Neutrosophic Sets and Systems, Vol. 49, 2022
University of New Mexico

Neutrosophic Entropy Based Fluoride Contamination Indices for Community Health Risk Assessment from Groundwater of Kangra County, North India

C. P. Gandhi[1], Simerjit Kaur[2] *, Rahul Dev[3] and Manoj Bali[4]

[1] Department of Life Sciences, Rayat Bahra University, Mohali, Punjab, India. Email: dr.simar@rayatbahrauniversity.edu.in
[2] Department of Mathematics, Rayat Bahra University, Mohali, Punjab, India. Email: cpgandhi@rayatbahrauniversity.edu.in
[3]Research Scholar, IKG Punjab Technical University, Punjab, India. Email: rahuldevanmole@gmail.com
[4]Department of Chemistry, Baba Hira Singh Bhattal, Institute of Engineering and Technology, Sunam-Jakhal Road, Lehragaga, Punjab 148031, India.

*Correspondence: dr.simar@rayatbahrauniversity.edu.in

Abstract: The underlying study intends to evaluate community health risk assessment from fluoride contamination of groundwater samples employing the proposed entropy variants of single valued neutrosophic sets. The symmetric fuzzy cross entropy numbers, which can represent the macroscopic view of fluoride contamination more effectively, are constructed in this study and then deployed to rank the seasonal parameters (pre-monsoon, rainy season and pre-monsoon) responsible for fluoride contamination in the study area. To quantify the non-linear relationship between seasonal parameters and sampling spots, the proposed neutrosophic entropy variants are fascinated for assigning weights to the monitored concentration reading of each seasonal parameter with respect to various sampling spots. Thereafter, these weights are coupled with the quality rating scale of each parameter, intended to establish new fuzzy and single valued neutrosophic entropy weighted fluoride contamination indices (FEFCI & NEFCI) respectively. The maximum (or minimum) FEFCI or NEFCI score at a particular sampling spot is designated to the "most (or least) contaminated" sampling spot accordingly.The underlying fluoride contaminated sampling spot identification methodology is efficacious for providing a better insight in assessing the community health risk from ground water of the study area.

Keywords: Fuzzy Entropy, Neutrosophic Entropy, Cross Entropy, Fluoride, Ground Water, Community health.

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1. Introduction

Fluoride contamination in groundwater affects public health and its excess is responsible for the spread of incurable but preventable disease called as fluorosis. Many health problems are associated with drinking of water contaminated with elevated level of fluoride ($2mg$ /day) and are responsible for various common diseases such as arthritis, brittle bones, Alzheimer, skeletal malformation etc. Excess concentration of fluoride in drinking water leads to low calcium, high alkalinity, fluoride poisoning and thus affects the individual.

The recommended concentration of fluoride **[9]** in drinking water quality $1.5\ mg/l$. However, optimum concentration of fluoride varies between 0.5 - $1.0\ mg/l$ **[10]** according to climatic conditions. Public across world dependent mostly on groundwater resources have been encountering issues with increased concentration of fluoride. Fluorosis has mostly affected India and China, the two most populated countries of the world. In Pakistan, fluoride analysis on 29 main cities **[12]** showed 34% of the cities with elevated fluoride levels having mean value greater than $1.5\ mg/l$. In this study, Lahore, Quetta and Tehsil Mailsi were found with highest fluoride level values of 23.60, 24.48, 5.5 mg/l respectively. Fluoride epidemic has been reported in upwards of 19 Indian states and union regions. India is among the 23 countries in the world where fluoride sullied ground water is making medical issues. Recent studies from state Andhra Pradesh (India) have shown that fluoride level ranges from 0.4-5.8 mg/l with a mean value of 1.98 mg/l and villagers have been suffering severely from Fluorosis **[3]**. The province of Art Report of UNICEF affirms the fluoride issues in 177 locales of 20 states in India **[1]**. Fluoride content in local water well-springs of Dungapur area of Rajasthan was examined by Choubisa **[2]** and they revealed the fluoride content in open wells up to 10 mg/l. Fluoride focus in ground water of Prakasham area (Andhra Pradesh) in India was observed by **[4]** and they found the convergence of fluoride in surface and ground water tests shifted between 0.5 mg/l to 9.0 mg/l. A detailed instance of fluoride was in the Tekelangjun region, Karbi Anglong area, where fluoride fixations, in May 2019, were ranged between 5-23 mg/l. The profoundly fluoride influenced zones of Assam viz. Kamrup, Nagaon were investigated. Fluoride focuses in these zones were accounted for to be substantially higher than the BIS reasonable cutoff points of 1.5 mg/l. Extreme sullying of fluoride in groundwater of Karbi Anglong and Nagaon locale of Assam and its appearance has been accounted for fluorosis **[5-7]**.

Recently, Adimalla et al. **[19]** constructed entropy water quality index (EWQI) and assessed the overall quality of ground water for irrigation and domestic purposes. Singh et al. **[20]** deployed Shannon's information entropy for constructing entropy weighted heavy metal contamination index (EHCI) and performed spatial assessment of water quality in some tributaries of Brahmaputra River. Unfortunately, the additive and probabilistic Shannon's entropy is facing a major drawback as it is based on the fancy presumption $0 \times log 0 = 0$ and hence indicates major conflicts in water treatment strategies. Under such problematic situation, Zadeh's **[17]** fuzzy set theory can handle the complexity of contamination level from macroscopic point of view. Dubois and Prade **[16]** developed the first non-additive and non-probabilistic entropy measure for elaborating some measurements of membership functions, intended to develop uncertainty modeling. In the existing literature, many equivalents of fuzzy sets are available and can be deployed to tackle fluoride contamination issues for quality evaluation. Subsequently, Smarandache **[18]** neutrosophic set theory can represent the macroscopic state of fluoride contamination of ground water in a broader way. A neutrosophic set contains more quantified information than any fuzzy set and can be characterized by the forms of truism membership, indeterminacy membership and fallacy membership functions respectively. To the our best knowledge, no neutrosophic entropy measure, till so far, has been developed and deployed for quantifying the non-linear relationship of fluoride contamination between seasonal parameters and sampling spots. Subsequently, an effort has been accomplished in this pathway by constructing symmetric fuzzy cross entropy numbers (SFCNs) followed by fuzzy entropy and single valued neutrosophic entropy weighted fluoride contamination indices (FEFCI and NEFCI) consecutively. A schematic flow chart of the underlying methodology is depicted in **Fig 1** and the rest of the proposed research work is organized as follows.

Section 2 provides the details of study area, materials and the procedure employed in collecting ground water samples under investigation. **Section 3** discusses in brief the basic terminology of Information theory, required for understanding the underlying study. **Section 4** deals with the establishment of a novel hyperbolic fuzzy entropy measure followed by symmetric fuzzy cross entropy (FCE) as well as single valued neutrosophic entropy measures. The efficaciousness of the proposed symmetric fuzzy cross entropy numbers (SFCNs) is validated in **Section 5** by classifying each seasonal parameter responsible for fluoride contamination in ground water samples. **Section 6** introduces a novel HFE and HNE based methodology for constructing fuzzy entropy and single valued neutrosophic entropy weighted fluoride contamination indices (FEFCI and NEFCI). **Section 7** provides the applicability and effectiveness of the underlying methodology by reckoning the most contaminated sampling spot along with community health risk assessment related to ground water quality whereas **Section 8** finally summarizes the concrete conclusions of this study.

2. Materials and Procedure

2.1 Study Area The area under investigation lies between 78.91 and 79.13 N longitude and 18.00 and Kangra-the most populated district of Himachal Pradesh, India, with a population of 15,07,223, (2011 Census)- is located on the southern ridge of the Himalaya between 31°2 to 32°5 N and 75° to 77°45 E. This district is surrounded by mountain altitude of the Shivaliks, Dhauladhar and the Himalayas from north-west to south-east. The district has a geographical area of 5,739 km. The altitude varies from 500 meters above the average sea level to 5000 meters. Due to its ideal location, Kangra is renowned for tourism activities and therefore the district's economy is centered mainly on tourism apart from agriculture and industrial resources. While Kangra is gifted with ample freshwater resources such as River Beas, Dal and Kareri Lakes, Pong reservoir etc.; along with numerous ground water sources such as dug wells, hand pumps, tube wells and springs. Due to environmental degradation, population overgrowth, pollution, tourism and various developmental activities affect overall water quality (MWR, 2016). CGWB (2018) surveyed annual fluctuation in water level of GWMS during different monitoring periods were analyzed. The climate of the district Kangra varies from sub-tropical to sub-humid. Winter varies from December to February and summer extends from March to June while July to September are rainy months. The average rainfall in the district during 2005 was 1765.1 mm. Snow fall is received in the higher reaches of Dhauladhar mountain ranges. Average minimum and maximum temperature ranges from 3°C and 45°C. In this study, the details of the sampling spots along with sampling codes are mentioned below:
S_1 = Shahpur, S_2 = Samlana Jawali, S_3 = Indora, S_4 = Mata Rani Chowk, Haripur, S_5 = Sukka Talab Chowk, Haripur, S_6 = Garli, S_7 = Sapadi, S_8 = Jawalaji, S_9 = Dehra, S_{10} = Nagrota Bagwan, S_{11} = Dharamsala, S_{12} = Bod, S_{13} = Thural, S_{14} = Baijnath, S_{15} = Chougan, Bir, S_{16} = Palampur and S_{17} = Main Bazar Kangra.

2.2 Spectrophotometric Method A compound of a metal such as aluminum, iron, thorium, zirconium, lanthanum or cerium reacts with an indicator dye to form a complex of low dissociation constant. This complex reacts with fluoride to give a new complex. Due to the change in the structure of the complex, the absorption spectrum also shifts relative to the spectrum for the fluoride-free reagent solutions. This change can be detected by using a spectrophotometer. One of the important dyes used is trisodium 2-(parasulfophenylazo-1), 8- dihydroxy-3, 6- naphthalene disulfonate, commonly known as SPADNS. The dye reacts with metal ions to give a colored complex. In the SPADNS method, zirconium reacts with SPADNS to form a red coloured complex.

Fluoride bleaches the red color of the complex and hence the change in absorbance can be measured using a spectrophotometer.

2.3 Procedure Preparation of the reagent: 958 mg of SPADNS was dissolved in distilled water and diluted to 500 ml. 133 mg of zirconyl chloride octahydrate ($ZrCl_2.8H_2O$) was dissolved in 25 ml distilled water and 350 ml of conc. HCl was added and diluted to 500 ml with distilled water. SPADNS solution and Zirconyl acid solutions were mixed in equal volume.

To prepare the calibration curve, 0.221 g of anhydrous sodium fluoride was dissolved in water and diluted up to one liter and further diluted to get standard solution having 10 mg per liter of fluoride.1, 2, 3, 4, 5 and 6 ml of this solution was pipetted out into 50 ml standard flasks. 10 ml of Zirconyl-SPADNS reagent and one drop of $NaAsO_2$ were added to each of the solutions and was diluted up to the mark and mixed well.

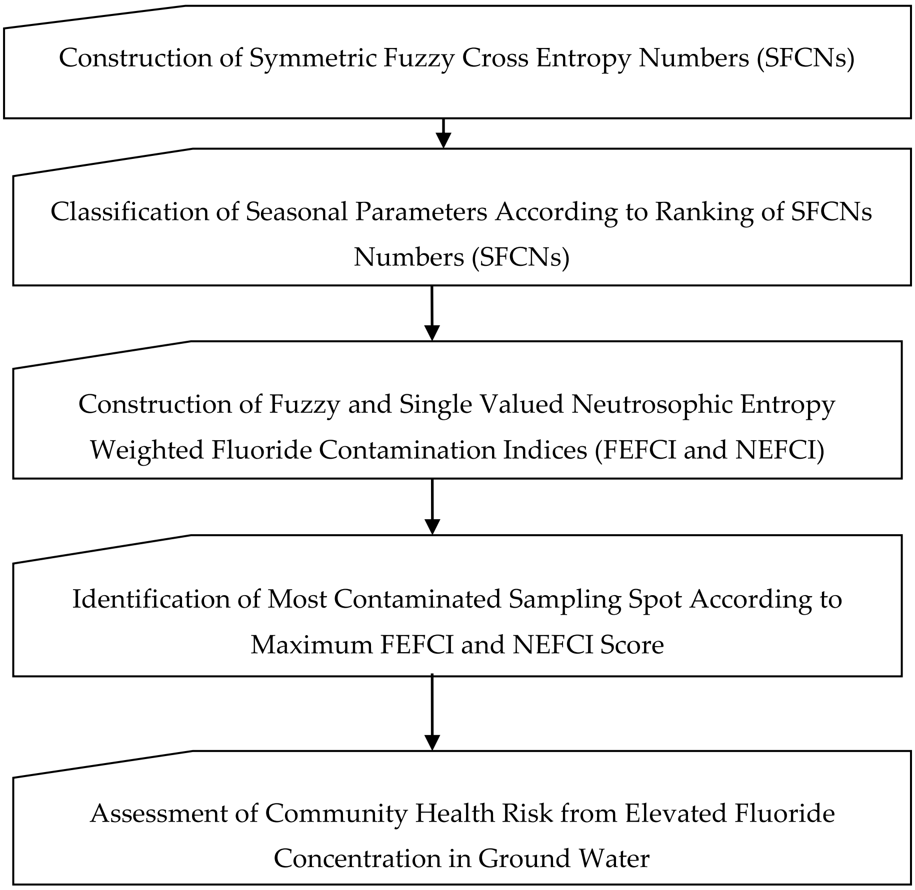


Fig.1 Schematic Flow Chart For Identifying the Most Contaminated Sampling Spot Responsible for Fluoride Contamination in Ground Water

The absorbance of the solutions was measured at 570 nm against a reagent blank and a calibration plot was constructed by plotting absorbance against concentrations using colorimeter. Suitable aliquot of water sample was taken and repeated the step. The concentration of F^-/l was calculated after using the calibration curve.

