

On the Association with Intelligent Reflecting Surfaces in Spatially Random Networks

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Abstract—Intelligent reflecting surfaces (IRSs) have the potential of increasing the coverage and energy efficiency of future wireless networks. In this paper, we study the performance of an IRS-aided network in the presence of blockages from a large-scale point-of-view, where multiple IRSs are randomly distributed. We model the blockages and IRSs with a line segment process and take into account the fading channels from both the direct and indirect links. The performance of a receiver is studied based on specific association policies, i.e. which IRS(s) should be activated to reflect the signal to the receiver. Specifically, we consider the association with a random IRS, the closest IRS or with all available IRSs in the cell. A complete analytical framework in terms of the outage probability is developed using tools from stochastic geometry. We show that each association policy has its advantages, which heavily depend on the network's parameters such as the cell-radius, the number of IRS elements and the blockage density.

Index Terms—Intelligent reflecting surface, stochastic geometry, outage probability.

I. INTRODUCTION

An intelligent reflecting surface (IRS) refers to a planar array of flat - and mostly passive - elements, which reflect the incident signal with the help of a dedicated controller [1]. Through these reflections, an IRS provides the means to intelligently control the propagation environment and boost the received signal at the designated receivers. Furthermore, as these elements are passive, the network's overall power consumption can be reduced and thus increase the energy efficiency [2].

An important aspect of IRS-aided communications is how to jointly design the access point's (AP) precoder and the IRS's phase shifts, e.g. [2], [3]. The work in [2] consider a multi-user scenario and maximizes the energy efficiency by designing the AP's transmit power allocation and the IRS reflections. In [3], the authors maximize the secrecy rate by jointly taking into account the AP's active transmit beamforming and the IRS's passive reflect beamforming. However, these approaches assume full channel state information (CSI), which is not always practical. As such, there are some works that focus on partial to no CSI approaches [4], [5]. In [4], the authors propose low-complexity coding-based and selection-based schemes, which can achieve high energy efficiency as well as full diversity order. The work in [5] proposes an IRS-aided opportunistic

beamforming scheme and derive the achieved sum-rate scaling laws; it is shown that the proposed scheme outperforms the conventional one.

Most works in the literature take into account simple point-to-point topologies, e.g. [2]–[5]. However, the study from a large-scale point-of-view has been limited. Using stochastic geometry, the work in [6] evaluates the spatial throughput of IRS-aided networks by also considering the interference generated by IRSs. In [7], the performance of an IRS-aided millimeter wave network is investigated for a two-step association policy, in terms of rate and energy efficiency. Both [6] and [7], assume that the IRSs are distributed in space based on a Poisson point process. Due to its flat planar surface, a more realistic approach is to model the IRSs in space as a line segment process [8], [9]. Based on this model, an analytical framework is developed in [8], which provides the probability a random line segment will reflect the AP's signal. The work in [9] considers the line segment model to study the deployment of IRSs in cellular networks and shows how it improves the coverage region of a base station.

Similar to the work in [9], in this paper, we consider the performance of IRS-aided networks, where the IRSs are assumed to be employed on the facades of buildings. In contrast to [9], we take into account the fading channels from both the direct and indirect links. Moreover, unlike the works in [6] and [7], we investigate how the association policy with the employed IRSs affects the performance of the network. The association policy determines which IRS(s) will be activated to reflect the signal to the receiver. In particular, we study the association with a random IRS, with the closest IRS as well as with all IRSs available in the cell. All policies are of low-complexity as they require no CSI and the IRSs employs random phase shifts. We provide a complete analytical framework in terms of the outage probability, using tools from stochastic geometry. Our results show that each of the studied association policies has its merits and its consideration heavily depends on the cell's size and the blockage density.

II. SYSTEM MODEL

A. Topology

Consider an IRS-aided single-cell network, modeled by a circular disk of radius ρ . The AP, located at the origin, attempts to serve a single receiver, located at a distance r_0 and at some random direction θ_0 ; without loss of generality, we assume

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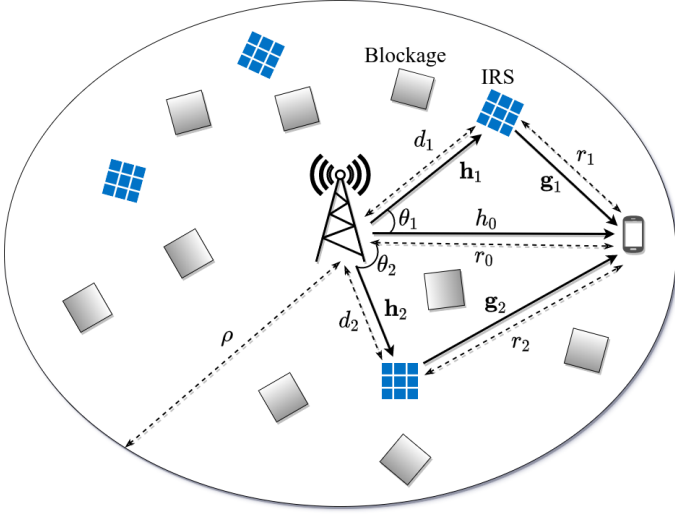


Fig. 1. The considered system model with $N = 3$ and $K = 2$.

