Methodology

of

LINI Test Rig Measurements

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1 Introduction

This methodology report is part of the paper:

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All data and supplementary documentation is published in the data repository:

https://doi.org/10.5281/zenodo.5802409

Software for data analysis can be found in the software repository:

https://doi.org/10.5281/zenodo.5802407

If not indicated otherwise, the references in square brackets refer to the manuscript references.

All calculations shown in this report are performed in the MATLAB script “Analyze_Discussion” in the software repository and are partially based on data in the data repository (all .mat-files containing the results of the different experiments). All heat transfer correlations are packed separately into different function files, to be found in the software repository as well. The variable names are as close as possible to the variable names in the MATLAB script and data files while ensuring their readability in this report.

2 Correlations from Literature

2.1 Andeen / Glicksman [2]

This heat transfer correlation depends on the parameters shown in Table 1.

Index	Variable	Description	Required for
1	d_{p}	Particle diameter	Bed porosity ε Fluidization velocity w Nusselt number Nu_{T}
2	ρ_{p}	Particle density	Bed porosity ε Fluidization velocity w Nusselt number Nu_{T}
3	FG	Fluidization grade	Bed porosity ε Fluidization velocity w
4	d_{tube}	Tube diameter	Nusselt number Nu_{T}
5	p_{A}	Fluidization air pressure	Bed porosity ε Fluidization velocity w
6	T_{A}	Fluidization air temperature	Bed porosity ε Fluidization velocity w Nusselt number Nu_{T}
7	ε	Bed porosity	Nusselt number Nu_{T}
8	w	Fluidization velocity	Nusselt number Nu_{T}

Table 1: Function parameters of the Andeen / Glicksman correlation and their uses

One can see that many of the parameters only influence the main result of the correlation, the Nusselt number Nu_{T} , indirectly by having an impact on the bed porosity ε or the fluidization velocity w . Calculation of the bed porosity is described in section 3.1 , and the calculation of (superficial) fluidization velocity based on Richardson's correlation is described in section 3.2 .

The tube diameter d_{tube} is the characteristic length of the Nusselt number Nu_{T} .

Calculations

$\lambda_{\mathrm{A}}=f\left(T_A\right)$	Fluidization air dynamic viscosity as a function of temperature. Property function of dry air at ambient pressure according to VDI Heat Atlas[1].
$c_{\mathrm{p,A}}=f\left(T_{\mathrm{A}}\right)$	Specific isobaric heat capacity of air as a function of temperature, as above
$\eta_{\mathrm{A}}=f\left(T_{\mathrm{A}}\right)$	Dynamic viscosity of air as a function of

1 https://materials.springer.com/lb/docs/sm_nlb_978-3-540-77877-6_11

	temperature, as above
$w = f(d_{p}, \rho_{p}, p_{A}, T_{A}, \mathrm{FG})$	Fluidization velocity as function of particle diameter, particle density, air pressure, air temperature, and degree of fluidization, see section 3.2
$\mathrm{Nu}_{T} = 900(1-\varepsilon)\left(\frac{w\, d_{\mathrm{tube}}\, \eta_{A}}{d_{p}^{3}\rho_{p} g}\right)^{0.326}\left(\frac{\eta_{A} c_{p,A}}{\lambda_{A}}\right)^{0.3}$	Nusselt number
$\alpha = \frac{\mathrm{Nu}_{T}\lambda_{A}}{d_{\mathrm{tube}}}$	Heat transfer coeffcient

2.2 Gelperin / Einstein [21]

This heat transfer correlation depends on the parameters shown in Table 2.

Index	Variable	Description	Required for
1	d_p	Particle diameter	Archimedes number Ar
2	ρ_p	Particle density	Archimedes number Ar
4	d_{tube}	Tube diameter	Nusselt number Nu_p
5	$pitch_{horz}$	Horizontal tube pitch	Nusselt number Nu_p
6	$pitch_{vert}$	Vertical tube pitch	Nusselt number Nu_p
7	p_A	Fluidization air pressure	Archimedes number Ar
8	T_A	Fluidization air temperature	Archimedes number Ar Nusselt number Nu_p
9	Ar	Archimedes number	Nusselt number Nu_p

Table 2: Function parameters of the Gelperin / Einstein correlation and their uses

It is important to note that this correlation does not depend on bed porosity or fluidization velocity.