3. Preliminaries: -

Def.3.1 Fuzzy Entropy Measure [16] Let $W(U)$ represents the collection of all fuzzy sets in a space of discourse U generated by generic elements $(x_1, x_2, x_3, .., x_n)$. Let

$R_1=\left(\prec x_i,\mu_R(x_i)\succ \forall x_i\in U\right)(i=1,2,...n)$ be any fuzzy set in U which is quantified by its truth membership functions $\mu_R(x_i):U\to[0,1]$ and satisfy $0\le\mu_R(x_i)\le 1\forall i$ .Then a function $T(R_1):W(U)\to R^+$ (the set of non-negative real numbers) is called as fuzzy entropy measure if $(i)\mathrm{T}(R_1)\ge 0\forall R_1\in W(U)$ with equality if $\mu_R(x_i)=0$ or 1. $(ii)\mathrm{T}(R_1)$ is a concave function with respect to each $\mu_R(x_i)$ and $(iii)\mathrm{T}(R_1^c)=\mathrm{T}(R_1)\forall R_1\in W(U)$ where R_1^c denotes the complement of the fuzzy set R_1 and is defined as $R_1^c=\left(\prec x_i,1-\mu_R(x_i)\succ\forall x_i\in U\right)$.

Def.3.2 Symmetric Cross Entropy Measure [16] Let $R_1=\left(\prec x_i,\mu_{R_1}(x_i)\succ\forall x_i\in U\right)$ and $R_2=\left(\prec x_i,\mu_{R_2}(x_i)\succ\forall x_i\in U\right)$ represent two fuzzy sets in U, quantified by their truth membership functions $\mu_{R_1}(x_i),\mu_{R_2}(x_i):U\to[0,1]$ and satisfy $0\le\mu_{R_1}(x_i),\mu_{R_2}(x_i)\le 1$. Then a function $F(R_1,R_2):W(U)\times W(U)\to R^+$ is called as symmetric fuzzy cross entropy (FCE) or discrimination information measure between two FSs R_1 and R_2 if $(i)F(R_1,R_2)\ge 0$ with equality if $R_1=R_2$ $(ii)F(R_1,R_2)=F(R_2,R_1)$. In other words, $F(R_1,R_2)$ is symmetric in nature $(iii)F\left(R_1^c,R_2^c\right)=F(R_1,R_2)\forall R_1,R_2\in W(U)$ which means $F(R_1,R_2)$ remains unchanged on interchanging $\mu_{R_1}(x_i),\mu_{R_2}(x_i)$ with their counter parts $1-\mu_{R_1}(x_i),1-\mu_{R_2}(x_i)$.

Def.3.3 Single Valued Neutrosophic Set [18] A single valued neutrosophic set S_1 in $U=(x_1,x_2,x_3,..x_n)$ is an entity of the form $S_1=\left(\prec x_i,\mu_{S_1}(x_i),i_{S_1}(x_i),f_{S_1}(x_i)\succ\forall x_i\in U\right)$ where each $\mu_{S_1}(x_i),i_{S_1}(x_i),f_{S_1}(x_i):U\to[0,1]$ satisfy $0\le\mu_{S_1}(x_i)+i_{S_1}(x_i)+f_{S_1}(x_i)\le 3$ and are characterized by (i) truth membership function $\mu_{S_1}(x_i)$ (ii) indeterminacy function $i_{S_1}(x_i)$ and (iii) falsity membership function $f_{S_1}(x_i)$ respectively where each $x_i\in U$ is associated to a unique real number in the closed interval [0,1].

Def.3.4 Single Valued Neutrosophic Entropy Measure [18] Let $T(U)$ represents the collection of all single valued neutrosophic sets (SVNSs) in U. Then a function $T(S_1):S(U)\to R^+$ is called as single valued neutrosophic entropy measure if

$(i)T(S_1)\ge 0\forall S_1\in T(U)$ $(ii)T(S_1)=0$ whenever either $\mu_{S_1}(x_i)=1,i_{S_1}(x_i)=0,f_{S_1}(x_i)=0$ or $\mu_{S_1}(x_i)=0,i_{S_1}(x_i)=0,f_{S_1}(x_i)=1$. $(iii)T(S_1^c)=T(S_1)$ where S_1^c denotes the complement of S_1 and is defined as $S_1^c=\left(\prec x_i,f_{S_1}(x_i),1-i_{S_1}(x_i),\mu_{S_1}(x_i)\succ\forall x_i\in U\right)$ and (iv) $T(S_1)$ exhibits the concavity property with respect to each $\mu_{S_1}(x_i),i_{S_1}(x_i),f_{S_1}(x_i)$. Also, $T(S_1)$ admits its maximum value, which arises when $\mu_{S_1}(x_i)=i_{S_1}(x_i)=f_{S_1}(x_i)=\frac{1}{2}$ and the maximum value is an increasing function of *n*.

4. Establishment of Single Valued Neutrosophic Entropy Measure

Our endeavor will be to develop a novel fuzzy entropy measure followed by symmetric fuzzy cross entropy measure hinged on two fuzzy sets. The aftermaths of which will be a backbone for the construction of proclaimed symmetric fuzzy cross entropy numbers (SFCNs), required for classifying the seasonal parameters responsible for fluoride contamination in ground water.

4.1 A Novel Hyperbolic Fuzzy Entropy Measure

We shall propose a novel hyperbolic fuzzy entropy (HFE) measure (**Theorem 4.1)**, the aftermaths of which will be a backbone for the proposed symmetric fuzzy cross entropy measure (**Theorem 4.2)**.

Theorem.4.1 Let $R_1 = \left(\prec x_i, \mu_{R_1}(x_i) \succ \forall x_i \in U \right)$ be any fuzzy set in U. Then

$$F(R_1) = -\sum_{i=1}^{n} \left[\tanh\left(\frac{1+\sqrt{\mu_{R_1}^2(x_i)+\left(1-\mu_{R_1}(x_i)\right)^2}}{2+\sqrt{\mu_{R_1}(x_i)}+\sqrt{1-\mu_{R_1}(x_i)}} \right) - \tanh\left(\frac{2}{3}\right) \right] \quad \ldots (1)$$

represents an authentic hyperbolic fuzzy entropy measure with minimum value zero and maximum value as $\left(\tanh\frac{2}{3} - \tanh\frac{1}{2} \right)n.$ Here, the generic element $'x_i'$ denotes the $'i^{th}'$ macroscopic level of fluoride contamination and $F(R_1)$ represents the fuzziness of fluoride contamination indicated by the fuzzy set R_1.

Proof (i) In view of **Def. 3.1,** $F(R_1) \geq 0$ since $0 \leq \mu_{R_1}(x_i) \leq 1 (i = 1, 2, ..., n)$. Also, $F(R_1)$ vanishes whenever $\mu_{R_1}(x_i) = 0 \text{ or } 1$.

(ii) $F(R_1)$ remains unchanged after replacing $\mu_R(x_i)$ with $1-\mu_R(x_i)$.

(iii) **Concavity:** The fact that hyperbolic fuzzy entropy $F(R_1)$ exhibits its concavity property with respect to each $\mu_R(x_i)$, can be seen from its 3-D rotational plot displayed in **Fig. 2.** Next, the positive term finite series **(1)** converges absolutely which motivates $F(R_1)$ to possess first order partial differentiation with respect to each $\mu_R(x_i)$ Set $T_0 = \sqrt{\mu_{R_1}^2(x_i)+\left(1-\mu_{R_1}(x_i)\right)^2}$, $T_1 = \sqrt{\mu_{R_1}(x_i)}$, and $T_2 = \sqrt{\left(1-\mu_{R_1}(x_i)\right)}$. Due to concavity, $F(R_1)$ affirms its maximum value which arises only when $\frac{\partial F(R_1)}{\partial \mu_{R_1}(x_i)} = 0$ which implies

$$Sech^2\left(\frac{1+T_0}{2+T_1+T_2} \right) \times \left(\frac{2\left(1-2T_1^2\right)}{2T_0\left(2+T_1+T_2\right)^2} + \frac{\left(T_1-T_2\right)\left(1+T_0\right)}{2T_1T_2\left(2+T_1+T_2\right)^2} \right) = 0 \quad \ldots (2)$$

The resulting expression **(2)** yields $\mu_{R_1}(x_i) = \frac{1}{2}$ and hence (1) returns

$$\text{Max.}F(R_1) = F(R_1)\Big|_{\mu_A(x_i)=\frac{1}{2}} = \left(\tanh\frac{2}{3} - \tanh\frac{1}{2} \right)n. \quad \ldots (3)$$

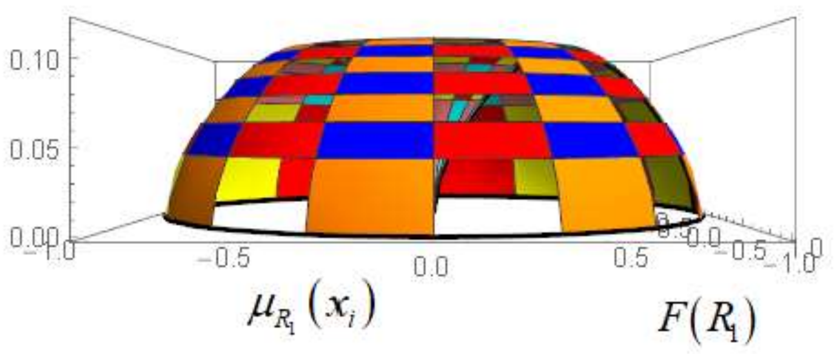


Fig.2 Concavity of $F(R_1)$ with respect to $\mu_{R_1}(x_i)$

Theorem.4.2 Let $R_1 = \left(\prec x_i, \mu_{R_1}(x_i) \succ \forall x_i \in U\right)$ and $R_2 = \left(\prec x_i, \mu_{R_2}(x_i) \succ \forall x_i \in U\right)$ be any two fuzzy sets in U. *Then* $F^{\mu}(R_1, R_2)$ is an authentic symmetric hyperbolic fuzzy cross entropy measure **(Def. 3.2)** hinged on two fuzzy sets R_1 and R_2 where

$$F^{\mu}(R_1,R_2)=\sum_{i=1}^{n}\left[\begin{array}{l} -6Tanh\,\dfrac{1}{2}+\left(2+\mu_{R_1}(x_i)+\mu_{R_2}(x_i)\right)Tanh\left(\dfrac{1+\sqrt{\mu_{R_1}^2(x_i)+\mu_{R_2}^2(x_i)}}{2+\left(\sqrt{\mu_{R_1}(x_i)}+\sqrt{\mu_{R_2}(x_i)}\right)\left(\sqrt{\mu_{R_1}(x_i)+\mu_{R_2}(x_i)}\right)}\right) \\ +\left(4-\mu_{R_1}(x_i)-\mu_{R_2}(x_i)\right)Tanh\left(\dfrac{1+\sqrt{\left(1-\mu_{R_1}(x_i)\right)^2+\left(1-\mu_{R_2}(x_i)\right)^2}}{2+\left(\sqrt{1-\mu_{R_1}(x_i)}+\sqrt{1-\mu_{R_2}(x_i)}\right)\left(\sqrt{2-\mu_{R_1}(x_i)-\mu_{R_2}(x_i)}\right)}\right)\end{array}\right]$$

… (4)

Here $F^{\mu}(R_1, R_2)$ indicates the mathematical value of true membership degree of symmetric discrimination of the fuzzy set R_1 against R_2

Proof. The conditions (ii) and (iii) of **Def. 3.2** are obvious. We shall, equally well, establish the following **Lemma 4.1,** intended to establish that non-negativity of symmetric fuzzy cross entropy measure $F^{\mu}(R_1, R_2)$

Lemma 4.1 If $P=\sqrt{\dfrac{\mu_{R_1}^2(x_i)+\mu_{R_2}^2(x_i)}{2}}$, $N=\left(\dfrac{\sqrt{\mu_{R_1}(x_i)}+\sqrt{\mu_{R_2}(x_i)}}{2}\right)\left(\sqrt{\dfrac{\mu_{R_1}(x_i)+\mu_{R_2}(x_i)}{2}}\right)$.

Then, there exists the inequality $P\left(\mu_{R_1}(x_i), \mu_{R_2}(x_i)\right) \geq N\left(\mu_{R_1}(x_i), \mu_{R_2}(x_i)\right)$ with equality if $\mu_{R_1}(x_i) = \mu_{R_2}(x_i) \forall \mu_{R_1}(x_i), \mu_{R_2}(x_i) \in [0,1]$.