$\theta_0 = 0$. Both the AP and the receiver are equipped with a single antenna and the AP transmits with fixed power P_t . The direct path between the AP and the receiver, may be blocked by blockages within the cell, e.g. buildings. The blockages are modeled by a line segment process of two dimensional lines with random lengths and orientations, where their midpoints are distributed according to a PPP Ψ of density λ_O [10]. We assume that a fraction $\kappa \in [0, 1]$ of the blockages, employ an IRS with M reflecting elements on one of their two sides [9]; we denote the density of these blockages by $\lambda_I = \kappa \lambda_O$. As a result, depending on the line-of-sight (LOS) paths, the AP can achieve communication with the receiver through a direct and/or multiple indirect links. Specifically, if there are N LOS indirect paths available, the AP associates with $K \leq N$ to serve the receiver. Therefore, the K IRSs are activated through their controller while the remaining $K - N$ remain inactive. The considered network model is illustrated in Fig. 1.

B. Reflection Probability

The probability of having a LOS path, at a distance d , in an area with blockages of density λ is [10]

$$p_L(d, \lambda) = \exp(-\beta(\lambda)d), \quad (1)$$

where $\beta(\lambda)$ is a non-negative function, which models the characteristics of the blockages (e.g. density, length), given by

$$\beta(\lambda) = \frac{2L}{\pi} \lambda, \quad (2)$$

where L is the length of the blockages. Furthermore, the probability of establishing an indirect LOS link through the k -th IRS, i.e., the reflection probability, is given by [9]

$$p_R(d_k, \theta_k) = \frac{1}{2\pi} p_L(d_k, (1 - \kappa)\lambda_O) p_L(r_k, \lambda_O) \times \left(\pi - \cos^{-1} \left(\frac{d_k - r_0 \cos(\theta_k)}{r_k} \right) \right), \quad (3)$$

where θ_k is the angle formed by the lines AP-IRS and AP-receiver (see Fig. 1), d_k the distance between the AP and the k -th IRS and r_k the distance between the k -th IRS and the receiver. By defining the function

$$\Delta(x, y, \theta) \triangleq \sqrt{x^2 + y^2 - 2xy \cos(\theta)}, \quad (4)$$

then by the law of cosines, $r_k = \Delta(d_k, r_0, \theta_k)$. Note that the reflection probability takes into account: *i*) the links from the AP to the k -th IRS and the k -th IRS to the receiver are in LOS, *ii*) the side of the blockage that faces the AP and the receiver is equipped with an IRS, and *iii*) the angle at which the signal will be reflected.

C. Channel Model

All wireless links suffer from both small-scale block fading and large-scale path loss effects. We consider Rayleigh fading¹ and define by $h_{k,i}$ and $g_{k,i}$ the fading coefficients from the AP to the i -th element of the k -th IRS and from the i -th element of the k -th IRS to the receiver, respectively. Furthermore, we define by h_0 the fading coefficient of the direct link between the AP and the receiver. The fading coefficients follow a complex Gaussian distribution with zero mean and unit variance, i.e. $h_{k,i}, g_{k,i}, h_0 \sim \mathcal{CN}(0, 1)$. The path loss model assumes that the received power is proportional to $d^{-\alpha}$, where $\alpha > 2$ is the path loss exponent. Hence, the path loss for the k -th indirect link is $(d_k r_k)^{-\alpha}$.