The particle diameter d_p is the characteristic length of the Nusselt number Nu_p .

Calculations

$\lambda_A = f(T_A)$ — Fluidization air dynamic viscosity as a function of temperature. Property function of dry air at ambient pressure according to VDI Heat Atlas[2].

$\eta_A = f(T_A)$ — Dynamic viscosity of air as a function of temperature, as above

$\rho_A = f(p_A, T_A)$ — Air density as a function of pressure and air temperature based on the ideal gas equation

$$\mathrm{Ar} = \frac{\rho_A d_p^3 (\rho_p - \rho_A) g}{\eta_A^2}$$ Archimedes number

$$\mathrm{Nu}_p = 0.74\, \mathrm{Ar}^{0.22} \left(1 - \frac{d_{tube}}{pitch_{vert}} \frac{1 + d_{tube}}{pitch_{horz} + d_{tube}}\right)^{0.25}$$ Nusselt number

$$\alpha = \frac{\mathrm{Nu}_p \lambda_A}{d_p}$$ Heat transfer coeffcient

2 https://materials.springer.com/lb/docs/sm_nlb_978-3-540-77877-6_11

2.3 Grewal [4]

This heat transfer correlation depends on the parameters shown in Table 3.

Index	Variable	Description	Required for
1	d_{p}	Particle diameter	Bed porosity ε Fluidization velocity w Nusselt number $\mathrm{Nu_T}$
2	ρ_{p}	Particle density	Bed porosity ε Fluidization velocity w Nusselt number $\mathrm{Nu_T}$
3	FG	Fluidization grade	Bed porosity ε Fluidization velocity w
4	d_{tube}	Tube diameter	Nusselt number $\mathrm{Nu_T}$
5	$pitch$	Tube pitch	Nusselt number $\mathrm{Nu_T}$
6	p_{A}	Fluidization air pressure	Bed porosity ε Fluidization velocity w
7	T_{A}	Fluidization air temperature	Bed porosity ε Fluidization velocity w Nusselt number $\mathrm{Nu_T}$
8	ε	Bed porosity	Nusselt number $\mathrm{Nu_T}$
9	w	Fluidization velocity	Nusselt number $\mathrm{Nu_T}$

Table 3: Function parameters of the Grewal correlation and their uses

This correlation is based on the one by Andeen and Glicksman, see section 2.1 , which is why it is very similar to it. The main difference is that it accounts for the effect of several tubes in a bundle, with $pitch$ being the distance between them.

The tube diameter d_{tube} is the characteristic length of the Nusselt number $\mathrm{Nu_T}$.

Calculations

$\lambda_{\mathrm{A}}=f(T_A)$	Fluidization air dynamic viscosity as a function of temperature. Property function of dry air at ambient pressure according to VDI Heat Atlas[3].
$\eta_{\mathrm{A}}=f(T_{\mathrm{A}})$	Dynamic viscosity of air as a function of temperature, as above
$c_{\mathrm{p,A}}=f(T_{\mathrm{A}})$	Specific isobaric heat capacity of air as a function of temperature, as above
$c_{\mathrm{p,p}}=f(T_{\mathrm{A}})$	Specific isobaric heat capacity of particles as a function of temperature. It is assumed that the

3 https://materials.springer.com/lb/docs/sm_nlb_978-3-540-77877-6_11

fluidization air temperature is equal to the bed temperature. Property functions of the particles (SiO_2) derived from Chase[4].

$$w = f(d_p, \rho_p, p_A, T_A, FG)$$

Fluidization velocity as function of particle diameter, particle density, air pressure, air temperature, and degree of fluidization, see section 3.2

Nusselt number:

$$Nu_T = 47(1-\varepsilon)\left(\frac{w\, d_{tube}\, \eta_A}{d_p^3 \rho_p\, g}\right)^{0.325}\left(\frac{\eta_A c_{p,A}}{\lambda_A}\right)^{0.3}\left(\frac{\rho_p c_{p,p} d_{tube}^{1.5}\sqrt{g}}{\lambda_A}\right)^{0.23}\left(1-0.21\left(\frac{pitch}{d_{tube}}\right)^{-1.75}\right)$$

$$\alpha = \frac{Nu_T \lambda_A}{d_{tube}}$$

Heat transfer coeffcient

4 Chase, M.W., Jr., NIST-JANAF Themochemical Tables, Fourth Edition, J. Phys. Chem. Ref. Data, Monograph 9, 1998, 1-1951. https://webbook.nist.gov/cgi/cbook.cgi?ID=C14808607&Type=JANAFS&Table=on

2.4 Martin [7]

This heat transfer correlation depends on the parameters shown in Table 4.