Proof. The undergoing inequality can be made true if

$$P^2 \geq N^2 \Rightarrow \frac{\mu_{R_1}^2(x_i)+\mu_{R_2}^2(x_i)}{2} \geq \left(\frac{\sqrt{\mu_{R_1}(x_i)}+\sqrt{\mu_{R_2}(x_i)}}{2}\right)^2 \left(\frac{\mu_{R_1}(x_i)+\mu_{R_2}(x_i)}{2}\right) \text{ or if}$$

$$3\mu_{R_1}^2(x_i)+3\mu_{R_2}^2(x_i)-2\mu_{R_1}(x_i)\mu_{R_2}(x_i) \geq 2\sqrt{\mu_{R_1}(x_i)}\sqrt{\mu_{R_1}(x_i)}\left(\mu_{R_1}(x_i)+\mu_{R_2}(x_i)\right) \text{ or if}$$

$$\left(3\mu_{R_1}^2(x_i)+3\mu_{R_2}^2(x_i)-2\mu_{R_1}(x_i)\mu_{R_2}(x_i)\right)^2 \geq 4\mu_{R_1}(x_i)+\mu_{R_2}(x_i)\left(\mu_{R_1}(x_i)+\mu_{R_2}(x_i)\right)^2 \text{ or if}$$

$9\mu_{R_1}^4(x_i)+9\mu_{R_2}^4(x_i)+14\mu_{R_1}^2(x_i)\mu_{R_2}^2(x_i) \geq 16\mu_{R_1}(x_i)\mu_{R_2}(x_i)\left(\mu_{R_1}^2(x_i)+\mu_{R_2}^2(x_i)\right)$ which is obviously true for each $\mu_{R_1}(x_i), \mu_{R_2}(x_i) \in [0,1]$.

Thus, in view of **Lemma 3.1,** the oncoming inequality can be re-scheduled as

$$P\left(\mu_{R_1}(x_i), \mu_{R_2}(x_i)\right)+1 \geq N\left(\mu_{R_1}(x_i), \mu_{R_2}(x_i)\right)+1$$

$$\Rightarrow \sqrt{\frac{\mu_{R_1}^2(x_i)+\mu_{R_2}^2(x_i)}{2}}+1 \geq \left(\frac{\sqrt{\mu_{R_1}(x_i)}+\sqrt{\mu_{R_1}(x_i)}}{2}\right)\left(\sqrt{\frac{\mu_{R_1}(x_i)+\mu_{R_2}(x_i)}{2}}\right)+1$$

$$\Rightarrow \frac{1+\sqrt{\mu_{R_1}^2(x_i)+\mu_{R_2}^2(x_i)}}{2+\left(\sqrt{\mu_{R_1}(x_i)}+\sqrt{\mu_{R_2}(x_i)}\right)\left(\sqrt{\mu_{R_1}(x_i)+\mu_{R_2}(x_i)}\right)} \geq \frac{1}{2} \qquad \ldots (5)$$

Since, the hyperbolic functions over $[0,1]$ are monotonic in nature, the foregoing inequality **(5)** can be re-designed as

$$\left(2+\mu_{R_1}(x_i)+\mu_{R_2}(x_i)\right)\tanh\left(\frac{1+\sqrt{\mu_{R_1}^2(x_i)+\mu_{R_2}^2(x_i)}}{2+\left(\sqrt{\mu_{R_1}(x_i)}+\sqrt{\mu_{R_2}(x_i)}\right)\left(\sqrt{\mu_{R_1}(x_i)+\mu_{R_2}(x_i)}\right)}\right) \geq \left(2+\mu_{R_1}(x_i)+\mu_{R_2}(x_i)\right)\tanh\left(\frac{1}{2}\right) \qquad \ldots (6)$$

After replacement of $\mu_{R_1}(x_i), \mu_{R_2}(x_i)$ with their counter parts $1-\mu_{R_1}(x_i), 1-\mu_{R_2}(x_i)$ into **(6)** yields

$$\left(4-\mu_{R_1}(x_i)-\mu_{R_2}(x_i)\right)\tanh\left(\frac{1+\sqrt{\left(1-\mu_{R_1}(x_i)\right)^2+\left(1-\mu_{R_2}(x_i)\right)^2}}{2+\left(\sqrt{1-\mu_{R_1}(x_i)}+\sqrt{1-\mu_{R_2}(x_i)}\right)\left(\sqrt{2-\mu_{R_1}(x_i)-\mu_{R_2}(x_i)}\right)}\right) \geq \left(4-\mu_{R_1}(x_i)-\mu_{R_2}(x_i)\right)\tanh\left(\frac{1}{2}\right) \qquad \ldots (7)$$

with equality if $\mu_{R_1}(x_i)=\mu_{R_2}(x_i) \forall i=1,2,\ldots,n.$

Simply adding the resulting inequalities **(6)** and **(7)** and summing over $i=1$ to $i=n$ yields $F^{\mu}(R_1,R_2) \forall \mu_{R_1}(x_i), \mu_{R_2}(x_i) \in [0,1]$ with equality if $\mu_{R_1}(x_i)=\mu_{R_2}(x_i) \forall i=1,2,\ldots,n.$

We next divert our attention to discuss the situation under which the proposed symmetric fuzzy cross entropy $F^{\mu}(R_1,R_2)$ admists its maximum and minimum values as follows.

Theorem 4.3 Let $R_1 = \left(\prec x_i, \mu_{R_1}(x_i) \succ \forall x_i \in U\right)$ and $R_2 = \left(\prec x_i, \mu_{R_2}(x_i) \succ \forall x_i \in U\right)$ be two fuzzy sets with same cardinality as of U. Then there exists the inequality: $0 \le F^{\mu}(R_1, R_2) \le 6\left(\tanh\frac{2}{3} - \tanh\frac{1}{2}\right)n,$ where n is the cardinality of U.

Proof. Replacement of R_2 with R_1^c into the resulting equality **(4)** yields

$$F^{\mu}\left(R_1, R_1^c\right) = \sum_{i=1}^{n}\left[-6\tanh\frac{1}{2} + 6\tanh\left(\frac{1+\sqrt{\mu_{R_1}^2(x_i) + \left(1-\mu_{R_2}(x_i)\right)^2}}{2+\left(\sqrt{\mu_{R_1}(x_i)} + \sqrt{1-\mu_{R_2}(x_i)}\right)}\right)\right]$$

$$= \sum_{i=1}^{n}\left[6\tanh\frac{2}{3} - 6\tanh\frac{1}{2} - 6\left(\tanh\frac{2}{3} - \tanh\left(\frac{1+\sqrt{\mu_{R_1}^2(x_i) + \left(1-\mu_{R_2}(x_i)\right)^2}}{2+\left(\sqrt{\mu_{R_1}(x_i)} + \sqrt{1-\mu_{R_2}(x_i)}\right)}\right)\right)\right]$$

$$= 6\text{Max.}F(R_1) - 6F(R_1) \qquad \dots (8)$$

Since $F(R_1) \ge 0 \forall \mu_{R_1}(x_i)$, the oncoming equality **(8)** yields

$$F(R_1) = \text{Max.}F(R_1) - \frac{1}{6}F^{\mu}\left(R_1, R_1^c\right) \ge 0 \Rightarrow 0 \le F^{\mu}\left(R_1, R_1^c\right) \le 6\left(\tanh\frac{2}{3} - \tanh\frac{1}{2}\right)n \qquad \dots (9)$$

Discussion. The undergoing inequality **(9)** clarifies the finiteness of $F^{\mu}\left(R_1, R_1^c\right)$ whenever n is a fixed natural number. On the same pattern, the users can establish that, $F^{\mu}\left(R_1, R_1^c\right)$ will also be finite and has the range value $0 \le F^{\mu}(R_1, R_2) \le 6\left(\tanh\frac{2}{3} - \tanh\frac{1}{2}\right)n.$ In view of **Theorem 4.2**, we have $\text{Max.}F^{\mu}(R_1, R_2) = 6\left(\tanh\frac{2}{3} - \tanh\frac{1}{2}\right)n$ for a fixed n and this value completely depends upon the cardinality of U. Also, the three-dimensional plot depicted in **Fig 3(a, b)** exhibits that $F^{\mu}(R_1, R_2)$ admits its minimum value zero. Furthermore, $F^{\mu}(R_1, R_2)$ increases with the increase in $\left|R_1 - R_2\right|$, attains it maximum value at the points $(1,0)$ & $(0,1)$ and minimum value zero whenever $R_1 = R_{2.}$.

We next switch to establish the proclaimed single valued neutrosophic entropy measure hinged on two single valued neutrosophic sets, the aftermaths of which will be utilized to understand the macroscopic state of fluoride contamination in ground water.

4.2 A Novel Hyperbolic Single Valued Neutrosophic Entropy Measure

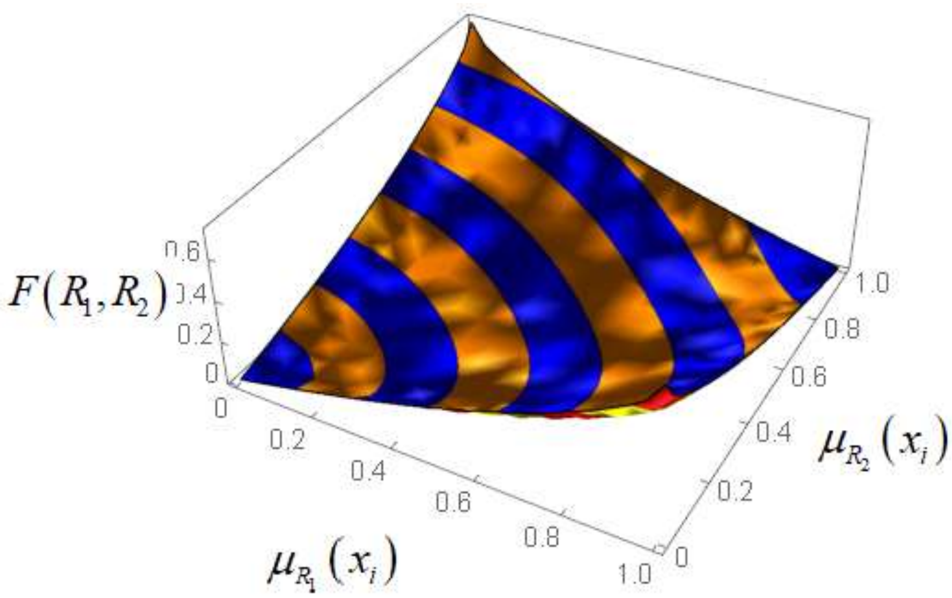
To meet the desired goal, we shall first extend the resulting symmetric hyperbolic fuzzy cross entropy measure **(Theorem 4.2)** hinged on two fuzzy sets to this measure hinged on two single valued neutrosophic sets as follows.

Def. 4.1 Let $S_1 = \left(\prec x_i, \mu_{S_1}(x_i), i_{S_1}(x_i), f_{S_1}(x_i) \succ \forall x_i \in U\right); S_2 = \left(\prec x_i, \mu_{S_2}(x_i), i_{S_2}(x_i), f_{S_2}(x_i) \succ \forall x_i \in U\right)$

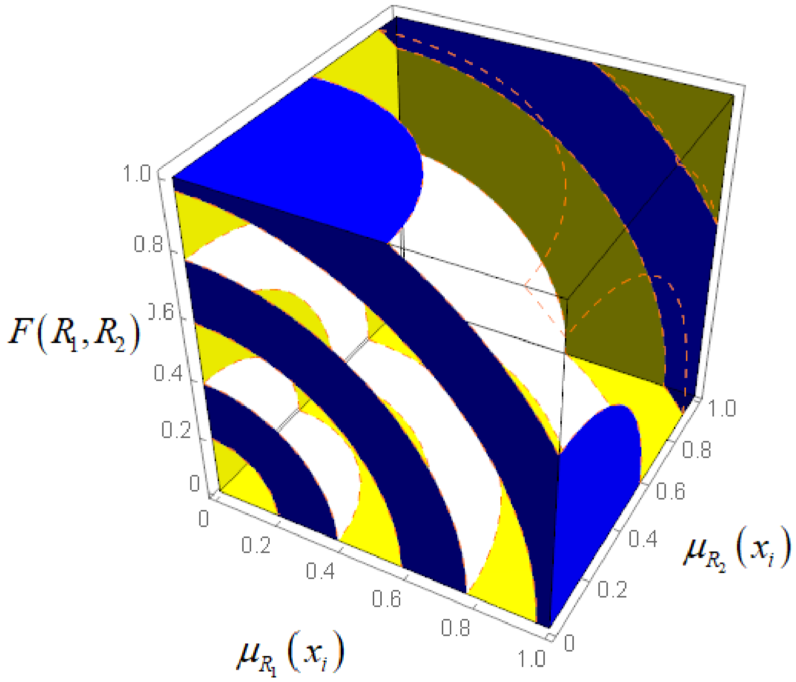
be any two single valued neutrosophic sets **(Def. 3.3).** In view of **Theorem 4.2,** the mathematical value of true membership degree of symmetric discrimination of S_1 against S_2 is given as

$$F^{\mu}(S_1,S_2) = \sum_{i=1}^{n} \left[\begin{array}{l} -6\tanh\frac{1}{2} + \left(2+\mu_{S_1}(x_i)+\mu_{S_2}(x_i)\right)\tanh\left(\dfrac{1+\sqrt{\mu_{S_1}^2(x_i)+\mu_{S_2}^2(x_i)}}{2+\left(\sqrt{\mu_{S_1}(x_i)}+\sqrt{\mu_{S_2}(x_i)}\right)\left(\sqrt{\mu_{S_1}(x_i)+\mu_{S_2}(x_i)}\right)}\right) \\ +\left(4-\mu_{S_1}(x_i)-\mu_{S_2}(x_i)\right)\tanh\left(\dfrac{1+\sqrt{\left(1-\mu_{S_1}(x_i)\right)^2+\left(1-\mu_{S_2}(x_i)\right)^2}}{2+\left(\sqrt{1-\mu_{S_1}(x_i)}+\sqrt{1-\mu_{S_2}(x_i)}\right)\left(\sqrt{2-\mu_{S_1}(x_i)-\mu_{S_2}(x_i)}\right)}\right) \end{array} \right]$$