D. Signal-to-Noise Ratio

Each element of the k -th IRS randomly rotates the phase of the incident signal and we denote by

$$\Phi_k = \text{diag}[\exp(j\phi_{k,1}) \exp(j\phi_{k,2}) \cdots \exp(j\phi_{k,M})], \quad (5)$$

the diagonal matrix containing the independent and identically distributed phase shift variables $\phi_{k,i} \in [0, 2\pi)$. Therefore, assuming there are $N > 0$ LOS IRSs and $K \leq N$ are selected, the signal at the receiver is

$$y = \sqrt{P_t} \left(\mathbb{1}_{D_L} r_0^{-\frac{\alpha}{2}} h_0 + \sum_{k=1}^K (d_k r_k)^{-\frac{\alpha}{2}} \mathbf{h}_k^\top \Phi_k \mathbf{g}_k \right) x + n, \quad (6)$$

where $n \sim \mathcal{CN}(0, \sigma^2)$ is the additive white Gaussian noise with variance σ^2 , $\mathbf{h}_k = [h_{k,1} \ h_{k,2} \ \cdots \ h_{k,M}]^\top$ and $\mathbf{g}_k = [g_{k,1} \ g_{k,2} \ \cdots \ g_{k,M}]^\top$. Also, $\mathbb{1}_{D_L}$ is an indicator function, describing whether or not there's an LOS link between the receiver and the AP (event D_L). Then, the instantaneous signal-to-noise ratio (SNR) at the receiver via K IRSs is

$$\Gamma = \frac{P_t}{\sigma^2} \left| \mathbb{1}_{D_L} r_0^{-\frac{\alpha}{2}} h_0 + \sum_{k=1}^K (d_k r_k)^{-\frac{\alpha}{2}} H_k \right|^2, \quad (7)$$

where

$$H_k = \sum_{i=1}^M h_{k,i} g_{k,i} \exp(j\phi_{k,i}), \quad (8)$$

is the channel from the M elements of the k -th IRS.

¹We consider Rayleigh fading for simplicity but this work could also be extended with other models.

III. IRS ASSOCIATION POLICIES

In this section, we present the main analytical results of this paper. Specifically, we derive the outage probability of each association policy, i.e. $\mathbb{P}(\Gamma < \tau)$, where τ is a pre-defined threshold.

We first state some preliminary results, which will assist in the derivation of the main analytical framework.

A. Preliminaries

Conditioned on an LOS path between the AP and the receiver and no indirect paths, the outage probability of the direct link is

$$P_D(\tau|D_L) = 1 - \exp\left(-\frac{\sigma^2 \tau r_0^\alpha}{P_t}\right), \quad (9)$$

which follows since $\mathbb{1}_{D_L} = 1$, $N = 0$ and $|h_0|^2 \sim \exp(1)$. Moreover, the intensity measure of the indirect LOS links is

$$\Lambda(d) = \lambda_I \int_{-\pi}^{\pi} \int_0^d p_R(r, \theta) r dr d\theta, \quad (10)$$

where $p_R(r, \theta)$ is the reflection probability given by (3). Therefore, the probability mass function (PMF) of N is

$$\Pi(N) = \exp(-\Lambda(\rho)) \frac{(\Lambda(\rho))^N}{N!}, \quad (11)$$

where $\Lambda(\rho)$ is the average number of indirect LOS links in a cell of radius ρ and is given by (10). Then, the probability of having at least one indirect LOS link, i.e. $N \geq 1$, is

$$p_I = 1 - \Pi(0) = 1 - \exp\left(-\lambda_I \int_{-\pi}^{\pi} \int_0^\rho p_R(r, \theta) r dr d\theta\right). \quad (12)$$

B. Random IRS Association

The random IRS association (RA) policy selects and activates a random IRS. This policy has very low complexity and does not require any CSI. The following proposition provides the outage probability through the direct link and a random indirect link.

Proposition 1. *Conditioned on a direct path and $N \geq 1$ indirect paths, the outage probability via the direct link and a randomly selected IRS is*

$$P_{RA}^B(\tau|D_L) \approx 1 - \frac{1}{\pi \rho^2} \int_0^\rho \int_{-\pi}^\pi r \exp\left(-\frac{\sigma^2 \tau}{P_t \Xi(r, \theta)}\right) dr d\theta, \quad (13)$$

where

$$\Xi(r, \theta) \triangleq r_0^{-\alpha} + M r^{-\alpha} \Delta(r, r_0, \theta)^{-\alpha}, \quad (14)$$

and $\Delta(r, r_0, \theta)$ is given by (4).

Proof. See Appendix A. \square

It is clear that when there is no IRS available, i.e. communication is done only through the direct path, (13) is reduced to

(9) as $\Xi(r, \theta) = r_0^{-\alpha}$. Also, when there is no direct path, the outage probability via the indirect path is

$$P_{RA}^I(\tau) = P_B(\tau|\overline{D_L}), \quad (15)$$

with $\Xi(r, \theta) \triangleq M r^{-\alpha} \Delta(r, r_0, \theta)^{-\alpha}$.

