ON LAKE FORM, LAKE VOLUME AND LAKE HYPSOGRAPHIC SURVEY

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ABSTRACT. This work is based on a study of the relative hypsographic curves of 48 lakes. The five largest belong to the Great Lakes System, the rest of the lakes are Swedish. A method for determining a mean lake form as well as statistically defined deviations from this mean is introduced. A classification system for lakes based on the lake form is discussed. A method for determining the error in lake volume calculations is presented and formulas to correct for the volume error are derived. A first outline of a model for optimization of lake hypsographic surveys is discussed.

Introduction and aim of the work

When Strahler (1952) introduced the relative hypsographic curve for a drainage area, as an approach for determining the geological maturity of an area, new possibilities for geomorphological thinking were opened (Fig. 1). The method has become classical and it has been adopted for drainage areas from a variety of geographical, climatological and geological environments. Scheidegger (1963) has emphasized that the relative hypsographic curve can be defined for the whole earth and it can of course also be drawn for the smallest definable drainage area. In these extreme cases a genetic interpretation is meaningless. Nor can the hypsographic method be adopted for a general and direct genetical interpretation of single geomorphological constituents within a drainage area, such as a mountain range or a lake. However, in the present work we shall, from the standpoint of physical limnology, adopt the method of relative hypsographic curves for lakes. Thus in this context we are not primarily interested in a genetical discussion. This may possibly be a spin-off effect. Instead, our present purpose is to make a rather detailed analysis of the concept "lake form", and to show that this is a meaningful task from theoretical as well as practical scientific viewpoints.

In the standard limnological work by Hutchinson (1957), "A Treatise on Limnology", examples are given, not only of a well-conducted descriptive classification of lakes, based on their formation history, but also of the impact of lake type on the productivity and life of the lake. In many recent reports further examples have been presented concerning the impact of lake morphometry on, for example, the load of nutritions (Ahl, 1973, Vollenweider, 1975), the heat budget (Timms, 1975) and the contamination of heavy metals (Håkanson, 1974a, 1975). A fundamental question in this context concerns the determination of the lake volume, which constitutes an important factor in all budget calculations. It is quite remarkable that, to the best of the author's knowledge, there does not at present exist any error calculation concerning determinations of lake volume. The introduction of such a method for error calculation is one of the main tasks in the present work.

No error calculation can, however, be conducted independently of the data collection. The difficulty faced by lake morphometry in obtaining a statistically workable material for an error calculation may well explain why no one has treated this problem. This task—the obtaining of lake morphometrical data which can be treated with statistical methods—is another object of the present paper. The results which will be introduced may also provide means for objectively, and with an a priori given aim, obtaining a method for hypsographic surveys which can be used for all lakes and which yields good comparability for the data obtained.

In summary, we may conclude that the aim of the present paper is:

— to derive and define a statistical mean form for a lake, based on the relative hypsographic curve, and to give statistically determined deviations from this mean lake form;
— to use this method to classify lakes according to their forms;
— to present a method for statistical error calculation of lake volume;

Table 1. Lakes studied.

Name	Area (km^2)	Max depth (m)	Source
1 Superior	821 000	405	Beeton, 1971
2 Huron	598 000	223	Beeton, 1971
3 Michigan	578 000	281	Beeton, 1971
4 Erie	25 700	64	Beeton, 1971
5 Ontario	19 000	244	Beeton, 1971
6 Vänern	5 650	106	Håkanson, 1975
7 Värmlandssjön	3 580	106	Håkanson, 1975
8 Dalbosjön	2 070	89	Håkanson, 1975
9 Vättern	1 910	119	Håkanson, 1974b
10 Mälaren	1 140	63	Lemming et al., 1971
11 Hjälmaren	484	22	Lemming et al., 1971
12 Skagen	128	72	Sahlström, 1916
13 Roxen	94.7	8.5	Håkanson, 1974b
14 Unden	92.0	86	von zur Mühlen, 1915
15 Mälaren (A)	63.8	18.0	Lemming et al., 1971
16 Ivösjön	54.2	50.0	Hamrin et al., 1974
17 Erken	23.8	20.7	IHD:s sjögrupp, 1975
18 Ekoln	18.6	38	Axelsson and Håkanson, 1972
19 Mälaren (D_4)	16.0	34.0	Lemming et al., 1971
20 Lilla Korslången	14.9	24	Ramberg et al., 1973
21 Laitaure	10.0	17.8	Axelsson, 1967
22 Saxen	7.10	41	Thanderz, 1963
23 Gimo damm	5.15	4.0	Coleman et al., 1975
24 Norra Vidöstern	4.96	38.5	Granéli and Leonardsson, 1974
25 Pajep Måskejaure	3.88	25.8	Andersson, 1974
26 Sparren	3.28	14.2	Nordlander and Segerfeldt, 1967
27 Velen	2.80	18	IHD-report No. 6, 1971
28 Norrviken	2.66	12.5	Ahlgren, 1967
29 Tullingesjön	2.00	32.0	Bengtsson et al., 1972
30 Lilla Ullevifjärden	1.90	52	Ryding and Borg, 1973
31 Björken	1.80	18	Axelsson and Håkanson, 1973
32 Fiolen	1.60	10	Åberg und Rodhe, 1942
33 Oxundasjön	1.59	6.0	Ahlgren, 1970
34 Trehörningen	1.47	6.2	Eriksson et al., 1974
35 Testen	1.13	2.1	James et al., 1975
36 Munksjön	1.1	25	Håkanson, 1974b
37 Edssjön	1.02	5.4	Ahlgren, 1970
38 Örnässjön	0.91	7.4	Wallsten, 1972
39 Växsjösjön	0.87	6.5	Bengtsson et al., 1972
40 Järlasjön	0.84	24	Bengtsson et al., 1972
41 Fansjön	0.57	6.4	Avd. för Hydrologi, 1974
42 Lillsjön	0.56	10.0	Wallsten, 1972
43 Skärshultsjön	0.36	14	Åberg und Rodhe, 1942
44 Drängsjön	0.32	11.1	Avd. för Hydrologi, 1975
45 Skärsjön	0.31	9.1	Abrahamsson et al., 1975
46 Ramsjön	0.17	4.2	Andersson et al., 1974
47 Botjärn	0.097	14.4	Ramberg et al., 1973
48 Vitalampa	0.029	10.1	Ramberg et al., 1973

— to give an outline of a model for optimization of lake hydrographic surveys, in which levels of ambition concerning acceptable errors in lake volume and contour line layout can be discussed and determined a priori.

Presuppositions

The base data used in the present work has been obtained from published sources. 48 lakes have been studied (Table 1). The lakes differ in size, varying from Lake Superior (with an area of

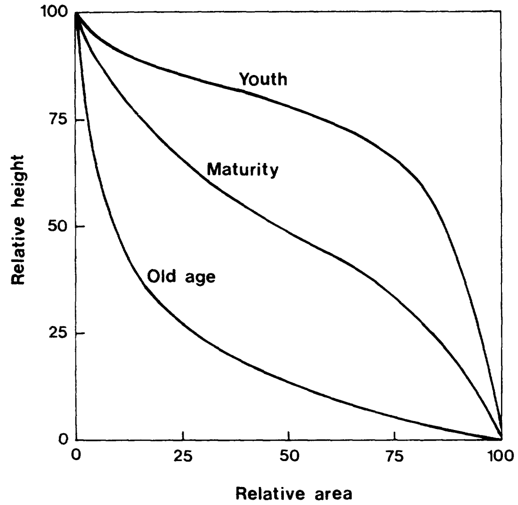

Fig. 1. Characteristic relative hypsographic curves of drainage basins of different maturity (after Strahler, 1952).

Table 2. Hutchinson's classification of lakes.

1	Tectonic basins
2	Lakes associated with volcanic activity
3	Lakes formed by landslides
4	Lakes formed by glacial activity
	a) Lakes held by ice or by moraine in contact with existing ice
	b) Glacial rock basins
	c) Moraine and outwash dams
	d) Drift basins
5	Solution basins
6	Lakes due to fluviatile action
	a) Plunge-pool lakes
	b) Fluviatile dams
	c) Lakes of mature flood plains
7	Lake basins formed by wind
8	Lakes associated with shore lines
9	Lakes formed by organic accumulation
10	Lakes produced by the complex behaviour of higher organisms
11	Lakes produced by meteorite impact

821 000 km²) to Lake Vitalampa (with an area of only 0.029 km²). The maximum depths vary from 405 m to 4 m. The five largest of the lakes examined constitute the Great Lakes system in North America. The remaining lakes are Swedish. Consequently, the lakes investigated all belong to a comparatively homogeneous climatic zone and were all directly influenced by the last great glaciation. According to Hutchinson's (1957) classification (Table 2), they may all be connected to the type "Lakes formed by glacial activity". The lakes examined can thus only be linked to one, or perhaps two (tectonic basins), of all existing lake types. This implies that the following discussion cannot be considered a general one. The analytical principles to be introduced, however, may be generally used for all lakes, independent of their genesis.

All figures given in Table 1 have been adopted directly from the given references. Some figures are therefore given in a more exact way than others. This also illustrates the fact that the lakes have been surveyed differently, with different methods, different levels of ambition and from different premises. Consequently, the material does not provide a high degree of quantitative comparison. The problem of comparability will be discussed subsequently; the results to be presented make no claim to be either general or quantitative, as only glacially influenced lakes have been studied, as the lakes examined are few compared to all existing glacial lakes, and as the lakes examined have been surveyed differently. The results should therefore be considered for their theoretical, qualitative merits; our present ambition has been to introduce new ideas and new methods which hopefully may lead to better quantitative results in the future.

A relevant question in this context is: how have the lakes examined here been chosen? Quite simply, in the first place, the lakes have been chosen from those of the glacial type because it has been part of the author's work at the National Swedish Environment Protection Board's Limnological Survey (NLU) to conduct morphometrical and sedimentological investigations in Swedish lakes; secondly, the number of lakes to be studied was determined, after consultation with statistical experts, to be between 30 and 50, for present purposes. The methods developed in this work have been processed manually, without the aid of a computer, thus yielding the upper limit of 50. A larger material would be quite impossible to handle, as well as being quite unnecessary for a qualitative approach. Since the quantitative aspects are subordinate to the qualitative ones, the discussion of statistical sampling is of secondary importance. The demand placed upon the material has instead been that it should contain a great variety of lakes, large and small, deep and shallow. This demand may have implied that the spread in this particular sample is greater than would have been the case if a regular, random statistical selection had governed the choice of the lakes to be studied.

Methods

Measurement of length, area and volume

A general presentation of morphometrical methods has been given by, for example, Welch (1948). Since all raw data in the present work emanates from published sources, we may refer to them for information concerning measurements of length and area. The problem of measuring length is, however, important and intricate; Scheidegger (1970) has pointed out that "The problem of measuring the length of a line is an old one. In general, not much thought is given to this problem by geomorphologists, inasmuch as it is simply assumed that the length be measured by some type of integrating device. However, the fact that natural geomorphological lines are wiggly, introduces certain complications. It is clear that more and more "wiggles" tend to disappear, the smaller the scale of the map is. Thus, the "length" of a geomorphic line does not have any meaning per se, inasmuch as it depends very much on the scale of map used: The better the map, the greater the length of a given natural feature." The same type of argumentation is of course also valid for determinations of area, which in the present context, that of lakes, is generally conducted with a planimeter.

The volume of a lake is, in the ideal case, determined from the integral

$$V = \int_0^{z_{\max}} f(A, z)dz \tag{1}$$

where

z = the depth

$f(A, z)$ = the hypsographic curve.

As one in practice can never obtain a continous hypsographic curve, which is exactly defined for all points, but only a certain number of points along the curve, corresponding to the existing contour lines, some formula to approximate the volume is generally used. The two formulas most frequently used may be called the *linear* and the *parabolic* approximations. The formula for the linear approximation (V_l) is given by (2) and for the parabolic (V_p) by (4).

$$V_l = \sum_{j=1}^{n} \frac{e}{2}\left(a_j^k + a_{j-1}^k\right) \tag{2}$$

where

e = the contour-line interval

n = the number of contour-lines

a_j^k = the cumulative area at contour-line j

The following relationship is valid between n, e and the maximum depth ($z_{\max}$):

$$n \times e \approx z_{\max} \tag{3}$$

For practical and pedagogical reasons, it is reasonable to assign the e-values positive integers (Håkanson, 1974b). The parabolic approximation is given by

$$V_p = \sum_{j=1}^{n} \frac{e}{3}\left(a_j^k + a_{j-1}^k + \sqrt{a_j^k \times a_{j-1}^k}\right) \tag{4}$$

Hypsographic curves

Welch (1948) defines hypsographic curves as follows: "A hypsographic curve for subsurface horizontal areas is constructed by plotting depth along the vertical axis (ordinate) and area along the horizontal axis (abscissa). Such a curve not only represents certain elements in the form of the lake basin but it also provides a means whereby areas of any depth level may be determined. If actual areas within contours are plotted, the curve may be called *a direct hypsographic curve*. If, on the other hand, the areas within submerged contours are expressed in terms of per cent of surface area of lake, then the curve is referred to as a *percentage hypsographic curve*." In the discussion to follow, we shall not employ either of these two types of hypsographic curves, which are the most commonly used; in fact, the author has never, in any lake morphological work, seen the third alternative, which is the *relative hypsographic curve*, where both the abscissa and the ordinate are on a relative scale. The direct hypsographic curve and the percentage hypsographic curve give, as Welch puts it, "a certain element of form". The comprehension of the form will, for these two cases, however, depend completely on the choice of the scale on the axis, so that these methods cannot be used if the pure form is desired. With the relative hypsographic curve, the element of form, and nothing else, is emphasized, and lakes with different dimensions may be fully and directly compared.