… (10)



(a)



(b)

Fig. 3 Maximum and Minimum Value of $F^{\mu}(R_1,R_2)$

Similarly, the mathematical values of indeterminacy and falsity membership degrees of symmetric discrimination of S_1 against S_2 are given as

$$F^{i}(S_1,S_2)=\sum_{i=1}^{n}\left[\begin{array}{l}-6\tanh\frac{1}{2}+\left(2+i_{S_1}(x_i)+i_{S_2}(x_i)\right)\tanh\left(\dfrac{1+\sqrt{i_{S_1}^2(x_i)+i_{S_2}^2(x_i)}}{2+\left(\sqrt{i_{S_1}(x_i)}+\sqrt{i_{S_2}(x_i)}\right)\left(\sqrt{i_{S_1}(x_i)+i_{S_2}(x_i)}\right)}\right)\\+\left(4-i_{S_1}(x_i)-i_{S_2}(x_i)\right)\tanh\left(\dfrac{1+\sqrt{\left(1-i_{S_1}(x_i)\right)^2+\left(1-i_{S_2}(x_i)\right)^2}}{2+\left(\sqrt{1-i_{S_1}(x_i)}+\sqrt{1-i_{S_2}(x_i)}\right)\left(\sqrt{2-i_{S_1}(x_i)-i_{S_2}(x_i)}\right)}\right)\end{array}\right]$$

... (11)

$$F^{f}(S_1,S_2)=\sum_{i=1}^{n}\left[\begin{array}{l}-6\tanh\frac{1}{2}+\left(2+f_{S_1}(x_i)+f_{S_2}(x_i)\right)\tanh\left(\dfrac{1+\sqrt{f_{S_1}^2(x_i)+f_{S_2}^2(x_i)}}{2+\left(\sqrt{f_{S_1}(x_i)}+\sqrt{f_{S_2}(x_i)}\right)\left(\sqrt{f_{S_1}(x_i)+f_{S_2}(x_i)}\right)}\right)\\+\left(4-f_{S_1}(x_i)-f_{S_2}(x_i)\right)\tanh\left(\dfrac{1+\sqrt{\left(1-f_{S_1}(x_i)\right)^2+\left(1-f_{S_2}(x_i)\right)^2}}{2+\left(\sqrt{1-f_{S_1}(x_i)}+\sqrt{1-f_{S_2}(x_i)}\right)\left(\sqrt{2-f_{S_1}(x_i)-f_{S_2}(x_i)}\right)}\right)\end{array}\right]$$

... (12)

Hence, the proclaimed single valued neutrosophic cross entropy measure hinged on two SVNSs S_1 and S_2 can be easily established by adding the resulting expressions **(10), (11)** and **(12).** Thus,

$$T(S_1,S_2)=F^{\mu}(S_1,S_2)+F^{i}(S_1,S_2)+F^{f}(S_1,S_2) \qquad \dots (13)$$

Here, $T(S_1,S_2)$ represents the true, indeterminacy and falsity membership degrees indicated by the symmetric discrimination of SVNS S_1 against S_2

Theorem.4.4 Let $S_1=(\prec x_i,\mu_{S_1}(x_i),i_{S_1}(x_i),f_{S_1}(x_i)\succ)$ and $S_2=(\prec x_i,\mu_{S_2}(x_i),i_{S_2}(x_i),f_{S_2}(x_i)\succ)$ be any two single valued neutrosophic sets, with same cardinality as of U. Then there exists the inequality $0\le T(S_1,S_2)\le 18\left(\tanh\frac{2}{3}-\tanh\frac{1}{2}\right)n.$

Proof. Replacement of S_2 with its counterpart S_1^c into the expression **(13)** yields

$$T(S_1,S_1^c)=\sum_{i=1}^{n}\left[\begin{array}{l}18\tanh\frac{2}{3}-18\tanh\frac{1}{2}\\ -6\left(\begin{array}{l}3\tanh\frac{2}{3}-\left(\frac{2+\mu_{S_1}(x_i)+f_{S_1}(x_i)}{3}\right)\tanh\left(\frac{1+\sqrt{\mu_{S_1}^2(x_i)+f_{S_1}^2(x_i)}}{2+\left(\sqrt{\mu_{S_1}(x_i)}+\sqrt{f_{S_1}(x_i)}\right)\left(\sqrt{\mu_{S_1}(x_i)+f_{S_1}(x_i)}\right)}\right)\\ -\left(\frac{4-\mu_{S_1}(x_i)-f_{S_1}(x_i)}{3}\right)\tanh\left(\frac{1+\sqrt{\left(1-\mu_{S_1}(x_i)\right)^2+\left(1-f_{S_1}(x_i)\right)^2}}{2+\left(\sqrt{1-\mu_{S_1}(x_i)}+\sqrt{1-f_{S_1}(x_i)}\right)\left(\sqrt{2-\mu_{S_1}(x_i)-f_{S_1}(x_i)}\right)}\right)\\ -\tanh\left(\frac{1+\sqrt{i_{S_1}^2(x_i)+\left(1-i_{S_1}(x_i)\right)^2}}{2+\sqrt{i_{S_1}(x_i)}+\sqrt{1-i_{S_1}(x_i)}}\right)\end{array}\right)\end{array}\right]$$

$$=6\text{Max.T}(S_1)-6T(S_1);\quad \text{where} \qquad \dots (14)$$

$$T(S_1)=\sum_{i=1}^{n}\left[\begin{array}{l}3\tanh\frac{2}{3}-\left(\frac{2+\mu_{S_1}(x_i)+f_{S_1}(x_i)}{3}\right)\tanh\left(\frac{1+\sqrt{\mu_{S_1}^2(x_i)+f_{S_1}^2(x_i)}}{2+\left(\sqrt{\mu_{S_1}(x_i)}+\sqrt{f_{S_1}(x_i)}\right)\left(\sqrt{\mu_{S_1}(x_i)+f_{S_1}(x_i)}\right)}\right)\\ -\left(\frac{4-\mu_{S_1}(x_i)-f_{S_1}(x_i)}{3}\right)\tanh\left(\frac{1+\sqrt{\left(1-\mu_{S_1}(x_i)\right)^2+\left(1-f_{S_1}(x_i)\right)^2}}{2+\left(\sqrt{1-\mu_{S_1}(x_i)}+\sqrt{1-f_{S_1}(x_i)}\right)\left(\sqrt{2-\mu_{S_1}(x_i)-f_{S_1}(x_i)}\right)}\right)\\ -\tanh\left(\frac{1+\sqrt{i_{S_1}^2(x_i)+\left(1-i_{S_1}(x_i)\right)^2}}{2+\sqrt{i_{S_1}(x_i)}+\sqrt{1-i_{S_1}(x_i)}}\right)\end{array}\right]$$

… (15)

The mathematical expression **(15)** is the desired hyperbolic single valued neutrosophic entropy measure since it meets all the essential conditions laid down in **Def. 3.4.** With the aid of non-negativity of $S(R_1)$, the equality **(14)** can be re-scheduled as

$$T(S_1)=\text{Max.T}(S_1)-\frac{1}{6}T(S_1,S_1^c)\geq 0 \Rightarrow 0\leq T(S_1,S_1^c)\leq 18\left(\tanh\frac{2}{3}-\tanh\frac{1}{2}\right)n \quad \ldots (16)$$

The resulting inequality equality **(14)** clarifies that $T(S_1,S_1^c)$ is a finite quantity for a fixed $n\in N.$ Following the similar pattern, the users can easily establish that $0\leq T(S_1,S_2)\leq 18\left(Tanh\frac{2}{3}-Tanh\frac{1}{2}\right)n$ where $n\in N$ is the cardinality of S_1. Thus,

$\text{Max.}T(S_1,S_2)=18\left(Tanh\frac{2}{3}-Tanh\frac{1}{2}\right)n$, $\text{Min.}T(S_1,S_2)=0$ The fact that $T(S_1)$ affirms its minimum value zero can also been experiences from its three-dimensional contour plot shown in **Fig. 4.**

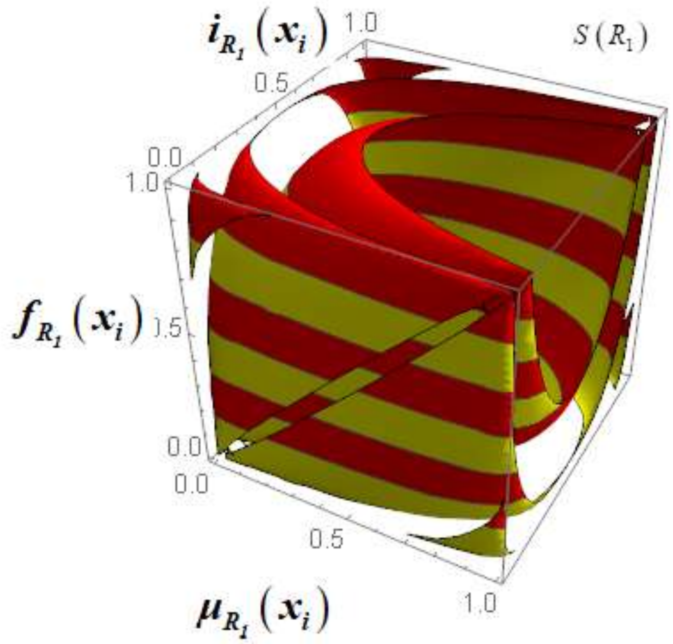


Fig. 4 Three- Dimensional Contour Plot Exhibiting the Minimum Value of $T(S_1)$

To evaluate the impact of elevated levels of fluoride concentration, we shall first customize or rank seasonal parameters employing the proposed possibility fuzzy cross entropy degree measure as follows.

5. Ranking of Seasonal Parameters

To reckon the quality of river water for drinking or irrigation purposes, it is mandatory to represent fluoride concentration of seasonal parameters by the set $P=(P_1,P_2,P_3,...,P_n)$. A symmetric fuzzy cross entropy number(SFCN), denoted b f_{rs}, is an object of the from $f_{rs}=\prec F(P_r,P_1),F(P_r,P_2),....,F(P_r,P_s)\succ;1\leq r,s\leq n$ under the assumption $F(P_r,P_s)=0\Leftrightarrow r=s.$

where each pair $F(P_r,P_s)$ indicates the mathematical value of true membership degree of symmetric discrimination of the seasonal parameter P_r against P_s and can be evaluated employing **(4).** Let f_{rs} and f_{ts} be any two symmetric fuzzy cross entropy numbers (SFCNs). Then the inclusion-comparison fuzziness of two SFCNs $f_{rs} \geq f_{ts}$ for $r = 1, 2, ..., n$ and fixed s, is denoted by $\eta(f_{rs} \geq f_{ts})$ and is known as possibility fuzzy cross entropy degree measure. Let the matrix representation of $\eta(f_{rs} \geq f_{ts})$ is denoted by $N = (\eta_{rt})_{n\times n}$ where $\eta_{rt} = \eta(f_{rs} \geq f_{ts})$ and

$$N = \begin{pmatrix} \eta_{11} & \eta_{12} \cdots & \eta_{1n} \\ \eta_{21} & \eta_{22} \cdots & \eta_{2n} \\ \vdots & \ddots & \vdots \\ \eta_{m1} & \eta_{n2} \cdots & \eta_{nn} \end{pmatrix} \quad \dots (17)$$

Then N is called as possibility fuzzy cross entropy degree measure matrix. The optimal fuzzy cross entropy membership degree, denoted by s_k, is defined

$$s_k = \frac{1}{n(n+1)}\left(\sum_{t=1}^{n} \eta_{kt} + \frac{n}{2} - 1\right); k \in N \quad \dots (18)$$

The ranking of each seasonal parameter $P_k\,(k = 1, 2, .., n)$ is obtained according to the corresponding decreasing ordered value of s_k. For convenience, the symmetric fuzzy cross entropy numbers f_{rs} for $r = 1, 2, 3$ and $s = 3$ are given as

$$f_{13} = \prec F(R_1, R_1), F(R_1, R_2), F(R_1, R_3) \succ \quad \dots (19)$$

$$f_{23} = \prec F(R_2, R_1), F(R_2, R_2), F(R_2, R_3) \succ \quad \dots (20)$$

$$f_{31} = \prec F(R_3, R_1), F(R_3, R_2), F(R_3, R_3) \succ \quad \dots (21)$$

The corresponding possibility fuzzy cross entropy degree measures are proposed as

$$\eta(f_{13} \geq f_{23}) = \text{Min}\left(\text{Max}\left(\frac{F(R_1, R_2) + F(R_2, R_2)}{1 + F(R_1, R_1) - 2F(R_2, R_1) - F(R_2, R_3)}, 0\right), 1\right) \quad \dots (22)$$

$$\eta(f_{13} \geq f_{33}) = \text{Min}\left(\text{Max}\left(\frac{F(R_1, R_2) + F(R_3, R_2)}{1 + F(R_1, R_1) - 2F(R_3, R_1) - F(R_3, R_3)}, 0\right), 1\right) \quad \dots (23)$$

$$\eta(f_{23} \geq f_{33}) = \text{Min}\left(\text{Max}\left(\frac{F(R_2, R_2) + F(R_3, R_2)}{1 + F(R_2, R_1) - 2F(R_3, R_1) - F(R_3, R_3)}, 0\right), 1\right) \quad \dots (24)$$