By taking into account the above analytical expressions, we can derive the system's outage probability under the RA policy. In particular, the system's outage probability when an IRS is randomly selected is given by

$$\begin{aligned} P_{RA}(\tau) &= p_I P_L(r_0, \lambda_O) P_{RA}^B(\tau|D_L) \\ &\quad + (1 - p_L(r_0, \lambda_O)) P_{RA}^I(\tau) \\ &\quad + p_L(r_0, \lambda_O) (1 - p_I) P_D(\tau|D_L) \\ &\quad + (1 - p_I) (1 - p_L(r_0, \lambda_O)). \end{aligned} \quad (16)$$

The proof for the above result is given in Appendix B. It is important to note that the outage probability asymptotically converges to a constant floor for $P_t \rightarrow \infty$. In this case, the outage is dominated by the probability of no direct and no indirect paths, i.e.

$$\lim_{P_t \rightarrow \infty} P_{RA}(\tau) = (1 - p_I) (1 - p_L(r_0, \lambda_O)). \quad (17)$$

In what follows, we consider the case $r_0 \gg \rho$, i.e., the receiver is located far from the cell's boundary. Then, $\Delta(r, r_0, \theta) \approx r_0$ and (13) becomes

$$P_{RA}^B(\tau|D_L) \approx 1 - \frac{2}{\rho^2} \int_0^\rho r \exp\left(-\frac{\sigma^2 \tau r_0^\alpha}{P_t (M r^{-\alpha} + 1)}\right) dr. \quad (18)$$

We now turn our attention to the outage probability via the indirect path (expression (15)). By considering the case $r_0 \gg \rho$, we have

$$\begin{aligned} P_{RA}^I(\tau) &\approx 1 - \frac{2}{\rho^2} \int_0^\rho \exp\left(-\frac{\sigma^2 \tau r_0^\alpha}{M P_t}\right) r dr \\ &= 1 - \frac{2}{\alpha \rho^2 r_0^2} \left(\frac{M P_t}{\sigma^2 \tau}\right)^{2/\alpha} \gamma\left(\frac{2}{\alpha}, \frac{\sigma^2 \tau r_0^\alpha}{M P_t} \rho^\alpha\right), \end{aligned} \quad (19)$$

which follows from [12, 3.381] and $\gamma(\cdot, \cdot)$ is the lower incomplete gamma function [12]. By setting $\alpha = 2$, we get

$$P_{RA}^I(\tau) \approx 1 - \frac{M P_t}{\sigma^2 \tau \rho^2 r_0^2} \left(1 - \exp\left(-\frac{\sigma^2 \tau r_0^2 \rho^2}{M P_t}\right)\right), \quad (20)$$

which follows from $\gamma(1, x) = 1 - \exp(-x)$.

C. Closest IRS Association

A more practical approach, is to associate the AP with the IRS which is closest to it. Again, the closest IRS association (CA) policy does not need CSI. The only knowledge the AP requires, is the locations of the IRSs, which can be obtained a-priori. Based on this policy, we can state the following.

Proposition 2. *Conditioned on a direct path, the outage probability via the direct link and the closest IRS is*

$$\begin{aligned} P_{CA}^B(\tau|D_L) &\approx \int_0^\rho \int_{-\pi}^\pi \left(1 - \exp\left(-\frac{\sigma^2 \tau}{P_t \Xi(r, \theta)}\right)\right) \\ &\quad \times f(r, \theta) dr d\theta, \end{aligned} \quad (21)$$

where

$$f(r, \theta) = \lambda_I p_R(r, \theta) r \exp(-\Lambda(r)), \quad (22)$$

and $\Xi(r, \theta)$ is given by (14).

Proof. See Appendix C. \square

As above, when no IRS is available, (21) is reduced to (9) with $\Xi(r, \theta) = r_0^{-\alpha}$. Moreover, the outage probability only via the closest available indirect path is

$$P_{CA}^I(\tau) = P_{CA}^B(\tau | \overline{D_L}), \quad (23)$$

with $\Xi(r, \theta) \triangleq M r^{-\alpha} \Delta(r, r_0, \theta)^{-\alpha}$. Therefore, the overall outage probability achieved by the CA policy is given by

$$\begin{aligned} P_{CA}(\tau) &= p_L(r_0, \lambda_O) P_{CA}^B(\tau | D_L) + P_{CA}^I(\tau) \\ &\quad + p_L(r_0, \lambda_O) (1 - p_I) P_D(\tau | D_L) \\ &\quad + (1 - p_I) (1 - p_L(r_0, \lambda_O)). \end{aligned} \quad (24)$$

The proof is omitted as it follows similar arguments to the proof for the RA policy (see Appendix B). It is easy to see that, for high SNRs, $P_{CA}(\tau)$ converges to the same error floor as $P_{RA}(\tau)$, given by (17).