Index	Variable	Description	Required for
1	d_p	Particle diameter	Bed porosity ε Nusselt number $Nu_{WP,max}$ Nusselt number Nu_p Archimedes number Ar
2	ρ_p	Particle density	Bed porosity ε Nusselt number Nu_p Archimedes number Ar
3	λ_p	Particle thermal conductivity	Nusselt number Nu_p
4	FG	Fluidization grade	Bed porosity ε
5	p_A	Fluidization air pressure	Bed porosity ε Nusselt number $Nu_{WP,max}$ Archimedes number Ar
6	T_A	Fluidization air temperature	Bed porosity ε Nusselt number $Nu_{WP,max}$ Nusselt number Nu_p Archimedes number Ar Nusselt number Nu_g Radiative HTC α_R
7	ε_{mf}	Bed porosity at minimum fluidization	Nusselt number Nu_p
8	ε_R	Emissivity between surface and bed	Nusselt number Nu_R
9	ε	Bed porosity	Nusselt number Nu_p
10	$Nu_{WP,max}$	Maximum Nusselt number between wall and particle	Nusselt number Nu_p
11	Ar	Archimedes number	Nusselt number Nu_g

Table 4: Function parameters of the Martin correlation and their uses

This correlation does not depend on the superficial fluidization velocity directly, only the bed porosity that depends on the fluidization grade has an influence in this regard. It uses the difference between the bed porosity at operating conditions and at minimum fluidization, which is why the minimum bed porosity ε_{mf} and the actual bed porosity ε have great influence on the results. While the minimum bed porosity was measured before the experiments (0.45), the actual bed porosity was estimated using the correlation presented in section 3.1 .

Martin's correlation considers three different parts of heat transfer, that are all contributing to the total heat transfer coefficient (HTC) α : a particle convective part α_p , a gas convective part α_g , and a radiative part α_R. They are all based on the respective Nusselt numbers Nu_p, Nu_g, and Nu_R.

The particle diameter d_p is the characteristic length of all Nusselt numbers, and they are all calculated with the air (gas) thermal conductivity λ_A.

Calculations

$\lambda_A = f(T_A)$	Fluidization air dynamic viscosity as a function of temperature. Property function of dry air at ambient pressure according to VDI Heat Atlas[5].
$\eta_A = f(T_A)$	Dynamic viscosity of air as a function of temperature, as above
$c_{p,A} = f(T_A)$	Specific isobaric heat capacity of air as a function of temperature, as above
$\mathrm{Pr}_A = f(T_A)$	Prandtl number of air as a function of temperature, as above
$\rho_A = f(p_A, T_A)$	Air density as a function of pressure and air temperature based on the ideal gas equation
$c_{p,p} = f(T_A)$	Specific isobaric heat capacity of particles as a function of temperature. It is assumed that the fluidization air temperature is equal to the bed temperature. Property functions of the particles (SiO_2) derived from Chase[6].
$\log\left(\frac{1}{\gamma} - 1\right) = 0.6 - \left(\frac{1\,000\,\mathrm{K}}{T_A} + 1\right) / C_A$	Function to iteratively calculate the accomodation coefficient γ, which considers the “incompleteness of energy transfer during a molecule-wall collision” [7] (p. 1305). The constant C_A is 2.8 for air.
$\Lambda = \sqrt{2\pi R_A T_A}\ \frac{\lambda_A}{p_A\left(2 c_{p,A} - R_A\right)}$	Effective mean free path, from kinetic gas theory. The ideal gas constant of air R_A is 287.058 J/kgK[7]
$l = 2\Lambda\left(\frac{2}{\gamma} - 1\right)$	Modified mean free path
$\mathrm{Nu}_{WP,max} = 4\left(\left(1 + \frac{2l}{d_p}\right)\ln\left(1 + \frac{d_p}{2l} - 1\right)\right)$	Maximum Nusselt number between wall and particle, according to Schlünder[8]