After collecting ground water samples during sampling year 2014-15 and 2015-16, we have done a lot of data comparison and experimental investigations to extract the lower and upper bounds from monitored fluoride concentration reading of each $P_K\,(K = 1, 2, 3)$ where P_1 = Pre-Monsoon, P_2 = Rainy Season and P_3 = Post-Monsoon respectively. Suppose $\mu_{P_K}(x)$ denotes the lower bound of K^{th} seasonal parameter, then the set $P = (P_1, P_2, P_3)$ can be constructed for both the sampling years under study and the results are displayed in **Table.1.**

Table 1 Possibility Fuzzy Cross Entropy Degree Measure Values (2014-15, 2015-16)

	2015-16				2014-15			
Parameter	Lower Bound	FCE Measure Values			Lower Bound	FCE Measure Values		
		P_1	P_2	P_3		P_1	P_2	P_3
P_1	0.0282	0.0000	0.0112	0.0187	0.0301	0.0000	0.0119	0.0125
P_2	0.0000	0.0112	0.0000	0.0472	0.0000	0.0119	0.0000	0.0395
P_3	0.1162	0.0187	0.0472	0.0000	0.0978	0.0125	0.0395	0.0000

For the sampling year **2015-16,** the various symmetric fuzzy cross entropy numbers

$$f_{13} = \prec 0.0000, 0.0112, 0.0187 \succ,\ f_{23} = \prec 0.0112, 0.0000, 0.0472 \succ,\ f_{33} = \prec 0.0187, 0.0472, 0.0000 \succ \quad (25)$$

can be evaluated employing equations **(19-21)** and the results are shown in the first row of **Table.1.** Next, the various possibility fuzzy cross entropy degree measures can be computed employing **(22-24)** as follows.

$$\eta_{11} = 0, \eta_{12} = \eta\left(f_{13} \geq f_{23}\right) = \mathrm{Min}\left(\mathrm{Max}\left(\frac{0.0112+0}{1+0-2\times 0.0112-0.0472}, 0\right), 1\right)$$

$$= \mathrm{Min}\left(\mathrm{Max}\left(\frac{0.0112-0}{0.9304}, 0\right), 1\right) = \mathrm{Min}(0.0120, 1)$$

$$\eta_{13} = 0.0606, \eta_{21} = 0.0113, \eta_{22} = 0, 0.0000,$$

$$\eta_{23} = \eta\left(f_{23} \geq f_{33}\right) = \mathrm{Min}\left(\mathrm{Max}\left(\frac{0+0.0472}{1+0.0112-2\times 0.0187-0}, 0\right), 1\right)$$

$$= \mathrm{Min}\left(\mathrm{Max}\left(\frac{0.0472}{0.9738}, 0\right), 1\right) = \mathrm{Min}(0.0485, 1)$$

$$\eta_{31} = \eta\left(f_{33} \geq f_{13}\right) = \mathrm{Min}\left(\mathrm{Max}\left(\frac{0.0472+0.0112}{1+0.0187-2\times 0.0000-0.0187}, 0\right), 1\right)$$

$$= \mathrm{Min}\left(\mathrm{Max}(0.0584, 0), 1\right) = \mathrm{Min}(0.0584, 1) = 0.0584,$$

$$\eta_{32} = 0.0497, \eta_{33} = 0.0000.$$

Hence, the required possibility fuzzy cross entropy measure degree matrix in this case is given as

$$N = \begin{pmatrix} \eta_{11} & \eta_{12} & \eta_{13} \\ \eta_{21} & \eta_{22} & \eta_{23} \\ \eta_{31} & \eta_{32} & \eta_{33} \end{pmatrix} = \begin{pmatrix} 0.0000 & 0.0120 & 0.0606 \\ 0.0113 & 0.0000 & 0.0485 \\ 0.0584 & 0.0497 & 0.0000 \end{pmatrix} \quad \ldots (26)$$

For the sampling year **2014-15,** the various symmetric fuzzy cross entropy numbers

$$f_{13} = \prec 0.0000, 0.0119, 0.0125 \succ,\ f_{23} = \prec 0.0119, 0.0000, 0.0395 \succ,\ f_{33} = \prec 0.0125, 0.0395, 0.0000 \succ \quad (27)$$

can also be evaluated employing **(19-21)** and the results are shown in the first row of **Table.1.** The corresponding possibility fuzzy cross entropy measure degree matrix, say M, is given as

$$M=\begin{pmatrix} 0.0000 & 0.0127 & 0 \\ 0.0119 & 0.0000 & 0 \\ 0.0514 & 0.0416 & 0 \end{pmatrix} \quad \dots (28)$$

For the sampling year **2015-16, t**he optimal fuzzy cross entropy membership degrees $s_k\,(k=1,2,3)$ for $n=3$ can be computed employing (**18)** and the results are as under.

$s_1=\frac{1}{12}\left(\sum_{t=1}^{3}\eta_{kt}+\frac{n}{2}-1\right)=\frac{1}{12}(0+0.0120+0.0606+0.5)=0.0477, s_2=0.0466, s_3=0.0507.$ For the sampling year **2014-15, t**he corresponding values of $s_k\,(k=1,2,3)$ for $n=3$ are $s_1=0.0471, s_2=0.0460, s_3=0.0494.$

Since the ranking order of $s_k\,(k=1,2,3)$ for both the sampling years 2014-15 and 2015-16 is $s_3 \succ s_1 \succ s_2$, therefore, the classification of seasonal parameters should be $P_3 \succ P_1 \succ P_2$.

Results and Discussions.

Based upon experimental investigations, it has been found that during 2015-16, fluoride concentration of groundwater samples varied from 0.065 to 0.91 mg/l during pre-monsoon season. Fluoride concentration varied from 0.025 to 0.42 mg/l (lowest)during rainy season whereas during post-monsoon season it varied from 0.19 to 1.42 mg/l(highest). During 2014-15, fluoride concentration varied from 0.06 to 0.85 mg/l during pre-monsoon season. In rainy season, fluoride ranged from 0.02 to 0.36 mg/l (lowest)whereas during post-monsoon season it varied from 0.15 to 1.33 mg/l(highest). The classification of seasonal parameters $P_3 \succ P_1 \succ P_2$ also exhibit that the fluoride concentration was highest in post monsoon season, owing to the highest fuzzy cross entropy membership degree (0.0507,0.0494).

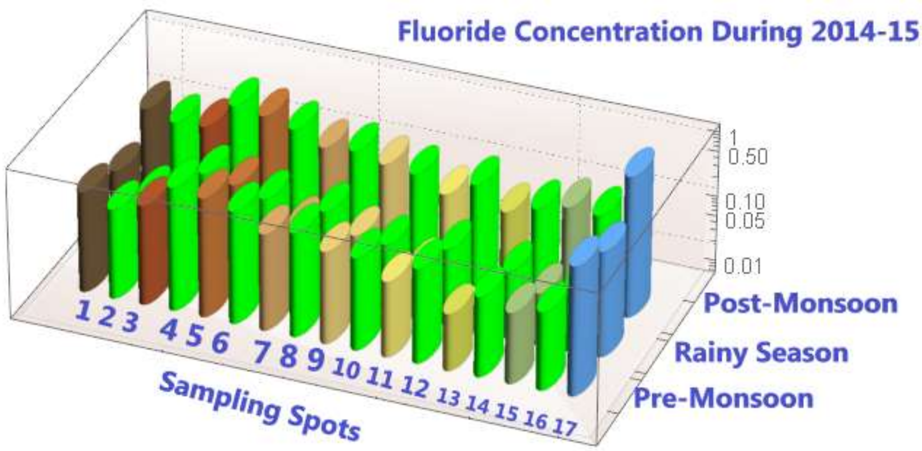


Fig. 5 Seasonal Variations in Fluoride Concentration of Groundwater (2014-15)

5.1 Experimental Assessment of Fluoride Concentration (2014-15)

The results depicted in **Fig.5** dictate that in during **2014-15**, fluoride concentration varied from 0.06 to 0.85 mg/l during pre-monsoon season. In rainy season, it ranged from 0.02 to 0.36 mg/l

whereas during post-monsoon season it varied from 0.15 to 1.33 mg/l It was found to be highest at sampling spot $S_{17}(0.85 mg/l)$ followed by $S_4(0.70\ mg/l)$, $S_5(0.60\ mg/l)$, $S_6(0.47\ mg/l)$ and lowest concentration was observed at $S_{13}(0.06\ mg/l)$. During rainy season, fluoride concentration was found to be highest at sampling spot $S_{17}(0.36 mg/l)$ followed by $S_5(0.25 mg/l)$ and lowest concentration was observed at $S_{13}(0.02\ mg/l)$. During post-monsoon season, fluoride has shown highest concentration at $S_{17}(1.33\ mg/l)$ followed by $S_4(0.94\ mg/l)$ & $S_4(0.80\ mg/l)$. Likewise during pre-monsoon & rainy season, fluoride has shown lowest concentration at $S_{13}(0.15\ mg/l)$ during post-monsoon also (**Fig.5)**.

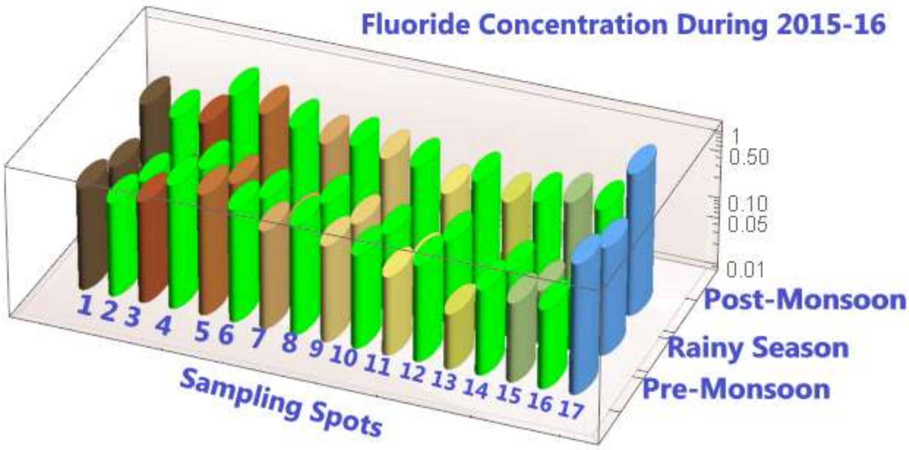


Fig. 6 Seasonal Variations in Fluoride Concentration of Groundwater (2015-16)

5.2 Experimental Assessment of Fluoride Concentration (2015-16)

The results depicted in **Fig.6** indicate that during **2015-16**, fluoride concentration of groundwater samples varied from 0.065 to 0.91 mg/l during pre-monsoon season. Fluoride concentration varied from 0.025 to 0.42 mg/l during rainy season whereas during post-monsoon season it varied from 0.19 to 1.42 mg/l. It was observed highest at sampling spot $S_{17}(0.91\ mg/l)$ followed by $S_4(0.75\ mg/l)$ & $S_5(0.68 mg/l)$ and lowest concentration was observed at $S_{13}(0.065 mg/l)$. During rainy season, fluoride concentration was found to be highest at sampling spot $S_{17}(0.42\ mg/l)$ followed by $S_4(0.36\ mg/l)$ & $S_5(0.27\ mg/l)$ and lowest was observed at $S_{13}(0.025\ mg/l)$. During post-monsoon, fluoride concentration has shown highest concentration at $S_{17}(1.42\ mg/l)$

followed by $S_4(1.30\ mg/l)$, $S_5(0.85\ mg/l)$, $S_6(0.65\ mg/l)$ & lowest fluoride concentration was observed at $S_{13}(0.19\ mg/l)$ **(Fig.6)**. Furthermore, if fluoride concentration is below 0.6 mg/l drinking water should be rejected. Maximum limit of fluoride is extended up to 1.5 mg/l. During these investigations, fluoride concentration was found to be highest during post-monsoon season followed by pre-monsoon and rainy season. The most contaminated sampling spot was identified as S_{17} and least contaminated site was discovered as S_{13}. Most of the sampling spots have shown fluoride concentration below 0.6 mg/l in **2014-15.** Fluoride concentration increased during **2015-16** at sampling spots S_4, S_5, S_6, S_{11} and S_{17} but was found to be with in permissible limits.

7. Methodology for the Identification of "most" contaminated sampling spot

We next switch to construct the proclaimed fuzzy entropy and single valued neutrosophic entropy weighted fluoride contamination indices (FEFCI and NEFCI), intended to identify the most contaminated sampling spot responsible for fluoride contamination in ground water samples as follows.