Lake form

Working hypothesis

The relative hypsographic curves for the 48 lakes studied are given in Figures 2, 3, 4, 5 and 6.

The working hypothesis is that, using these curves, we may determine a statistically mean form for a lake and also define statistical deviation forms from this mean form.

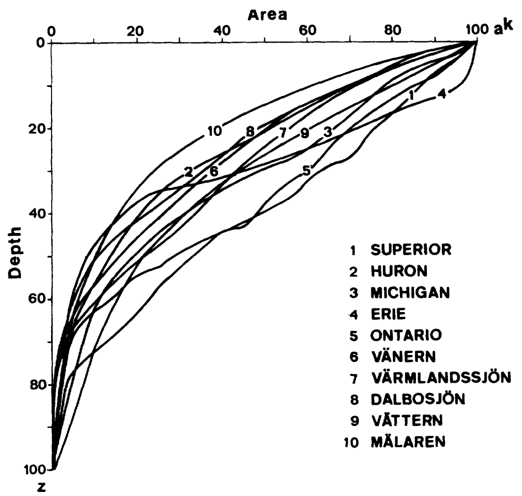

Fig. 2. Relative hypsographic curves for lakes Superior, Huron, Michigan, Erie, Ontario, Vänern, Värmlandssjön, Dalbosjön, Vättern and Mälaren.

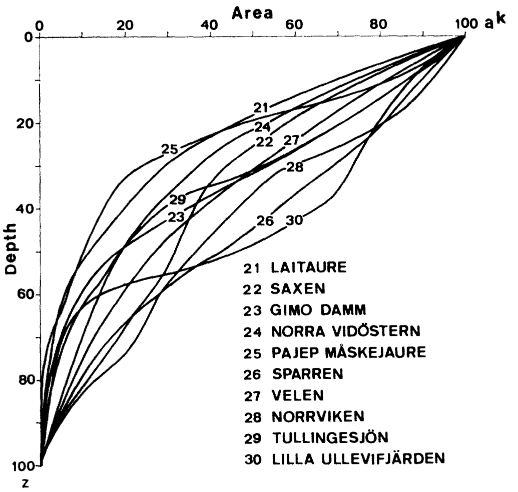

Fig. 4. Relative hypsographic curves for lakes Laitaure, Saxen, Gimo damm, Norra Vidöstern, Pajep Måskejaure, Sparren, Velen, Norrviken, Tullingesjön and Lilla Ullevifjärden.

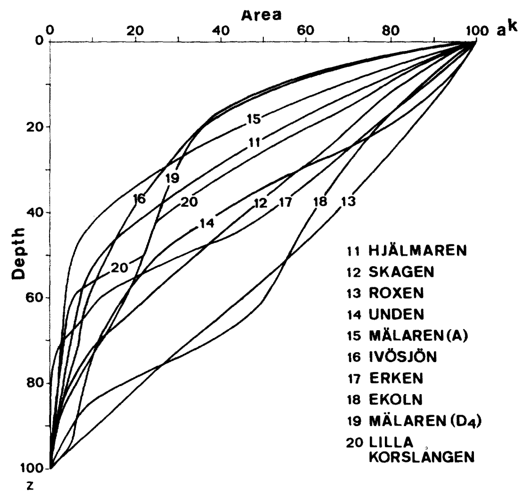

Fig. 3. Relative hypsographic curves for lakes Hjälmaren, Skagen, Roxen, Unden, Mälaren (A), Ivösjön, Erken, Ekoln, Mälaren (D_4) and Lilla Korslången.

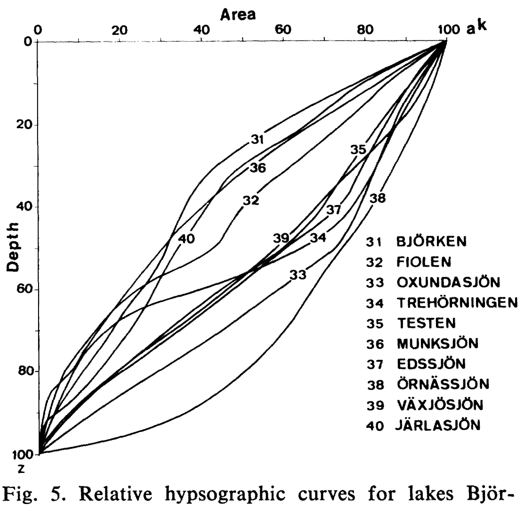

Fig. 5. Relative hypsographic curves for lakes Björken, Fiolen, Oxundasjön, Trehörningen, Testen, Munksjön, Edssjön, Örnässjön, Växjösjön and Järlasjön.

For some lakes, such as Lake Ekoln, there exists a considerable and welldefined body of echo-sounded lines, which provide great accuracy for this relative hypsographic curve. In this case the curve presented is very close to the exact, ideal one. For most lakes, however, the reliability is poorer. In any case, the given curves provide an accuracy which is sufficient for the present approach; even in the poorest cases, we may accept the curves given as at least representing probable examples of lakes from the glacial lake type environment, and the latter is, in this context, what is most important.

The verification or the rejection of the working hypothesis consequently starts from these given relative hypsographic curves.

The statistical approach

A natural starting point in our analysis is to draw a line from the point ($a^k = 100$, $z = 0$) to the point ($a^k = 0$, $z = 100$), that is

$$z = -a^k + 100 \tag{5}$$

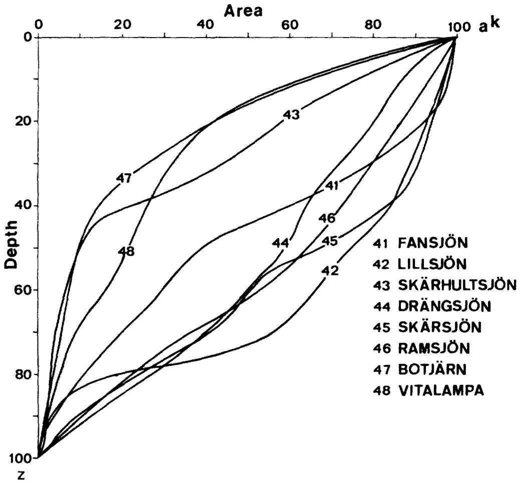

Fig. 6. Relative hypsographic curves for lakes Fansjön, Lillsjön, Skärshultsjön, Drängsjön, Skärsjön, Ramsjön, Botjärn and Vitalampa.

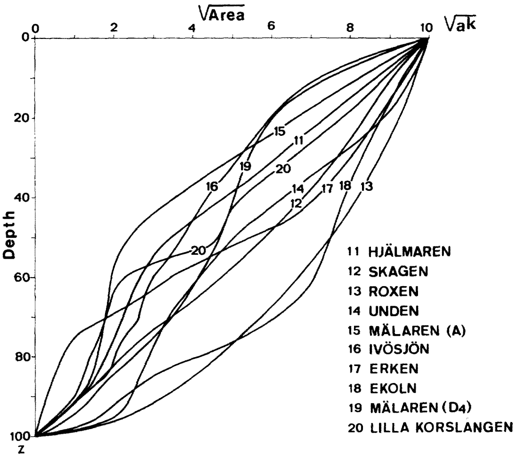

Fig. 8. TRH-curves for lakes Hjälmaren, Skagen, Roxen, Unden, Mälaren (A), Ivösjön, Erken, Ekoln, Mälaren (D_4) and Lilla Korslången.

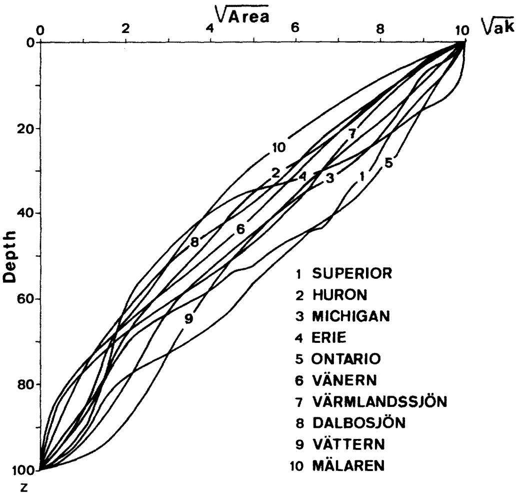

Fig. 7. TRH-curves for lakes Superior, Huron, Michigan, Erie, Ontario, Vänern, Värmlandssjön, Dalbosjön, Vättern and Mälaren.

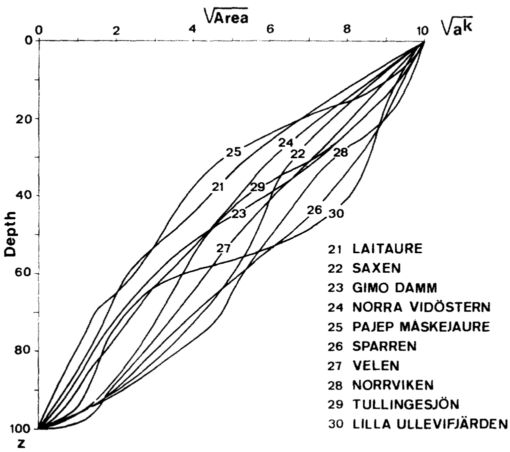

Fig. 9. TRH-curves for lakes Laitaure, Saxen, Gimo damm, Norra Vidöstern, Pajep Måskejaure, Sparren, Velen, Norrviken, Tullingesjön and Lilla Ullevifjärden.

Relative hypsographic curves above this line will, unless otherwise specified, be called convex and curves below this line called concave. If we now study the distribution of the 48 relative hypsographic curves compared to this reference line, we find that 35, or approx 73 %, are clearly convex, 10, or approx 20 % are concave, and that 3 are rather in between and more difficult to determine. Consequently there exists an unsymmetric distribution of the relative hypsographic curves compared to the reference line. It should be possible, even at this early stage, with mathematical-statistical methods to determine this unsymmetrical distribution and obtain a statistically well-defined mean hypsographic curve. This simple pilot test indicates, however, that the analysis may be done in a simpler and perhaps also more sophisticated manner. An appealing and obvious approach is, then, to take the square root of the a^k-values, i.e., to plot $\sqrt{a^k}$ on the horizontal axis. This procedure implies, in the first place, that we obtain the same

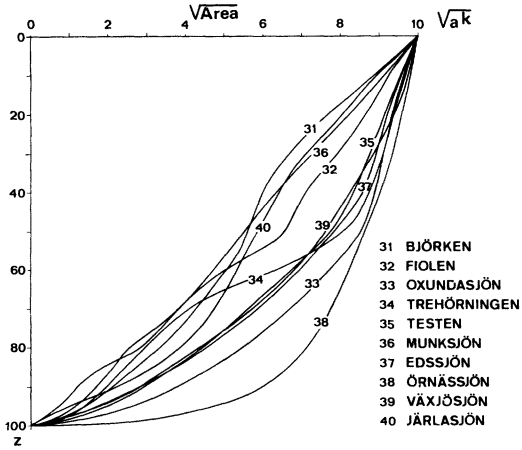

Fig. 10. TRH-curves for lakes Björken, Fiolen, Oxundasjön, Trehörningen, Testen, Munksjön, Edssjön, Örnässjön, Växjösjön and Järlasjön.

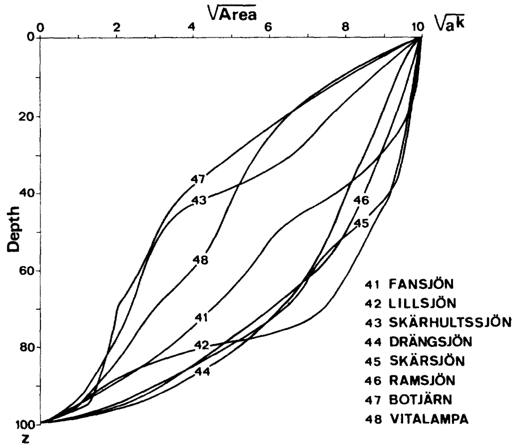

Fig. 11. TRH-curves for lakes Fansjön, Lillsjön, Skärshultsjön, Drängsjön, Skärsjön, Ramsjön, Botjärn and Vitalampa.