5 https://materials.springer.com/lb/docs/sm_nlb_978-3-540-77877-6_11
6 Chase, M.W., Jr., NIST-JANAF Themochemical Tables, Fourth Edition, J. Phys. Chem. Ref. Data, Monograph 9, 1998, 1-1951. https://webbook.nist.gov/cgi/cbook.cgi?ID=C14808607&Type=JANAFS&Table=on
7 Span, R. Properties of Dry Air. In VDI Heat Atlas, 2nd ed.; Stephan, P., Kabelac, S., et al., Eds.; Springer: Berlin Heidelberg, Germany, 2010; pp. 172–191. https://doi.org/10.1007/978-3-540-77877-6_11
8 Schlünder EU (1971) Wärmeübergang an bewegte Kugelschüttungen bei kurzfristigem Kontakt. Chemie-Ing-Techn 43(11):651–654

$$Z = \frac{\rho_p c_{p,p}}{6 \lambda_A} \sqrt{\frac{g d_p^3 (\varepsilon - \varepsilon_{mf})}{5 (1 - \varepsilon_{mf})(1 - \varepsilon)}}$$

Scaling factor for $\mathrm{Nu}_{WP,max}$ to operating conditions

$$\mathrm{Nu}_{WP} = \left(\frac{1}{\mathrm{Nu}_{WP,max}} + \frac{\lambda_A / \lambda_p}{4 \left(1 + \sqrt{\frac{3 C_K \lambda_A}{2 \pi \lambda_p} Z} \right)} \right)^{-1}$$

Nusselt number between wall and particle at operating conditions. The constant C_K is 2.6

$$N = \frac{\mathrm{Nu}_{WP}}{C_K Z}$$

Exponent for particle Nusselt number function

$$\mathrm{Nu}_p = (1 - \varepsilon) Z (1 - e^{-N})$$

Particle Nusselt number

$$\alpha_p = \frac{\mathrm{Nu}_p \lambda_A}{d_p}$$

Particle heat transfer coeffcient

$$\mathrm{Ar} = \frac{\rho_A d_p^3 (\rho_p - \rho_A) g}{\eta_A^2}$$

Archimedes number

$$\mathrm{Nu}_g = 0.009 \, \mathrm{Pr}_A^{1/3} \mathrm{Ar}^{1/2}$$

Gas Nusselt number

$$\alpha_g = \frac{\mathrm{Nu}_g \lambda_A}{d_p}$$

Gas heat transfer coeffcient

$$\mathrm{Nu}_R = 4 \varepsilon_R \sigma T_A^3 (d_p / \lambda_A)$$

Radiation Nusselt number, linearized. σ Is the Stefan Boltzmann constant

$$\alpha_R = \frac{\mathrm{Nu}_R \lambda_A}{d_p} = 4 \varepsilon_R \sigma T_A^3$$

Radiative heat transfer coeffcient

$$\alpha = \alpha_p + \alpha_g + \alpha_R$$

Total heat transfer coeffcient as linear combination of all the other coeffcients

2.5 Molerus [5]

This heat transfer correlation depends on the parameters shown in Table 5.

Index	Variable	Description	Required for
1	ρ_p	Particle density	Characteristic length l
2	p_A	Fluidization air pressure	Characteristic length l
3	T_A	Fluidization air temperature	Characteristic length l
4	l	Characteristic length	Heat transfer coeffcient α

Table 5: Function parameters of the Molerus correlation and their uses

This correlation uses its own definition of a characterstic length l , which is the main variable.

Similar to Gelperin / Einstein, this correlation does not depend on bed porosity or fluidization velocity.

Calculations

$\lambda_A = f(T_A)$ Fluidization air dynamic viscosity as a function of temperature. Property function of dry air at ambient pressure according to VDI Heat Atlas[9].

