 | 1100 | 1120 | 1096 | 1122 | 1092 | 1126 | 1086 | 1504 | 1509 | 1505 | 1525 | 1501 | 1527 | 1497 | 1531 | 1491 | 1819 | 1824 | 1820 | 1840 | 1816 | 1842 | 1812 | 1846 | 1806 | 2161 | 51935 |
| 47 | 397 | 395 | 367 | 378 | 385 | 392 | 403 | 375 | 373 | 712 | 710 | 682 | 693 | 700 | 707 | 718 | 690 | 688 | 1117 | 1115 | 1087 | 1098 | 1105 | 1112 | 1123 | 1095 | 1093 | 1522 | 1520 | 1492 | 1503 | 1510 | 1517 | 1528 | 1500 | 1498 | 1837 | 1835 | 1807 | 1818 | 1825 | 1832 | 1843 | 1815 | 1813 | 2163 | 51935 |
| 2167 | 404 | 364 | 398 | 368 | 394 | 370 | 390 | 386 | 391 | 719 | 679 | 713 | 683 | 709 | 685 | 705 | 701 | 706 | 1124 | 1084 | 1118 | 1088 | 1114 | 1090 | 1110 | 1106 | 1111 | 1529 | 1489 | 1523 | 1493 | 1519 | 1495 | 1515 | 1511 | 1516 | 1844 | 1804 | 1838 | 1808 | 1834 | 1810 | 1830 | 1826 | 1831 | 43 | 51935 |
| 2169 | 363 | 377 | 399 | 396 | 401 | 387 | 389 | 365 | 388 | 678 | 692 | 714 | 711 | 716 | 702 | 704 | 680 | 703 | 1083 | 1097 | 1119 | 1116 | 1121 | 1107 | 1109 | 1085 | 1108 | 1488 | 1502 | 1524 | 1521 | 1526 | 1512 | 1514 | 1490 | 1513 | 1803 | 1817 | 1839 | 1836 | 1841 | 1827 | 1829 | 1805 | 1828 | 41 | 51935 |
| 2171 | 607 | 630 | 606 | 608 | 594 | 599 | 596 | 618 | 632 | 1867 | 1890 | 1866 | 1868 | 1854 | 1859 | 1856 | 1878 | 1892 | 1417 | 1440 | 1416 | 1418 | 1404 | 1409 | 1406 | 1428 | 1442 | 427 | 450 | 426 | 428 | 414 | 419 | 416 | 438 | 452 | 1192 | 1215 | 1191 | 1193 | 1179 | 1184 | 1181 | 1203 | 1217 | 39 | 51935 |
| 2173 | 604 | 609 | 605 | 625 | 601 | 627 | 597 | 631 | 591 | 1864 | 1869 | 1865 | 1885 | 1861 | 1887 | 1857 | 1891 | 1851 | 1414 | 1419 | 1415 | 1435 | 1411 | 1437 | 1407 | 1441 | 1401 | 424 | 429 | 425 | 445 | 421 | 447 | 417 | 451 | 411 | 1189 | 1194 | 1190 | 1210 | 1186 | 1212 | 1182 | 1216 | 1176 | 37 | 51935 |
| 2175 | 622 | 620 | 592 | 603 | 610 | 617 | 628 | 600 | 598 | 1882 | 1880 | 1852 | 1863 | 1870 | 1877 | 1888 | 1860 | 1858 | 1432 | 1430 | 1402 | 1413 | 1420 | 1427 | 1438 | 1410 | 1408 | 442 | 440 | 412 | 423 | 430 | 437 | 448 | 420 | 418 | 1207 | 1205 | 1177 | 1188 | 1195 | 1202 | 1213 | 1185 | 1183 | 35 | 51935 |
| 2177 | 629 | 589 | 623 | 593 | 619 | 595 | 615 | 611 | 616 | 1889 | 1849 | 1883 | 1853 | 1879 | 1855 | 1875 | 1871 | 1876 | 1439 | 1399 | 1433 | 1403 | 1429 | 1405 | 1425 | 1421 | 1426 | 449 | 409 | 443 | 413 | 439 | 415 | 435 | 431 | 436 | 1214 | 1174 | 1208 | 1178 | 1204 | 1180 | 1200 | 1196 | 1201 | 33 | 51935 |
| 2179 | 588 | 602 | 624 | 621 | 626 | 612 | 614 | 590 | 613 | 1848 | 1862 | 1884 | 1881 | 1886 | 1872 | 1874 | 1850 | 1873 | 1398 | 1412 | 1434 | 1431 | 1436 | 1422 | 1424 | 1400 | 1423 | 408 | 422 | 444 | 441 | 446 | 432 | 434 | 410 | 433 | 1173 | 1187 | 1209 | 1206 | 1211 | 1197 | 1199 | 1175 | 1198 | 31 | 51935 |
| 2181 | 472 | 495 | 471 | 473 | 459 | 464 | 461 | 483 | 497 | 517 | 540 | 516 | 518 | 504 | 509 | 506 | 528 | 542 | 1912 | 1935 | 1911 | 1913 | 1899 | 1904 | 1901 | 1923 | 1937 | 1327 | 1350 | 1326 | 1328 | 1314 | 1319 | 1316 | 1338 | 1352 | 1282 | 1305 | 1281 | 1283 | 1269 | 1274 | 1271 | 1293 | 1307 | 29 | 51935 |
| 2183 | 469 | 474 | 470 | 490 | 466 | 492 | 462 | 496 | 456 | 514 | 519 | 515 | 535 | 511 | 537 | 507 | 541 | 501 | 1909 | 1914 | 1910 | 1930 | 1906 | 1932 | 1902 | 1936 | 1896 | 1324 | 1329 | 1325 | 1345 | 1321 | 1347 | 1317 | 1351 | 1311 | 1279 | 1284 | 1280 | 1300 | 1276 | 1302 | 1272 | 1306 | 1266 | 27 | 51935 |
| 2185 | 487 | 485 | 457 | 468 | 475 | 482 | 493 | 465 | 463 | 532 | 530 | 502 | 513 | 520 | 527 | 538 | 510 | 508 | 1927 | 1925 | 1897 | 1908 | 1915 | 1922 | 1933 | 1905 | 1903 | 1342 | 1340 | 1312 | 1323 | 1330 | 1337 | 1348 | 1320 | 1318 | 1297 | 1295 | 1267 | 1278 | 1285 | 1292 | 1303 | 1275 | 1273 | 25 | 51935 |
| 2187 | 494 | 454 | 488 | 458 | 484 | 460 | 480 | 476 | 481 | 539 | 499 | 533 | 503 | 529 | 505 | 525 | 521 | 526 | 1934 | 1894 | 1928 | 1898 | 1924 | 1900 | 1920 | 1916 | 1921 | 1349 | 1309 | 1343 | 1313 | 1339 | 1315 | 1335 | 1331 | 1336 | 1304 | 1264 | 1298 | 1268 | 1294 | 1270 | 1290 | 1286 | 1291 | 23 | 51935 |
| 2189 | 453 | 467 | 489 | 486 | 491 | 477 | 479 | 455 | 478 | 498 | 512 | 534 | 531 | 536 | 522 | 524 | 500 | 523 | 1893 | 1907 | 1929 | 1926 | 1931 | 1917 | 1919 | 1895 | 1918 | 1308 | 1322 | 1344 | 1341 | 1346 | 1332 | 1334 | 1310 | 1333 | 1263 | 1277 | 1299 | 1296 | 1301 | 1287 | 1289 | 1265 | 1288 | 21 | 51935 |
| 2191 | 1462 | 1485 | 1461 | 1463 | 1449 | 1454 | 1451 | 1473 | 1487 | 2047 | 2070 | 2046 | 2048 | 2034 | 2039 | 2036 | 2058 | 2072 | 652 | 675 | 651 | 653 | 639 | 644 | 641 | 663 | 677 | 1147 | 1170 | 1146 | 1148 | 1134 | 1139 | 1136 | 1158 | 1172 | 202 | 225 | 201 | 203 | 189 | 194 | 191 | 213 | 227 | 19 | 51935 |
| 2193 | 1459 | 1464 | 1460 | 1480 | 1456 | 1482 | 1452 | 1486 | 1446 | 2044 | 2049 | 2045 | 2065 | 2041 | 2067 | 2037 | 2071 | 2031 | 649 | 654 | 650 | 670 | 646 | 672 | 642 | 676 | 636 | 1144 | 1149 | 1145 | 1165 | 1141 | 1167 | 1137 | 1171 | 1131 | 199 | 204 | 200 | 220 | 196 | 222 | 192 | 226 | 186 | 17 | 51935 |
| 2195 | 1477 | 1475 | 1447 | 1458 | 1465 | 1472 | 1483 | 1455 | 1453 | 2062 | 2060 | 2032 | 2043 | 2050 | 2057 | 2068 | 2040 | 2038 | 667 | 665 | 637 | 648 | 655 | 662 | 673 | 645 | 643 | 1162 | 1160 | 1132 | 1143 | 1150 | 1157 | 1168 | 1140 | 1138 | 217 | 215 | 187 | 198 | 205 | 212 | 223 | 195 | 193 | 15 | 51935 |
| 2197 | 1484 | 1444 | 1478 | 1448 | 1474 | 1450 | 1470 | 1466 | 1471 | 2069 | 2029 | 2063 | 2033 | 2059 | 2035 | 2055 | 2051 | 2056 | 674 | 634 | 668 | 638 | 664 | 640 | 660 | 656 | 661 | 1169 | 1129 | 1163 | 1133 | 1159 | 1135 | 1155 | 1151 | 1156 | 224 | 184 | 218 | 188 | 214 | 190 | 210 | 206 | 211 | 13 | 51935 |
| 2199 | 1443 | 1457 | 1479 | 1476 | 1481 | 1467 | 1469 | 1445 | 1468 | 2028 | 2042 | 2064 | 2061 | 2066 | 2052 | 2054 | 2030 | 2053 | 633 | 647 | 669 | 666 | 671 | 657 | 659 | 635 | 658 | 1128 | 1142 | 1164 | 1161 | 1166 | 1152 | 1154 | 1130 | 1153 | 183 | 197 | 219 | 216 | 221 | 207 | 209 | 185 | 208 | 11 | 51935 |
| 2201 | 2092 | 2115 | 2091 | 2093 | 2079 | 2084 | 2081 | 2103 | 2117 | 247 | 270 | 246 | 248 | 234 | 239 | 236 | 258 | 272 | 562 | 585 | 561 | 563 | 549 | 554 | 551 | 573 | 587 | 1372 | 1395 | 1371 | 1373 | 1359 | 1364 | 1361 | 1383 | 1397 | 1237 | 1260 | 1236 | 1238 | 1224 | 1229 | 1226 | 1248 | 1262 | 9 | 51935 |
| 2203 | 2089 | 2094 | 2090 | 2110 | 2086 | 2112 | 2082 | 2116 | 2076 | 244 | 249 | 245 | 265 | 241 | 267 | 237 | 271 | 231 | 559 | 564 | 560 | 580 | 556 | 582 | 552 | 586 | 546 | 1369 | 1374 | 1370 | 1390 | 1366 | 1392 | 1362 | 1396 | 1356 | 1234 | 1239 | 1235 | 1255 | 1231 | 1257 | 1227 | 1261 | 1221 | 7 | 51935 |
| 2205 | 2107 | 2105 | 2077 | 2088 | 2095 | 2102 | 2113 | 2085 | 2083 | 262 | 260 | 232 | 243 | 250 | 257 | 268 | 240 | 238 | 577 | 575 | 547 | 558 | 565 | 572 | 583 | 555 | 553 | 1387 | 1385 | 1357 | 1368 | 1375 | 1382 | 1393 | 1365 | 1363 | 1252 | 1250 | 1222 | 1233 | 1240 | 1247 | 1258 | 1230 | 1228 | 5 | 51935 |
| 2207 | 2114 | 2074 | 2108 | 2078 | 2104 | 2080 | 2100 | 2096 | 2101 | 269 | 229 | 263 | 233 | 259 | 235 | 255 | 251 | 256 | 584 | 544 | 578 | 548 | 574 | 550 | 570 | 566 | 571 | 1394 | 1354 | 1388 | 1358 | 1384 | 1360 | 1380 | 1376 | 1381 | 1259 | 1219 | 1253 | 1223 | 1249 | 1225 | 1245 | 1241 | 1246 | 3 | 51935 |
| 2209 | 2073 | 2087 | 2109 | 2106 | 2111 | 2097 | 2099 | 2075 | 2098 | 228 | 242 | 264 | 261 | 266 | 252 | 254 | 230 | 253 | 543 | 557 | 579 | 576 | 581 | 567 | 569 | 545 | 568 | 1353 | 1367 | 1389 | 1386 | 1391 | 1377 | 1379 | 1355 | 1378 | 1218 | 1232 | 1254 | 1251 | 1256 | 1242 | 1244 | 1220 | 1243 | 1 | 51935 |
| 48 | 92 | 90 | 88 | 86 | 84 | 82 | 80 | 78 | 76 | 74 | 72 | 70 | 68 | 66 | 64 | 62 | 60 | 58 | 56 | 54 | 52 | 50 | 2165 | 2166 | 2168 | 2170 | 2172 | 2174 | 2176 | 2178 | 2180 | 2182 | 2184 | 2186 | 2188 | 2190 | 2192 | 2194 | 2196 | 2198 | 2200 | 2202 | 2204 | 2206 | 2208 | 46 | 51935 |
| 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 |