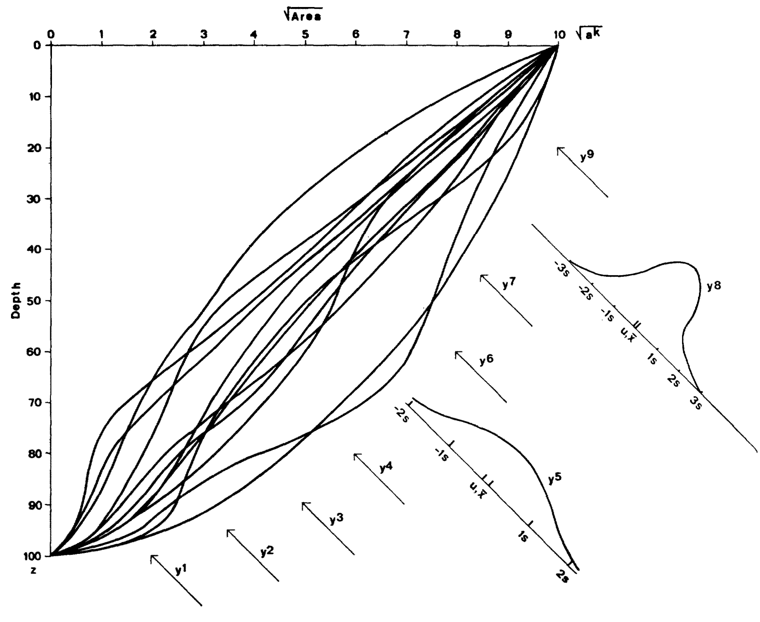

Fig. 12. Illustration to the statistical approach. TRH-curves for lakes Vänern, Vättern, Mälaren, Hjälmaren, Skagen, Roxen, Unden, Ekoln, Saxen, Velen, Björken and Munksjön.

dimension (length units) on both axes; secondly, it leads to a better symmetry along a new reference line, which is given by

$$z = -10\sqrt{a^k} + 100 \tag{6}$$

This square-root procedure indicates a way to determine a statistical mean form; all the relative hypsographic curves in Figures 2–6 have been recalculated into a new form which may be called *transformed relative hypsographic curves* (TRH-curves), shown in Figures 7, 8, 9, 10 and 11. We shall now proceed to justify this transformation, showing that the material will approximately be symmetrical and normally distributed around the new reference line (6).

The statistical methods that will be adopted

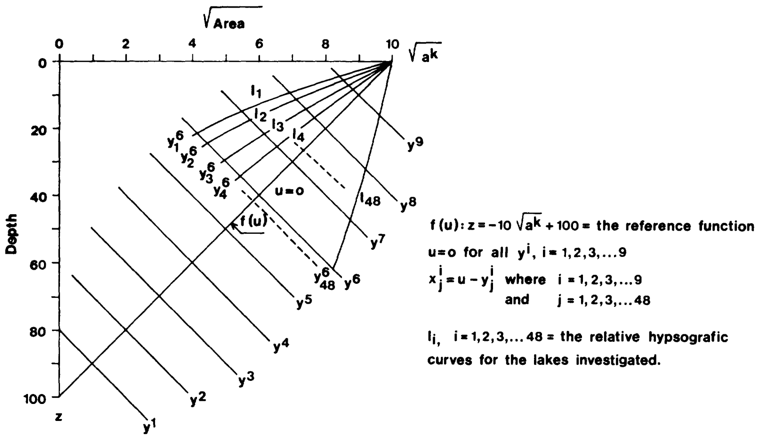

Fig. 13. Schematical illustration to the statistical approach.

are quite basic and may be obtained from any statistical handbook. Here we have used Hyrenius (1957). The normal distribution is given by

$$f(u) = \frac{1}{\sqrt{2\pi}} \times e^{-\frac{u^2}{2}} \tag{7}$$

and

$$F(u) = 0.5000 + \int_0^u f(x)dx \tag{8}$$

Symbols, notations and part of the methods to be used are illustrated in Figures 12 and 13. For the sake of clarity, only 12 of the 48 TRH-curves are illustrated in Fig. 12.

The distribution around reference line (6) has been determined from nine equally spaced positions on the reference line. Position 1 corresponds to the point with coordinates (1,90), position 2 to (2, 80), position 3 to (3, 70), and so on. The right-angle distances, measured on the diagram in mm, from the reference line to the various intersectional points on the different TRH-curves have been determined. These values, 48 for each position, constitute the samples for which the mean and the standard deviation have been determined. Sample 1, corresponding to position 1 and coordinates (1, 90), is called y^1, sample 2, y^2, and so on.

For sample 1, distribution y^1, the following mean ($\bar{x}^1$) and standard deviation (s^1) were obtained: 0.635 and 1.019, respectively (Fig. 14).

The mean and the standard deviation for the other 8 samples are also given in Fig. 14. If the distribution had been exactly normal around the reference line, all means would have been 0, i.e., $u = 0$ (Fig. 14). The difference, however, is quite small, and the reason for using the normal distribution is emphasized if we study all the TRH-curves in one diagram. This, however, becomes difficult, due to a resultant lack of clarity; therefore, the entire material has been "concentrated" into Fig. 15. In the latter figure, which contains the entire sample, i.e., $\sum y_j^i$, where $i = 1, 2, 3, \ldots, 9$ and $j = 1, 2, 3, \ldots, 48$, the reason for using the normal distribution to describe the material is clearly illustrated.

Returning to Fig. 14, we see that this figure also illustrates that the deviation from the mean is largest for the positions at the centre of the reference line and smallest for positions at the ends. The mean values, for all positions, are larger than 0, varying between 0.306 and 0.741, which is a rather small deviation. This small deviation from the reference line is also evident from Fig. 15. The statistical mean lake form for the existing base material, expressed as a TRH-curve, may now be obtained as the connection line between the 9 different $\bar{x}^i$-values. This statistical mean curve is indicated as $f(\bar{x})$. We may now also determine curves corresponding to statistically well-defined deviations from the mean. The connection line between the 9 different points corresponding to $+1$ standard deviation ($1s$), for example, is indicated as $f(1)$, the connection line between $+2s$ will be represented as $f(2)$, and so on.

From formula (8), or directly from common normal distribution tables, we may now express the probability that a TRH-curve for any given lake falls between different limitation curves,

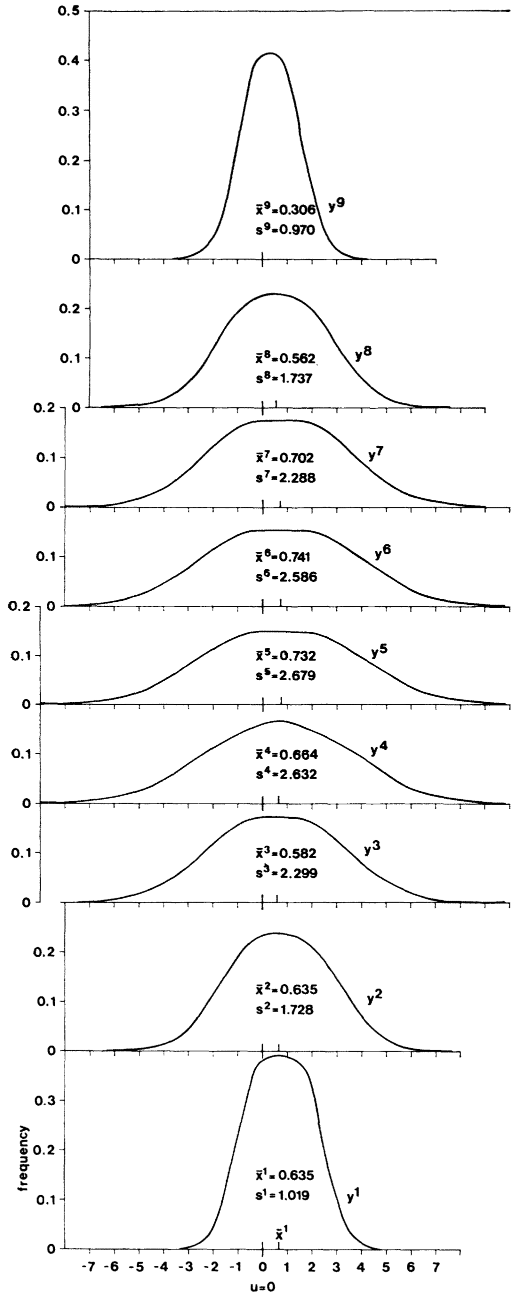

Fig. 14. Mean, standard deviation and normal distribution for the samples from the nine positions on the reference line.

such as $f(-3), f(-2), f(-1.5), f(-1), f(-0.5), f(\bar{x}), f(0.5), f(1), f(1.5), f(2)$ and $f(3)$ (see Table 3). From Table 3 we may see that there

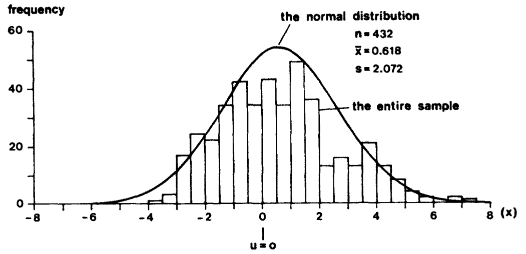

Fig. 15. A comparison between the normal distribution curve and the entire sample from the nine positions of the reference line.

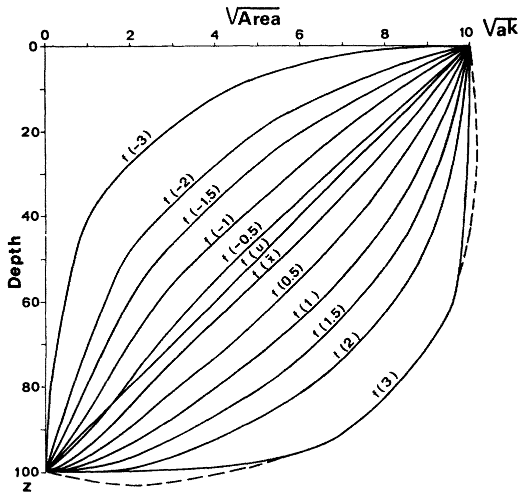

Fig. 16. The TRH-curves for the statistically derived mean lake form, f(x), and the deviation forms.

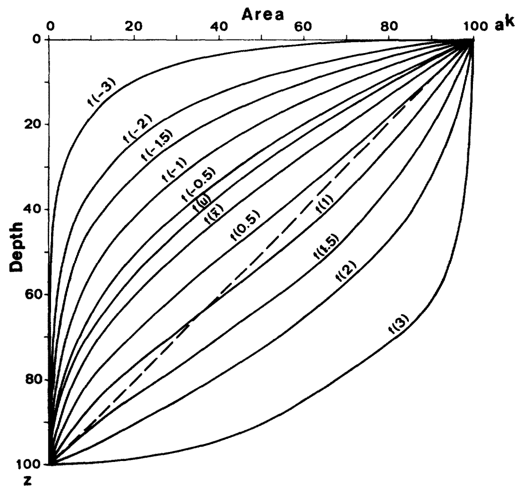

Fig. 17. Relative hypsographic curves for the statistically derived mean lake form, f(x), and the deviation forms.

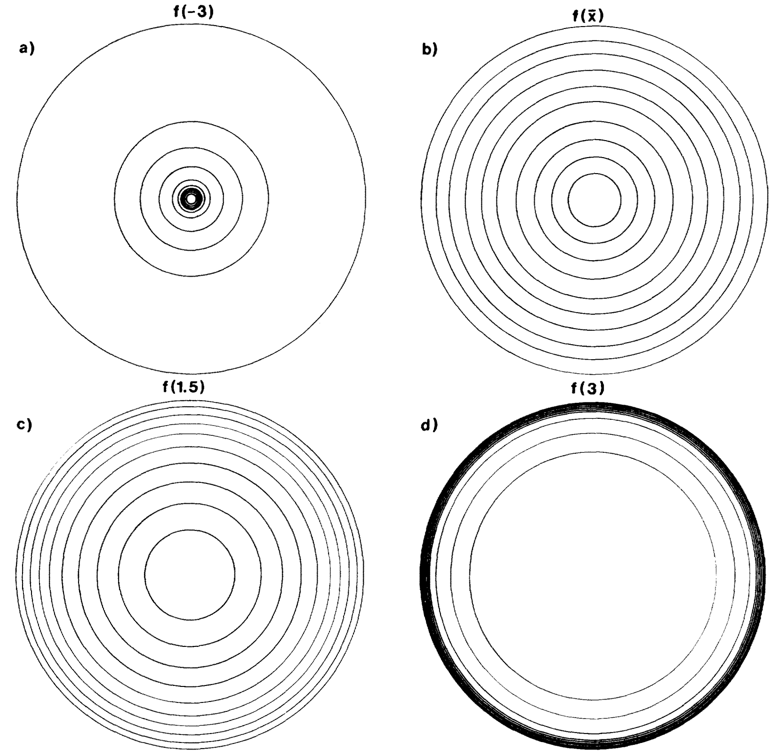

Fig. 18. Schematical bathymetric interpretation of four statistical lake forms.

exists, for example, a 4.4 % probability that a TRH-curve for an unknown lake will fall between the $f(-2)$ and $f(-1.5)$ or between the $f(1.5)$ and $f(2)$ lines. We also see that there exists less than a 0.3 % probability that a lake will have a TRH-curve which lies outside $f(\pm 3)$.