Step: -1 Collection of Ground Water Samples

Present investigations were carried out in District Kangra, Himachal Pradesh. The reason for this area selection was because of its position in relation to groundwater morphometric. In this study, seventeen sampling spots of groundwater were sampled in pre-monsoon, rainy and post-monsoon season in the specified area for two sampling years **2014-15** and **2015-16.**

Step: -2 Normalization of Monitored Fluoride Concentration Reading

Suppose the number of seasonal parameters (seasons) to be studied is denoted by "n". Let the number of sampling spots under study is denoted by "m". Let l_{ji} denotes the monitored fluoride concentration reading of j^{th} season at i^{th} sampling spot. The normalization of concentration reading is essential for the purpose of reducing the errors created by various factors. If p_{ji} denotes the normalization construction function for j^{th} season at i^{th} sampling spot, then

$$p_{ji} = \frac{l_{ji} - \text{Min.}l_{ji}}{\text{Max.}l_{ji} - \text{Min.}l_{ji}};\ j = 1,2,...,n, i = 1,2,...,m. \quad \dots (29)$$

Step:- 3 Determination of Fuzzy Entropy Weights

Deluca and Termini **[16]** suggested the following first non-additive and non-probabilistic equivalent associate of Shannon's entropy.

$$H(R_1) = -\frac{1}{\log m}\sum_{j=1}^{n}\left[\mu_{R_1}(x_j)\log\mu_{R_1}(x_j) + \left(1-\mu_{R_1}(x_j)\right)\log\left(1-\mu_{R_1}(x_j)\right)\right] \quad \dots (30)$$

where $R_1 = \left(< x_j, \mu_{R_1}(x_j) > | x_j \in U\right)$ is a fuzzy set (**Def. 3.1)**

Let T_{ji} denotes the amount of fuzziness based on the true membership concentration of j^{th} seasonal parameter at i^{th} sampling spot. Then,

$$T_{ji} = \frac{p_{ji}}{\sum_{j=1}^{n} p_{ji}} \quad \ldots (31)$$

(a) The fuzzy entropy weights $w_{ji}^{(0)}$ of j^{th} seasonal parameter at i^{th} sampling spot employing Deluca and Termini **(30)** can be evaluated as follows; Let "m" be the number of sampling spots, then

$$w_{ji}^{(0)} = \frac{1-E_{ji}^{(0)}}{\sum_{j=1}^{n} E_{ji}^{(0)}}, \text{ where} \quad \ldots (32)$$

$$E_{ji}^{(0)} = -\frac{1}{\log m} \sum_{j=1}^{n} \left[T_{ji} \log T_{ji} + \left(1-T_{ji}\right) \log\left(1-T_{ji}\right) \right] \quad \ldots (33)$$

However, the fuzzy entropy measure **(30)** is facing a major drawback as it is based on the fancy presumption $0 \times log 0 = 0$ and hence indicates major conflicts in water treatment strategies. To overcome these barricades and problematic situations, the proposed hyperbolic fuzzy and single valued neutrosophic entropy measures (HFE and HNE) can play a crucial role for handling the complexity of contamination level in a macroscopic point of view.

(b) The fuzzy entropy weights $w_{ji}^{(1)}$ of j^{th} seasonal parameter at i^{th} sampling spot employing the proposed hyperbolic fuzzy entropy measure (**1)** can be evaluated as follows: Let "m" be the number of sampling spots, then

$$w_{ji}^{(1)} = \frac{1-E_{ji}^{(1)}}{\sum_{j=1}^{n} E_{ji}^{(1)}}, \text{ where } E_{ji}^{(1)} = -\tanh\left(m^{-1}\right) \sum_{j=1}^{n} \left[\tanh\left(\frac{1+\sqrt{T_{ji}^2 + \left(1-T_{ji}\right)^2}}{2+\sqrt{T_{ji}}+\sqrt{1-T_{ji}}} \right) - \tanh\left(\frac{2}{3}\right) \right] \quad \ldots (34)$$

(c) The fuzzy entropy weights $w_{ji}^{(2)}$ of j^{th} seasonal parameter at i^{th} sampling spot employing the proposed single valued neutrosophic entropy measure **(15)** can be evaluated as follows: Let $F_{ji} = 1-T_{ji}$ and $I_{ji} = 1-T_{ji}-F_{ji}$ denote the amount of fuzziness based on the indeterminacy and falsity membership concentration of j^{th} seasonal parameter at i^{th} sampling. Here, the values of I_{ji} are restricted to $0 \cdot 001$ if it is less than or equal to zero. Then,

$$w_{ji}^{(2)} = \frac{1-E_{ji}^{(2)}}{\sum_{j=1}^{n} E_{ji}^{(2)}} \quad \ldots (35)$$

$$E_{ji}^{(2)}=\tanh\left(m^{-1}\right)\sum_{j=1}^{n}\left[\begin{array}{l}3\tanh\dfrac{2}{3}-\tanh\left(\dfrac{1+\sqrt{I_{ji}^{2}+\left(1-I_{ji}\right)^{2}}}{2+\sqrt{I_{ji}}+\sqrt{1-I_{ji}}}\right)-\left(\dfrac{2+T_{ji}+F_{ji}}{3}\right)\tanh\left(\dfrac{1+\sqrt{T_{ji}^{2}+F_{ji}^{2}}}{2+\left(\sqrt{T_{ji}}+\sqrt{F_{ji}}\right)\left(\sqrt{T_{ji}+F_{ji}}\right)}\right)\\-\left(\dfrac{4-T_{ji}-F_{ji}}{3}\right)\tanh\left(\dfrac{1+\sqrt{\left(1-T_{ji}\right)^{2}+\left(1-F_{ji}\right)^{2}}}{2+\left(\sqrt{1-T_{ji}}+\sqrt{1-F_{ji}}\right)\left(\sqrt{2-T_{ji}-F_{ji}}\right)}\right)\end{array}\right] \quad \dots (36)$$

Step: -4 Quality Rating Scales of Seasonal Parameters

To describe the quality of ground water parameters, eminent researchers have been employing two types of quality rating scales-absolute and relative. Since absolute quality rating does not depend upon water quality standards, therefore, relative quality rating approach has been empowered in this study. Let $Q_{ji} =$ Relative Quality Scale, $S_{ji} =$ Maximum permissible fluoride concentration limit and $l_{ji} =$ Monitored fluoride concentration reading, of j^{th} seasonal parameter at i^{th} sampling spot consecutively. Then

$$Q_{ji}=\left(\frac{l_{ji}}{S_{ji}}\times 100\right)\left(\frac{7-S_{pH}}{7-l_{pH}}\right);\, j=1,2,...,n, i=1,2..,m. \quad \dots (37)$$

where $(i) S_{ji} = 1.5\left(mg/L\right)$ is the maximum permissible limit of fluoride concentration (WHO Standards) of j^{th} seasonal parameter at i^{th} sampling spot. (ii) S_{pH} is the permissible limit of pH (varies from 6.5 to 8.5) values and is defined as $S_{pH}=\begin{cases}6.5, \text{if } l_{pH}<7\\ 8.5, \text{if } l_{pH}>7\end{cases}$

(iii) l_{pH} is the pH value in ground water samples (Table)

Step: -5 Construction of f FEFCI and NEFCI

The existing Deluca and Termini fuzzy entropy **(33)** and the proposed hyperbolic fuzzy entropy and single valued neutrosophic entropy weighted fluoride contamination indices (DEFCI, FEFCI and NEFCI) can be computed as follows:

$$\text{DEFCI at } i^{th} \text{ Sampling Spot} = \sum_{j=1}^{n} w_{ji}^{(0)} Q_{ji} \quad \dots (38)$$

$$\text{FEFCI at } i^{th} \text{ Sampling Spot} = \sum_{j=1}^{n} w_{ji}^{(1)} Q_{ji} \quad \dots (39)$$

$$\text{NEFCI at } i^{th} \text{ Sampling Spot} = \sum_{j=1}^{n} w_{ji}^{(2)} Q_{ji} \quad \dots (40)$$

Step: -6 Identifying the Most Contaminated Sampling Spot

The maximum (or minimun) DEFCI, FEFCI or NEFCI scores among various sampling spot is designated to the "most (or least) contaminated" sampling spot.

7. Application of HFE and HNE Based Method

To predict the contamination impact of each sampling spot, the DEFCI, FEFCI and NEFCI score at various sampling spots $S_1, S_2, ..., S_{17}$ can be evaluated employing as follows.

7.1 Identification of Most Contaminated Sampling Spot Based on DEFCI

Based upon Deluca and Termini entropy **(30)**, the existing fuzzy entropy weighted fluoride contamination index (DEFCI) scores at 17 sampling spots can be calculated employing the proposed methodology explained in **Section. 6.** The steps involved in the calculation of DEFCI scores at various sampling spots during **2014-15** and **2015-16** are depicted in **Table 2(a, b).** The monitored fluoride concentration readings of each seasonal parameter are expressed in terms of mg/l. The number of seasonal parameters (seasons) in this study, is three $(n=3)$ and the number of sampling spots is seventeen $(m=17)$ .The normalization construction function $p_{ji}\left(j=1,2,3; i=1,2,..,17\right)$ of all the three seasonal parameters at 17 sampling spots is calculated employing **(29).**

Observations The tabulated values of **Table 2(a, b)** as well as trend of DEFCI score **(Fig. 7) i**ndicate that during **2014-15**, the sampling spot S_{17} was found to be most contaminated owing to its maximum DEFCI score (882) whereas the least contaminated sampling spot was observed as $S_{16}(54)$. During **2015-16**, the sampling spot S_{17} was again found to be most contaminated owing to its maximum DEFCI score (1244) and $S_{16}(73)$ was the least contaminated **(Fig 8)**.

7.2 Identification of Most Contaminated Sampling Spot Based on FEFCI

The proposed fuzzy entropy weighted fluoride contamination index (FEFCI) scores at 17 sampling spots can be calculated employing the proposed methodology explained in **Section. 6.** The steps involved in the calculation of FEFCI scores at various sampling spots during **2014-15** and **2015-16** are depicted in **Table 3(a, b).**

Observations The resulting values of **Table 3(a, b)** and trend of FEFCI score **(Fig 7) i**ndicate that during **2014-15**, the sampling spot S_{17} was found to be most contaminated owing to its maximum

Table 2: Calculation of DEFCI Score Employing Deluca and Termini Entropy [ ] (C.F.=Construction Function, FVs=Fuzzy Values, EVS=Entropy Values, Aws=Assigned Weights, RSIs=Relative Sub-Indices)

Seasons		C.F.	FVs	EVs	AWs	RSIs	DEFCI Score	C.F.	FVs	EVs	AWs	RSIs	DEFCI Score
		p_{ji}	T_{ji}	$E_{ji}^{(0)}$	$w_{ji}^{(0)}$	Q_{ji}		p_{ji}	T_{ji}	$E_{ji}^{(0)}$	$w_{ji}^{(0)}$	Q_{ji}	
		2014-2015						2015-2016					
Pre-M	**S_1**	0.21	0.33	0.22	1.22	76.92	299	0.23	0.33	0.22	1.22	145.8	522
RS		0.11	0.18	0.17	1.31	60.71		0.12	0.18	0.17	1.31	133.3	
Post-M		0.31	0.49	0.24	1.19	104.88		0.33	0.49	0.24	1.19	142.8	
Pre-M	**S_2**	0.14	0.32	0.22	1.30	34.48	147	0.17	0.32	0.22	1.24	57.78	230
RS		0.05	0.13	0.13	1.45	25.71		0.09	0.17	0.16	1.33	51.72	

Post-M		0.23	0.55	0.24	1.27	50.77		0.26	0.50	0.24	1.20	75.00	
Pre-M		0.20	0.40	0.24	1.25	31.52		0.22	0.38	0.23	1.20	47.22	
RS	S_3	0.06	0.12	0.13	1.43	19.61	102	0.10	0.17	0.16	1.31	37.78	147
Post-M		0.25	0.49	0.24	1.24	27.56		0.26	0.45	0.24	1.18	34.78	
Pre-M		0.51	0.36	0.23	1.23	85.37		0.51	0.31	0.22	1.29	104.1	
RS	S_4	0.21	0.15	0.15	1.36	66.67	318	0.24	0.14	0.15	1.41	109.0	490
Post-M		0.69	0.49	0.24	1.21	101.08		0.90	0.55	0.24	1.25	162.5	
Pre-M		0.44	0.36	0.23	1.24	82.19		0.46	0.38	0.23	1.23	103.0	
RS	S_5	0.17	0.14	0.15	1.37	49.02	302	0.17	0.14	0.14	1.37	60.00	363
Post-M		0.59	0.49	0.24	1.21	109.59		0.58	0.48	0.24	1.21	126.8	
Pre-M		0.34	0.37	0.23	1.24	47.96		0.33	0.37	0.23	1.24	61.73	
RS	S_6	0.13	0.14	0.14	1.38	33.93	172	0.12	0.14	0.14	1.39	42.55	209
Post-M		0.45	0.49	0.24	1.22	53.91		0.44	0.49	0.24	1.22	60.75	
Pre-M		0.18	0.35	0.23	1.32	29.89		0.19	0.35	0.23	1.31	39.47	
RS	S_7	0.05	0.10	0.12	1.51	12.33	104	0.06	0.11	0.12	1.49	16.42	138
Post-M		0.29	0.55	0.24	1.29	36.04		0.31	0.55	0.24	1.28	48.42	
Pre-M		0.24	0.35	0.23	1.20	73.91		0.30	0.36	0.23	1.18	132.3	
RS	S_8	0.13	0.19	0.17	1.29	86.36	264	0.17	0.20	0.18	1.26	136.8	418
Post-M		0.32	0.46	0.24	1.18	54.32		0.36	0.43	0.24	1.17	76.81	
Pre-M		0.15	0.33	0.22	1.28	26.83	91	0.18	0.32	0.22	1.24	35.90	
RS		0.06	0.13	0.14	1.42	13.51		0.10	0.17	0.16	1.33	21.62	117
Post-M	S_9	0.24	0.53	0.24	1.25	29.82		0.28	0.50	0.24	1.20	36.84	
Pre-M		0.15	0.36	0.23	1.26	20.95		0.16	0.32	0.22	1.25	19.38	
RS	S_{10}	0.05	0.13	0.13	1.42	6.12	148	0.08	0.16	0.16	1.35	9.52	180
Post-M		0.21	0.51	0.24	1.24	90.91		0.26	0.52	0.24	1.21	118.1	
Pre-M		0.08	0.34	0.23	1.32	17.91		0.09	0.38	0.23	1.26	22.39	
RS	S_{11}	0.02	0.10	0.12	1.50	20.00	116	0.03	0.12	0.13	1.43	26.00	128
Post-M		0.12	0.55	0.24	1.29	48.65		0.12	0.50	0.24	1.24	51.35	
Pre-M		0.17	0.35	0.23	1.22	16.00		0.18	0.34	0.23	1.20	62.22	
RS	S_{12}	0.08	0.16	0.16	1.34	8.00	56	0.10	0.19	0.17	1.28	48.57	171
Post-M		0.23	0.48	0.24	1.20	21.33		0.24	0.46	0.24	1.17	29.84	
Pre-M		0.03	0.24	0.19	2.10	10.07		0.03	0.17	0.16	2.61	46.43	
RS	S_{13}	0.00	0.00	0.00*	2.60	4.63	462	0.00	0.00	0.00*	3.11	2.53	341
Post-M		0.10	0.76	0.19	2.10	204.69		0.14	0.83	0.16	2.61	81.48	
Pre-M		0.09	0.32	0.22	1.24	24.14		0.12	0.34	0.23	1.26	41.30	
RS	S_{14}	0.05	0.18	0.17	1.32	7.44	75	0.05	0.14	0.14	1.39	7.36	95
Post-M		0.14	0.50	0.24	1.20	29.17		0.18	0.52	0.24	1.23	26.67	
Pre-M	S_{15}	0.06	0.22	0.18	1.64	25.64		0.09	0.26	0.20	1.53	44.12	
RS		0.02	0.08	0.10	1.81	6.67	75	0.02	0.07	0.09	1.74	5.26	165