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*In this case the magic sums are* $S_{47\times 47} := 51935$ *and* $S_{45\times 45} := 49725$*. The magic rectangles of order* $5 \times 9$ *are of* ***different magic sums****.*

**Example 3.18.** *A magic square of order* $47 \times 47$ *is constructed based on magic squares of order* $45 \times 45$ *given in Examples 3.16. It is given by*

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 51935 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2164 | 2118 | 2120 | 2122 | 2124 | 2126 | 2128 | 2130 | 2132 | 2134 | 2136 | 2138 | 2140 | 2142 | 2144 | 2146 | 2148 | 2150 | 2152 | 2154 | 2156 | 2158 | 2160 | 45 | 44 | 42 | 40 | 38 | 36 | 34 | 32 | 30 | 28 | 26 | 24 | 22 | 20 | 18 | 16 | 14 | 12 | 10 | 8 | 6 | 4 | 2 | 2162 | 51935 |
| 91 | 93 | 120 | 118 | 96 | 101 | 119 | 117 | 104 | 127 | 128 | 109 | 114 | 135 | 136 | 108 | 1758 | 1785 | 1783 | 1761 | 1766 | 1784 | 1782 | 1769 | 1792 | 1793 | 1774 | 1779 | 1800 | 1801 | 1773 | 1398 | 1425 | 1423 | 1401 | 1406 | 1424 | 1422 | 1409 | 1432 | 1433 | 1414 | 1419 | 1440 | 1441 | 1413 | 2119 | 51935 |
| 89 | 130 | 131 | 132 | 133 | 123 | 124 | 125 | 115 | 105 | 106 | 107 | 97 | 98 | 99 | 100 | 1795 | 1796 | 1797 | 1798 | 1788 | 1789 | 1790 | 1780 | 1770 | 1771 | 1772 | 1762 | 1763 | 1764 | 1765 | 1435 | 1436 | 1437 | 1438 | 1428 | 1429 | 1430 | 1420 | 1410 | 1411 | 1412 | 1402 | 1403 | 1404 | 1405 | 2121 | 51935 |
| 87 | 122 | 94 | 95 | 116 | 121 | 102 | 103 | 126 | 113 | 111 | 129 | 134 | 112 | 110 | 137 | 1787 | 1759 | 1760 | 1781 | 1786 | 1767 | 1768 | 1791 | 1778 | 1776 | 1794 | 1799 | 1777 | 1775 | 1802 | 1427 | 1399 | 1400 | 1421 | 1426 | 1407 | 1408 | 1431 | 1418 | 1416 | 1434 | 1439 | 1417 | 1415 | 1442 | 2123 | 51935 |
| 85 | 1308 | 1335 | 1333 | 1311 | 1316 | 1334 | 1332 | 1319 | 1342 | 1343 | 1324 | 1329 | 1350 | 1351 | 1323 | 1803 | 1830 | 1828 | 1806 | 1811 | 1829 | 1827 | 1814 | 1837 | 1838 | 1819 | 1824 | 1845 | 1846 | 1818 | 138 | 165 | 163 | 141 | 146 | 164 | 162 | 149 | 172 | 173 | 154 | 159 | 180 | 181 | 153 | 2125 | 51935 |
| 83 | 1345 | 1346 | 1347 | 1348 | 1338 | 1339 | 1340 | 1330 | 1320 | 1321 | 1322 | 1312 | 1313 | 1314 | 1315 | 1840 | 1841 | 1842 | 1843 | 1833 | 1834 | 1835 | 1825 | 1815 | 1816 | 1817 | 1807 | 1808 | 1809 | 1810 | 175 | 176 | 177 | 178 | 168 | 169 | 170 | 160 | 150 | 151 | 152 | 142 | 143 | 144 | 145 | 2127 | 51935 |
| 81 | 1337 | 1309 | 1310 | 1331 | 1336 | 1317 | 1318 | 1341 | 1328 | 1326 | 1344 | 1349 | 1327 | 1325 | 1352 | 1832 | 1804 | 1805 | 1826 | 1831 | 1812 | 1813 | 1836 | 1823 | 1821 | 1839 | 1844 | 1822 | 1820 | 1847 | 167 | 139 | 140 | 161 | 166 | 147 | 148 | 171 | 158 | 156 | 174 | 179 | 157 | 155 | 182 | 2129 | 51935 |
| 79 | 1218 | 1245 | 1243 | 1221 | 1226 | 1244 | 1242 | 1229 | 1252 | 1253 | 1234 | 1239 | 1260 | 1261 | 1233 | 1848 | 1875 | 1873 | 1851 | 1856 | 1874 | 1872 | 1859 | 1882 | 1883 | 1864 | 1869 | 1890 | 1891 | 1863 | 183 | 210 | 208 | 186 | 191 | 209 | 207 | 194 | 217 | 218 | 199 | 204 | 225 | 226 | 198 | 2131 | 51935 |
| 77 | 1255 | 1256 | 1257 | 1258 | 1248 | 1249 | 1250 | 1240 | 1230 | 1231 | 1232 | 1222 | 1223 | 1224 | 1225 | 1885 | 1886 | 1887 | 1888 | 1878 | 1879 | 1880 | 1870 | 1860 | 1861 | 1862 | 1852 | 1853 | 1854 | 1855 | 220 | 221 | 222 | 223 | 213 | 214 | 215 | 205 | 195 | 196 | 197 | 187 | 188 | 189 | 190 | 2133 | 51935 |
| 75 | 1247 | 1219 | 1220 | 1241 | 1246 | 1227 | 1228 | 1251 | 1238 | 1236 | 1254 | 1259 | 1237 | 1235 | 1262 | 1877 | 1849 | 1850 | 1871 | 1876 | 1857 | 1858 | 1881 | 1868 | 1866 | 1884 | 1889 | 1867 | 1865 | 1892 | 212 | 184 | 185 | 206 | 211 | 192 | 193 | 216 | 203 | 201 | 219 | 224 | 202 | 200 | 227 | 2135 | 51935 |
| 73 | 228 | 255 | 253 | 231 | 236 | 254 | 252 | 239 | 262 | 263 | 244 | 249 | 270 | 271 | 243 | 1893 | 1920 | 1918 | 1896 | 1901 | 1919 | 1917 | 1904 | 1927 | 1928 | 1909 | 1914 | 1935 | 1936 | 1908 | 1128 | 1155 | 1153 | 1131 | 1136 | 1154 | 1152 | 1139 | 1162 | 1163 | 1144 | 1149 | 1170 | 1171 | 1143 | 2137 | 51935 |
| 71 | 265 | 266 | 267 | 268 | 258 | 259 | 260 | 250 | 240 | 241 | 242 | 232 | 233 | 234 | 235 | 1930 | 1931 | 1932 | 1933 | 1923 | 1924 | 1925 | 1915 | 1905 | 1906 | 1907 | 1897 | 1898 | 1899 | 1900 | 1165 | 1166 | 1167 | 1168 | 1158 | 1159 | 1160 | 1150 | 1140 | 1141 | 1142 | 1132 | 1133 | 1134 | 1135 | 2139 | 51935 |
| 69 | 257 | 229 | 230 | 251 | 256 | 237 | 238 | 261 | 248 | 246 | 264 | 269 | 247 | 245 | 272 | 1922 | 1894 | 1895 | 1916 | 1921 | 1902 | 1903 | 1926 | 1913 | 1911 | 1929 | 1934 | 1912 | 1910 | 1937 | 1157 | 1129 | 1130 | 1151 | 1156 | 1137 | 1138 | 1161 | 1148 | 1146 | 1164 | 1169 | 1147 | 1145 | 1172 | 2141 | 51935 |
| 67 | 453 | 480 | 478 | 456 | 461 | 479 | 477 | 464 | 487 | 488 | 469 | 474 | 495 | 496 | 468 | 1443 | 1470 | 1468 | 1446 | 1451 | 1469 | 1467 | 1454 | 1477 | 1478 | 1459 | 1464 | 1485 | 1486 | 1458 | 1353 | 1380 | 1378 | 1356 | 1361 | 1379 | 1377 | 1364 | 1387 | 1388 | 1369 | 1374 | 1395 | 1396 | 1368 | 2143 | 51935 |