In Fig. 16, all the above-mentioned statistically deduced limitation lines are illustrated as TRH-curves. This figure summarizes the results and shows that the reference line, $f(u)$, has a position approximately halfway between the statistical mean, the $f(\bar{x})$-curve, and the curve which represents $-0.5s$, the $f(-0.5)$-curve. This figure also illustrates that the deviation from the mean is largest at the centre and least at the ends of the reference line.

Table 3. Probability in per cent for arbitrary TRH-curves falling between given limitation lines.

Limitation lines	Probability (%)
$-f(-3)$	0.135
$f(-3)-f(-2)$	2.145
$f(-2)-f(-1.5)$	4.400
$f(-1.5)-f(-1)$	9.190
$f(-1)-f(-0.5)$	14.980
$f(-0.5)-f(\bar{x})$	19.150
$f(\bar{x})-f(0.5)$	19.150
$f(0.5)-f(1)$	14.980
$f(1)-f(1.5)$	9.190
$f(1.5)-f(2)$	4.400
$f(2)-f(3)$	2.145
$f(3)-$	0.135
	100

We may now perform a transformation such that the TRH-curves from Fig. 16 will become relative hypsographic curves (Fig. 17), i.e., we have retransformed $\sqrt{a^k}$ back to a^k.

In Fig. 17 we now have a nomogram of statistically defined relative hypsographic curves which permits us to classify all types of lake forms. This figure shows that the mean form is convex and that the majority of the lakes studies also have convex form.

The present results show that it is possible with statistical methods to determine a mean form for a lake as well as deviations from this mean. One specific question then arise: how do these statistical forms look on a map?

In Fig. 18, four of the relative hypsographic curves, $f(-3)$, $f(\bar{x})$, $f(1.5)$ and $f(3)$, are illustrat-

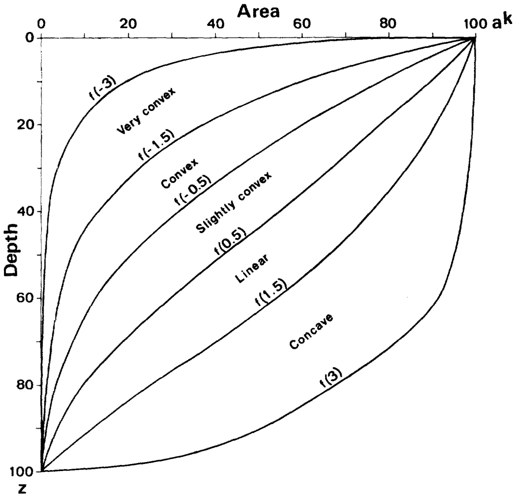

Fig. 19. Terminology and class limits for the classification of lake forms.

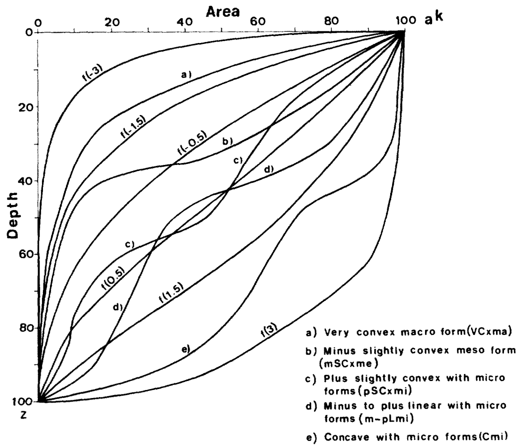

Fig. 20. Five hypothetical relative hypsographic curves illustrating the classfication of lakes according to form.

ed as schematical bathymetric maps. A lake of the $f(-3)$ type will, given a certain area and a certain depth, have the bathymetric form shown in Fig. 18a. This lake type has one (or more) areally limited deep hole (s). It is on the average shallow, in the terminology previously used, very convex. The mean lake, $f(\bar{x})$, Fig. 18b, has an even depth profile. A lake with a relative hypsographic curve corresponding to $f(3)$ is trough-like, with steep inclining walls and a very plane, areally dominating bottom.

Classification of lakes according to form

From the results summarized in Fig. 17 and from Figs. 2–6, it is now possible to classify the lakes investigated according to their form, expressed by the relative hypsographic curves. Such a classification will, like all classifications, be a matter of terminology as well as a matter of how to choose the class limits. The main advantages with a classification are, in this context, that it simplifies descriptions and that it may yield objects for statistical analysis, provided that the chosen class limits are well defined. These statements will be substantiated in the next chapter, which deals with calculations of the lake volume and the impact of the lake form on lake volume calculations.

During the progress of the present work, it has been natural to use certain labels for the different lake forms, for example, concave and convex. We shall now proceed to define and name certain statistically deduced lake forms, namely, the forms corresponding to the following 7 functions: $f(-3), f(-1.5), f(-0.5), f(\bar{x}), f(0.5), f(1.5)$ and $f(3)$.

The functions we have chosen are illustrated in Fig. 19. Relative hypsographic curves for lakes which completely fall between the given lines are called: *Very Convex, Convex, Slightly Convex, Linear* and *Concave*. From eq (8) we may calculate the theoretical probability that a given lake falls within the given classes. The results are given in Table 4.

Since not all lake forms are *pure* (i.e., the relative hypsographic curve lies within one and the same class), the hypsographs can to a greater or lesser extent belong to two or more classes. For these "unpure" cases, we can introduce a terminological rule system based on the following premises:

Table 4. Terminology and class limits for lake forms.

Class limits	Name	Probability
f(−3)−f(−1.5)	Very Convex	6.545 %
f(−1.5)−f(−0.5)	Convex	24.170 %
f(−0.5)−f(0.5)	Slightly Convex	38.300 %
f(0.5)−f(1.5)	Linear	24.170 %
f(1.5)−f(3)	Concave	6.545 %

Table 5. Nomenclature for classification of lake forms.

Macro, ma: no point of inflexion
Meso, me: one point of inflexion
Micro, mi: two or more points of inflexion

1 Pure forms: area cover > 95 % in one class
   Very Convex, VCx
   Convex, Cx
   Slightly Convex, SCx
   Linear, L
   Concave, C
   Examples: VCxma, Cxme, SCxmi, etc

2 Simple forms:

area cover $\leqslant 5$ % to < 40 %	area cover $\geqslant 60$ %
plus, p	
minus, m	
p-m (if p > m)	{VCx, Cx, SCx, L, C}
m-p (if m > p)	

   Examples: pVCxma, mCxme, p-mSCxmi, etc

3 Complex forms:
   a) two main classes with $\geqslant 40$ % area cover
      VCx − Cx (if VCx > Cx)
      Cx − SCx
      SCx − L
      L − C
      Examples: VCx-Cxma, pSCx-Cxme, m-p L-Cmi, etc
   b) three main classes with $\geqslant 25$ % area cover
      VCx-Cx-SCx (if VCx > Cx > SCx)
      Cx-SCx-L
      SCx-L-C
   c) four classes with $\geqslant 20$ % area cover
      VCx-Cx-SCx-L
      Cx-SCx-L-C

Table 6. Forms of the lakes studied.

Lake	Form
Superior	mSCxmi
Huron	Cxma
Michigan	mSCxmi
Erie	pCx-SCxmi
Ontario	SCxmi
Vänern	Cxma
Värmlandssjön	pCxmi
Dalbosjön	Cxma
Vättern	mSCxma
Mälaren	Cxma
Hjälmaren	Cxma
Skagen	SCxma
Roxen	Lma
Unden	pSCxmi
Mälaren (A)	mCxma
Ivösjn	pVCxma
Erken	mSCxmi
Ekoln	mLmi
Mälaren (Dy)	pVCxmi
Lilla Korslången	pCxmi
Laitaure	Cxma
Saxen	pCxme
Gimo damm	mSCxme
Norra Vidöstern	pCxma
Pajep Måskejaure	pCxme
Sparren	mLme
Velen	SCxma
Norrviken	SCx-Lmi
Tullingesjön	mSCxme
Lilla Ullevifjärden	mL-SCxmi
Björken	pCx-SCxme
Fiolen	pSCxmi
Oxundasjön	mCme
Trehörningen	m-pLme
Testen	Lme
Munksjön	SCxma
Edssjön	Lmi
Örnässjön	Cma
Växjösjön	Lme
Järlasjön	pSCxmi
Fansjön	mL-Cmi
Lillsjön	mCmi
Skärshultsjön	Cmi
Drängsjön	p-mLmi
Skärsjön	L-Cmi
Ramsjön	Lme
Botjärn	Cxma
Vitalampa	pCxmi

— if the relative hypsograhic curve to more than 95 % of the area belongs to only one of the classes, it will be called *pure*;
— if between 5 and 40 % of the hypsograph falls into one or more classes other than the main class, the former class(es) is (are) called *secondary*, and the class which contains more than 40 % is called the *main* class.
— curves with secondary classes can be of *minus* and/or *plus* types. The curve is called minus if it belongs to a more convex secondary class and plus if it belongs to a more concave type than the main type;
— if the curve does not have any point of inflexion (i.e., is parabolic), it is denoted as a *macro* type;
— if the curve has *one* point of inflexion, it is called a *meso* type, and if it has two or more points of inflexion, it is called a *micro* type;
— if the curve belongs to two classes with more than 40 % of the area in each, it is called a *complex* curve. It is also called complex if it belongs to three classes with more than 25 % in each or to four classes with more than 20 % in each class. Curves which are not com-

Table 7.

Simple forms	ma	me	mi		Found (%)	Expected (%)
VCx	1	0	1	2	5	6
Cx	9	2	3	14	33	24
SCx	4	2	7	13	31	38
L	1	5	3	9	21	24
C	1	1	2	4	10	6
	16	10	16	42		

Complex forms	ma	me	mi	
VCx-Cx	—	—	—	0
Cx-SCx	—	1	1	2
SCx-L	—	—	2	2
L-C	—	—	2	2
	0	1	5	6

Table 8.

	pure	m	p	m-p	
Simple	18	11	11	2	42
Complex	2	2	2	0	6
	20	13	13	2	48

plex or are pure called *simple*. The rule system with abbreviations is summarized in Table 5.

Fig. 20 illustrates the use of the terminology for 5 hypothetical relative hypsographic curves, *VCxma*, *mSCxme*, *pSCxmi*, *m-pLmi* and *Cmi*.

Table 6 gives the forms for all 48 lakes studied. We shall now discuss the distribution and frequency of these lake forms. A summary of the results is given in Tables 7, 8, and 9.

Of the lakes studied, 42, or 87.5 %, have simple forms, i.e., with the given rule system, they can be defined as belonging to one definite class. Of these 42, 18 are pure. Six of the lakes, or 12.5 %, have complex form; i.e., they have more than one main class. None of the lakes studied belong to the more complex forms with three or four main classes.

Table 7 shows that there are two lakes of the *VCx* type. This is approximately 5 %, which may be compared with the expected 6 % (see Table 5). Most lakes are of the *Cx* and *SCx* types, 14 and 18, respectively. Compared to statistical expectations, the number of the convex lakes is a little high in this material. The difference, however, is not remarkable, 33 % compared to 24 %. In general the agreement is good between found and expected number. From Table 7 we also see that the macro forms dominate among the convex lakes, while the meso and micro forms are more frequent among the *SCx*, *L* and *C* types. This is understandable for morphological reasons. For the convex, and especially for the very convex lakes, there is not enough "space" for fully developed inflexion points to appear. This is clear if we regard the lakes in bathymetric maps. A slight modification of the position of the contour lines, which can cause an inflexion point in the relative hypso-

Table 9.

	Simple pure	m	p	m-p	*Complex* pure	m	p	m-p	
ma	12	2	2	0	0	0	0	0	16
me	3	4	2	1	0	0	1	0	11
mi	3	5	7	1	2	2	1	0	21
	18	11	11	2	2	2	2	0	48

Table 10. The relationship between lake size and lake form for the 48 lakes studied.

	Area (km^2)	n	$\bar{a}$ (km^2)	$\bar{Z}_{max}$ (m)	VCx	Cx	Form SCx	L	C
Continental seas	>10,000	5	49,000	240	—	1 [1]	3	—	—
Large lakes	100–10,000	7	2,100	82	—	5	2	—	—
Lakes	1–100	25	17	24	2	6 [1]	7 [2]	6	1
Small lakes	0.01–1	11	0.5	11	—	2	1	4 [1]	3

graphic curve, does not influence the hypsograph significantly if the contour interval is very small or if the spacing is very large (see Fig. 18), while, on the other hand, a change in the position of the contour lines will have a relatively large impact on the hypsograph if the spacing between the contour lines is constant or nearly constant. This is valid for lakes of the *SCx* and *L* form, that is, for hypsographs which lie close to the reference line (5). Thus, the results are both logical and in good agreement with what can be morphologically understood.

The lakes of the complex type also have hypsographic curves around the reference line (5). A majority of these complex lakes, logically enough, also have micro forms.