For the case where $r_0 \gg \rho$, the reflection probability can be approximated as

$$p_R(r, \theta) = \frac{|\theta|}{2\pi} p_L(r, (1 - \kappa)\lambda_O) p_L(r_0, \lambda_O), \quad (25)$$

since $\Delta(r, r_0, \theta) \approx r_0$ and $r/r_0 \approx 0$ (as $r < \rho \ll r_0$), which results in $\pi - \cos^{-1}\left(\frac{r}{r_0} - \cos(\theta)\right) = |\theta|$. Then, the intensity measure can be written as

$$\Lambda(r) = \frac{\lambda_I}{2\pi} p_L(r_0, \lambda_O) \int_{-\pi}^{\pi} \int_0^r p_L(v, (1 - \kappa)\lambda_O) v dv |\theta| d\theta \quad (26)$$

$$= \frac{\lambda_I \pi p_L(r_0, \lambda_O)}{2\beta((1 - \kappa)\lambda_O)^2} \left(1 - \frac{1 + r\beta((1 - \kappa)\lambda_O)}{\exp(r\beta((1 - \kappa)\lambda_O))}\right). \quad (27)$$

As such, we can write (21) as

$$\begin{aligned} P_{CA}^B(\tau | D_L) &\approx \frac{\lambda_I \pi}{2} p_L(r_0, \lambda_O) \int_0^\rho \frac{p_L(r, (1 - \kappa)\lambda_O)}{\exp(\Lambda(r))} \\ &\quad \times \left(1 - \exp\left(-\frac{\sigma^2 \tau r_0^\alpha}{P_t(Mr^{-\alpha} + 1)}\right)\right) r dr. \end{aligned} \quad (28)$$

Note that, for the case $\kappa = 1$, i.e. when all the blockages are equipped with an IRS, (26) reduces to

$$\Lambda(r) = \frac{\lambda_I \pi}{4} p_L(r_0, \lambda_O) r^2, \quad (29)$$

which in turn, reduces (21) to

$$\begin{aligned} P_{CA}^B(\tau | D_L) &\approx 1 - \exp\left(-\frac{\lambda_I \pi}{4} p_L(r_0, \lambda_O) \rho^2\right) \\ &\quad - \frac{\lambda_I \pi}{2} p_L(r_0, \lambda_O) \int_0^\rho \exp\left(-\frac{\sigma^2 \tau r_0^\alpha}{P_t(Mr^{-\alpha} + 1)} - \Lambda(r)\right) r dr. \end{aligned} \quad (30)$$

The approximation for $P_{CA}^I(\tau)$ follows similar steps and so it is omitted.

D. Distributed Reflections

The distributed reflections (DR) policy employs all available IRSs to reflect the AP's signal. Therefore, it can potentially increase the channel gain at the receiver. On the other hand, it has much higher power consumption compared to the RA and CA policies as more IRSs are activated. Moreover, in a multi-cell scenario, this policy will generate more interference.

Proposition 3. *Conditioned on a direct path and K indirect paths, the outage probability via the direct link and all K IRSs is*

$$\begin{aligned} P_{DR}^B(\tau, K | D_L) &\approx 1 - \frac{1}{\pi^K \rho^{2K}} \int \cdots \int_R \int \cdots \int_\Theta r_1 \cdots r_K \\ &\quad \times \exp\left(-\frac{\sigma^2 \tau}{P_t \Xi(K, r_0)}\right) dr_1 \cdots dr_K d\theta_1 \cdots d\theta_K, \end{aligned} \quad (31)$$

where $R = (0, \rho)^K$, $\Theta = (-\pi, \pi)^K$ and

$$\Xi(K, r_0) \triangleq r_0^{-\alpha} + M \sum_{k=1}^K r_k^{-\alpha} \Delta(r_k, r_0, \theta_k)^{-\alpha}, \quad (32)$$

with $\Delta(r_k, r_0, \theta_k)$ given by (4).