| 65 | 490 | 491 | 492 | 493 | 483 | 484 | 485 | 475 | 465 | 466 | 467 | 457 | 458 | 459 | 460 | 1480 | 1481 | 1482 | 1483 | 1473 | 1474 | 1475 | 1465 | 1455 | 1456 | 1457 | 1447 | 1448 | 1449 | 1450 | 1390 | 1391 | 1392 | 1393 | 1383 | 1384 | 1385 | 1375 | 1365 | 1366 | 1367 | 1357 | 1358 | 1359 | 1360 | 2145 | 51935 |
| 63 | 482 | 454 | 455 | 476 | 481 | 462 | 463 | 486 | 473 | 471 | 489 | 494 | 472 | 470 | 497 | 1472 | 1444 | 1445 | 1466 | 1471 | 1452 | 1453 | 1476 | 1463 | 1461 | 1479 | 1484 | 1462 | 1460 | 1487 | 1382 | 1354 | 1355 | 1376 | 1381 | 1362 | 1363 | 1386 | 1373 | 1371 | 1389 | 1394 | 1372 | 1370 | 1397 | 2147 | 51935 |
| 61 | 1263 | 1290 | 1288 | 1266 | 1271 | 1289 | 1287 | 1274 | 1297 | 1298 | 1279 | 1284 | 1305 | 1306 | 1278 | 1488 | 1515 | 1513 | 1491 | 1496 | 1514 | 1512 | 1499 | 1522 | 1523 | 1504 | 1509 | 1530 | 1531 | 1503 | 498 | 525 | 523 | 501 | 506 | 524 | 522 | 509 | 532 | 533 | 514 | 519 | 540 | 541 | 513 | 2149 | 51935 |
| 59 | 1300 | 1301 | 1302 | 1303 | 1293 | 1294 | 1295 | 1285 | 1275 | 1276 | 1277 | 1267 | 1268 | 1269 | 1270 | 1525 | 1526 | 1527 | 1528 | 1518 | 1519 | 1520 | 1510 | 1500 | 1501 | 1502 | 1492 | 1493 | 1494 | 1495 | 535 | 536 | 537 | 538 | 528 | 529 | 530 | 520 | 510 | 511 | 512 | 502 | 503 | 504 | 505 | 2151 | 51935 |
| 57 | 1292 | 1264 | 1265 | 1286 | 1291 | 1272 | 1273 | 1296 | 1283 | 1281 | 1299 | 1304 | 1282 | 1280 | 1307 | 1517 | 1489 | 1490 | 1511 | 1516 | 1497 | 1498 | 1521 | 1508 | 1506 | 1524 | 1529 | 1507 | 1505 | 1532 | 527 | 499 | 500 | 521 | 526 | 507 | 508 | 531 | 518 | 516 | 534 | 539 | 517 | 515 | 542 | 2153 | 51935 |
| 55 | 1173 | 1200 | 1198 | 1176 | 1181 | 1199 | 1197 | 1184 | 1207 | 1208 | 1189 | 1194 | 1215 | 1216 | 1188 | 1533 | 1560 | 1558 | 1536 | 1541 | 1559 | 1557 | 1544 | 1567 | 1568 | 1549 | 1554 | 1575 | 1576 | 1548 | 543 | 570 | 568 | 546 | 551 | 569 | 567 | 554 | 577 | 578 | 559 | 564 | 585 | 586 | 558 | 2155 | 51935 |
| 53 | 1210 | 1211 | 1212 | 1213 | 1203 | 1204 | 1205 | 1195 | 1185 | 1186 | 1187 | 1177 | 1178 | 1179 | 1180 | 1570 | 1571 | 1572 | 1573 | 1563 | 1564 | 1565 | 1555 | 1545 | 1546 | 1547 | 1537 | 1538 | 1539 | 1540 | 580 | 581 | 582 | 583 | 573 | 574 | 575 | 565 | 555 | 556 | 557 | 547 | 548 | 549 | 550 | 2157 | 51935 |
| 51 | 1202 | 1174 | 1175 | 1196 | 1201 | 1182 | 1183 | 1206 | 1193 | 1191 | 1209 | 1214 | 1192 | 1190 | 1217 | 1562 | 1534 | 1535 | 1556 | 1561 | 1542 | 1543 | 1566 | 1553 | 1551 | 1569 | 1574 | 1552 | 1550 | 1577 | 572 | 544 | 545 | 566 | 571 | 552 | 553 | 576 | 563 | 561 | 579 | 584 | 562 | 560 | 587 | 2159 | 51935 |
| 49 | 588 | 615 | 613 | 591 | 596 | 614 | 612 | 599 | 622 | 623 | 604 | 609 | 630 | 631 | 603 | 1083 | 1110 | 1108 | 1086 | 1091 | 1109 | 1107 | 1094 | 1117 | 1118 | 1099 | 1104 | 1125 | 1126 | 1098 | 1578 | 1605 | 1603 | 1581 | 1586 | 1604 | 1602 | 1589 | 1612 | 1613 | 1594 | 1599 | 1620 | 1621 | 1593 | 2161 | 51935 |
| 47 | 625 | 626 | 627 | 628 | 618 | 619 | 620 | 610 | 600 | 601 | 602 | 592 | 593 | 594 | 595 | 1120 | 1121 | 1122 | 1123 | 1113 | 1114 | 1115 | 1105 | 1095 | 1096 | 1097 | 1087 | 1088 | 1089 | 1090 | 1615 | 1616 | 1617 | 1618 | 1608 | 1609 | 1610 | 1600 | 1590 | 1591 | 1592 | 1582 | 1583 | 1584 | 1585 | 2163 | 51935 |
| 2167 | 617 | 589 | 590 | 611 | 616 | 597 | 598 | 621 | 608 | 606 | 624 | 629 | 607 | 605 | 632 | 1112 | 1084 | 1085 | 1106 | 1111 | 1092 | 1093 | 1116 | 1103 | 1101 | 1119 | 1124 | 1102 | 1100 | 1127 | 1607 | 1579 | 1580 | 1601 | 1606 | 1587 | 1588 | 1611 | 1598 | 1596 | 1614 | 1619 | 1597 | 1595 | 1622 | 43 | 51935 |
| 2169 | 1623 | 1650 | 1648 | 1626 | 1631 | 1649 | 1647 | 1634 | 1657 | 1658 | 1639 | 1644 | 1665 | 1666 | 1638 | 633 | 660 | 658 | 636 | 641 | 659 | 657 | 644 | 667 | 668 | 649 | 654 | 675 | 676 | 648 | 993 | 1020 | 1018 | 996 | 1001 | 1019 | 1017 | 1004 | 1027 | 1028 | 1009 | 1014 | 1035 | 1036 | 1008 | 41 | 51935 |
| 2171 | 1660 | 1661 | 1662 | 1663 | 1653 | 1654 | 1655 | 1645 | 1635 | 1636 | 1637 | 1627 | 1628 | 1629 | 1630 | 670 | 671 | 672 | 673 | 663 | 664 | 665 | 655 | 645 | 646 | 647 | 637 | 638 | 639 | 640 | 1030 | 1031 | 1032 | 1033 | 1023 | 1024 | 1025 | 1015 | 1005 | 1006 | 1007 | 997 | 998 | 999 | 1000 | 39 | 51935 |
| 2173 | 1652 | 1624 | 1625 | 1646 | 1651 | 1632 | 1633 | 1656 | 1643 | 1641 | 1659 | 1664 | 1642 | 1640 | 1667 | 662 | 634 | 635 | 656 | 661 | 642 | 643 | 666 | 653 | 651 | 669 | 674 | 652 | 650 | 677 | 1022 | 994 | 995 | 1016 | 1021 | 1002 | 1003 | 1026 | 1013 | 1011 | 1029 | 1034 | 1012 | 1010 | 1037 | 37 | 51935 |
| 2175 | 1668 | 1695 | 1693 | 1671 | 1676 | 1694 | 1692 | 1679 | 1702 | 1703 | 1684 | 1689 | 1710 | 1711 | 1683 | 678 | 705 | 703 | 681 | 686 | 704 | 702 | 689 | 712 | 713 | 694 | 699 | 720 | 721 | 693 | 903 | 930 | 928 | 906 | 911 | 929 | 927 | 914 | 937 | 938 | 919 | 924 | 945 | 946 | 918 | 35 | 51935 |
| 2177 | 1705 | 1706 | 1707 | 1708 | 1698 | 1699 | 1700 | 1690 | 1680 | 1681 | 1682 | 1672 | 1673 | 1674 | 1675 | 715 | 716 | 717 | 718 | 708 | 709 | 710 | 700 | 690 | 691 | 692 | 682 | 683 | 684 | 685 | 940 | 941 | 942 | 943 | 933 | 934 | 935 | 925 | 915 | 916 | 917 | 907 | 908 | 909 | 910 | 33 | 51935 |
| 2179 | 1697 | 1669 | 1670 | 1691 | 1696 | 1677 | 1678 | 1701 | 1688 | 1686 | 1704 | 1709 | 1687 | 1685 | 1712 | 707 | 679 | 680 | 701 | 706 | 687 | 688 | 711 | 698 | 696 | 714 | 719 | 697 | 695 | 722 | 932 | 904 | 905 | 926 | 931 | 912 | 913 | 936 | 923 | 921 | 939 | 944 | 922 | 920 | 947 | 31 | 51935 |