From Table 8 we can see that 18 of the lakes of the simple type are also pure. There are 11 lakes of the minus type, as well as 11 lakes of the plus type. This seems to be statistically understandable. Table 9 shows that 16 of the lakes, i.e., 1/3, are of the macro type, 11 lakes have one point of inflexion, and the majority, 21, have two or more points of inflexion. Most of the lakes studied, or 2/3, have inflexion points, which is quite logical, as most of the lakes belong to the *SCx* and *L* types, and it is for these lakes that micro and meso forms dominate.

It should, in this context, be emphasized that the results presented aim only at introducing a way of classifying lakes according to their form. The interpretation of the morphological significance of, for example, the meso and the micro forms cannot be adequately done until we have a base material where the lakes studied have been comparatively surveyed and randomly sampled. The terminology and the class limits can and should be further examined, and all numerical results given here should be regarded with a certain reservation. These results should be considered as a first attempt toward a qualitative and quantitative morphological analysis of the concept "lake form".

Table 11. Comparison between size and form.

Form	n	$\bar{a}$ (km^2)	s_a (km^2)	$\bar{z}_{max}$ (m)	$s_{z_{max}}$ (m)
VCx-Cx-SCx	31	8,400	21,000	74	94
L-C	15	8.4	24	9.9	8.9

Relationship between form and size

The next step is thus to study the relationship between lake size and lake form. It is evident that large glacial lakes generally are deeper than small ones. This statement is, of course, not valid for single objects. In Table 10 we have divided the base material used in this study into the following classes: (1) lakes > 10 000 km^2 (continental seas), (2) lakes of 100–10 000 km^2 (large lakes), (3) lakes of 1–100 km^2 (lakes), and small lakes with an area of 0.01–1 km^2. The number of objects within the four classes (n), the mean area in km^2 ($\bar{a}$), and the mean values of the maximum depths ($\bar{z}_{max}$) have been determined. The table clearly illustrates that the $\bar{z}_{max}$-values are related to the $\bar{a}$-values. The table also gives the number of objects with defined forms. The figures in brackets are numbers of complex forms. As can be seen from the table, no statistically significant trends can be obtained between the size and the form. There is, however, a certain indication from the table that small lakes and lakes seem more often than larger lakes to have a concave, linear and/or slightly convex form.

This indication is supported by the results given in Table 11. In this table, the number of objects (n), the mean area ($\bar{a}$), the standard deviation for the area (s_a), the mean value of the maximum depths ($\bar{z}_{max}$), and the standard deviation for the maximum depths $\left(s_{z_{max}}\right)$ have been determined for lakes belonging to the 3 classes $VCx-SCx$ and to the 2 classes $L-C$.

The figures concerning the standard deviations (in Table 11) show that it would not be meaningful to conduct an analysis of significance for the material. Nor would this be in agreement with the qualitative aim of the present paper. The present results concerning any possible relation between the size and the form have, in consequence, not been proved, but rather indicated.

Lake volume

Working hypothesis

Our working hypothesis can be stated as follows:

— the error (E) in the volume determination depends on the method of determination (M) and the form of the lake (F), i.e.,

$$E = f(M, F) \tag{9}$$

— this error can be established and hence also corrected for.

Determination of the error

The two formulas that will be tested are: the linear approximation

$$V_l = \sum_{j=1}^{n} \frac{e}{2}\left(a_j^k + a_{j-1}^k\right) \tag{2}$$

and the parabolic approximation

$$V_p = \sum_{j=1}^{n} \frac{e}{3}\left(a_j^k + a_{j-1}^k + \sqrt{a_j^k \times a_{j-1}^k}\right) \tag{4}$$

The very simple derivation of formula (2) and the significance of the various abbrevations are illustrated in Fig. 21. The error (E) for calculations according to the V_l-formula is illustrated in this figure as the shaded area. The fact that the error depends on the shape of the relative hypsographic curve is also quite evident from this figure.

For convex lakes, or rather, for lakes with hypsographs above the reference line (5), i.e.,

$$z = -a^k + 100 \tag{5}$$

we have

$$\frac{a_n^k + a_{n-1}^k}{2} > \sqrt{a_n^k \times a_{n-1}^k} \Rightarrow V_l > V_p$$

For concave lakes, the opposite is valid, i.e., $V_l < V_p$. This implies that the V_p method is

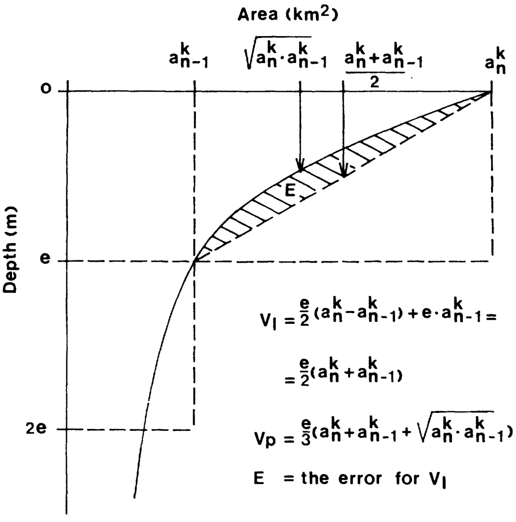

Fig. 21. Schematical illustration of the methods for determining lake volume.

generally better for convex lakes, which are in the majority (approximately 75 %), and that the V_l method is better for lakes with relative hypsograhic curves below the reference line (5).

In order to determine the impact of lake form on the volume determination according to these two methods, we may now use the lake form functions previously defined: $f(-3)$, $f(-2)$, $f(-1.5)$, $f(-0.5)$, $f(\bar{x})$, $f(0.5)$, $f(1.5)$, $f(2)$, and $f(3)$. Furthermore, the effect of the choice of the relative contour-line interval (e^r) has been tested for the following e^r-values: 100, 50, 20, 10, 4 and 2, i.e., for n-values 1, 2, 5, 10, 25 and 50, respectively, where

$$e^r = \frac{z_{max}^r}{n} \tag{10}$$

e^r = the relative contour-line interval
z_{max}^r = the relative maximum depth, 100 %
n = the number of contour-lines

The results of these tests are summarized in Table 12, where the numerical values signify the error in per cent. The true values for the volume have in all cases been determined by graphical integration according to formula (1). As can be seen from Table 12, the error can vary from over 700 % to zero, depending upon the choice of the e^r-value, the lake form and the determination method. Graphical illustrations of the results are also given in Figs. 22 and 23. In agreement with what could be expected, the table and the

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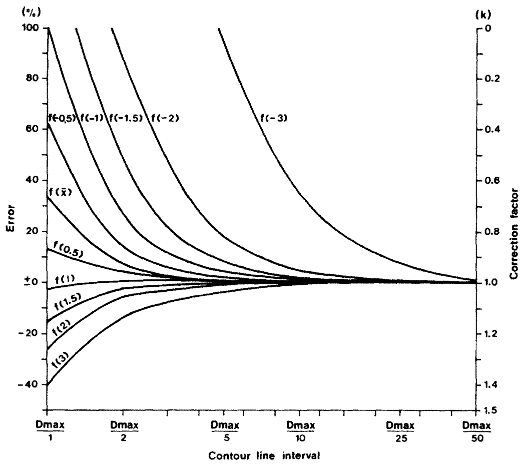

Fig. 22. Nomogram for the error and the correction of V_l. D_{max} = the maximum depth.

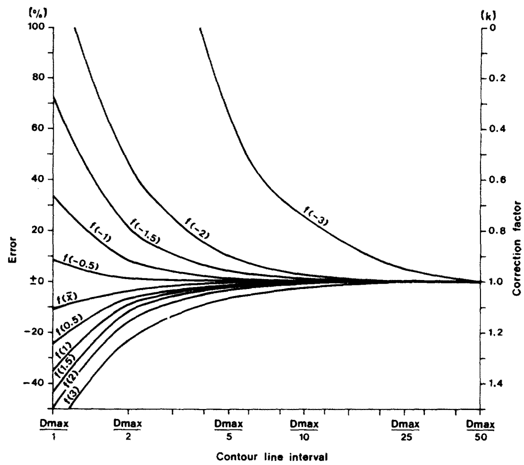

Fig. 23. Nomogram for the error and the correction of V_p.

figures show that the V_p method is better than the V_l method for all lakes with relative hypsographic curves on the convex side of the reference line (5) and worse for all concave hypsographs. The error is naturally smallest when the contour-line interval is smallest, independent of the method of determination. In general, the error is smallest for lakes with hypsographs close to the mean, and increases with increasing deviation from the mean.

The correction factor for lakes with macro form
Since we now can determine the error, we are also able to determine a correction factor which

Table 12. The relative error, its dependence of the lake form, the determination method and the relative contour-lines interval.

e^r	f(−3)		f(−2)		f(−1.5)		f(−1)		f(−0.5)	
	V_l	V_p	V_l	V_p	V_l	V_p	V_l	V_p	V_l	V_p
100	706	438	246	131	159	72.96	101	33.91	62.81	8.53
50	307	189	85.81	46.09	47.64	21.17	26.76	8.76	14.78	1.11
20	93.55	64.19	18.27	10.73	8.56	4.05	4.46	1.53	2.44	0.26
10	34.84	25.32	4.84	2.91	2.18	1.04	1.29	0.56	0.62	0.07
4	7.58	5.65	0.97	0.62	0.26	0.10	0.16	0.04	0.09	0
2	1.13	0.48	0.21	0.14	0.10	0.03	0.08	0	0.03	0

f(x̄)		f(0.5)		f(1)		f(1.5)		f(2)		f(3)	
V_l	V_p	V_l	V_p	V_l	V_p	V_l	V_p	V_l	V_p	V_l	V_p
33.99	−10.68	13.46	−24.37	−2.80	−35.21	−15.63	−43.76	−26.10	−50.74	−40.37	−60.25
8.00	−3.28	4.15	−6.13	+0.89	−8,98	−2.13	−11.90	−5.93	−15.62	−13.54	−22.98
1.35	−0.47	0.34	−1.34	+0.04	−1.69	−0.20	−2.09	−0.98	−3.12	−3.57	−6.31
0.28	−0.17	0.11	−0.32	+0.02	−0.43	−0.02	−0.54	−0.24	−0.87	−1.13	−2.17
0.07	−0.01	0.02	−0.05	0	−0.08	0	−0.10	−0.03	−0.16	−0.23	−0.50
0.01	0	0	0	0	0	0	−0.02	0	−0.03	−0.04	−0.13

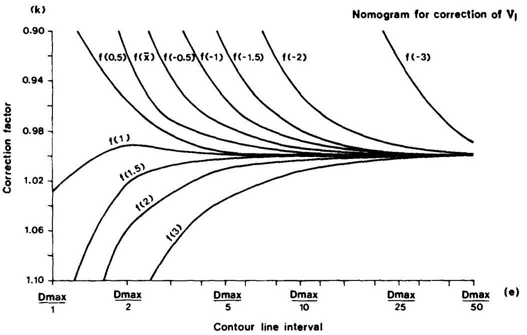

Fig. 24. Enlarged nomogram for the correction factor for V_l.

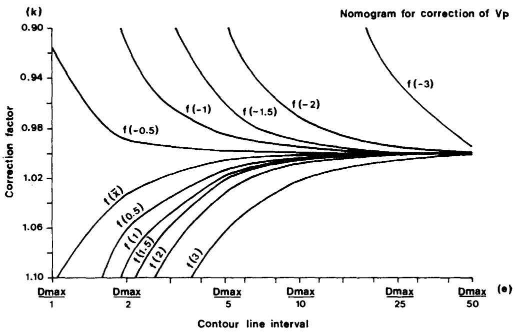

Fig. 25. Enlarged nomogram for the correction factor for V_p.

reduces the error. The correction factor (k) may be defined accordingly:

$$k = \frac{100 - E}{100} \tag{11}$$

where $k = 0$ for $|E| > 100$.

That is, for all errors larger than 100 %, the k-value is 0. In such cases, it may be considered meaningless to correct the error.

Correction factor tables for the two methods can now be established. They are given in Table 13. Correction nomograms for the V_l and V_p methods are illustrated in Figs. 22 and 23, and enlarged nomograms for k-values 0.9−1.10 are shown in Figs. 24 and 25.

The correction tables and correction nomograms will be put to detailed practical use later on. It is quite simple. For example, a lake with a reative hypsographic curve which visually may be associated with the $f(-1.5)$-line and which has been volume determined according to the V_l method with 10 contour-lines should accordingly have a correction factor of 0.979, i.e., the figure for the uncorrected volume should be multiplied by 0.979. If, now, we have 10 contour-lines and all associated points fall between, for example, the limitation lines $f(-2)$ and $f(-1)$, the volume error cannot be larger than what we would have obtained if we had multiplied our uncorrected volume by the k-values from the $f(-2)$ line and the $f(-1)$ line, i.e., 0.951 and 0.987, respectively. If the uncorrected volume is 100 km³, the corrected volume would be 97.9 km³. This figure cannot be lower than 95.1 km³ and higher than 98.7 km³. That is, the error is max. −2.9 % to +0.8 %.