$\eta_A = f(T_A)$ Dynamic viscosity of air as a function of temperature, as above

$\rho_A = f(p_A, T_A)$ Air density as a function of pressure and air temperature based on the ideal gas equation

$c_{p,p} = f(T_A)$ Specific isobaric heat capacity of particles as a function of temperature. It is assumed that the fluidization air temperature is equal to the bed temperature. Property functions of the particles (SiO_2) derived from Chase[10].

$$l = \left(\frac{\eta_A}{\sqrt{g}(\rho_p - \rho_A)} \right)^{2/3}$$ Characteristic length

$$\alpha = \left(\frac{1}{0.09\,\lambda_A} + \frac{1}{0.09\,c_{p,p}\,\eta_A} \right)^{-1}$$ Heat transfer coeffcient

9 https://materials.springer.com/lb/docs/sm_nlb_978-3-540-77877-6_11

10 Chase, M.W., Jr., NIST-JANAF Themochemical Tables, Fourth Edition, J. Phys. Chem. Ref. Data, Monograph 9, 1998, 1-1951. https://webbook.nist.gov/cgi/cbook.cgi?ID=C14808607&Type=JANAFS&Table=on

2.6 Zabrodsky [20]

This heat transfer correlation depends on the parameters shown in Table 6.

Index	Variable	Description	Required for
1	d_{p}	Particle diameter	Heat transfer coeffcient α
2	ρ_{P}	Particle density	Heat transfer coeffcient α
3	T_{A}	Fluidization air temperature	Heat transfer coeffcient α

Table 6: Function parameters of the Zabrodsky correlation and their uses

This is a very simple correlation that only depends on particle diameter d_{p}, particle density ρ_{P}, and fluidization air temperature T_{A}. It does not take bed porosity or fluidization velocity into account.

Calculations

$\lambda_{\mathrm{A}}=f\left(T_A\right)$ Fluidization air dynamic viscosity as a function of temperature. Property function of dry air at ambient pressure according to VDI Heat Atlas[11].

$\alpha=35.8\rho_{\mathrm{p}}^{0.2}\lambda_{\mathrm{A}}^{0.6}d_{\mathrm{p}}^{-0.36}$ Heat transfer coeffcient

11 https://materials.springer.com/lb/docs/sm_nlb_978-3-540-77877-6_11

3 Auxiliary Functions

3.1 Bed Porosity

The bed porosity function is based on a correlation by Goroshko[12]. Table 7 shows its parameters and uses.

Index	Variable	Description	Required for
1	d_p	Particle diameter	Fluidization velocity w Archimedes number Ar Reynolds number Re
2	ρ_p	Particle density	Fluidization velocity w Archimedes number Ar
4	FG	Fluidization grade	Fluidization velocity w
3	p_A	Fluidization air pressure	Fluidization velocity w
4	T_A	Fluidization air temperature	Fluidization velocity w
5	w	Fluidization velocity	Reynolds number Re
6	Ar	Archimedes number	Bed porosity ε
7	Re	Reynolds number	Bed porosity ε

Table 7: Function parameters of the Bed Porosity correlation and their uses

Fluidization air pressure and temperature are only needed to calculate the fluidization air properties (density and dynamic viscosity). The particle diameter represents the characteristic length of the Reynolds number.

Calculations

$\eta_\mathrm{A}=f(T_\mathrm{A})$

Fluidization air dynamic viscosity as a function of bed temperature. Property function of dry air at ambient pressure according to VDI Heat Atlas[13].

$\rho_\mathrm{A}=f(p_\mathrm{A},T_\mathrm{A})$

Fluidization air density as a function of pressure and air temperature based on the ideal gas equation

$w=f(d_\mathrm{p},\rho_\mathrm{p},p_\mathrm{A},T_\mathrm{A},\mathrm{FG})$

Fluidization velocity as function of particle diameter, particle density, air pressure, air temperature, and degree of fluidization, see section 3.2

12 Zabrodsky, S.S. Hydrodynamics and Heat Transfer in Fluidized Beds, 7th ed.; MIT Press: Cambridge, Massachusetts, United States, 1966; p. 71.
13 https://materials.springer.com/lb/docs/sm_nlb_978-3-540-77877-6_11

$$\mathrm{Ar} = \frac{\rho_\mathrm{A} d_\mathrm{p}^3 (\rho_\mathrm{p} - \rho_\mathrm{A}) g}{\eta_\mathrm{A}^2}$$

Archimedes number, where $g = 9.81\ \mathrm{m/s^2}$ is the gravitational acceleration

$$\mathrm{Re} = \frac{d_\mathrm{p} \rho_\mathrm{A} w}{\eta_\mathrm{A}}$$

Particle Reynolds number

$$\varepsilon = \left(\frac{0.36\,\mathrm{Re}^2 + 18\,\mathrm{Re}}{\mathrm{Ar}} \right)^{0.21}$$

Bed porosity

3.2 Fluidization velocity (Richardson's correlation) [17]

The fluidization velocity primarily depends on the minimum fluidization velocity, which is modeled after the correlation by Richardson [17]. Table 8 shows its parameters and uses.