| 2181 | 813 | 840 | 838 | 816 | 821 | 839 | 837 | 824 | 847 | 848 | 829 | 834 | 855 | 856 | 828 | 723 | 750 | 748 | 726 | 731 | 749 | 747 | 734 | 757 | 758 | 739 | 744 | 765 | 766 | 738 | 1713 | 1740 | 1738 | 1716 | 1721 | 1739 | 1737 | 1724 | 1747 | 1748 | 1729 | 1734 | 1755 | 1756 | 1728 | 29 | 51935 |
| 2183 | 850 | 851 | 852 | 853 | 843 | 844 | 845 | 835 | 825 | 826 | 827 | 817 | 818 | 819 | 820 | 760 | 761 | 762 | 763 | 753 | 754 | 755 | 745 | 735 | 736 | 737 | 727 | 728 | 729 | 730 | 1750 | 1751 | 1752 | 1753 | 1743 | 1744 | 1745 | 1735 | 1725 | 1726 | 1727 | 1717 | 1718 | 1719 | 1720 | 27 | 51935 |
| 2185 | 842 | 814 | 815 | 836 | 841 | 822 | 823 | 846 | 833 | 831 | 849 | 854 | 832 | 830 | 857 | 752 | 724 | 725 | 746 | 751 | 732 | 733 | 756 | 743 | 741 | 759 | 764 | 742 | 740 | 767 | 1742 | 1714 | 1715 | 1736 | 1741 | 1722 | 1723 | 1746 | 1733 | 1731 | 1749 | 1754 | 1732 | 1730 | 1757 | 25 | 51935 |
| 2187 | 1038 | 1065 | 1063 | 1041 | 1046 | 1064 | 1062 | 1049 | 1072 | 1073 | 1054 | 1059 | 1080 | 1081 | 1053 | 273 | 300 | 298 | 276 | 281 | 299 | 297 | 284 | 307 | 308 | 289 | 294 | 315 | 316 | 288 | 1938 | 1965 | 1963 | 1941 | 1946 | 1964 | 1962 | 1949 | 1972 | 1973 | 1954 | 1959 | 1980 | 1981 | 1953 | 23 | 51935 |
| 2189 | 1075 | 1076 | 1077 | 1078 | 1068 | 1069 | 1070 | 1060 | 1050 | 1051 | 1052 | 1042 | 1043 | 1044 | 1045 | 310 | 311 | 312 | 313 | 303 | 304 | 305 | 295 | 285 | 286 | 287 | 277 | 278 | 279 | 280 | 1975 | 1976 | 1977 | 1978 | 1968 | 1969 | 1970 | 1960 | 1950 | 1951 | 1952 | 1942 | 1943 | 1944 | 1945 | 21 | 51935 |
| 2191 | 1067 | 1039 | 1040 | 1061 | 1066 | 1047 | 1048 | 1071 | 1058 | 1056 | 1074 | 1079 | 1057 | 1055 | 1082 | 302 | 274 | 275 | 296 | 301 | 282 | 283 | 306 | 293 | 291 | 309 | 314 | 292 | 290 | 317 | 1967 | 1939 | 1940 | 1961 | 1966 | 1947 | 1948 | 1971 | 1958 | 1956 | 1974 | 1979 | 1957 | 1955 | 1982 | 19 | 51935 |
| 2193 | 1983 | 2010 | 2008 | 1986 | 1991 | 2009 | 2007 | 1994 | 2017 | 2018 | 1999 | 2004 | 2025 | 2026 | 1998 | 318 | 345 | 343 | 321 | 326 | 344 | 342 | 329 | 352 | 353 | 334 | 339 | 360 | 361 | 333 | 948 | 975 | 973 | 951 | 956 | 974 | 972 | 959 | 982 | 983 | 964 | 969 | 990 | 991 | 963 | 17 | 51935 |
| 2195 | 2020 | 2021 | 2022 | 2023 | 2013 | 2014 | 2015 | 2005 | 1995 | 1996 | 1997 | 1987 | 1988 | 1989 | 1990 | 355 | 356 | 357 | 358 | 348 | 349 | 350 | 340 | 330 | 331 | 332 | 322 | 323 | 324 | 325 | 985 | 986 | 987 | 988 | 978 | 979 | 980 | 970 | 960 | 961 | 962 | 952 | 953 | 954 | 955 | 15 | 51935 |
| 2197 | 2012 | 1984 | 1985 | 2006 | 2011 | 1992 | 1993 | 2016 | 2003 | 2001 | 2019 | 2024 | 2002 | 2000 | 2027 | 347 | 319 | 320 | 341 | 346 | 327 | 328 | 351 | 338 | 336 | 354 | 359 | 337 | 335 | 362 | 977 | 949 | 950 | 971 | 976 | 957 | 958 | 981 | 968 | 966 | 984 | 989 | 967 | 965 | 992 | 13 | 51935 |
| 2199 | 2028 | 2055 | 2053 | 2031 | 2036 | 2054 | 2052 | 2039 | 2062 | 2063 | 2044 | 2049 | 2070 | 2071 | 2043 | 363 | 390 | 388 | 366 | 371 | 389 | 387 | 374 | 397 | 398 | 379 | 384 | 405 | 406 | 378 | 858 | 885 | 883 | 861 | 866 | 884 | 882 | 869 | 892 | 893 | 874 | 879 | 900 | 901 | 873 | 11 | 51935 |
| 2201 | 2065 | 2066 | 2067 | 2068 | 2058 | 2059 | 2060 | 2050 | 2040 | 2041 | 2042 | 2032 | 2033 | 2034 | 2035 | 400 | 401 | 402 | 403 | 393 | 394 | 395 | 385 | 375 | 376 | 377 | 367 | 368 | 369 | 370 | 895 | 896 | 897 | 898 | 888 | 889 | 890 | 880 | 870 | 871 | 872 | 862 | 863 | 864 | 865 | 9 | 51935 |
| 2203 | 2057 | 2029 | 2030 | 2051 | 2056 | 2037 | 2038 | 2061 | 2048 | 2046 | 2064 | 2069 | 2047 | 2045 | 2072 | 392 | 364 | 365 | 386 | 391 | 372 | 373 | 396 | 383 | 381 | 399 | 404 | 382 | 380 | 407 | 887 | 859 | 860 | 881 | 886 | 867 | 868 | 891 | 878 | 876 | 894 | 899 | 877 | 875 | 902 | 7 | 51935 |
| 2205 | 768 | 795 | 793 | 771 | 776 | 794 | 792 | 779 | 802 | 803 | 784 | 789 | 810 | 811 | 783 | 408 | 435 | 433 | 411 | 416 | 434 | 432 | 419 | 442 | 443 | 424 | 429 | 450 | 451 | 423 | 2073 | 2100 | 2098 | 2076 | 2081 | 2099 | 2097 | 2084 | 2107 | 2108 | 2089 | 2094 | 2115 | 2116 | 2088 | 5 | 51935 |
| 2207 | 805 | 806 | 807 | 808 | 798 | 799 | 800 | 790 | 780 | 781 | 782 | 772 | 773 | 774 | 775 | 445 | 446 | 447 | 448 | 438 | 439 | 440 | 430 | 420 | 421 | 422 | 412 | 413 | 414 | 415 | 2110 | 2111 | 2112 | 2113 | 2103 | 2104 | 2105 | 2095 | 2085 | 2086 | 2087 | 2077 | 2078 | 2079 | 2080 | 3 | 51935 |
| 2209 | 797 | 769 | 770 | 791 | 796 | 777 | 778 | 801 | 788 | 786 | 804 | 809 | 787 | 785 | 812 | 437 | 409 | 410 | 431 | 436 | 417 | 418 | 441 | 428 | 426 | 444 | 449 | 427 | 425 | 452 | 2102 | 2074 | 2075 | 2096 | 2101 | 2082 | 2083 | 2106 | 2093 | 2091 | 2109 | 2114 | 2092 | 2090 | 2117 | 1 | 51935 |
| 48 | 92 | 90 | 88 | 86 | 84 | 82 | 80 | 78 | 76 | 74 | 72 | 70 | 68 | 66 | 64 | 62 | 60 | 58 | 56 | 54 | 52 | 50 | 2165 | 2166 | 2168 | 2170 | 2172 | 2174 | 2176 | 2178 | 2180 | 2182 | 2184 | 2186 | 2188 | 2190 | 2192 | 2194 | 2196 | 2198 | 2200 | 2202 | 2204 | 2206 | 2208 | 46 | 51935 |
| 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 | 51935 |