It is of course relatively simple to place the relative hypsographic curve of a lake correctly and with good statistical accuracy if we have many and well-established contour-lines at our disposal. In such cases it is simple to perform a good error calculation and correction directly

Table 13. Correction factor table for the V_l and the V_p methods.

	er	f(−3)	f(−2)	f(−1.5)	f(−1)	f(−0.5)	f(x̄)	f(0.5)	f(1)	f(1.5)	f(2)	f(3)
V_l	Dmax/1	0	0	0	0	0.3719	0.6601	0.8654	1.0208	1.1563	1.2610	1.4037
	Dmax/2	0	0.1419	0.5236	0.7324	0.8522	0.9200	0.9585	0.9911	1.0213	1.0593	1.1354
	Dmax/5	0.0645	0.8173	0.9144	0.9554	0.9756	0.9865	0.9966	0.9966	1.0020	1.0098	1.0357
	Dmax/10	0.6516	0.9516	0.9782	0.9871	0.9938	0.9972	0.9989	0.9998	1.0002	1.0024	1.0113
	Dmax/25	0.9242	0.9903	0.9974	0.9984	0.9991	0.9993	0.9998	1.0000	1.0000	1.0003	1.0023
	Dmax/50	0.9887	0.9979	0.9990	0.9992	0.9997	0.9999	1.0000	1.0000	1.0000	1.0000	1.0004
V_p	Dmax/1	0	0	0.2704	0.6609	0.9147	1.1068	1.2437	1.3521	1.4376	1.5074	1.6025
	Dmax/2	0	0.5391	0.7883	0.9124	0.9889	1.0328	1.0613	1.0898	1.1190	1.1562	1.2298
	Dmax/5	0.3581	0.8927	0.9595	0.9847	0.9974	1.0047	1.0134	1.0169	1.0209	1.0312	1.0631
	Dmax/10	0.7468	0.9709	0.9896	0.9944	0.9993	1.0017	1.0032	1.0043	1.0054	1.0087	1.0217
	Dmax/25	0.9435	0.9938	0.9990	0.9996	1.0000	1.0001	1.0005	1.0008	1.0010	1.0016	1.0050
	Dmax/50	0.9952	0.9986	0.9997	1.0000	1.0000	1.0000	1.0000	1.0000	1.0002	1.0003	1.0013

Table 14. Special correction factor table for lakes classified according to the classification system.

		$\frac{D_{max}}{2}$	$\frac{D_{max}}{3}$	$\frac{D_{max}}{4}$	$\frac{D_{max}}{5}$	$\frac{D_{max}}{6}$	$\frac{D_{max}}{8}$	$\frac{D_{max}}{10}$
V_l	VCx	0.276	0.619	0.776	0.851	0.892	0.938	0.961
	Cx	0.760	0.893	0.936	0.960	0.972	0.982	0.989
	SCx	0.920	0.971	0.981	0.986	0.990	0.994	0.997
	L	0.984	0.992	0.997	0.999	0.999	1.000	1.000
	C	1.046	1.023	1.012	1.007	1.006	1.003	1.002
V_p	VCx	0.627	0.799	0.878	0.916	0.945	0.966	0.977
	Cx	0.930	0.969	0.981	0.988	0.990	0.993	0.996
	SCx	1.033	1.017	1.010	1.005	1.004	1.003	1.002
	L	1.083	1.046	1.028	1.016	1.012	1.007	1.004
	C	1.143	1.073	1.045	1.028	1.019	1.011	1.008

from the nomograms. The primary advantage of the method introduced here may perhaps lie in the fact that we can, even with rather restricted base information, perform an error calculation and obtain a comparatively good statistical accuracy in the volume determination. It is consequently valid for lakes where the hypsographic survey has been limited and where it is not meaningful to use more than 2—10 contour-lines. Another example, from the work conducted by NLU in Uppsala, may be mentioned as a case where only a certain number of contour-lines can be used. A hypsographic survey of Lake Vänern was carried out in 1972. Three boats located at 500 m intervals and with echo-sounding equipment traversed the lake in a predominantly west-east direction. Positions were given by Decca and radar. Echograms were worked up to contour-line maps on a scale of 1:60 000, with a contour-line interval of 5 m. The new maps show that the bottom is very rough and irregular (Håkanson, 1974b). The bottom roughness and the methodological errors (interpolation of depth between lines 500 m apart) imply several problems. The interpolation creates false topographical structures. In the case of Lake Vänern, it would seem that a contour-line interval of 5 m will yield too many of these illogical structures, and if the contour-line interval selected is too large, it will imply that also the real topographical elements of form will be obscured. The optimum lies at about 10 m contour-line intervals (NLU, Uppsala, unpublished material). In all cases where it is not possible to obtain such a large number of contour-lines that the relative hypsographic curve of the lake can be drawn with such an accuracy ($n > 10$) as allows the nomograms of Figs. 24 and 25 to be directly used, it seems logical to adopt the previously-introduced classification system for lake forms. This will provide us with a method of determining whether the hypsograph of the lake lies between some of the limits given, i.e., if some of the classes *VCx*, *Cx*, *SCx*, *L* or *C* can be utilized. The points, corresponding to the existing contour-lines, may then be plotted onto Fig. 19 and the class or the class limits may be determined. One may then proceed to the special correction factor table (Table 14) and obtain the appropriate correction factor. This table is determined by assuming that the material is normally distributed(from eq (8); see Table 15).

According to our theory, we find, for example, that 50 % of all relative hypsographic curves within the *VCx* class fall on the convex side of the mean class function $f(-1.824)$. The correction factor table (Table 14) is to be used in the same manner as the other correction factor table (Table 13). In a later chapter, we shall give examples of this.

It should be noted that all figures hitherto

Table 15.

Class	Class limits	Mean class function
VCx	$f(-3)-f(-1.5)$	$f(-1.824)$
Cx	$f(-1.5)-f(-0.5)$	$f(-0.886)$
SCx	$f(-0.5)-f(0.5)$	$f(\bar{x})$
L	$f(0.5)-f(1.5)$	$f(0.886)$
C	$f(1.5)-f(3)$	$f(1.824)$

given concerning the error (E) and the correction factor (k) have been attributed to lakes with macro form, i.e., for lakes with no point of inflexion in the relative hypsographic curve. Since these lakes with macro form, according to Tables 7 and 9, only constitute 33 % of the lakes studied, it is of vital importance to establish the impact of meso and micro forms on the applicability of the method.

Volume correction for lakes with meso and micro forms

Our working hypothesis is that meso and/or micro forms do not have any particular significance on the error calculation and the correction methodology. It would seem logical, and in agreement with the results previously discussed, to assume that a "convex" error would be balanced by a "concave" error, around a point of inflexion, so that the final results should be in good agreement with the results given by the curves with macro forms.

The working hypothesis has been tested in the following manner:

— five hypothetical but plausible relative hypsographic curves have been drawn within two fixed class limits;

— all possible class limits have been tested, i.e., those of the "first order": VCx, Cx, SCx, L and C,
those of the "second order": $VCx-Cx$, $Cx-SCx$, $SCx-L$ and $L-C$,
those of the "third order": $VCx-SCx$, $Cx-L$ and $SCx-C$,
those of the "forth order": $VCx-L$ and $Cx-C$,
and that of the "fifth order": $VCx-C$;

— both volume determination methods, V_1 and V_p, have been tested.

In Fig. 26 two of these possible fifteen cases are illustrated, namely the test set 1.1., 2.1., 3.1., 4.1., and 5.1., which is of the "second order" with the class limits $SCx-L$, and the test set 1.3., 2.3., 3.3., 4.3., and 5.3., which is of the "first order", C, type. The results from these two test sets for the V_1 method are summarized in Tables 16 and 17.

The mean of the five k-values obtained in each test set is indicated by $\bar{k}$. The relative contour-line interval is e^r. The expected k-values given in Tables 16 and 17 are derived from Table 13. Tables 16 and 17 clearly indicate that the k-values obtained are well within the expected tolerance limits. It may also be pointed out that only four out of a total number of 75 determified k-values from all the test sets were on the wrong side of the tolerance limits and in all four cases the "error" lay in the fourth decimal. The working hypothesis can thus be considered as valid.

The error and correction methods introduced for lakes with macro forms may consequently

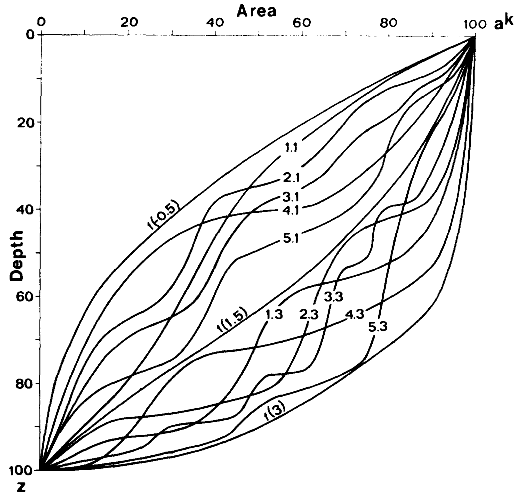

Fig. 26. Hypothetical relative hypsographic curves from two series of tests to study the impact of meso and micro forms on the determination of lake volume.

Table 16. Test set SCx-L.

e^r	$\bar{k}$	expected k-value
100	0.8419	0.3719 – 1.1563
50	0.9732	0.8522 – 1.0213
20	0.9985	0.9756 – 1.0020
10	0.9965	0.9938 – 1.0002
4	0.9997	0.9991 – 1.0000

Table 17. Test set C.

e^r	$\bar{k}$	expected k-value
100	1.2973	1.1563 – 1.4037
50	1.0898	1.0213 – 1.1354
20	1.0188	1.0020 – 1.0357
10	1.0054	1.0002 – 1.0113
4	1.0002	1.0000 – 1.0023

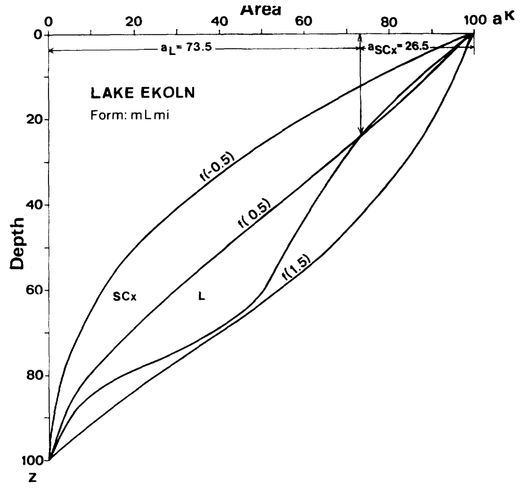

Fig. 27. The relative hypsographic curve for Lake Ekoln illustrated to show one type of correction method for the volume determination.

be considered as also being valid for lakes with meso and/or micro forms. The correction factor for lakes with *pure* relative hypsographic curves can now be determined directly from Table 13, independent of whether the curves are of macro, meso or micro form. If the hypsograph is not pure, i.e., if it is of the *simple* or the *complex* type, this correction factor table cannot be used directly. We shall now show how the correction factor can be determined for these types of lakes, as well.

Determination of the correction factor for simple and complex hypsographs

The volume of Lake Ekoln was first determined by Axelsson and Håkanson (1972), using the graphic integral method, formula (1). After repeated checks on their base material, it has been shown that the correct value, according to formula (1), of Lake Ekoln's volume should be 361.1×10^6 m^3 and not 356×10^6 m^3, as given by Axelsson and Håkanson. The reason for this discrepancy, which in fact is very small and of minor interest in all contexts other than the present one, was that one point from the contour-line intervals was erroneously placed on the graphic diagram from which the integral was determined. In this particular case, it is of importance to have a correct figure for the volume, and we may now execute an error determination for Lake Ekoln, also showing how the correction factor can be derived for lakes with simple and complex relative hypsographic curves.

The correction factor

For pure relative hypsographic curves, that is, for curves where more than 95 % of the area falls within one of the actual classes *VCx*, *Cx*, *SCx*, *L* or *C*, we can determine the correction factor (k) from an empirical relationship between the contour-line interval (e) and the lake form ($f(u)$), where

$$k = g(e, f(u)) \tag{12}$$

For simple hypsographs of the minus and/or plus type, i.e., when the hypsograph to more than 5 % but less than 40 % falls into some class other than the main class, we can normalize the correction factor according to the following general formula:

$$k_n = a_1 k_1 + a_m k_m + a_p k_p \tag{13}$$

where

k_n = the normalized correction factor
a_1 = the area within the main class
k_1 = the correction factor of the main class
a_m = the area within the "minus" (convex) class
k_m = the correction factor of the "minus" class
a_p = the area within the "plus" (concave) class
k_p = the correction factor of the "plus" class

For Lake Ekoln, for example, which is of the minus linear micro form (*mLmi*), we do not have any "plus" class—merely the "minus" class. The main class is linear. The "minus" class is *SCx*. Formula (13) can, for this lake, thus be written as:

$$k_n = a_L k_L + a_{SCx} k_{SCx} \tag{14}$$

In the case of Lake Ekoln (see Fig. 27), we have $a_L = 73.5$ %, i.e., 73.5 % of the area falls within the main *L*-class. The percentage area of the "minus" class consequently is 26.5. The correction factor for the *L*-class is, according to the V_p method, when $n = 19$, 1.0020 (see Fig. 25). The correction factor for the *SCx*-class is, in this example, 1.0007. The normalized correc-

Table 18. Base data on depth and area for Lake Ekoln.

