

Failure Oriented Accelerated Testing (FOAT), Boltzmann Arrhenius Zhurkov Equation (BAZ), and their Application in Aerospace Microelectronics and Photonics Reliability Engineering

Research Article

E. Suhir^{1,2*}

¹ Portland State University, Portland, USA.

² Technical University, Vienna, Austria.

Abstract

The best engineering product is, in effect, the best compromise between its cost, time-to-market and reliability. No successful compromise can be achieved nor can an adequate performance of an electronic or photonic product be assured, if the product's reliability is not quantified. Since nothing is perfect, and the difference between a highly reliable product and an insufficiently reliable one is "merely" in the difference between the levels of their never-zero probabilities of failure, reliability of such products should be evaluated on the probability basis. In the analysis that follows it is shown how this could be done using highly focused and highly cost effective failure-oriented accelerated testing (FOAT) geared to a physically meaningful and flexible Boltzmann-Arrhenius-Zhurkov (BAZ) constitutive equation. FOAT and BAZ are the core of the recently suggested probabilistic design for reliability (PDfR) concept. The following practically important problems are addressed, as suitable examples of the general concept: assessment of the static fatigue lifetime of an optical specialty-fiber intended for high-temperature operations; predicted time-to-failure (TTF) of an electron device subjected to temperature cycling; TTF of a solder material subjected to the combined action of low-temperatures and random-vibrations; and TTF of an electron device under the combined action of an elevated humidity and voltage. Future work should include experimental verification of the theoretical findings, as well as new applications of the suggested technique.

Abbreviations: FOAT: Failure-Oriented Accelerated Testing; BAZ: Boltzmann-Arrhenius-Zhurkov; PDfR: Probabilistic Design for Reliability; TTF: Time-to-Failure; HALT: Highly Accelerated Life Testing; MTTF: Mean Time to Failure.

Introduction

The almost forty years old highly accelerated life testing (HALT) is currently widely employed, in different modifications, to determine, as it is believed, an electronic product's reliability weaknesses, assess its reliability limits, and ruggedized the product by applying elevated stresses (not necessarily mechanical and not necessarily limited to the anticipated field stresses) that could cause field failures. HALT provides large, although, actually, unknown, safety margins over expected in-use conditions. HALT tries to "kill many unknown birds with one big stone" and is a "test-fail-fix" process. Its end point is defined by the predetermined number or percent of failures, if any, and its follow up activity is failure (root cause) analysis. In an ideal HALT, no failures

occur in a long time.

FOAT, on the other hand, is aimed at understanding the physics of failure, to confirm the use of a particular physically meaningful predictive model and assess the probability of failure. Its end point is also defined by the predetermined number or percent (typically 50%) of failures. The follow-up activity is failure analysis and probabilistic analyses of the test data. An ideal FOAT generates numerous failures in a short time. A highly focused and highly cost effective FOAT is the "heart" of the PDfR concept and is a solid experimental foundation of the PDfR approach. FOAT could be viewed as an extension or a modification of HALT. While HALT is a "black box", i.e., a methodology, which can be perceived in terms of its inputs and outputs, without a clear knowledge of the underlying physics and the likelihood of

*Corresponding Author:

E. Suhir,
Portland State University, Portland, USA and Technical University, Vienna, Austria and ERS Co., 727 Alvina Ct., Los Altos, CA 94024, USA.
Tel: 650.969.1530/408-410-0886
Email: suhire@aol.com

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failure, FOAT is a “white box”, a methodology that clearly identifies its objective to confirm the anticipated physics of failure and to determine the probability of failure. HALT does not measure/quantify reliability, FOAT does. The FOAT based approach could be viewed as a “quantified and reliability physics oriented HALT”. HALT can be used for “rough tuning” of product’s reliability, while FOAT should be employed when “fine tuning” is needed, i.e., when there is a need to quantify, assure and even specify the operational reliability of a material or a device. Since the principle of superposition does not work in reliability engineering, both HALT and FOAT use, when appropriate, combined stressing under various stimuli (stressors). These types of accelerated testing could be carried out separately, or might be partially combined in a particular accelerated test effort. FOAT should be implemented, whenever feasible and appropriate, in addition to HALT, when quantification of the product’s reliability is imperative. In some cases FOAT could be conducted even instead of HALT, especially for new materials and products, whose operational reliability is unclear and for which no experience is accumulated and no best practices exist yet. New products present natural reliability concerns, as well as significant challenges at all the stages of their design, manufacture and use, and an appropriate combination of HALT and FOAT could be especially useful for quantifying reliability of such products.