*In this case the magic sums are* $S_{47\times 47} := 51935$ *and* $S_{45\times 45} := 49725$*. The magic rectangles of order* $3 \times 15$ *are of* ***different magic sums****.*

# 4 Appendix: Magic Rectangles

Magic rectangles are well known in literature. Below are few examples of magic rectangles.

## 4.1 Multiples of 2

### 4.1.1 Magic Rectangle of Order $2 \times 4$

**Example 4.1.** *Let's consider a magic rectangle of order* $2 \times 4$ *given by*

| | | | | |
|---|---|---|---|---|
| 1 | 7 | 6 | 4 | 18 |
| 8 | 2 | 3 | 5 | 18 |
| 9 | 9 | 9 | 9 | |

### 4.1.2 Magic Rectangle of Order $2 \times 6$

**Example 4.2.** *Let's consider a magic rectangle of order* $2 \times 6$ *given by*

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 | 11 | 3 | 9 | 8 | 7 | 39 |
| 12 | 2 | 10 | 4 | 5 | 6 | 39 |
| 13 | 13 | 13 | 13 | 13 | 13 | |

### 4.1.3 Magic Rectangle of Order $2 \times 8$

**Example 4.3.** *Let's consider a magic rectangle of order* $2 \times 8$ *given by*

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1 | 15 | 3 | 13 | 12 | 6 | 10 | 8 | 68 |
| 16 | 2 | 14 | 4 | 5 | 11 | 7 | 9 | 68 |
| 17 | 17 | 17 | 17 | 17 | 17 | 17 | 17 | |

### 4.1.4 Magic Rectangle of Order $2 \times 10$

**Example 4.4.** *Let's consider a magic rectangle of order* $2 \times 10$ *given by*

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 19 | 18 | 4 | 16 | 6 | 14 | 8 | 9 | 10 | 105 |
| 20 | 2 | 3 | 17 | 5 | 15 | 7 | 13 | 12 | 11 | 105 |
| 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | 21 | |

### 4.1.5 Magic Rectangle of Order $2 \times 12$

**Example 4.5.** *Let's consider a magic rectangle of order* $2 \times 12$ *given by*

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 23 | 3 | 21 | 20 | 19 | 7 | 8 | 9 | 15 | 11 | 13 | 150 |
| 24 | 2 | 22 | 4 | 5 | 6 | 18 | 17 | 16 | 10 | 14 | 12 | 150 |
| 25 | 25 | 25 | 25 | 25 | 25 | 25 | 25 | 25 | 25 | 25 | 25 | |

### 4.1.6 Magic Rectangle of Order $2 \times 14$

**Example 4.6.** *Let's consider a magic rectangle of order* $2 \times 14$ *given by*

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 27 | 3 | 25 | 5 | 23 | 7 | 21 | 9 | 19 | 18 | 17 | 13 | 15 | 203 |
| 28 | 2 | 26 | 4 | 24 | 6 | 22 | 8 | 20 | 10 | 11 | 12 | 16 | 14 | 203 |
| 29 | 29 | 29 | 29 | 29 | 29 | 29 | 29 | 29 | 29 | 29 | 29 | 29 | 29 | |

## 4.2 Multiples of 4

### 4.2.1 Magic Rectangle of Order $4 \times 6$

**Example 4.7.** *Let's consider a magic rectangle of order* $4 \times 6$ *given by*

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 22 | 23 | 24 | 75 |
| 19 | 20 | 21 | 4 | 5 | 6 | 75 |
| 18 | 17 | 16 | 9 | 8 | 7 | 75 |
| 12 | 11 | 10 | 15 | 14 | 13 | 75 |
| 50 | 50 | 50 | 50 | 50 | 50 | |

### 4.2.2 Magic Rectangle of Order $4 \times 8$

**Example 4.8.** *Let's consider a magic rectangle of order* $4 \times 8$ *given by*

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 29 | 30 | 31 | 32 | 132 |
| 25 | 26 | 27 | 28 | 5 | 6 | 7 | 8 | 132 |
| 24 | 23 | 22 | 21 | 12 | 11 | 10 | 9 | 132 |
| 16 | 15 | 14 | 13 | 20 | 19 | 18 | 17 | 132 |
| 66 | 66 | 66 | 66 | 66 | 66 | 66 | 66 | |