Proof. Since a direct link is available together with K indirect links, we define $Z \triangleq r_0^{-\frac{\alpha}{2}} h_0 + \sum_{k=1}^K (d_k \Delta(d_k, r_0, \theta_k))^{-\frac{\alpha}{2}} H_k$. Then, the final result can be derived using similar steps given in Appendix A. \square

Therefore, the average outage probability achieved by the DR policy is

$$\begin{aligned} P_{DR}(\tau) &= p_L(r_0, \lambda_O) \sum_{K=1}^{\infty} \Pi(K) P_{DR}^B(\tau, K | D_L) \\ &\quad + (1 - p_L(r_0, \lambda_O)) \sum_{K=1}^{\infty} \Pi(K) P_{DR}^I(\tau, K) \\ &\quad + p_L(r_0, \lambda_O) (1 - p_I) P_D(\tau | D_L) \\ &\quad + (1 - p_I) (1 - p_L(r_0, \lambda_O)), \end{aligned} \quad (33)$$

where $P_{DR}^I(\tau, K) = P_{DS}^B(\tau, K | \overline{D_L})$, $P_D(\tau | D_L)$ is given by (9) and $\Pi(K)$ is given by (11). Note that, in this case, we need to consider the probability of having K IRSs available. As such, the outage probability is evaluated over all possible values of K . For high SNRs, the DR policy converges to the same error floor as the other two policies, given by (17).

Finally, to simplify (31), we consider the case $r_0 \gg \rho$. Therefore, we have

$$\begin{aligned} P_{DR}^B(\tau, K | D_L) &\approx 1 - \frac{2^K}{\rho^{2K}} \int \cdots \int_R r_1 \cdots r_K \\ &\quad \times \exp\left(-\frac{\sigma^2 \tau r_0^\alpha}{P_t(1 + M \sum_{k=1}^K r_k^{-\alpha})}\right) dr_1 \cdots dr_K. \end{aligned} \quad (34)$$

Moreover, as $r_k < \rho \ll r_0$, we assume $\sum_{k=1}^K r_k^{-\alpha} \approx K r_1^{-\alpha}$, and so

$$\begin{aligned} P_{DR}^B(\tau, K | D_L) &\approx 1 - \frac{2}{\rho^2} \int_0^\rho \exp\left(-\frac{\sigma^2 \tau r_0^\alpha}{P_t(1 + MK r_1^{-\alpha})}\right) \\ &\quad \times r_1 dr_1. \end{aligned} \quad (35)$$

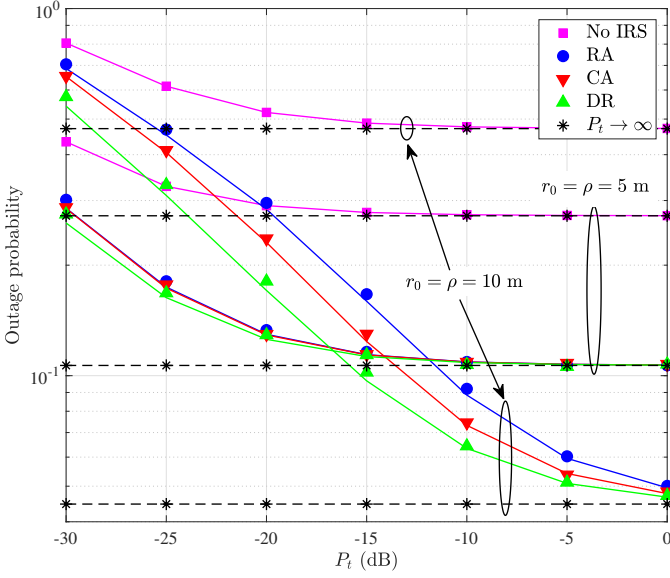


Fig. 2. Outage probability versus transmit power P_t ; theoretical and simulation results are depicted with lines and markers, respectively.

We show in the following section that this is a tight approximation.

IV. NUMERICAL RESULTS

In this section, we present our numerical results. The performance of the considered setup is compared to the case where there is no IRS, which we will refer to as the conventional case. The outage probability for the conventional case is

$$P_C(\tau) = 1 - p_L(r_0, \lambda_O) \exp\left(-\frac{\sigma^2 \tau r_0^\alpha}{P_t}\right). \quad (36)$$

The following parameters are considered, unless otherwise stated: $\alpha = 2$, $r_0 = 10$ m, $\lambda_O = 0.1$, $\kappa = 0.6$, $L = 1$, $M = 8$, $\sigma^2 = -50$ dB and $\rho = 10$ m. These parameters were chosen for the sake of presentation and a different choice of values will provide similar observations/conclusions.

Fig. 2 depicts the outage probability of each association policy in terms of the transmit power P_t . We consider the case where the receiver is at the edge of the cell, i.e. $r_0 = \rho$. It is clear that the employment of IRSs provides significant gains to the performance. With regards to the association policies, for $r_0 = \rho = 10$ m, the DR policy provides the best performance whereas the RA policy provides the worst. However, in the case of a smaller cell ($\rho = 5$ m), all association policies achieve the same performance; smaller cells have fewer available IRSs and so the performance is dominated by a single IRS. In both cases, the policies converge to the same error floor. A larger cell provides a lower error floor as more IRSs are employed. In contrast, a smaller cell is preferable when only the direct link is available. Finally, our theoretical results (lines) are tight approximations of the simulation results (markers), which validates our analysis.