BAZ model [1] formula (1) could be employed withing the framework of an appropriate FOAT effort. This model is the generalization of the well known and still widely used Arrhenius equation [2] introduced in 1889 in the kinetic theory of chemical reactions (1903 Nobel Prize in chemistry). The model formula (2) considers the role of the ratio U_0/kT of the activation energy U_0 (the term was introduced by Arrhenius to characterize the material’s propensity to get engaged into a chemical reaction), to the thermal energy kT evaluated as the product of the Boltzmann’s constant $k = 8.6173303 \times 10^{-5} \text{ eV/K}$ and the absolute temperature T . In the equations (1) and (2), τ is the mean time to failure (MTTF), τ_0 is the time constant, and σ (in (1)) is the applied stress per unit volume. Arrhenius’ equation (2) is formally not different of what is known as Boltzmann’s or Maxwell-Boltzmann’s statistics [3] in the kinetic theory of gases. This theory postulates that the absolute temperature of a gas, when it is in thermodynamic equilibrium with the environment, is determined by the average probability of the collisions of the gas particles (atoms or molecules): the higher this probability, the higher is the gas temperature. Chemist Arrhenius was member of physicist Boltzmann’s team in the University of Graz in Austria in 1887 and proposed that Boltzmann’s equation (2) be used to assess the height of the energy barrier determined by the activation energy of the given material to get over this barrier to commence a chemical reaction. The effective activation energy formula (3) plays in the BAZ model (1) the role of the stress-free energy U_0 plays in the Arrhenius model (2). Zhurkov and his associates used the model (1) to determine the fracture toughness of materials experiencing combined action of elevated temperature and external loading. It has been shown [4] that the models (1) and (2) can be obtained as steady-state solutions to the Fokker-Planck equation in the theory of Markovian processes (see, e.g., [5]), and that these solutions represent the worst case scenarios, so that the reliability predictions based on the steady-state BAZ model (1) are conservative and, hence, advisable in engineering practice.

$$\tau = \tau_0 \exp\left(\frac{U_0 - \gamma\sigma}{kT}\right) \text{ ---- (1)}$$

$$\tau = \tau_0 \exp\left(\frac{U_0}{kT}\right) \text{ ---- (2)}$$

$$U = kT \ln \frac{\tau}{\tau_0} = U_0 - \gamma\sigma \text{ ---- (3)}$$

In Zhurkov’s tests the loading σ was always a constant mechanical tensile stress applied to notched specimens. It has been suggested [6] that, when the reliability of an electronic or a photonic material is being evaluated, any other loading (stressor, stimulus) of importance (voltage, current, thermal stress, humidity, vibrations, radiation, light output, etc.) can also be used as a stressor, and that, since the principle of superposition does not work in reliability physics, an appropriate combination of relevant stimuli can be considered [7].

The τ value is viewed in the BAZ model as the MTTF. Such an assumption suggests that if the exponential law of the probability of non-failure is used, the MTTF corresponds to the moment of time when the entropy $H(P)$ of the distribution formula (4) reaches its maximum value. Indeed, from the formula $H(P) = -P \ln P$ it could be concluded that the maximum entropy $H(P)$ is equal to e^{-1} and takes place for $P = e^{-1} = 0.3679$. In such a situation the equation (4) yields:

$$P = \exp(-\lambda t) = \exp\left(-\frac{t}{\tau}\right) = \exp\left[-\frac{t}{\tau_0} \exp\left(-\frac{U_0 - \gamma\sigma}{kT}\right)\right] \text{ ---- (4)}$$