Post-M		0.20	0.70	0.21	1.58	13.15		0.22	0.66	0.23	1.48	59.65	
Pre-M		0.08	0.27	0.21	1.36	9.15		0.09	0.26	0.20	1.43	11.03	
RS	S_{16}	0.04	0.14	0.14	1.47	9.15	54	0.04	0.11	0.12	1.57	11.11	72
Post-M		0.18	0.59	0.24	1.30	21.49		0.21	0.63	0.23	1.37	28.95	
Pre-M		0.62	0.33	0.22	1.27	146.55		0.62	0.33	0.22	1.26	178.4	
RS	S_{17}	0.26	0.14	0.14	1.41	150.00	882	0.28	0.15	0.15	1.38	233.3	1244
Post-M		0.98	0.53	0.24	1.24	391.18		0.98	0.52	0.24	1.23	568.0	
		***At S_{13}, the entropy value of Rainy Season is based on the assumption:** $0\times\log 0 = 0.$											

FEFCI score (38904) whereas the least contaminated sampling spot was observed as $S_{16}(2356)$. During **2015-16**, the sampling spot S_{17} was again found to be most contaminated owing to its maximum FEFCI score (57943) and $S_{16}(3165)$ was the least contaminated **(Fig 8).**

7.3 Identification of Most Contaminated Sampling Spot Based on NEFCI

The steps involved in the computation of single valued neutrosophic entropy weighted fluoride contamination index (NEFCI) scores at 17 sampling spots during **2014-15** and **2015-16** are depicted in **Table 4(a, b).**

Observations The tabulated values exhibited by **Table 4(a, b)** and trend of NEFCI score **(Fig 7) i**ndicate that during **2014-15**, the sampling spot S_{17} was found to be most contaminated owing to its maximum NEFCI score (81596) whereas the least contaminated sampling spot was observed as $S_{16}(4773)$.

During **2015-16**, the sampling spot S_{17} was again found to be most contaminated owing to its maximum NEFCI score (115995) and $S_{16}(6193)$ was the least contaminated **(Fig 8).**

Discussions. The accumulated trend of DEFCI, FEFCI and NEFCI scores at 17 sampling spot has finally put us in a culminative situation to wind-up the conclusion that the quality of ground water was " impeccable" and " favourable".

Table 3: Calculation of FEFCI Score Employing Proposed Fuzzy Entropy Measure (C.F.=Construction Function, FVs=Fuzzy Values, EVS=Entropy Values, Aws=Assigned Weights, RSIs=Relative Sub-Indices)

Seasons		**C.F.**	**FVs**	**EVs**	**AWs**	**RSIs**	**FEFCI Score**	**C.F.**	**FVs**	**EVs**	**AWs**	**RSIs**	**FEFCI Score**
		p_{ji}	T_{ji}	$E_{ji}^{(1)}$	$w_{ji}^{(1)}$	Q_{ji}		p_{ji}	T_{ji}	$E_{ji}^{(1)}$	$w_{ji}^{(1)}$	Q_{ji}	
		2014-2015						**2015-2016**					
Pre-M		0.21	0.33	0.0064	54.51	76.92		0.23	0.33	0.0064	54.47	145.83	
RS	S_1	0.11	0.18	0.0047	54.61	60.71	13222	0.12	0.18	0.0047	54.57	133.33	22997
Post-M		0.31	0.49	0.0071	54.48	104.88		0.33	0.49	0.0071	54.44	142.86	

Pre-M	S_2	0.14	0.32	0.0063	57.76	34.48	6412	0.17	0.32	0.0064	55.00	57.78	10150
RS		0.05	0.13	0.0038	57.91	25.71		0.09	0.17	0.0046	55.10	51.72	
Post-M		0.23	0.55	0.0070	57.73	50.77		0.26	0.50	0.0071	54.96	75.00	
Pre-M	S_3	0.20	0.40	0.0068	56.33	31.52	4435	0.22	0.38	0.0067	54.00	47.22	6473
RS		0.06	0.12	0.0037	56.51	19.61		0.10	0.17	0.0046	54.12	37.78	
Post-M		0.25	0.49	0.0071	56.31	27.56		0.26	0.45	0.0070	53.99	34.78	
Pre-M	S_4	0.51	0.36	0.0066	55.28	85.37	13997	0.51	0.31	0.0062	56.97	104.17	21414
RS		0.21	0.15	0.0042	55.41	66.67		0.24	0.14	0.0042	57.09	109.09	
Post-M		0.69	0.49	0.0071	55.25	101.08		0.90	0.55	0.0070	56.93	162.50	
Pre-M	S_5	0.44	0.36	0.0067	55.43	82.19	13351	0.46	0.38	0.0067	55.31	103.03	16040
RS		0.17	0.14	0.0042	55.57	49.02		0.17	0.14	0.0041	55.45	60.00	
Post-M		0.59	0.49	0.0071	55.40	109.59		0.58	0.48	0.0071	55.29	126.87	
Pre-M	S_6	0.34	0.37	0.0067	55.62	47.96	7557	0.33	0.37	0.0067	55.66	61.73	9191
RS		0.13	0.14	0.0041	55.77	33.93		0.12	0.14	0.0041	55.81	42.55	
Post-M		0.45	0.49	0.0071	55.60	53.91		0.44	0.49	0.0071	55.64	60.75	
Pre-M	S_7	0.18	0.35	0.0065	58.52	29.89	4580	0.19	0.35	0.0065	58.18	39.47	6070
RS		0.05	0.10	0.0034	58.71	12.33		0.06	0.11	0.0035	58.36	16.42	
Post-M		0.29	0.55	0.0070	58.49	36.04		0.31	0.55	0.0070	58.15	48.42	
Pre-M	S_8	0.24	0.35	0.0066	53.85	73.91	11563	0.30	0.36	0.0067	53.20	132.35	18417
RS		0.13	0.19	0.0048	53.94	86.36		0.17	0.20	0.0050	53.28	136.84	
Post-M		0.32	0.46	0.0071	53.82	54.32		0.36	0.43	0.0070	53.18	76.81	
Pre-M	S_9	0.15	0.33	0.0064	56.83	26.83	3988	0.18	0.32	0.0064	55.01	35.90	5191
RS		0.06	0.13	0.0040	56.97	13.51		0.10	0.17	0.0046	55.10	21.62	
Post-M		0.24	0.53	0.0071	56.79	29.82		0.28	0.50	0.0071	54.97	36.84	
Pre-M	S_{10}	0.15	0.36	0.0066	56.38	20.95	6651	0.16	0.32	0.0063	55.59	19.38	8173
RS		0.05	0.13	0.0039	56.54	6.12		0.08	0.16	0.0045	55.70	9.52	
Post-M		0.21	0.51	0.0071	56.36	90.91		0.26	0.52	0.0071	55.55	118.18	
Pre-M	S_{11}	0.08	0.34	0.0065	58.47	17.91	5063	0.09	0.38	0.0067	56.42	22.39	5631
RS		0.02	0.10	0.0034	58.65	20.00		0.03	0.12	0.0038	56.59	26.00	
Post-M		0.12	0.55	0.0070	58.44	48.65		0.12	0.50	0.0071	56.40	51.35	
Pre-M	S_{12}	0.17	0.35	0.0066	54.83	16.00	2486	0.18	0.34	0.0065	53.72	62.22	7558
RS		0.08	0.16	0.0044	54.95	8.00		0.10	0.19	0.0049	53.80	48.57	
Post-M		0.23	0.48	0.0071	54.80	21.33		0.24	0.46	0.0071	53.69	29.84	
Pre-M	S_{13}	0.03	0.24	0.0055	91.03	10.07	19973	0.03	0.17	0.0046	108.66	46.43	14174
RS		0.00	0.00	0.0000	91.53	4.63		0.00	0.00	0.0000	109.16	2.53	
Post-M		0.10	0.76	0.0055	91.03	204.69		0.14	0.83	0.0046	108.66	81.48	
Pre-M	S_{14}	0.09	0.32	0.0063	54.91	24.14	3335	0.12	0.34	0.0065	56.17	41.30	4232
RS		0.05	0.18	0.0047	55.00	7.44		0.05	0.14	0.0041	56.30	7.36	
Post-M		0.14	0.50	0.0071	54.87	29.17		0.18	0.52	0.0071	56.14	26.67	
Pre-M	S_{15}	0.06	0.22	0.0052	69.36	25.64		0.09	0.26	0.0058	65.95	44.12	

RS		0.02	0.08	0.0030	69.52	6.67	3153	0.02	0.07	0.0028	66.15	5.26	7189
Post-M		0.20	0.70	0.0061	69.30	13.15		0.22	0.66	0.0065	65.91	59.65	
Pre-M	S_{16}	0.08	0.27	0.0059	59.22	9.15	2356	0.09	0.26	0.0057	61.95	11.03	3165
RS		0.04	0.14	0.0040	59.33	9.15		0.04	0.11	0.0036	62.08	11.11	
Post-M		0.18	0.59	0.0069	59.16	21.49		0.21	0.63	0.0067	61.89	28.95	
Pre-M	S_{17}	0.62	0.33	0.0064	56.56	146.55	38904	0.62	0.33	0.0064	56.07	178.43	54943
RS		0.26	0.14	0.0041	56.70	150.00		0.28	0.15	0.0042	56.19	233.33	
Post-M		0.98	0.53	0.0071	56.52	391.18		0.98	0.52	0.0071	56.03	568.00	

Table 4: Calculation of NEFCI Score (C.F.=Construction Function, FVs=Fuzzy Values, EVS=Entropy Values, Aws=Assigned Weights, RSIs=Relative Sub-Indices)

Seasons		**C.F.**	**FVs**	**EVs**	**AWs**	**RSIs**	**NEFCI Score**	**C.F.**	**FVs**	**EVs**	**AWs**	**RSIs**	**NEFCI Score**
		p_{ji}	T_{ji}	$E_{ji}^{(2)}$	$w_{ji}^{(2)}$	Q_{ji}		p_{ji}	T_{ji}	$E_{ji}^{(2)}$	$w_{ji}^{(2)}$	Q_{ji}	
		2014-2015						**2015-2016**					
Pre-M	S_1	0.21	0.33	0.0029	117.5	76.92	28429	0.23	0.33	0.0029	117.54	145.83	49606
RS		0.11	0.18	0.0027	117.5	60.71		0.12	0.18	0.0027	117.56	133.33	
Post-M		0.31	0.49	0.0029	117.5	104.88		0.33	0.49	0.0029	117.53	142.86	
Pre-M	S_2	0.14	0.32	0.0029	117.8	34.48	13233	0.17	0.32	0.0029	117.82	57.78	21739
RS		0.05	0.13	0.0026	117.8	25.71		0.09	0.17	0.0027	117.85	51.72	
Post-M		0.23	0.55	0.0029	117.8	50.77		0.26	0.50	0.0029	117.81	75.00	
Pre-M	S_3	0.20	0.40	0.0029	117.2	31.52	9328	0.22	0.38	0.0029	117.28	47.22	14050
RS		0.06	0.12	0.0025	117.3	19.61		0.10	0.17	0.0027	117.31	37.78	
Post-M		0.25	0.49	0.0029	117.2	27.56		0.26	0.45	0.0029	117.28	34.78	
Pre-M	S_4	0.51	0.36	0.0029	118.8	85.37	28697	0.51	0.31	0.0028	118.85	104.17	44661
RS		0.21	0.15	0.0026	118.8	66.67		0.24	0.14	0.0026	118.88	109.09	
Post-M		0.69	0.49	0.0029	118.8	101.08		0.90	0.55	0.0029	118.84	162.50	
Pre-M	S_5	0.44	0.36	0.0029	117.9	82.19	28510	0.46	0.38	0.0029	117.99	103.03	34207
RS		0.17	0.14	0.0026	118.0	49.02		0.17	0.14	0.0026	118.03	60.00	
Post-M		0.59	0.49	0.0029	117.9	109.59		0.58	0.48	0.0029	117.99	126.87	
Pre-M	S_6	0.34	0.37	0.0029	118.1	47.96	16047	0.33	0.37	0.0029	118.18	61.73	19504
RS		0.13	0.14	0.0026	118.2	33.93		0.12	0.14	0.0026	118.22	42.55	
Post-M		0.45	0.49	0.0029	118.1	53.91		0.44	0.49	0.0029	118.17	60.75	
Pre-M	S_7	0.18	0.35	0.0029	119.4	29.89	9361	0.19	0.35	0.0029	119.46	39.47	12462
RS		0.05	0.10	0.0025	119.5	12.33		0.06	0.11	0.0025	119.50	16.42	
Post-M		0.29	0.55	0.0029	119.4	36.04		0.31	0.55	0.0029	119.45	48.42	
Pre-M	S_8	0.24	0.35	0.0029	116.8	73.91	25152	0.30	0.36	0.0029	116.83	132.35	40427
RS		0.13	0.19	0.0027	116.8	86.36		0.17	0.20	0.0027	116.85	136.84	
Post-M		0.32	0.46	0.0029	116.8	54.32		0.36	0.43	0.0029	116.83	76.81	
Pre-M	S_9	0.15	0.33	0.0029	117.8	26.83	8335	0.18	0.32	0.0029	117.82	35.90	11118
RS		0.06	0.13	0.0026	117.8	13.51		0.10	0.17	0.0027	117.85	21.62	
Post-M				0.0029	117.8	29.82		0.28	0.50	0.0029	117.81	36.84	