Fig. 3 illustrates the effect of the parameter κ on the outage probability. Note that $\kappa = 0$ refers to the case where no IRS is

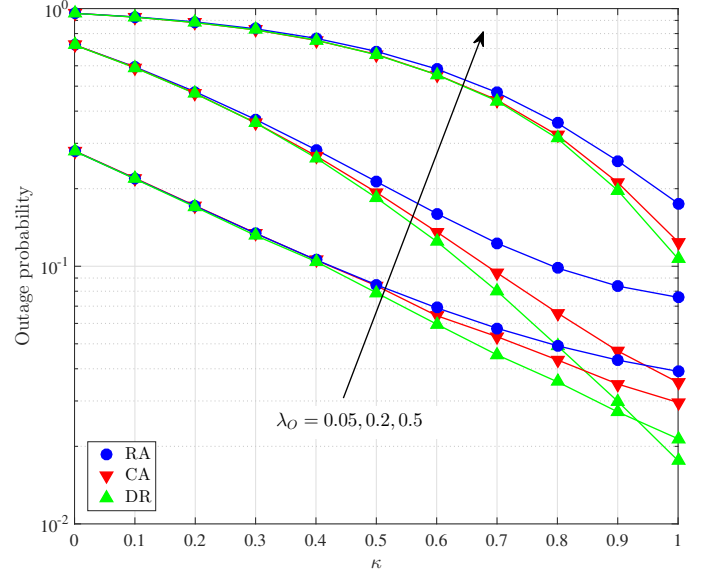


Fig. 3. Outage probability versus parameter κ ; $P_t = -10$ dB.

available. As expected, as κ increases (more IRSs employed), the outage probability decreases. For small values of κ , the choice of policy makes no difference to the performance. This is because the reflection probability in these cases is small, and so the performance is dominated by a single IRS. This is also true for highly dense networks ($\lambda_O = 0.5$), where the difference in performance is relatively small for any value of κ . On the other hand, for $\lambda_O = 0.2$ and $\kappa = 1$, it is obvious that the DR policy provides significant gains compared to the other policies.

Finally, Fig. 4 shows the outage probability with respect to the number of elements M at the IRSs. As expected, the outage probability decreases with M . Moreover, as $M \rightarrow \infty$, the performance converges to the case $P_t \rightarrow \infty$. We consider the scenario where the receiver is located further away from the edge of the cell, i.e. $r_0 > \rho$. It is obvious that as r_0 increases, the policies achieve similar performances. The figure also depicts the analytical approximations for the case $r_0 \gg \rho$. It is important to point out that the approximations are very close to the actual values, even for $r_0 = 15$ m.

V. CONCLUSION

In this paper, we considered the performance of IRS-aided networks, where the IRSs are assumed to be employed on the facades of buildings. By taking into account the fading channels from both the direct and indirect links, we investigated several low-complexity association policies: with a random IRS, the closest IRS and with all available IRSs in the cell. The derived analytical framework in terms of the outage probability employed tools from stochastic geometry and showed that each association policy has its merits. Specifically, for small coverage areas or large blockage density, activating multiple IRSs is not efficient and suffices to activate a single one. This can result in higher energy efficiency and also reduce interference in multi-cell scenarios.

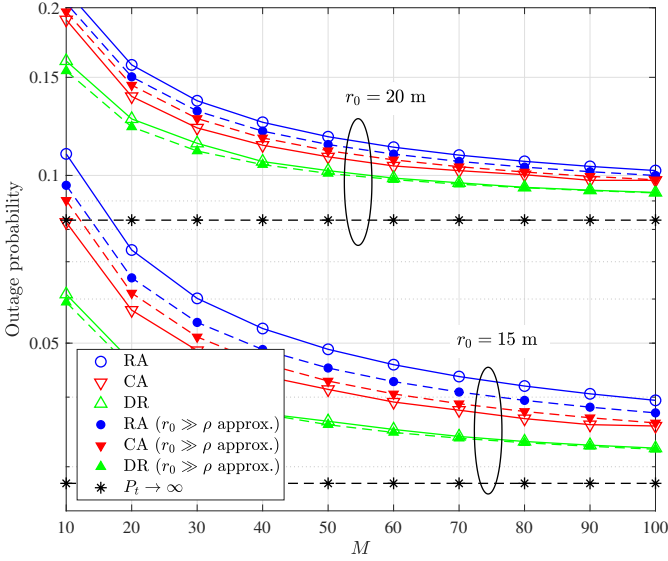


Fig. 4. Outage probability versus number of elements M ; $P_t = -10$ db, $\kappa = 1$.