$$t = \tau_0 \exp\left(\frac{U}{kT}\right) \text{ ---- (5)}$$

Comparing this result with the equation (1) we conclude that the MTTF expressed by this equation corresponds to the moment of time when the entropy of the time-depending process $P = P(t)$ is the largest and is equal to e^{-1} . Another modification that has been recently introduced to the model (1) has to do with the probabilistic design-for-reliability (PDFR) concept [9-17] and its experimental basis - highly focused and highly cost-effective FOAT [18-21]. Such testing should always be geared to a physically meaningful predictive model, and flexible and easy-to-use BAZ model can be effectively employed in this capacity. It has been suggested also that when a suitable FOAT is considered, the time constant τ_0 in the distribution (4) is replaced by the quantity $(\gamma_c C)^{-1}$ where t is time, C is a suitable criterion of failure (such as, say, elevated leakage current or high electrical resistance) and γ_c is the sensitivity factor. Then the distribution (4) can be written, considering its application in FOAT, as

$$P = \exp\left[-\gamma_c C t \exp\left(-\frac{U_0 - \gamma\sigma}{kT}\right)\right] \text{ ---- (6)}$$

In the analysis below this expression or its multi-parametric extension formula (7) are employed to assess the probability of non-failure and time-to-failure (TTF) 1) of an optical fiber proof-tested at an elevated temperature and experiencing static fatigue [22], 2) of an electron device subjected to temperature cycling [23], 3) of a solder material subjected to the low-temperature/random-vibrations bias [24], and 4) of a device subjected to elevated-humidity/elevated-voltage bias [25].

$$P = \exp \left[-\gamma_c C t \exp \left(-\frac{1}{kT} \left(U_0 - \sum_{i=1}^n \gamma_i \sigma_i \right) \right) \right] \text{ ---- (7)}$$

Analysis

Static fatigue lifetime of an optical fiber proof-tested at an elevated temperature

There is a significant research on reliability and proof-testing of optical fibers (see, e.g., [26-36]). Using the BAZ model, the condition at failure is obvious, and the equation (6) can be assumed in the form:

$$P = \exp \left[-\gamma_t t \exp \left(-\frac{U_0 - \gamma \sigma}{kT} \right) \right] \text{ ---- (8)}$$

where γ_t is the sensitivity factor for the testing time. The equation (8) has three unknowns: the activation energy U_0 and two sensitivity factors: the applied stress factor γ and the testing time factor γ_t .

At the first step the FOAT should be conducted for two different temperatures T_1 and T_2 keeping the level of the applied stress σ the same in both tests. After recording the percentages P_1 and P_2 of non-failed samples by the long enough times t_1 and t_2 the following relationships can be obtained:

$$P_{1,2} = \exp \left[-\gamma_t t_{1,2} \exp \left(-\frac{U_0 - \gamma \sigma}{kT_{1,2}} \right) \right] \text{ ---- (9)}$$

Since the numerator $U_0 - \gamma \sigma$ was kept the same, the following equation must be fulfilled for the sought time sensitivity factor γ_t :

$$\ln \left(\frac{n_1}{\gamma_t} \right) - \theta \ln \left(\frac{n_2}{\gamma_t} \right) = 0, \text{ ---- (10)}$$

where the notations

$$n_{1,2} = \frac{-\ln P_{1,2}}{t_{1,2}}, \quad \theta = \frac{T_2}{T_1} \text{ ---- (11)}$$

are used. Here t_1 and t_2 are the times, at which the failures fiber breaks occurred. The equation (10) has the following solution:

$$\gamma_t = \exp \left[\frac{1}{\theta - 1} \ln \left(\frac{n_2^\theta}{n_1} \right) \right] \text{ ---- (12)}$$

At the second step, FOAT at two stress levels, σ_1 and σ_2 , should be conducted for the same temperature. This leads to the following formula for the ratio of the stress sensitivity factor to the thermal energy:

$$\frac{\gamma}{kT} = \frac{\ln \left(\frac{n_1}{n_2} \right)}{\sigma_1 - \sigma_2} \text{ ---- (13)}$$

Note that the stress sensitivity factor γ is independent of the temperature sensitivity factor γ_t . The ratio of the activation energy U_0 to the thermal energy can be computed for any consistent levels

of stress, temperature and time as:

$$\frac{U_0}{kT} = \frac{\gamma \sigma}{kT} \sigma_1 - \ln \left(\frac{n_1}{\gamma_t} \right) = \frac{\gamma \sigma}{kT} \sigma_2 - \ln \left(\frac{n_2}{\gamma_t} \right) \text{ ---- (14)}$$

The expected fatigue lifetime can be found from (8) for the given (specified) probability P as:

$$t = -\frac{\ln P}{\gamma_t} \exp \left(\frac{U_0}{kT} - \frac{\gamma \sigma}{kT} \right) \text{ ---- (15)}$$

Clearly, the predicted fatigue lifetime depends on the expected (accepted, specified) probability of non-failure of the fiber subjected to the given loading and temperature. If the acceptable probability of non-failure is low, the estimated fatigue lifetime can be rather long.