Index	Variable	Description	Required for
1	d_p	Particle diameter	Archimedes number Ar Reynolds number Re
2	ρ_P	Particle density	Archimedes number Ar
3	p_A	Fluidization air pressure	Archimedes number Ar Minimum fluidization velocity w_mf
4	T_A	Fluidization air temperature	Archimedes number Ar Minimum fluidization velocity w_mf
5	Ar	Archimedes number	Reynolds number Re
6	Re	Reynolds number	Minimun fluidization velocity w_mf

Table 8: Function parameters of the Minimum Fluidization Velocity correlation and their uses

Fluidization air pressure and temperature are only needed to calculate the fluidization air properties (density and dynamic viscosity). The particle diameter represents the characteristic length of the Reynolds number.

Calculations

$\eta_\mathrm{A}=f(T_\mathrm{A})$ — Fluidization air dynamic viscosity as a function of bed temperature. Property function of dry air at ambient pressure according to VDI Heat Atlas[14].

$\rho_\mathrm{A}=f(p_\mathrm{A},T_\mathrm{A})$ — Fluidization air density as a function of pressure and air temperature based on the ideal gas equation

$$\mathrm{Ar}=\frac{\rho_\mathrm{A}d_\mathrm{p}^3(\rho_\mathrm{P}-\rho_\mathrm{A})g}{\eta_\mathrm{A}^2}$$

Archimedes number, where $g=9.81\,\mathrm{m/s^2}$ is the gravitational acceleration

$$\mathrm{Re}=\sqrt{C_1^2+C_2\,\mathrm{Ar}}-C_1$$

Reynolds number with $C_1=25.7$ and $C_2=0.0365$ according to Richardson [17]

$$w_\mathrm{mf}=\mathrm{Re}\frac{\eta_\mathrm{A}}{d_\mathrm{p}\rho_\mathrm{A}}$$

Minimum fluidization velocity

The (superficial) fluidization velocity then only depends on the degree of fluidization FG :

$$w=w_\mathrm{mf}\,\mathrm{FG}$$

14 https://materials.springer.com/lb/docs/sm_nlb_978-3-540-77877-6_11

4 Comparison of Results with Literature

4.1 General

The following properties of the fluidized bed were considered to be constant for all calculations, Table 9.

Variable	Value	Description
$pitch_{\mathrm{horz,ST}}$	$2\,d_{\mathrm{tube}}$	Horizontal sandTES pitch
$pitch_{\mathrm{vert,ST}}$	$2.5\,d_{\mathrm{tube}}$	Vertical sandTES pitch
$pitch_{\mathrm{horz,Reg}}$	$3.1\,d_{\mathrm{tube}}$	Horizontal Regular pitch
$pitch_{\mathrm{vert,Reg}}$	$3.1\,d_{\mathrm{tube}}$	Vertical Regular pitch
d_{p}	$146\,\mu\mathrm{m}$	Regular mean particle diameter (LINI, TRINI)
d_{p1}	$87.141\,\mu\mathrm{m}$	Smaller mean particle diameter (MICRO)
d_{p2}	$210\,\mu\mathrm{m}$	Larger mean particle diameter (MICRO)
d_{tube}	$25\,\mathrm{mm}$	Tube diameter
$\varepsilon_{\mathrm{mf}}$	0.45	Bed porosity at minimum fluidization, from measurements
ε_{R}	0.9	Emissivity tube – fluidized bed
λ_{p}	$3\,\mathrm{W/mK}$	Thermal conductivity of SiO_2
ρ_{p}	$2650\,\mathrm{kg/m^3}$	Particle density of SiO_2

Table 9: General boundary conditions for all experiments

Since the Grewal correlation (section 2.3) only accepts a single pitch value, thereby in effect assuming an even pitch in both the horizontal and vertical directions, the mean value of the pitches was used in the case of the sandTES pitch:

$$pitch_{\mathrm{ST}} = \frac{pitch_{\mathrm{horz,ST}} + pitch_{\mathrm{vert,ST}}}{2}$$

The same can be done for the regular pitch $pitch_{\mathrm{Reg}}$, but it obviously has no impact on the results.