		0.24	0.53									
Pre-M	S_{10}	0.15	0.36	0.0029	118.1	20.95	13987	0.16	0.32	0.0029	118.14	19.38	17375
RS		0.05	0.13	0.0026	118.1	6.12		0.08	0.16	0.0026	118.16	9.52	
Post-M		0.21	0.51	0.0029	118.1	90.91		0.26	0.52	0.0029	118.13	118.18	
Pre-M	S_{11}	0.08	0.34	0.0029	118.5	17.91	10353	0.09	0.38	0.0029	118.58	22.39	11827
RS		0.02	0.10	0.0025	118.6	20.00		0.03	0.12	0.0026	118.62	26.00	
Post-M		0.12	0.55	0.0029	118.5	48.65		0.12	0.50	0.0029	118.57	51.35	
Pre-M	S_{12}	0.17	0.35	0.0029	117.1	16.00	5337	0.18	0.34	0.0029	117.12	62.22	16472
RS		0.08	0.16	0.0026	117.1	8.00		0.10	0.19	0.0027	117.14	48.57	
Post-M		0.23	0.48	0.0029	117.1	21.33		0.24	0.46	0.0029	117.12	29.84	
Pre-M	S_{13}	0.03	0.24	0.0028	134.4	10.07	29861	0.03	0.17	0.0027	134.47	46.43	17540
RS		0.00	0.00	0.0021	134.5	4.63		0.00	0.00	0.0021	134.55	2.53	
Post-M		0.10	0.76	0.0028	134.4	204.69		0.14	0.83	0.0027	134.47	81.48	
Pre-M	S_{14}	0.09	0.32	0.0029	118.4	24.14	7154	0.12	0.34	0.0029	118.44	41.30	8923
RS		0.05	0.18	0.0027	118.4	7.44		0.05	0.14	0.0026	118.47	7.36	
Post-M		0.14	0.50	0.0029	118.4	29.17		0.18	0.52	0.0029	118.43	26.67	
Pre-M	S_{15}	0.06	0.22	0.0027	122.9	25.64	5648	0.09	0.26	0.0028	122.94	44.12	13403
RS		0.02	0.08	0.0025	122.9	6.67		0.02	0.07	0.0024	122.98	5.26	
Post-M		0.20	0.70	0.0028	122.9	13.15		0.22	0.66	0.0029	122.93	59.65	
Pre-M	S_{16}	0.08	0.27	0.0028	121.2	9.15	4773	0.09	0.26	0.0028	121.22	11.03	6193
RS		0.04	0.14	0.0026	121.2	9.15		0.04	0.11	0.0025	121.25	11.11	
Post-M		0.18	0.59	0.0029	121.2	21.49		0.21	0.63	0.0029	121.21	28.95	
Pre-M	S_{17}	0.62	0.33	0.0029	118.3	146.55	81596	0.62	0.33	0.0029	118.39	178.43	115995
RS		0.26	0.14	0.0026	118.4	150.00		0.28	0.15	0.0026	118.42	233.33	
Post-M		0.98	0.53	0.0029	118.3	391.18		0.98	0.52	0.0029	118.38	568.00	

A careful analysis of tabulated values of **Table.2 (a, b)** reveals that, while calculating DEFCI score, the values $E_{13}^{(0)}$ at sampling spots S_{13} is based on the fancy assumption $0 \times \log 0 = 0$ which creates uncertainty in the quantification of information contained in fluoride concentration of ground water samples. However, the identification of most and least contaminated sampling spots based on Deluca and Termini **[16]** and proposed fuzzy and single valued neutrosophic entropy measures identical. This justifies the feasibility and compatibility of the proposed methodology of identifying the most and least contaminated sampling spots.

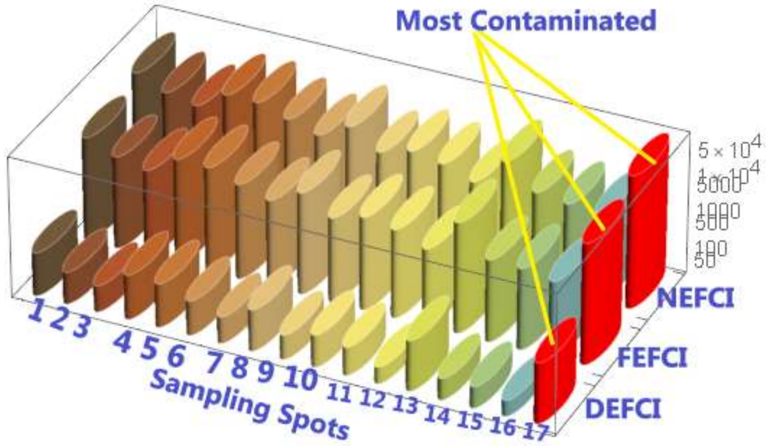


Fig.7 Identification of Most Contaminated Sampling Spot Based on DEFCI, FEFCI and NEFCI Scores at Seventeen Sampling Spots (2014-15)

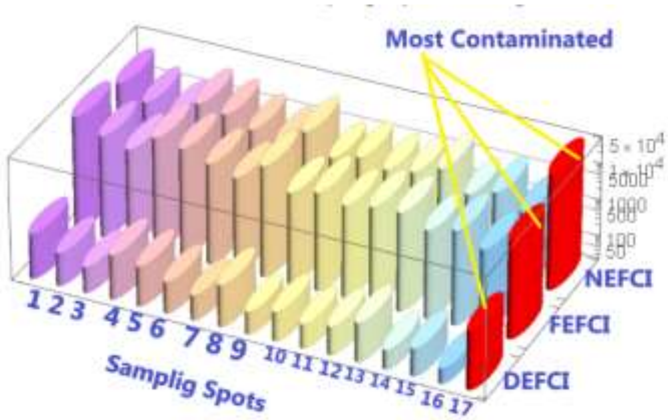


Fig.8 Identification of Most Contaminated Sampling Spot Based on DEFCI, FEFCI and NEFCI Scores at Seventeen Sampling Spots (2015-16)

7.4 Impact of elevated Fluoride concentration on Community Health

According to **[15]** and **[13]**, drinking water containing high concentrations of fluoride is one of the main sources of fluorosis. As per American Dental Association (ADA), fluoride in water is beneficial to people as it protects against cavities and reduces tooth decay by 20-40%. On contrary, just like any other substance we are exposed to in our everyday lives, fluoride carries toxic effects in certain quantities. Acute toxicity can occur after ingesting one or more doses of fluoride over a short time period which then leads to poisoning. The stomach is the first organ that is affected. First signs and symptoms are nausea, abdominal pain, bloody vomiting and diarrhea. Based on extensive studies, probable toxic dose (PTD) was defined at 5 mg/kg of body mass. The PTD is the minimal dose that could trigger serious and life-threatening signs and symptoms and requires immediate treatment and hospitalization **[11]**.

To evaluate the impact of elevated levels of Fluoride on public health, a survey was conducted in the selected areas and interaction with the public was done. To verify the facts, local Hospitals/Clinics and public health department were visited and authorities were consulted to understand the nature of health problem people have been suffering. During these investigations, it was found out that residents, who have been using unfiltered/untreated groundwater for drinking, have been suffering from dental Fluorosis or skeletal fluorosis, which mostly damage their bones & joints. Many residents were observed with white streaks or specks in their teeth enamel. In skeletal fluorosis, bones become hardened and less elastic that increases the risk of fracture. Residents were found to be complaining about pain in bones and joints. Though this data could not be considered as a base for medical investigations yet it could be measured as a connecting link between fluorosis and drinking water with higher fluoride concentration. Similar kind of studies were conducted on Factors influencing the relationship between fluoride in drinking water and dental fluorosis and results of the systematic review have shown that dental fluorosis affects individuals of all ages, with the highest prevalence below 11, while the impact of other factors (gender, environmental conditions, diet and dental caries) was inconclusive. Meta-regression analysis, based on information collected through systematic review, indicates that both fluoride in drinking water and temperature influence dental fluorosis significantly and that these studies might be affected by publication bias. Findings show that fluoride negatively affects people's health in less developed countries **[14]**.

Besides, fluoride acts as neurotoxin that could carry adverse impact on human development. As per, International Association of Oral medicine and Toxicology (IAOMT), excessive use of added fluoride may create skin problems, arteriosclerosis, arterial calcification, high blood pressure, myocardial damage and some reproductive issues such as lower fertility and early puberty in girls.

CONCLUSION

It has been concluded that in Kangra district fluoride concentration in groundwater has been increased since 2014 to 2016 and higher concentration has been observed during post-monsoon season consecutively for both years. Although elevated levels of fluoride in drinking water have shown adverse impact on people residing in this region; however no consistent pattern has been observed during these studies for these health problems. Many other factors like nutrition can play a significant role in weakening health condition also. By considering elevated levels of fluoride in drinking water & health related issues, it is advisable for the public to treat the water before drinking to avoid any health complications. State pollution control board should intervene in this matter and to make sure that guidelines laid down by pollution board has been followed up regularly by the industries before disposing off any wastewater in to any adjacent water body or open field.

In 2014-15, fluoride concentration varied from 0.06 to 0.85 mg/l during pre-monsoon; varied from 0.02 to 0.36 mg/l during rainy season; varied from 0.15 to 1.33 mg/l during post-monsoon. In 2015-16, fluoride concentration of groundwater samples varied from 0.065 to 0.91 mg/l during pre-monsoon; varied from 0.025 to 0.42 mg/l during rainy season; varied from 0.19 to 1.42 mg/l during post-monsoon. Most of the sampling spots during all seasons have shown marginal value of fluoride. Elevated levels of fluoride in groundwater for prolonged time cause many negative impacts on public health such as fluorosis, discoloration, osteoporosis, cardiovascular disorders and skeletal deformities. These studies have shown that local residents have been suffering from these kinds of health issues due to elevated fluoride level in groundwater and advised to use proper water purification techniques to avoid any health complications.

ACKNOWLEDGEMENT

Authors are highly grateful to Health department, Kangra and HPSPCB, Parwanoo, Himachal, for supporting in conducting the current investigations.

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Annexure.1Table 1 pH value of Seasonal Parameters Collected from Various Sampling Spots

Sampling	**Pre-Monsoon**		**Rainy Season**		**Post-Monsoon**	
Spots	*2014-2015*	2015-2016	*2014-2015*	2015-2016	*2014-2015*	2015-2016
S_1	7.39	7.24	7.28	7.15	7.41	7.35
S_2	7.58	7.45	7.35	7.29	7.65	7.52
S_3	7.92	7.72	7.51	7.45	8.27	8.15
S_4	7.82	7.72	7.45	7.33	7.93	7.8
S_5	7.73	7.66	7.51	7.45	7.73	7.67
S_6	7.98	7.81	7.56	7.47	8.15	8.07
S_7	7.87	7.76	7.73	7.67	8.11	7.95
S_8	7.46	7.34	7.22	7.19	7.81	7.69
S_9	7.82	7.78	7.74	7.74	8.14	8.14
S_{10}	6.65	6.57	6.51	6.51	6.89	6.89
S_{11}	7.67	7.67	7.25	7.25	7.37	7.37
S_{12}	7.58	7.45	7.45	7.35	8.31	8.24
S_{13}	7.24	7.14	6.73	6.67	7.02	6.91
S_{14}	7.58	7.46	6.61	6.57	6.76	6.65
S_{15}	6.87	7.34	6.75	6.62	6.29	6.81
S_{16}	8.42	8.36	7.82	7.72	8.21	8.14
S_{17}	7.58	7.51	7.24	7.18	7.34	7.25

Received: Dec. 20, 2021. Accepted: April 2, 2022.