Let, e.g., the following input FOAT information is obtained at the first step of FOAT for a coated specialty fiber intended for elevated temperature operations:

1) After $t_1 = 10h$ of testing at the temperature of $T_1 = 300^\circ C = 573^\circ K$, under the stress of $\sigma = 420 \text{ kg/mm}^2$, 10% of the tested specimens failed, so that the probability of non-failure is $P_1 = 0.9$;

2) After $t_2 = 8.0h$ of testing at the temperature of $T_2 = 350^\circ C = 623^\circ K$ under the same stress, 25% of the tested samples failed, so that the probability of non-failure is $P_2 = 0.75$

Then the formulas (11) yield:

$$n_1 = -\frac{\ln P_1}{t_1} = -\frac{\ln 0.9}{10.0} = 0.010536h^{-1},$$

$$n_2 = -\frac{\ln P_2}{t_2} = -\frac{\ln 0.75}{8.0} = 0.035960h^{-1},$$

$$\theta = \frac{T_2}{T_1} = \frac{623}{573} = 1.08726$$

and the formula (12) yields:

$$\gamma_t = \exp \left[\frac{1}{\theta - 1} \ln \left(\frac{n_2^\theta}{n_1} \right) \right] = \exp \left[\frac{1}{1.08726 - 1} \ln \left(\frac{0.035960^{1.08726}}{0.010536} \right) \right] = 46307.5136h^{-1}$$

At the second step FOAT has been conducted at the stress levels of $\sigma_1 = 420 \text{ kg/mm}^2$ and $\sigma_2 = 320 \text{ kg/mm}^2$ at the temperature of $T = 350^\circ C = 623^\circ K$ and it has been confirmed that 10% of the tested samples under the stress level of $\sigma_1 = 420 \text{ kg/mm}^2$ failed after $t_1 = 10.0h$ of testing, so that $P_1 = 0.9$. The percentage of failed samples tested at the stress level of $\sigma_2 = 320 \text{ kg/mm}^2$ was 5% after $t_2 = 24h$ of testing, so that $P_2 = 0.95$. Then the formula (13) results in the following ratio of the stress sensitivity factor to the thermal energy:

$$\frac{\gamma}{kT} = \frac{\ln \left(\frac{n_1}{n_2} \right)}{\sigma_2 - \sigma_1} = \frac{1}{100} \ln \left(\frac{0.035960}{0.010536} \right) = 0.0122761 \text{ mm}^2 / \text{kg}$$

After the sensitivity factors for the time and the stress are deter-

mined, the ratio of the stress activation energy to the thermal energy can be found as:

$$\frac{U_0}{kT} = \frac{\gamma}{kT} \sigma - \ln \left(-\frac{\ln P}{t\gamma_i} \right) = 0.0122761\sigma - \ln \left(-2.1595 \times 10^{-5} \frac{\ln P}{t} \right)$$

If, e.g., the stress $\sigma = 320 \text{ kg/mm}^2$ is applied for $t = 24h$ and the acceptable probability of non-failure at the end of this time is, say, $P = 0.99$ then

$$\frac{U_0}{kT} = \frac{\gamma}{kT} \sigma - \ln \left(-\frac{\ln P}{t\gamma_i} \right) = 0.0122761 \times 320 - \ln \left(-\frac{\ln 0.99}{24 \times 46307.5136} \right) = 3.9284 + 18.5213 = 22.4496$$

This result indicates particularly that the activation energy U_0 is determined primarily by the property of the silica material (second term in the above equation), but is affected also, in this approach, by the applied stress: the higher the applied stress is, the higher is the activation energy.