4.2 MICRO

The necessary boundary conditions of the MICRO experiments are shown in Table 10.

Variable	Value	Description
p_A	$1013.35\,\text{mbar} + 4335\,\text{Pa}$	Fluidization air pressure, see MICRO methodology report

Table 10: MICRO boundary conditions

The different correlations for comparison are then calculated for a range of fluidization grades FG at the bed temperature T_{BED1} as follows (see section 2 for details on the individual functions):

Calculations

Grewal, for both pitches and 87 µm sand:

$$Grewal_{\mathrm{ST,87\,\mu m}} = f\left(d_{\mathrm{p1}}, \rho_{\mathrm{p}}, \mathrm{FG}, d_{\mathrm{tube}}, pitch_{\mathrm{ST}}, p_{\mathrm{A}}, T_{\mathrm{BED1}}\right)$$

$$Grewal_{\mathrm{Reg,87\,\mu m}} = f\left(d_{\mathrm{p1}}, \rho_{\mathrm{p}}, \mathrm{FG}, d_{\mathrm{tube}}, pitch_{\mathrm{Reg}}, p_{\mathrm{A}}, T_{\mathrm{BED1}}\right)$$

Martin, 87 µm sand:

$$Martin_{\mathrm{87\,\mu m}} = f\left(d_{\mathrm{p1}}, \rho_{\mathrm{p}}, \lambda_{\mathrm{p}}, \mathrm{FG}, p_{\mathrm{A}}, T_{\mathrm{BED1}}, \varepsilon_{\mathrm{mf}}, \varepsilon_{\mathrm{R}}\right)$$

Molerus:

$$Molerus = f\left(\rho_{\mathrm{p}}, p_{\mathrm{A}}, T_{\mathrm{BED1}}\right)$$

It is important to note that Molerus' correlation does not depend on particle diameter. The results of the Grewal and Martin correlations for 210 µm sand can be calculated by substituting d_{p1} with d_{p2} in the equations above.

4.3 LINI

There are no relevant boundary conditions other than the ones listed in section 4.1 .

For the calculation of the relevant fluidization air pressure p_{A} and temperature T_{A} , see the LINI methodology report. Only the sandTES-pitch $pitch_{\mathrm{ST}}$ was used in the LINI experiments, and only the plain tube results are compared to the different correlations.

The different correlations for comparison are then calculated for a range of fluidization grades FG as follows (see section 2 for details on the individual functions):

Calculations

$Grewal = f(d_{\mathrm{p}}, \rho_{\mathrm{p}}, \mathrm{FG}, d_{\mathrm{tube}}, pitch_{\mathrm{ST}}, p_{\mathrm{A}}, T_{\mathrm{A}})$	Grewal's correlation
$Martin = f(d_{\mathrm{p}}, \rho_{\mathrm{p}}, \lambda_{\mathrm{p}}, \mathrm{FG}, p_{\mathrm{A}}, T_{\mathrm{A}}, \varepsilon_{\mathrm{mf}}, \varepsilon_{\mathrm{R}})$	Martin's correlation
$Molerus = f(\rho_{\mathrm{p}}, p_{\mathrm{A}}, T_{\mathrm{A}})$	Molerus' correlation

4.4 TRINI

There are no relevant boundary conditions other than the ones listed in section 4.1 .

For the calculation of the relevant fluidization air pressure p_{A} and temperature T_{A} , see the TRINI methodology report. Only the sandTES-pitch $pitch_{\mathrm{ST}}$ was used in the TRINI experiments, and only the plain tube results are compared to the different correlations.

The different correlations for comparison are then calculated for a range of fluidization grades FG as follows (see section 2 for details on the individual functions):

Calculations

$Grewal = f(d_{\mathrm{p}}, \rho_{\mathrm{p}}, \mathrm{FG}, d_{\mathrm{tube}}, pitch_{\mathrm{ST}}, p_{\mathrm{A}}, T_{\mathrm{A}})$	Grewal's correlation
$Martin = f(d_{\mathrm{p}}, \rho_{\mathrm{p}}, \lambda_{\mathrm{p}}, \mathrm{FG}, p_{\mathrm{A}}, T_{\mathrm{A}}, \varepsilon_{\mathrm{mf}}, \varepsilon_{\mathrm{R}})$	Martin's correlation
$Molerus = f(\rho_{\mathrm{p}}, p_{\mathrm{A}}, T_{\mathrm{A}})$	Molerus' correlation

5 Dependence of Air Mass Flow for Fluidization on Temperature

This is a simple investigation of how the air mass flow changes with temperature at constant degrees of fluidization. The following boundary conditions were chosen, Table 11.

