Cap Floor Explained

Pricing Cap

- An interest rate cap is designed to provide insurance against the interest cap on an underlying floating-rate note rising above a certain level (cap rate), and is modelled as a portfolio of European interest rate call options (caplets).
- A cap is a contract between two parties that provides an interest rate ceiling or cap on the floating rate payments.
- The buyer receives payments at the end of each period when the interest rate exceeds the strike. The payment frequency could be monthly, quarterly or semiannually.

Cap

- An interest rate floor provides a payoff when the interest rate on the underlying floating-rate note falls below a certain level, and is modelled as a portfolio of European put options (floorlets).
- A floor is a contract between two parties that provides an interest rate floor on the floating rate payments.
- The buyer receives payments at the end of each period when the interest rate falls below the strike. The payment frequency could be monthly, quarterly or semiannually.

Cap

- Caps are frequently purchased by issuers of floating rate debt who wish to protect themselves from the increased financing costs that would result from a rise in interest rates.
- Floors are frequently purchased by purchasers of floating rate debt who wish to protect themselves from the loss of income that would result from a decline in interest rates.
- Investors use caps and floors to hedge against the risk associated with floating interest rate.

Cap

- The payoff of a caplet

$$Payoff = N * \tau * max(R - K, 0)$$

where N – notional; R – realized interest rate; K – strike; τ – day count fraction.

- The payoff of a floorlet

$$Payoff = N * \tau * max(K - R, 0)$$

where N – notional; R – realized interest rate; K – strike; τ – day count fraction.

Cap

- The present value of a cap is given by

$$PV(0) = N\sum_{i=1}^{n} \tau_i D_i (F_i \Phi(d_1) - K\Phi(d_2))$$

where

$D_i = D(0, T_i)$ – the discount factor;

$F_i = F(t; T_{i-1}, T_i) = \left(\frac{D_{i-1}}{D_i} - 1\right)/\tau_i$ – the forward rate for period (T_{i-1}, T_i).

Φ – the accumulative normal distribution function

$$d_{1,2} = \frac{\ln(\frac{F_i}{K}) \pm 0.5\sigma_i^2 T_i}{\sigma_i \sqrt{T_i}}$$

Cap

- The present value of a floor is given by

$$PV(0) = N \sum_{i=1}^{n} \tau_i D_i \left(K\Phi(-d_2) - F_i\Phi(-d_1)\right)$$

where

$D_i = D(0, T_i)$ – the discount factor;

$F_i = F(t; T_{i-1}, T_i) = \left(\frac{D_{i-1}}{D_i} - 1\right) / \tau_i$ – the forward rate for period (T_{i-1}, T_i).

Φ – the accumulative normal distribution function

$$d_{1,2} = \frac{\ln(\frac{F_i}{K}) \pm 0.5\sigma_i^2 T_i}{\sigma_i \sqrt{T_i}}$$

Cap

- For a simple compounding rate, the discount factor is

$$df(t,t_i)=\frac{1}{1+R(t,t_i)\cdot(t_i-t)}$$

- For a compounding rate, the discount factor is

$$df(t,t_i)=\frac{1}{\left(1+\frac{R(t,t_i)}{m_R}\right)^{m_R(t_i-t)}}$$

Cap

- For continuous compounding rate, the discount factor is

$$df(t,t_i) = e^{-R(t,t_i)\cdot(t_i - t)}$$

- The non-arbitrage relationship is

$$df(t_i,t_{i+1}) = \frac{df(t,t_{i+1})}{df(t,t_i)}$$

Thanks!

Reference:

https://finpricing.com/lib/EqQuanto.html