The fatigue lifetime of the fiber can be determined for the acceptable (specified) probability of non-failure using the formula (15). If, e.g., the acceptable probability of non-failure is as low as $P = 0.8$, the applied temperature is $T = 325^\circ\text{C} = 598^\circ\text{K}$ and the applied stress is 5.0 kg/mm^2 , then the expected fatigue lifetime of the fiber is

$$t = -\frac{\ln P}{\gamma_i} \exp \left(\frac{U_0}{kT} - \frac{\gamma}{kT} \sigma \right) = -\frac{\ln 0.8}{46307.3146} \exp(22.4496 - 0.0122761 \times 5.0) = 25469.42h = 2.9075 \text{ years}$$

If the acceptable/specified probability of non-failure is $P = 0.99$, then the predicted lifetime is only

$$t = -\frac{\ln 0.99}{46307.3146} \exp(22.4496 - 0.0122761 \times 5.0) = 1147.15h = 47.8 \text{ days}$$

Time to failure (TTF) of an electron device subjected to temperature cycling

Using the BAZ model (6), the probability of non-failure of a vulnerable material, such as, e.g., solder joint interconnection experiencing inelastic strains during temperature cycling can be sought in the form:

$$P = \exp \left[-\gamma R t \exp \left(-\frac{U_0 - nW}{kT} \right) \right] \text{ ---- (16)}$$

Here U_0 , eV is the activation energy and is the characteristic of the solder material's propensity to fracture, W , eV is the damage caused by a single temperature cycle and measured, in accordance with Hall's concept, by the hysteresis loop area of a single temperature cycle for the strain of interest [37], T , K is the absolute temperature (say, the cycle's mean temperature), n is the number of cycles, k is Boltzmann's constant, t , sec, is time, R , Ω is the measured (monitored) electrical resistance at the peripheral joint location, and γ is the sensitivity factor for the resistance.

The above equation makes physical sense. Indeed, the probability P of non-failure is zero at the initial moment of time $t = 0$ and when the electrical resistance R of the joint material is zero; this probability decreases, because of material aging and/or structural degradation with time, and not necessarily only because of temperature cycling; it is lower for higher electrical resistance (a resist-

ance as high as, say, 450Ω can be viewed as an indication of an irreversible mechanical failure of the joint); materials with higher activation energy U_0 have a lower probability of possible failure; the increase in the number of cycles n leads to lower effective activation energy $U = U_0 - nW$, and so does the level of the energy W of a single cycle.

It could be shown that the maximum entropy of the distribution (16) takes place at the MTTF τ expressed as:

$$\tau = \frac{1}{\gamma R} \exp \left(\frac{U_0 - nW}{kT} \right) \text{ ---- (17)}$$

Mechanical failure in solder joints, associated with temperature cycling, occurs, when the number of cycles n is $n_f = U_0/W$. When this condition takes place, the temperature in the denominator in the parentheses of the equation (17) becomes irrelevant, and this equation yields: $P_f = \exp(-t_f/\tau_f)$ where P_f is the measured probability of non-failure and $\tau_f = 1/\gamma R_f$ is the MTTF.

If, e.g., 20 devices have been temperature cycled and the high resistance $R_f = 450\Omega$ considered as an indication of failure was detected in 15 of them, then $P_f = 0.25$. If the number of cycles during such FOAT was, say, $n_f = 2000$, and each cycle lasted for 20min = 1200sec., then the time at failure is $t_f = 2000 \times 1200 = 24 \times 10^5$ sec and

$$\gamma = \frac{-\ln P_f}{R_f t_f} = \frac{-\ln 0.25}{450 \times 24 \times 10^5} = 1.2836 \times 10^{-9} \Omega^{-1} \text{ sec}^{-1},$$

$$\tau_f = \frac{1}{1.2836 \times 10^{-9} \times 450} \text{ sec} = 480.9 \text{ hrs} = 20.0 \text{ days}$$

According to Hall's concept, the energy W of a single cycle should be measured, by running a specially designed test, using strain gages. Let, e.g., the measured area of the hysteresis loop was $W = 2.5 \times 10^{-4} \text{ eV}$. Then the stress-free activation energy is $U_0 = n_f W = 2000 \times 4.5 \times 10^{-4} = 0.9 \text{ eV}$. In order to assess the number of cycles to failure in actual operation conditions one could assume that the temperature range in these conditions is, say, half the accelerated test range, and that the area W of the hysteresis loop is proportional to the temperature range.