This magic rectangle can also be written as two pandiagonal equal sum magic squares of order 4. See below

**Example 4.9.** *Let's consider a magic rectangle of order* $4 \times 8$ *given by*

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 7 | 28 | 1 | 30 | 15 | 20 | 9 | 22 | 132 |
| 2 | 29 | 8 | 27 | 10 | 21 | 16 | 19 | 132 |
| 32 | 3 | 26 | 5 | 24 | 11 | 18 | 13 | 132 |
| 25 | 6 | 31 | 4 | 17 | 14 | 23 | 12 | 132 |
| 66 | 66 | 66 | 66 | 66 | 66 | 66 | 66 | |

### 4.2.3 Magic Rectangle of Order $4 \times 10$

**Example 4.10.** *Let's consider a magic rectangle of order* $4 \times 10$ *given by*

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 36 | 37 | 38 | 39 | 40 | 205 |
| 31 | 32 | 33 | 34 | 35 | 6 | 7 | 8 | 9 | 10 | 205 |
| 30 | 29 | 28 | 27 | 26 | 15 | 14 | 13 | 12 | 11 | 205 |
| 20 | 19 | 18 | 17 | 16 | 25 | 24 | 23 | 22 | 21 | 205 |
| 82 | 82 | 82 | 82 | 82 | 82 | 82 | 82 | 82 | 82 | |

### 4.2.4 Magic Rectangle of Order $4 \times 14$

**Example 4.11.** *Let's consider a magic rectangle of order* $4 \times 14$ *given by*

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 50 | 51 | 52 | 53 | 54 | 55 | 56 | 399 |
| 43 | 44 | 45 | 46 | 47 | 48 | 49 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 399 |
| 42 | 41 | 40 | 39 | 38 | 37 | 36 | 21 | 20 | 19 | 18 | 17 | 16 | 15 | 399 |
| 28 | 27 | 26 | 25 | 24 | 23 | 22 | 35 | 34 | 33 | 32 | 31 | 30 | 29 | 399 |
| 114 | 114 | 114 | 114 | 114 | 114 | 114 | 114 | 114 | 114 | 114 | 114 | 114 | 114 | |

## 4.3 Multiples of 6

Below are few examples of magic rectangles multiples of 6.

### 4.3.1 Magic Rectangle of Order $6 \times 10$

**Example 4.12.** *Let's consider a magic rectangle of order* $6 \times 10$ *given by*

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 55 | 54 | 12 | 13 | 47 | 46 | 45 | 14 | 18 | 305 |
| 2 | 56 | 53 | 11 | 42 | 20 | 40 | 21 | 23 | 37 | 305 |
| 3 | 57 | 52 | 10 | 36 | 35 | 27 | 28 | 32 | 25 | 305 |
| 58 | 4 | 9 | 51 | 30 | 26 | 33 | 34 | 29 | 31 | 305 |
| 59 | 5 | 8 | 50 | 19 | 38 | 22 | 39 | 41 | 24 | 305 |
| 60 | 6 | 7 | 49 | 43 | 17 | 15 | 16 | 44 | 48 | 305 |
| 183 | 183 | 183 | 183 | 183 | 183 | 183 | 183 | 183 | 183 | |

### 4.3.2 Magic Rectangle of Order $6 \times 14$

**Example 4.13.** *Let's consider a magic rectangle of order* $6 \times 14$ *given by*

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 79 | 78 | 12 | 13 | 67 | 66 | 24 | 25 | 59 | 58 | 57 | 26 | 30 | 595 |
| 2 | 80 | 77 | 11 | 14 | 68 | 65 | 23 | 54 | 32 | 52 | 33 | 35 | 49 | 595 |
| 3 | 81 | 76 | 10 | 15 | 69 | 64 | 22 | 48 | 47 | 39 | 40 | 44 | 37 | 595 |
| 82 | 4 | 9 | 75 | 70 | 16 | 21 | 63 | 42 | 38 | 45 | 46 | 41 | 43 | 595 |
| 83 | 5 | 8 | 74 | 71 | 17 | 20 | 62 | 31 | 50 | 34 | 51 | 53 | 36 | 595 |
| 84 | 6 | 7 | 73 | 72 | 18 | 19 | 61 | 55 | 29 | 27 | 28 | 56 | 60 | 595 |
| 255 | 255 | 255 | 255 | 255 | 255 | 255 | 255 | 255 | 255 | 255 | 255 | 255 | 255 | |

## 4.4 Multiples of 3

### 4.4.1 Magic Rectangle of Order $3 \times 5$

**Example 4.14.** *Let's consider a magic rectangle of order* $3 \times 5$ *given by*

| | | | | | |
|---|---|---|---|---|---|
| 14 | 10 | 4 | 5 | 7 | 40 |
| 1 | 3 | 8 | 13 | 15 | 40 |
| 9 | 11 | 12 | 6 | 2 | 40 |
| 24 | 24 | 24 | 24 | 24 | |

### 4.4.2 Magic Rectangle of Order $3 \times 7$

**Example 4.15.** *Let's consider a magic rectangle of order* $3 \times 7$ *given by*

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1 | 12 | 13 | 6 | 17 | 20 | 8 | 77 |
| 18 | 19 | 15 | 11 | 7 | 3 | 4 | 77 |
| 14 | 2 | 5 | 16 | 9 | 10 | 21 | 77 |
| 33 | 33 | 33 | 33 | 33 | 33 | 33 | |

### 4.4.3 Magic Rectangle of Order $3 \times 9$

**Example 4.16.** *Let's consider a magic rectangle of order* $3 \times 9$ *given by*

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 15 | 5 | 16 | 21 | 22 | 9 | 26 | 11 | 126 |
| 24 | 25 | 18 | 20 | 14 | 8 | 10 | 3 | 4 | 126 |
| 17 | 2 | 19 | 6 | 7 | 12 | 23 | 13 | 27 | 126 |
| 42 | 42 | 42 | 42 | 42 | 42 | 42 | 42 | 42 | |

### 4.4.4 Magic Rectangle of Order $3 \times 11$

**Example 4.17.** *Let's consider a magic rectangle of order* $3 \times 11$ *given by*

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 22 | 29 | 3 | 7 | 24 | 9 | 26 | 13 | 16 | 32 | 6 | 187 |
| 1 | 20 | 30 | 23 | 19 | 17 | 15 | 11 | 4 | 14 | 33 | 187 |
| 28 | 2 | 18 | 21 | 8 | 25 | 10 | 27 | 31 | 5 | 12 | 187 |
| 51 | 51 | 51 | 51 | 51 | 51 | 51 | 51 | 51 | 51 | 51 | |

### 4.4.5 Magic Rectangle of Order $3 \times 13$

**Example 4.18.** *Let's consider a magic rectangle of order* $3 \times 13$ *given by*

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 34 | 2 | 21 | 26 | 8 | 22 | 10 | 31 | 16 | 33 | 37 | 5 | 15 | 260 |
| 1 | 23 | 36 | 27 | 28 | 29 | 20 | 11 | 12 | 13 | 4 | 17 | 39 | 260 |
| 25 | 35 | 3 | 7 | 24 | 9 | 30 | 18 | 32 | 14 | 19 | 38 | 6 | 260 |
| 60 | 60 | 60 | 60 | 60 | 60 | 60 | 60 | 60 | 60 | 60 | 60 | 60 | |

### 4.4.6 Magic Rectangle of Order $3 \times 15$

**Example 4.19.** *Let's consider a magic rectangle of order* $3 \times 15$ *given by*

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 28 | 26 | 4 | 9 | 27 | 25 | 12 | 35 | 36 | 17 | 22 | 43 | 44 | 16 | 345 |
| 38 | 39 | 40 | 41 | 31 | 32 | 33 | 23 | 13 | 14 | 15 | 5 | 6 | 7 | 8 | 345 |
| 30 | 2 | 3 | 24 | 29 | 10 | 11 | 34 | 21 | 19 | 37 | 42 | 20 | 18 | 45 | 345 |
| 69 | 69 | 69 | 69 | 69 | 69 | 69 | 69 | 69 | 69 | 69 | 69 | 69 | 69 | 69 | |