Variable	Value	Description
T_0	$40\,°C$	Base temperature for relative comparison
p_A	$1\,bar$	Fluidization air pressure
w_{mf0}	$w_{mf}(d_p, \rho_p, p_A, T_0)$	Minimum fluidization velocity at base temperature T_0, based on Richardson's correlation, section 3.2
ρ_{A0}	$f(p_A, T_0)$	Fluidization air density as a function of pressure and air temperature based on the ideal gas equation at base temperature T_0

Table 11: Boundary conditions for the temperature dependence of the fluidization air mass flow

All other boundary conditions are the same as in Table 9 of section 4.1 . The degree of fluidization is irrelevant for this calculation. The relative requirement for fluidization air $\dot{m}_{rel}$ can then be calculated for a range of temperatures T_A as follows:

Calculations

$\rho_A = f(p_A, T_A)$	Fluidization air density as a function of pressure and air temperature based on the ideal gas equation
$w_{mf} = f(d_p, \rho_p, p_A, T_A)$	Minimum fluidization velocity as function of particle diameter, particle density, air pressure, air temperature, and degree of fluidization, see section 3.2
$\dot{m}_{rel} = \frac{w_{mf}\rho_A}{w_{mf0}\rho_{A0}}$	Relative mass flow requirement for fluidization

6 Temperature Dependence of Heat Transfer Correlations

This section investigates how the different heat transfer correlations presented in section 2 predicted relative changes in the heat transfer coefficients when increasing the temperature. The boundary conditions shown in Table 12 were chosen:

Variable	Value	Description
T_0	$40\,°\mathrm{C}$	Base temperature for relative comparison
FG	4	Degree of fluidization
p_A	$1\,\mathrm{bar}$	Fluidization air pressure

Table 12: Boundary conditions for the temperature dependence of heat transfer correlations

All other boundary conditions are the same as in Table 9 of section 4.1 . The sandTES-pitch $pitch_\mathrm{ST}$ was used for all calculations. The different correlations for comparison are then calculated for a range of fluidization air temperatures T_A as follows (see section 2 for details on the individual functions):

Calculations

$\alpha_\mathrm{Andeen} = f(d_\mathrm{p}, \rho_\mathrm{p}, \mathrm{FG}, d_\mathrm{tube}, p_\mathrm{A}, T_\mathrm{A})$ Andeen / Glicksman's function

$\alpha_\mathrm{Grewal} = f(d_\mathrm{p}, \rho_\mathrm{p}, \mathrm{FG}, d_\mathrm{tube}, pitch_\mathrm{ST}, p_\mathrm{A}, T_\mathrm{A})$ Grewal's correlation

$\alpha_\mathrm{Molerus} = f(\rho_\mathrm{p}, p_\mathrm{A}, T_\mathrm{A})$ Molerus' correlation

$\alpha_\mathrm{Zabrodsky} = f(d_\mathrm{p}, \rho_\mathrm{p}, T_\mathrm{A})$ Zabrodsky's correlation

$\alpha_\mathrm{Martin} = f(d_\mathrm{p}, \rho_\mathrm{p}, \lambda_\mathrm{p}, \mathrm{FG}, p_\mathrm{A}, T_\mathrm{A}, \varepsilon_\mathrm{mf}, \varepsilon_\mathrm{R})$ Martin's correlation

Gelperin / Einstein's correlation:

$\alpha_\mathrm{Gelperin} = f(d_\mathrm{p}, \rho_\mathrm{p}, d_\mathrm{tube}, pitch_\mathrm{horz,ST}, pitch_\mathrm{vert,ST}, p_\mathrm{A}, T_\mathrm{A})$

$\alpha_\mathrm{rel,x} = \dfrac{\alpha_\mathrm{x}(T_\mathrm{A})}{\alpha_\mathrm{x}(T_0)}$ Relative heat transfer coefficient. The x stands for any of the previous heat transfer correlations

It is important to note that the correlation by Gelperin / Einstein is the only one distinguishing between a horizontal and a vertical pitch.