Depth (m)	Cum depth (%)	Cum area (km²)	Cum area (%)
0– 2	0	18.60	100
2– 4	5.3	17.24	92.7
4– 6	10.5	16.13	86.7
6– 8	15.8	15.08	81.1
8–10	21.1	14.12	75.9
10–12	26.3	13.32	71.6
12–14	31.6	12.57	67.6
14–16	36.8	11.92	64.1
16–18	42.1	11.27	60.6
18–20	47.4	10.69	57.5
20–22	52.6	10.16	54.6
22–24	57.9	9.58	51.6
24–26	63.2	8.83	47.5
26–28	68.4	7.50	40.3
28–30	73.7	5.67	30.5
30–32	78.9	3.62	19.5
32–34	84.2	1.95	10.5
34–36	89.5	1.08	5.8
36–38	94.7	0.54	2.9
38	100	0	0

tion factor (k_n) can now be determined accordingly:

$$k_n = \frac{73.5}{100} \times 1.0020 + \frac{26.5}{100} \times 1.0007 = 1.0017$$

The V_p-volume for Lake Ekoln can be determined, from the values given in Table 18, as 360.6×10^6 m³.

The error and correction determination procedure for Lake Ekoln can now be summarized.

Form: *mLmi*
n: 19
Graphic integral volume: 361.1×10^6 m³
Uncorrected volume, according to V_p method: 360.6×10^6 m³
Maximum error for the V_p method: -0.16 % to -0.28 %
Correction factor: 1.0017
Corrected V_p-volume: 361.2×10^6 m³
Maximum error: -0.01 % to $+0.11$ %

For lakes with complex hypsographs, i.e., for hypsographs where more than 40 % of the area falls within two classes, or 25 % of the area falls within three classes, we may rewrite formula (13) accordingly:

$$k_n = a_1k_1 + a_2k_2 + a_3k_3 + a_mk_m + a_pk_p \quad (15)$$

where

a_1 = the area within the first main class
k_1 = the correction factor of the first main class
a_2 = the area within the second main class and so on.

To summarize, we may say that the error calculation for Lake Ekoln has shown that the V_p method can give a result that corresponds to more than 99.9 % of the exact value, as given by the very time-consuming graphical integration method. Furthermore, we may conclude that the V_p method, as well as the V_l method (which really would have been preferable for Lake Ekoln), provide us with the possibility of performing an error calculation which is not possible for the integration method, as the latter method is based upon a subjective and statistically incontrollable element, namely the very drawing of the hypsographic curve from a limited number of fixed points.

Further examples of the applicability of the results

Lake Mälaren

The determination of the volume of Lake Mälaren was first conducted by Lemming et al. (1971). Two alternative methods concerning error calculation and correction will be given.

(a) Uncorrected volume (according to Lemming et al., 1971): 14.306×10^6 m³
Determination method: V_l
Number of contour-lines (n): 6
Form: *Cxma*
Maximum error: $+6.1$ % to $+2.0$ %
Correction factor: 0.972
Corrected V_l-volume: 14.000×10^6 m³
Maximum error: $+1.0$ % to -3.1 %

(b) Assume that the relative hypsographic curve for Lake Mälaren on the average can be expressed by the line $f(-1.4)$; see Fig. 28.
Correction factor: 0.950
Corrected volume: 13.600×10^6 m³
Maximum error (within the limitation lines $f(-1.5)$ and $f(-1.0)$: ± 1.1 %

The first method is the conventional one when we have six defined contour-lines. Since the form is convex, it would have been more appropriate to use the V_p formula. The corrected volume for Lake Mälaren is, with the V_l formula

Table 19

Form: *Cxma*

(1) $e = 10 \rightarrow n = 10$

Uncorrected volume: $\begin{cases} V_1 = 154.48 \text{ km}^3 \\ V_p = 153.45 \text{ km}^3 \end{cases} \Rightarrow \begin{cases} E_1 = \text{max } +2.18\ \% \text{ to } +0.62\ \% \\ E_p = \text{max } +1.04\ \% \text{ to } +0.07\ \% \end{cases}$

Correction factor: $\begin{cases} k_1 = 0.989 \\ k_p = 0.996 \end{cases}$

Corrected volume: $\begin{cases} V_1 = 152.78 \\ V_p = 152.83 \end{cases} \Rightarrow \begin{cases} E_1 = \text{max } +0.51\ \% \text{ to } -0.92\ \% \\ E_p = \text{max } +0.30\ \% \text{ to } -0.60\ \% \end{cases}$

(2) $e = 20 \rightarrow n = 5$

Uncorrected volume: $\begin{cases} V_1 = 159.59 \\ V_p = 155.51 \end{cases} \Rightarrow \begin{cases} E_1 = \text{max } +8.56\ \% \text{ to } +2.44\ \% \\ E_p = \text{max } +4.05\ \% \text{ to } +0.26\ \% \end{cases}$

Correction factor: $\begin{cases} k_1 = 0.960 \\ k_p = 0.988 \end{cases}$

Corrected volume: $\begin{cases} V_1 = 153.21 \\ V_p = 153.64 \end{cases} \Rightarrow \begin{cases} E_1 = \text{max } -4.79\ \% \text{ to } +1.67\ \% \\ E_p = \text{max } -2.95\ \% \text{ to } +1.40\ \% \end{cases}$

(3) $e = 50 \rightarrow n = 2$

Uncorrected volume: $\begin{cases} V_1 = 192.50 \\ V_p = 168.78 \end{cases} \Rightarrow \begin{cases} E_1 = \text{max } -47.64\ \% \text{ to } +14.78\ \% \\ E_p = \text{max } +21.17\ \% \text{ to } +1.11\ \% \end{cases}$

Correction factor: $\begin{cases} k_1 = 0.760 \\ k_p = 0.930 \end{cases}$

Corrected volume: $\begin{cases} V_1 = 146.30 \\ V_p = 156.97 \end{cases} \Rightarrow \begin{cases} E_1 = \text{max } -31.05\ \% \text{ to } +36.96\ \% \\ E_p = \text{max } -15.27\ \% \text{ to } +6.34\ \% \end{cases}$

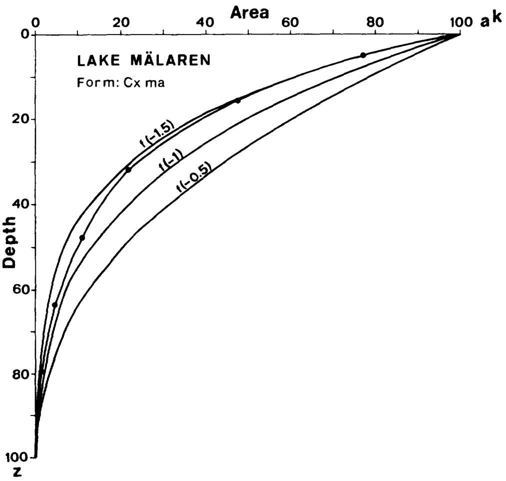

Fig. 28. The relative hypsographic curve for Lake Mälaren illustrated to show one type of correction method for the volume determination.

Table 20. Base data on depth and area for Lake Vänern (from NLU, Uppsala, unpublished material).

Depth (m)	Area (km^2)	Cum. area (km^2)
0–10	1652.838	5648.008
10–20	1037.722	3995.170
20–30	723.785	2957.448
30–40	629.809	2233.663
40–50	578.024	1603.854
50–60	480.971	1025.830
60–70	335.916	544.859
70–80	160.006	208.943
80–90	43.576	48.937
90–100	4.987	5.361
100–106	0.374	0.374
106	0	0

which was originally used, 14.000×10^6 m^3 and the maximum error is -3.1 %. This should be compared with the results first given by Lemming et al. (1971): 14.306×10^6 m^3 and no determination of the error.

The second method, which is a direct comparison between the points available on the hypsograph with known and statistically established reference lines, should in principle only be used if 10 or more points on the hypsograph are available. The example given here is consequently meant only to serve as an illustration. In general, as well as in this particular case, this method is better than the first one, which is more schematic. The corrected volume is here given as 13.6000×10^6 m^3. The maximum error as determined by the given limitation lines $f(-1.5)$ and $f(-1.0)$ is ± 1.1 %.

Lake Vänern

We shall now give examples of error and correction calculations where the impact of different contour-line intervals (e) on the V_p and the V_l formulas are studied. Base data on depth and area for Lake Vänern are given in Table 20.

These results show, in agreement with the previous discussion, for lakes with relative hypsographic curves on the convex side of the reference line (5), like Lake Vänern:

— that the V_p formula gives better results than the linear approximation for all contour-line intervals;

— that the maximum error depends on the choice of the e-value (decreasing with decreasing e-value);

— that even a limited level of ambition concerning the survey, as given by the number of contour-lines, can reveal good reliability in the volume determination after correction.

In the case of Lake Vänern, we have, with 5 contour lines with the V_p formula, an error which is at the maximum less than 3 % for the volume determination. With 10 contour-lines, the maximum error has decreased to less than 1 %, independent of the formula used.

Comments

The methods presented for the error calculations in lake volume determinations are flexible and provide possibilities for alternatives that can be obtained in relation to the presuppositions given by the hypsographic survey. The parabolic approximation, the V_p formula, is best for convex lakes and the linear approximation, the V_l formula, for concave hypsographs. The results show that even limited hypsographic surveys can reveal good volume determinations and above all that the results can be statistically examined and corrected. It has already been repeatedly emphasized that the figures given cannot be considered as definite. The entire approach is qualitative. Quantitative results can be obtained only if we examine a material which is randomly chosen from lakes which have been surveyed in a comparative manner. In the following chapter we shall discuss a first outline of such a new, objective hypsographic methodology.

A pilot outline for a method of optimization of hypsographic surveys for lakes

The purpose of the following discussion is to establish the first outline of a methodology by which all lakes, regardless of their size, configuration, and genesis, can be surveyed in an objective and optimal manner, so that the volume and the bathymetric map can be obtained with a statistical reliability which can be a priori discussed. This is indeed an ambitious approach, and the following discussion is only meant to "open the door", and to give a first brief theoretical outline.

A natural basic assumption for such a model, which is intended to be a general one, is that the starting point, the base, is definable. In the model, we cannot assume that there exists any information whatsoever about the depth conditions of the lake. Instead, the starting point is that we have information about the area, the two-dimensional form, and the shoreline configuration of the lake. We shall also start from the very reasonable assumption that the lake is to be surveyed with echo-sounding equipment. The purpose of the survey is to determine the volume and the bathymetric map with a level of ambition, a statistical reliability, that can a priori be discussed. For example, we may wish to determine the volume with an error that is less than ± 2.5 % and with an areal distribution of the contour lines of the bathymetric map which is at least 95 % correct. The model is, in principle, based on the results previously discussed concerning the lake form and the error calculation, as well as on an earlier publication (Håkanson, 1974b) concerning the relation between the lake

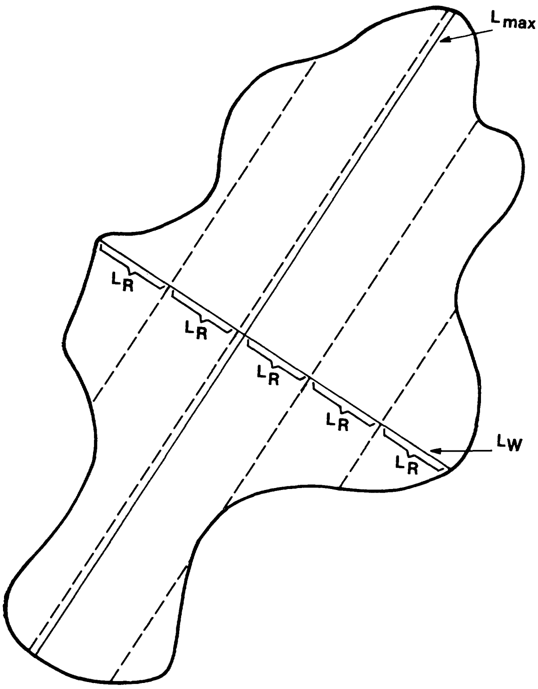

Fig. 29. Schematical illustration of the proposed method for lake hypsographic surveys. The dotted lines indicate routes for echo sounding.

bottom roughness (R) and the shore development (F), where R and F are given by

$$R = \frac{0.165\,(e+2)\sum_{i=1}^{n} S(z_i)}{D_{\text{median}}\sqrt{a}} \qquad (16)$$

and

$$F = \frac{s}{2\sqrt{a\pi}} \qquad (17)$$

where R = the topographical lake bottom roughness, dimensionless
e = the contour-line interval, in metres
$S(z_i)$ = the length of the contour-line at the depth z_i, in metres
D_{median} = the median depth, in metres
a = the area in km²
F = the shore development, dimensionless
s = the length of the shoreline in km.