### 4.4.7 Magic Rectangle of Order $3 \times 17$

**Example 4.20.** *Let's consider a magic rectangle of order* $3 \times 17$ *given by*

| | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 31 | 29 | 4 | 9 | 32 | 11 | 28 | 39 | 40 | 15 | 42 | 18 | 25 | 49 | 50 | 19 | 442 |
| 44 | 45 | 46 | 47 | 35 | 36 | 30 | 38 | 26 | 14 | 22 | 16 | 17 | 5 | 6 | 7 | 8 | 442 |
| 33 | 2 | 3 | 27 | 34 | 10 | 37 | 12 | 13 | 24 | 41 | 20 | 43 | 48 | 23 | 21 | 51 | 442 |
| 78 | 78 | 78 | 78 | 78 | 78 | 78 | 78 | 78 | 78 | 78 | 78 | 78 | 78 | 78 | 78 | 78 | |

### 4.4.8 Magic Rectangle of Order $3 \times 19$

**Example 4.21.** *Let's consider a magic rectangle of order* $3 \times 19$ *given by*

| | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 38 | 49 | 3 | 32 | 5 | 11 | 35 | 13 | 42 | 15 | 44 | 25 | 46 | 21 | 28 | 54 | 24 | 56 | 10 | 551 |
| 1 | 36 | 50 | 51 | 52 | 39 | 40 | 41 | 31 | 29 | 27 | 17 | 18 | 19 | 6 | 7 | 8 | 22 | 57 | 551 |
| 48 | 2 | 34 | 4 | 30 | 37 | 12 | 33 | 14 | 43 | 16 | 45 | 23 | 47 | 53 | 26 | 55 | 9 | 20 | 551 |
| 87 | 87 | 87 | 87 | 87 | 87 | 87 | 87 | 87 | 87 | 87 | 87 | 87 | 87 | 87 | 87 | 87 | 87 | 87 | |

In a similar way we can write further cases.

## 4.5 Multiples of 5

### 4.5.1 Magic Rectangle of Order $5 \times 7$

**Example 4.22.** *Let's consider a magic rectangle of order* $5 \times 7$ *given by*

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 15 | 26 | 13 | 6 | 20 | 24 | 22 | 126 |
| 1 | 33 | 27 | 11 | 31 | 19 | 4 | 126 |
| 28 | 2 | 29 | 18 | 7 | 34 | 8 | 126 |
| 32 | 17 | 5 | 25 | 9 | 3 | 35 | 126 |
| 14 | 12 | 16 | 30 | 23 | 10 | 21 | 126 |
| 90 | 90 | 90 | 90 | 90 | 90 | 90 | |

### 4.5.2 Magic Rectangle of Order $5 \times 9$

**Example 4.23.** *Let's consider a magic rectangle of order* $5 \times 9$ *given by*

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 20 | 43 | 19 | 21 | 7 | 12 | 9 | 31 | 45 | 207 |
| 17 | 22 | 18 | 38 | 14 | 40 | 10 | 44 | 4 | 207 |
| 35 | 33 | 5 | 16 | 23 | 30 | 41 | 13 | 11 | 207 |
| 42 | 2 | 36 | 6 | 32 | 8 | 28 | 24 | 29 | 207 |
| 1 | 15 | 37 | 34 | 39 | 25 | 27 | 3 | 26 | 207 |
| 115 | 115 | 115 | 115 | 115 | 115 | 115 | 115 | 115 | |

### 4.5.3 Magic Rectangle of Order $5 \times 11$

**Example 4.24.** *Let's consider a magic rectangle of order* $5 \times 11$ *given by*

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 23 | 51 | 27 | 7 | 19 | 9 | 15 | 13 | 53 | 36 | 55 | 308 |
| 50 | 25 | 18 | 21 | 26 | 17 | 48 | 11 | 4 | 54 | 34 | 308 |
| 44 | 42 | 40 | 24 | 46 | 28 | 10 | 32 | 16 | 14 | 12 | 308 |
| 22 | 2 | 52 | 45 | 8 | 39 | 30 | 35 | 38 | 31 | 6 | 308 |
| 1 | 20 | 3 | 43 | 41 | 47 | 37 | 49 | 29 | 5 | 33 | 308 |
| 140 | 140 | 140 | 140 | 140 | 140 | 140 | 140 | 140 | 140 | 140 | |

### 4.5.4 Magic Rectangle of Order $5 \times 13$

**Example 4.25.** *Let's consider a magic rectangle of order* $5 \times 13$ *given by*

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 30 | 62 | 52 | 54 | 9 | 10 | 11 | 16 | 40 | 63 | 43 | 38 | 429 |
| 25 | 61 | 32 | 27 | 8 | 22 | 20 | 18 | 42 | 59 | 45 | 64 | 6 | 429 |
| 51 | 49 | 47 | 53 | 29 | 31 | 33 | 35 | 37 | 13 | 19 | 17 | 15 | 429 |
| 60 | 2 | 21 | 7 | 24 | 48 | 46 | 44 | 58 | 39 | 34 | 5 | 41 | 429 |
| 28 | 23 | 3 | 26 | 50 | 55 | 56 | 57 | 12 | 14 | 4 | 36 | 65 | 429 |
| 165 | 165 | 165 | 165 | 165 | 165 | 165 | 165 | 165 | 165 | 165 | 165 | 165 | |

### 4.5.5 Magic Rectangle of Order $5 \times 15$

**Example 4.26.** *Let's consider a magic rectangle of order* $5 \times 15$ *given by*

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 31 | 33 | 3 | 71 | 61 | 27 | 11 | 12 | 13 | 19 | 47 | 52 | 41 | 74 | 75 | 570 |
| 30 | 28 | 70 | 37 | 32 | 34 | 25 | 23 | 21 | 66 | 67 | 72 | 50 | 7 | 8 | 570 |
| 60 | 58 | 56 | 54 | 59 | 62 | 36 | 38 | 40 | 14 | 17 | 22 | 20 | 18 | 16 | 570 |
| 68 | 69 | 26 | 4 | 9 | 10 | 55 | 53 | 51 | 42 | 44 | 39 | 6 | 48 | 46 | 570 |
| 1 | 2 | 35 | 24 | 29 | 57 | 63 | 64 | 65 | 49 | 15 | 5 | 73 | 43 | 45 | 570 |
| 190 | 190 | 190 | 190 | 190 | 190 | 190 | 190 | 190 | 190 | 190 | 190 | 190 | 190 | 190 | |

### 4.5.6 Magic Rectangle of Order $5 \times 17$

**Example 4.27.** *Let's consider a magic rectangle of order* $5 \times 17$ *given by*

| | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 36 | 79 | 3 | 81 | 69 | 10 | 39 | 41 | 13 | 24 | 15 | 16 | 77 | 82 | 6 | 55 | 85 | 731 |
| 33 | 38 | 29 | 42 | 35 | 32 | 30 | 72 | 26 | 74 | 22 | 49 | 52 | 59 | 46 | 84 | 8 | 731 |
| 67 | 65 | 63 | 61 | 68 | 66 | 11 | 28 | 43 | 58 | 75 | 20 | 18 | 25 | 23 | 21 | 19 | 731 |
| 78 | 2 | 40 | 27 | 34 | 37 | 64 | 12 | 60 | 14 | 56 | 54 | 51 | 44 | 57 | 48 | 53 | 731 |
| 1 | 31 | 80 | 4 | 9 | 70 | 71 | 62 | 73 | 45 | 47 | 76 | 17 | 5 | 83 | 7 | 50 | 731 |
| 215 | 215 | 215 | 215 | 215 | 215 | 215 | 215 | 215 | 215 | 215 | 215 | 215 | 215 | 215 | 215 | 215 | |

### 4.5.7 Magic Rectangle of Order $5 \times 19$

**Example 4.28.** *Let's consider a magic rectangle of order* $5 \times 19$ *given by*

| | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 39 | 87 | 43 | 4 | 90 | 77 | 12 | 13 | 31 | 15 | 27 | 25 | 18 | 85 | 91 | 7 | 93 | 60 | 95 | 912 |
| 86 | 41 | 34 | 32 | 47 | 40 | 35 | 33 | 46 | 29 | 82 | 17 | 54 | 59 | 66 | 51 | 8 | 94 | 58 | 912 |
| 76 | 74 | 72 | 70 | 68 | 75 | 73 | 44 | 80 | 48 | 16 | 52 | 23 | 21 | 28 | 26 | 24 | 22 | 20 | 912 |
| 38 | 2 | 88 | 45 | 30 | 37 | 42 | 79 | 14 | 67 | 50 | 63 | 61 | 56 | 49 | 64 | 62 | 55 | 10 | 912 |
| 1 | 36 | 3 | 89 | 5 | 11 | 78 | 71 | 69 | 81 | 65 | 83 | 84 | 19 | 6 | 92 | 53 | 9 | 57 | 912 |
| 240 | 240 | 240 | 240 | 240 | 240 | 240 | 240 | 240 | 240 | 240 | 240 | 240 | 240 | 240 | 240 | 240 | 240 | 240 | |