Then the number of cycles to failure is $n_f = U_0/W = (0.9 \times 2.0)/(2.5 \times 10^{-4}) = 7200$ and the time to failure will be $t_f = 7200 \text{ days} = 19.7 \text{ years}$, if the duration of one cycle in actual operation conditions is one day.

Time-to-failure for a solder material subjected to the low-temperature/random-vibrations bias

Although there exist promising ways to avoid inelastic strains in solder joints of the second level of interconnections in IC package designs [38], it still appears more typical than not that the peripheral joints of a package/PCB assembly experience inelastic strains. This takes place at low temperature conditions, when the deviation from the high fabrication temperature is the largest and the induced thermal stresses are the highest. On the other hand, it is well known that it is the combination of low temperatures and repetitive dynamic loading that accelerates dramatically the propagation of fatigue cracks, whether elastic or inelastic. A

modification of the BAZ model is developed for the evaluation of the time-to-failure of the second level solder joint interconnections whose peripheral joints experience inelastic strains. The suggested methodology is viewed as a possible, effective and attractive alternative to temperature cycling. The random vibrations are considered as a white noise of the given $(m/s^2)^2/Hz$ level - the ratio of the acceleration amplitudes squared to the vibration frequency. We use the BAZ equation in the form [39]:

$$P = \exp \left[-\gamma_R R_* t \exp \left(-\frac{U_0 - \gamma_S S}{kT} \right) \right] \text{ ---- (18)}$$

where S is the random vibration spectrum and R_* is the measured electrical resistance that is considered high enough, so that the peripheral solder joint(s) most likely lost their integrity. Using the FOAT procedure similar to the one in the previous sections, we obtain the following formula for the sensitivity factor γ_R :

$$\gamma_R = \exp \left[\frac{1}{\theta - 1} \ln \left(\frac{n_2^\theta}{n_1} \right) \right], \quad \theta = \frac{T_2}{T_1}; \quad n_{1,2} = -\frac{\ln P_{1,2}}{R_* t_{1,2}} \text{ ---- (19)}$$

the activation energy U_0 and the fatigue lifetime t can be calculated from the equation (18) as:

$$U_0 = \gamma_S S - kT \ln \left(-\frac{\ln P}{\gamma_R R_* t} \right) \text{ ---- (20)}$$

and

$$t = \frac{-\ln P}{\gamma_R R_*} \exp \left(\frac{U_0 - \gamma_S S}{kT} \right) \text{ ---- (21)}$$

Let, e.g., the first step of testing is conducted until the resistance threshold of $R_* = 450\Omega$ is reached. Half of the specimen population failed at the temperature of $T_1 = -50^\circ C = 223^\circ K$ after $t_1 = 100h$ of testing. When testing was conducted at the temperature of $T_2 = 0^\circ C = 273^\circ K$, half of the specimens population failed after $t_2 = 300h$ of testing. The level of the vibration power spectrum density S was kept the same in both sets of tests. The last two formulas in (19) yield:

$$\theta = \frac{T_2}{T_1} = \frac{273}{223} = 1.224215;$$

$$n_1 = -\frac{\ln P_1}{R_* t_1} = -\frac{\ln 0.5}{450 \times 100} = 1.540327 \times 10^{-5} \Omega^{-1} h^{-1};$$

$$n_2 = -\frac{\ln P_2}{R_* t_2} = -\frac{\ln 0.5}{450 \times 300} = 0.513442 \times 10^{-5} \Omega^{-1} h^{-1};$$

The first formula in (19) results in the following value of the factor γ_R :

$$\gamma_R = \exp \left[\frac{1}{\theta - 1} \ln \left(\frac{n_2^\theta}{n_1} \right) \right] = \exp \left[\frac{1}{0.224215} \ln \left(\frac{(5.13442 \times 10^{-5})^{1.224215}}{1.540327 \times 10^{-5}} \right) \right] = 0.0110289 \Omega^{-1} h^{-1},$$

Let the second step of testing be carried out until just 1% of the specimens failed, so that $P = 0.99$. This took place after $t_1 = 150h$ of testing at the temperature of $T_1 = -30^\circ C = 243^\circ K$ at the vibration level of $S_1 = 10^6 mm^2 sec^{-3}$ and after $t_2 = 50h$ of testing at the temperature of $T_2 = -50^\circ C = 223^\circ K$ at the vibration level of $S_2 =$