The working hypothesis is that the intensity of the survey (L) depends upon and is a function of the level of ambition of the survey concerning the volume determination (A_V), the bathymetric map reliability (A_A) and of the topographical lake bottom roughness (R), in such a manner that a high level of ambition and a high value of the bottom roughness will imply a high intensity of the survey, and vice versa, i.e.,

$$L = f(A_V, A_A, R) \qquad (18)$$

That a high level of ambition automatically will lead to a high intensity of the survey is evident. It should also be obvious that a high R-value will also lead to a high intensity of the survey, since for a lake with smooth and undramatic bottom conditions, one can with a high degree of probability foresee how the bottom configuration will look between echo-sounded tracks; the opposite is valid for lakes with rough bottom conditions (see Håkanson, 1974b).

The level of ambition concerning the volume (A_V) and the bathymetric map (A_A) can be discussed independently of the appearance of the lake in question. The topographical roughness, on the other hand, cannot be known in advance. Indirectly, however, the R-value may be determined from the following equation:

$$F = C_1R + C_2 \qquad (19)$$

The constants C_1 and C_2 have previously been determined for twelve of the lakes studied in the present work (Håkanson, 1974b). That these lakes represent a proper sample, at least from a qualitative viewpoint, is clear from Fig. 12, which gives the TRH-curves for the 12 lakes in question. The constants have been determined as $C_1 = 0.28$ and $C_2 = 1.53$, i.e.,

$$F = 0.28R + 1.53 \qquad (20)$$

or

$$R = 3.57F - 5.46 \qquad (21)$$

Admittedly, eq. (20) was determined from a very limited number of lakes, but the correlation coefficient was high (0.98), and from a geomorphological viewpoint, the relationship between the F- and the R-values is quite logical. Lakes with irregular shore lines should also have irregular bottom conditions.

Thus, we may now determine a priori all the factors that influence the intensity of the survey. Another question then arises: how should a given lake be surveyed?

In general geomorphology there exist a number of more or less sophisticated techniques based on both stochastical and deterministical approaches, from random Monte Carlo samplings to aerial photography (see, for example, Evans, 1972, or

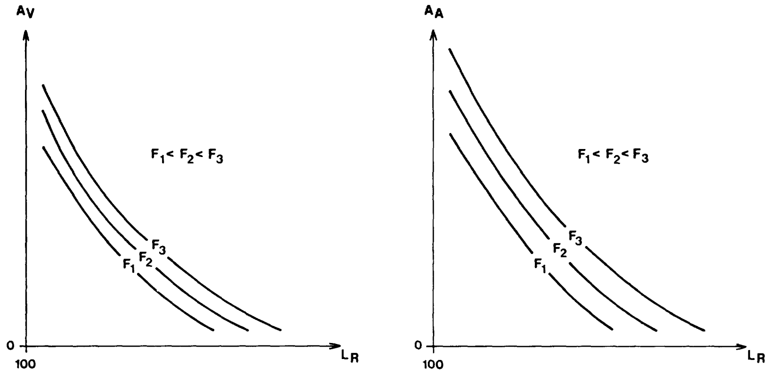

Fig. 30. Schematical curves illustrating the relationship between the level of ambition for the volume determination (A_V) and for the bathymetric map representation (A_A) to the relative distance between the echo-sounded lines (L_R) for lakes with different shore development (F).

Davies, 1973) to determine the land surface configuration. In lake morphology, however, the methods available are less developed. In this context, we shall base our approach on ordinary echo-sounding techniques. In principle, this provides us with two possible approaches. One is to orient the echo-sounding lines on certain definable fixed points in the terrain, for example, capes, sea marks and so on. The other alternative is to conduct the survey along some predetermined lines, for example, in a square net system, or using parallells or parabolic Decca lines.

The first method has the obvious drawback that it cannot be used in general, since the fixed points will never be identical for two lakes. Consequently, the second approach should be adopted in a model that claims to be general. The type of net system which is most suitable for these purposes can, of course, be discussed. It is probable, however, that the final results will be approximately the same, independent of the method adopted, provided that the lines can be significantly defined. Here, we shall choose a simple and very easily defined method, namely, that with lines parallell to the main direction of the axis of *maximum effective length* (see Fig. 29).

The maximum effective length can be defined as "length of straight line connecting most remote extremities of lake along which wind and wave action occur without any kind of land interruption" (Welch, 1948).

The *maximum width* is defined as the length of straight line connecting most remote transverse extremities at right angles to maximum effective length axis without crossing land. In this approach, the echo-sounding is conducted along lines parallell to the axis of maximum effective length ($L_{\max}$) and consequently also at right angles to the axis of maximum width (L_W).

The *relative distance* between the echo-sounding lines (L_R) is defined accordingly:

$$L_R = \frac{L^r_W}{n} \tag{22}$$

where

L^r_W = the relative maximum width, 100 %
n = the number of lines to be echo-sounded

L_R and n are here measurements of the intensity of the survey, corresponding to L in formula (17), which now can be rewritten as

$$L_R = f(A_V, A_A, F) \tag{23}$$

That is, the relative distance between the echo-sounded lines or the number of lines to be echo-sounded depends on the level of ambition and the shore development. All these independent parameters may be discussed and determined a priori. At this point, the problem becomes empirically to determine eq. (23) for different levels of ambition and for lakes with different F-values. This is a very comprehensive task, indeed, and one which only with difficulty can be done without the aid of computer techniques. In principle, it can be carried out such that various lakes with different sizes and F-values are echo-sounded with different intensities, according to the model given in Fig. 29. The results should be given as empirically derived curves like those shown in Fig. 30. They may be interpreted as follows: for a certain level of ambition, for example, a volume determination with a maximum error of less than 5 %, we must have a survey intensity (given by the L_R-value) which is comparatively high if the F-value is high. For several reasons, the establishment of the empirical relationships schematically illustrated in Fig. 30 is of great importance. Firstly, this will imply an optimization of both economic and scientific resources, since it provides us with a method for discussing a priori the level of ambition—i.e., what we want to obtain from the survey; the yield of the survey can be determined and discussed in statistical terms before the survey is started. We can carry out exactly what we want to, neither more nor less. Secondly, this will imply that we can obtain a material which permits comparisons between all types of lakes—and this is, as we have previously discussed, a condition that must be obtained before any real quantitative approach may be taken. This is furthermore a presupposition that must be fulfilled before any adequate genetical discussion concerning the lake forms can be commenced.

Summary

This work is based on the study of the morphometry of 48 lakes, as represented by relative hypsographic curves. The five largest of the lakes studied belong to the Great Lakes system, the rest are Swedish. Thus, all the lakes have a fairly similar climatological position and were all influenced by the last great glaciation. The lakes have not been selected randomly, but according to other criteria. The present work is qualitative rather quantitative, its purpose being to introduce new ideas and methods. The main aims have been:

— to discuss a statistical approach to the concept of lake form and to introduce a statistical mean form, as well as statistical deviation forms;
— to introduce a classification system for lakes based on their form;
— to discuss some methods for lake volume determinations and to introduce a methodology for error and correction calculations;
— finally, to introduce a first outline to a new approach to lake hypsographic surveys.

We may now summarize the resuls accordingly: (1) In order to present the base material, i.e., the 48 relative hypsographic curves, in a simple, attractive and statistically workable form, the curves have been transformed into what have been defined as TRH-curves (transformed relative hypsographic curves). The transformation consists quite simply of taking the square root of the relative cumulative area, i.e., $a^k \rightarrow \sqrt{a^k}$. The base material, represented by TRH-curves, has proved to be approximately normally distributed around the straight line given by

$$z = -10\sqrt{a^k} + 100$$

where

z = the relative depth
a^k = the relative cumulative area in percent.

The mean lake form has proved to correspond quite well to this line. Approximately 50 % of the TRH-curves for the lakes studied fall above this line and approximately 50 % below. Since the base material has proved to be roughly normally distributed around the mean, it has also been possible to determine different statistical deviation forms from the mean form. A number of statistically determined curves, the mean curve $f(\bar{x})$ and the deviation curves $f(s)$ (where the standard deviation $s = \pm 0.5$, ± 1.0, ± 1.5, ± 2.0 and ± 3.0) have then been retransformed to relative hypsographic curves, yielding a nomogram by means of which it is possible, with a certain definable statistical certainty, to place any given lake hypsographic curve. For example, there is less than a 0.3 % probability that an arbitrary hypsographic curve will fall outside the limits given by the lines $f(-3)$ and $f(3)$, and there exists a 68.26 % chance that a relative hypsographic curve will fall between the lines $f(-1)$ and $f(1)$, which represents ± 1 standard deviation. If we use the straight line

$$z = -a^k + 100$$

as a reference line for the relative hypsographic curves, we find that an overwhelming majority (approx 75 %) of the hypsographs for the lakes studied are convex and fall above this reference line.

(2) These results may now be used to classify lakes according to their form. The following terminology and class limits have been used: Very Convex (*VCx*)—lakes with relative hypsographic curves between the lines $f(-3)$ and $f(-1.5)$, Convex (*Cx*)—hypsographs between $f(-1.5)$ and $f(-0.5)$, Slightly Convex (*SCx*)—$f(-0.5)$ to $f(0.5)$, Linear (*L*)—$f(0.5)$ to $f(1.5)$ and Concave (*C*)—$f(1.5)$ to $f(3)$. Hypsographs with no point of inflexion are denoted as Macro (*ma*), curves with one point of inflexion are labelled Meso (*me*) and lakes with relative hypsographic curves with two or more "irregularities" have

been denoted as Micro (*mi*). These are the main definitions. Many lakes, however, have hypsographs which cannot be described by these simple rules. A special rule system has therefore been introduced, in order to obtain a general classification system by which all possible forms may be objectively described. The classification is descriptive rather than genetical. An analysis of the existing material indicates a probable relation between form and size, such that large lakes to a higher extent than small lakes seem to be convex rather than concave, and vice versa.

(3) A working hypothesis on a relation between volume determination method, lake form and determination error has been tested and verified. The following two determination methods have been used:

(a) The linear approximation

$$V_l = \sum_{j=1}^{n} \frac{3}{2}\left(a_j^k + a_{j-1}^k\right)$$

(b) The parabolic approximation

$$V_p = \sum_{j=1}^{n} \frac{e}{3}\left(a_j^k + a_{j-1}^k + \sqrt{a_j^k \times a_{j-1}^k}\right)$$

where

e = the contour-line interval, where $e \times n \approx z_{\max}$ (n = the number of contour-lines, $z_{\max}$ = the maximum depth)

a_j^k = the cumulative area at contour-line j

The results of the tests show that the parabolic approximation yields the least error for the volume for all convex lake forms, which are in the majority. The V_l formula is best for concave lakes. The error is relatively independent of whether the curve is of macro, meso, or micro type, i.e.. the small forms are less important in the volume determination than the main forms (*VCx*, *Cx*, *SCx*, *L* and *C*). This implies that a general correction system can be utilized. A correction factor (k) has been defined as

$$k = \frac{100-E}{100}$$

where

E = the error in per cent
$k = 0$ for $|E| > 100$

Tables and nomograms giving the k-value for the two volume formulas have been calculated. A formula for determination of the correction factor for lakes with complex hypsographs has been introduced and different examples of the applicability of the error and correction methods have been given.

To the best of the author's knowledge, the present work is the first attempt at obtaining an error calculation of lake volume determinations. It is remarkable that no previous work has been done in this field, as determinations of lake volume are essential for all budget calculations, as well as for all dynamic lake models. The results also show that even comparatively limited hypsographic surveys may yield a good accuracy of volume determinations.

(4) It has repeatedly been emphasized that the present work is qualitative rather than quantitative. In order to obtain quantitative results which permit comparisons to be made between lakes of different size and form, it is necessary, not only to have an adequate theoretical model, but also to have base data which has been collected objectively and in a morphologically and statistically definable way.

In a model for a lake hypsographic survey which aims at generality, a natural starting point is that one does not know anything about the morphometry of the lake to be studied. One should start "from scratch" and only use knowledge of the area and the shoreline configuration.

A number of arguments have been presented in favour of a survey with echo-sounding equipment by which a predetermined number of lines parallell to the axis of the maximum effective length are traversed. The density of the lines will, according to our theory, depend on the level of ambition of the survey, as well as on the degree of irregularity of the lake bottom. The level of ambition can be discussed a priori: for example, one may want to establish the volume with a 90 % certainty. The bottom roughness (R), which cannot be known in advance, may be determined indirectly from the following empirical relationship between the R-value and the shore development (F), which is a measure of the irregularity of the shoreline and which can be determined directly from a map:

$$R = 3.57F - 5.46$$

In order to establish the empirical relationship suggested between the level of ambition, the shore development and the density of the echo-sounded lines, it is necessary to carry out an ambitious program. Such work has just recently been started. The final results will hopefully be a model from which we shall be able to know how to execute a hypsographic survey in a well-defined, objective and optimizing manner. The results would permit comparisons between different types of lakes; they would also comprise the presuppositions for the establishment of quantitative form for the theoretical ideas presented in the present paper. This would be of great importance for future work in the field of physical limnology, from practical, economic and basic scientific viewpoints.

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