## 4.6 Multiples of 7

### 4.6.1 Magic Rectangle of Order $7 \times 9$

**Example 4.29.** *Let's consider a magic rectangle of order* $7 \times 9$ *given by*

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 58 | 7 | 63 | 2 | 14 | 8 | 55 | 21 | 60 | 288 |
| 42 | 48 | 10 | 53 | 5 | 47 | 18 | 52 | 13 | 288 |
| 19 | 33 | 23 | 34 | 39 | 40 | 27 | 44 | 29 | 288 |
| 15 | 61 | 36 | 38 | 32 | 26 | 28 | 3 | 49 | 288 |
| 35 | 20 | 37 | 24 | 25 | 30 | 41 | 31 | 45 | 288 |
| 51 | 12 | 46 | 17 | 59 | 11 | 54 | 16 | 22 | 288 |
| 4 | 43 | 9 | 56 | 50 | 62 | 1 | 57 | 6 | 288 |
| 224 | 224 | 224 | 224 | 224 | 224 | 224 | 224 | 224 | |

### 4.6.2 Magic Rectangle of Order $7 \times 11$

**Example 4.30.** *Let's consider a magic rectangle of order* $7 \times 11$ *given by*

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 75 | 73 | 4 | 67 | 10 | 17 | 2 | 33 | 8 | 71 | 69 | 429 |
| 58 | 16 | 52 | 22 | 57 | 6 | 65 | 12 | 59 | 18 | 64 | 429 |
| 44 | 51 | 25 | 29 | 46 | 31 | 48 | 35 | 38 | 54 | 28 | 429 |
| 23 | 42 | 63 | 1 | 41 | 39 | 37 | 77 | 15 | 36 | 55 | 429 |
| 50 | 24 | 40 | 43 | 30 | 47 | 32 | 49 | 53 | 27 | 34 | 429 |
| 14 | 60 | 19 | 66 | 13 | 72 | 21 | 56 | 26 | 62 | 20 | 429 |
| 9 | 7 | 70 | 45 | 76 | 61 | 68 | 11 | 74 | 5 | 3 | 429 |
| 273 | 273 | 273 | 273 | 273 | 273 | 273 | 273 | 273 | 273 | 273 | |

### 4.6.3 Magic Rectangle of Order $7 \times 13$

**Example 4.31.** *Let's consider a magic rectangle of order* $7 \times 13$ *given by*

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 84 | 9 | 82 | 1 | 90 | 3 | 20 | 11 | 80 | 39 | 88 | 5 | 86 | 598 |
| 21 | 70 | 62 | 78 | 15 | 76 | 7 | 68 | 25 | 66 | 17 | 74 | 19 | 598 |
| 60 | 28 | 47 | 52 | 34 | 48 | 36 | 57 | 42 | 59 | 63 | 31 | 41 | 598 |
| 27 | 49 | 23 | 79 | 54 | 55 | 46 | 37 | 38 | 13 | 69 | 43 | 65 | 598 |
| 51 | 61 | 29 | 33 | 50 | 35 | 56 | 44 | 58 | 40 | 45 | 64 | 32 | 598 |
| 73 | 18 | 75 | 26 | 67 | 24 | 85 | 16 | 77 | 14 | 30 | 22 | 71 | 598 |
| 6 | 87 | 4 | 53 | 12 | 81 | 72 | 89 | 2 | 91 | 10 | 83 | 8 | 598 |
| 322 | 322 | 322 | 322 | 322 | 322 | 322 | 322 | 322 | 322 | 322 | 322 | 322 | |

### 4.6.4 Magic Rectangle of Order $7 \times 15$

**Example 4.32.** *Let's consider a magic rectangle of order* $7 \times 15$ *given by*

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 102 | 99 | 6 | 101 | 15 | 92 | 13 | 23 | 3 | 104 | 1 | 35 | 10 | 97 | 94 | 795 |
| 79 | 22 | 70 | 20 | 76 | 29 | 78 | 8 | 88 | 17 | 90 | 26 | 81 | 24 | 87 | 795 |
| 31 | 58 | 56 | 34 | 39 | 57 | 55 | 42 | 65 | 66 | 47 | 52 | 73 | 74 | 46 | 795 |
| 68 | 69 | 85 | 11 | 61 | 62 | 63 | 53 | 43 | 44 | 45 | 95 | 21 | 37 | 38 | 795 |
| 60 | 32 | 33 | 54 | 59 | 40 | 41 | 64 | 51 | 49 | 67 | 72 | 50 | 48 | 75 | 795 |
| 19 | 82 | 25 | 80 | 16 | 89 | 18 | 98 | 28 | 77 | 30 | 86 | 36 | 84 | 27 | 795 |
| 12 | 9 | 96 | 71 | 105 | 2 | 103 | 83 | 93 | 14 | 91 | 5 | 100 | 7 | 4 | 795 |
| 371 | 371 | 371 | 371 | 371 | 371 | 371 | 371 | 371 | 371 | 371 | 371 | 371 | 371 | 371 | |

### 4.6.5 Magic Rectangle of Order $7 \times 17$

**Example 4.33.** *Let's consider a magic rectangle of order* $7 \times 17$ *given by*

| | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 110 | 11 | 108 | 13 | 119 | 2 | 117 | 4 | 26 | 14 | 105 | 16 | 103 | 39 | 114 | 7 | 112 | 1020 |
| 27 | 92 | 80 | 90 | 18 | 101 | 20 | 99 | 9 | 89 | 32 | 87 | 34 | 98 | 23 | 96 | 25 | 1020 |
| 35 | 65 | 63 | 38 | 43 | 66 | 45 | 62 | 73 | 74 | 49 | 76 | 52 | 59 | 83 | 84 | 53 | 1020 |
| 78 | 79 | 29 | 115 | 69 | 70 | 64 | 72 | 60 | 48 | 56 | 50 | 51 | 5 | 91 | 41 | 42 | 1020 |
| 67 | 36 | 37 | 61 | 68 | 44 | 71 | 46 | 47 | 58 | 75 | 54 | 77 | 82 | 57 | 55 | 85 | 1020 |
| 95 | 24 | 97 | 22 | 86 | 33 | 88 | 31 | 111 | 21 | 100 | 19 | 102 | 30 | 40 | 28 | 93 | 1020 |
| 8 | 113 | 6 | 81 | 17 | 104 | 15 | 106 | 94 | 116 | 3 | 118 | 1 | 107 | 12 | 109 | 10 | 1020 |
| 420 | 420 | 420 | 420 | 420 | 420 | 420 | 420 | 420 | 420 | 420 | 420 | 420 | 420 | 420 | 420 | 420 | |

### 4.6.6 Magic Rectangle of Order $7 \times 19$

**Example 4.34.** *Let's consider a magic rectangle of order* $7 \times 19$ *given by*

| | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 129 | 125 | 8 | 127 | 6 | 115 | 18 | 117 | 16 | 29 | 4 | 131 | 2 | 133 | 14 | 45 | 12 | 123 | 119 | 1273 |
| 100 | 28 | 88 | 26 | 109 | 38 | 97 | 36 | 99 | 10 | 111 | 22 | 113 | 20 | 101 | 32 | 103 | 30 | 110 | 1273 |
| 76 | 87 | 41 | 70 | 43 | 49 | 73 | 51 | 80 | 53 | 82 | 63 | 84 | 59 | 66 | 92 | 62 | 94 | 48 | 1273 |
| 39 | 74 | 107 | 13 | 90 | 77 | 78 | 79 | 69 | 67 | 65 | 55 | 56 | 57 | 44 | 121 | 27 | 60 | 95 | 1273 |
| 86 | 40 | 72 | 42 | 68 | 75 | 50 | 71 | 52 | 81 | 54 | 83 | 61 | 85 | 91 | 64 | 93 | 47 | 58 | 1273 |
| 24 | 104 | 31 | 102 | 33 | 114 | 21 | 112 | 23 | 124 | 35 | 98 | 37 | 96 | 25 | 108 | 46 | 106 | 34 | 1273 |
| 15 | 11 | 122 | 89 | 120 | 1 | 132 | 3 | 130 | 105 | 118 | 17 | 116 | 19 | 128 | 7 | 126 | 9 | 5 | 1273 |
| 469 | 469 | 469 | 469 | 469 | 469 | 469 | 469 | 469 | 469 | 469 | 469 | 469 | 469 | 469 | 469 | 469 | 469 | 469 | |

# 5 Author's Contributions to Magic Squares and Recreating Numbers

For author's contribution to **magic squares** and **recreation numbers** please see the links below:

- **Inder J. Taneja**, Magic Squares, *https://inderjtaneja.com/2019/06/27/publications-magic-squares/*
- **Inder J. Taneja**, Recreation of Numbers, *https://inderjtaneja.com/2019/06/27/publications-recreation-of-numbers/*

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