APPENDIX A

PROOF OF PROPOSITION 1

As there is a direct link and $K > 0$, the SNR can be written as $\Gamma = \frac{P_t}{\sigma^2} |Z|^2$, where

$$Z \triangleq r_0^{-\frac{\alpha}{2}} h_0 + (d_k \Delta(d_k, r_0, \theta_k))^{-\frac{\alpha}{2}} H_k, \quad (37)$$

where k denotes the index of the randomly selected IRS. The random variable $H_k = \sum_{i=1}^M h_{k,i} g_{k,i} \exp(j\phi_{k,i})$ can be approximated as a complex Gaussian distributed random variable with zero mean and variance M , i.e. $H_k \sim \mathcal{CN}(0, M)$ [4]. By the scaling property, we have $(d_k \Delta(d_k, r_0, \theta_k))^{-\frac{\alpha}{2}} H_k \sim \mathcal{CN}(0, (d_k \Delta(d_k, r_0, \theta_k))^{-\alpha} M)$. Using similar arguments, it follows that $Z \sim \mathcal{CN}(0, r_0^{-\alpha} + M(d_k \Delta(d_k, r_0, \theta_k))^{-\alpha})$.

Therefore, the outage can be evaluated as follows

$$\begin{aligned} P_{\text{RA}}^B(\tau|D_L) &= \mathbb{P}\{\Gamma < \tau|D_L\} = \mathbb{P}\left\{|Z|^2 < \frac{\tau\sigma^2}{P_t}\right\} \\ &\approx \mathbb{E}_\Psi \left\{1 - \exp\left(-\frac{\tau\sigma^2}{P_t} \frac{1}{\Xi(d, \theta)}\right)\right\}, \end{aligned} \quad (38)$$

where $\Xi(d, \theta) = r_0^{-\alpha} + M(d \Delta(d, r_0, \theta))^{-\alpha}$. Then, by taking the expectation over Ψ , we have

$$\begin{aligned} P_{\text{RA}}^B(\tau|D_L) &\approx 1 - \int_{-\pi}^{\pi} \int_0^{\rho} \exp\left(-\frac{\tau\sigma^2}{P_t} \frac{1}{\Xi(r, \theta)}\right) \\ &\quad \times f_d(r) p(\theta) dr d\theta, \end{aligned} \quad (39)$$

where $f_d(r) = 2r/\rho^2$ and $p(\theta) = 1/2\pi$ are the probability distribution functions (PDFs) of d and θ , respectively. After some algebraic manipulations, the proposition is proven.

APPENDIX B

The average outage probability is derived by considering all the possible events that result in an outage. In particular,

$$\begin{aligned} P_{\text{RA}}(\tau) &= \mathbb{P}\{N = 0, \overline{D_L}\} + \mathbb{P}\{N = 0, D_L, \Gamma < \tau|D_L\} \\ &\quad + \mathbb{P}\{N \geq 1, \overline{D_L}, \Gamma < \tau\} + \mathbb{P}\{N \geq 1, D_L, \Gamma < \tau|D_L\}, \end{aligned} \quad (40)$$

where $\overline{D_L}$ is the complementary of the event D_L , i.e. no direct link is available. The first term corresponds to the case where there are no available indirect or direct paths. The second term is the case without indirect path and the direct path is in outage. The third term is when there is no direct path but a randomly selected indirect path (from the ones available) is in outage. Finally, the last term is the case where both direct and indirect paths exist but the receiver is in outage. Since the events are mutually independent, each term is derived by the product of the probabilities of the corresponding events. Therefore, the result follows by considering expressions (1), (9), (12), (15) and Proposition 1.

APPENDIX C

PROOF OF PROPOSITION 2

Following the methodology from Appendix A, we have

$$P_{\text{CA}}^B(\tau|D_L) \approx \mathbb{E}_\Psi \left\{1 - \exp\left(-\frac{\tau\sigma^2}{P_t} \frac{1}{\Xi(d, \theta)}\right)\right\}. \quad (41)$$

The cumulative distribution function for the distance D from the origin is

$$\mathbb{P}\{D < d\} = 1 - \exp(-\Lambda(d)), \quad (42)$$

where $\Lambda(d)$ is the intensity measure given by (10). Then, the PDF is derived by the derivative of the above CDF, which gives (22), and the result follows.

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