$2 \times 10^6 mm^2 sec^{-3}$. The effective activation energy is

$$U_1 = U_0 - \gamma_S S_1 = -kT_1 \ln \left(-\frac{\ln P}{R_* t_1 \gamma_R} \right) = -8.6176 \times 10^{-5} \times 243 \ln \left(-\frac{\ln 0.99}{450 \times 150 \times 0.0110289} \right) = 0.234805 eV,$$

when testing is carried out at the temperature $T_1 = -30^\circ C = 243^\circ K$, and is

$$U_2 = U_0 - \gamma_S S_2 = -kT_2 \ln \left(-\frac{\ln P}{R_* t_2 \gamma_R} \right) = -8.6176 \times 10^{-5} \times 223 \ln \left(-\frac{\ln 0.99}{450 \times 50 \times 0.0110289} \right) = 0.194367 eV$$

when testing is carried out at the temperature $T_2 = -50^\circ C = 223^\circ K$. Requiring that the zero stress activation energy be loading independent, one can evaluate the vibration related sensitivity factor as

$$\gamma_S = \frac{U_1 - U_2}{S_2 - S_1} = \frac{0.234805 - 0.194367}{2 \times 10^6 - 10^6} = 4.0438 \times 10^{-8} eV mm^{-2} sec^3$$

Then the stress-free activation energy can be computed as

$$U_0 = U_1 + \gamma_S S_1 = 0.234805 + 4.0438 \times 10^{-8} = 0.23488 eV$$

The remaining useful life can be computed for any probability of non-failure, low temperature and vibration spectral density as

$$t = RUL = -\frac{\ln P}{\gamma_R R_*} \exp \left(\frac{U_0 - \gamma_S S}{kT} \right) = -\frac{\ln P}{0.0110289 \times 450} \exp \left(\frac{0.23488 - 4.0438 \times 10^{-8} S}{8.6176 \times 10^{-5} T} \right) = -0.201490 \ln P \exp \left(\frac{0.23488 - 4.0438 \times 10^{-8} S}{8.6176 \times 10^{-5} T} \right)$$

If, e.g., $P = 0.9$, $T = -20^\circ C = 253^\circ K$ and $S = 10^3 mm^2 sec^{-3}$ then

$$t = RUL = -0.201490 \ln 0.9 \exp \left(\frac{0.23488 - 4.0438 \times 10^{-8} S}{8.6176 \times 10^{-5} \times 253} \right) = 841.5131 h = 35.06 days$$

Multi-parametric BAZ model

Let us elaborate on the substance of the multi-parametric BAZ model using as an example a situation when the product of interest is subjected to the combined action of the elevated relative humidity H and elevated voltage V . The failure rate of a product is determined by the level of the leakage current: $\lambda = \gamma_I I$. Then the equation (7) can be written as

$$P = \exp \left[-\gamma_I I t \exp \left(-\frac{U_0 - \gamma_H H - \gamma_V V}{kT} \right) \right] \text{ ---- (22)}$$

Here the γ factors reflect the sensitivities of the device to the change in the corresponding stressors. Although only two stressors are selected – the relative humidity H and the elevated voltage V – the model can be easily made multiparametric, i.e., generalized for as many stimuli as necessary.

The sensitivity factors γ should be determined from the FOAT when the combined action of all the stimuli (stressors) of importance is considered. Because of that the structure of the multi-parametric BAZ should not be interpreted as a superposition of the effects of different stressors (as is known, superposition principle does not work in reliability engineering), but rather as a convenient and physically meaningful representation of the FOAT data. The physical meaning of the distribution (22) could be seen from the formulas

$$\frac{\partial P}{\partial I} = -\frac{H(P)}{I}, \quad \frac{\partial P}{\partial t} = -\frac{H(P)}{t}, \quad \frac{\partial P}{\partial U_0} = \frac{H(P)}{kT},$$

$$\frac{\partial P}{\partial H} = -\frac{H(P)}{kT} \gamma_H = -\gamma_H \frac{\partial P}{\partial U_0},$$

$$\frac{\partial P}{\partial V} = -\frac{H(P)}{kT} \gamma_V = -\gamma_V \frac{\partial P}{\partial U_0}, \quad \text{---- (23)}$$

where $H(P) = -P \ln P$ is the entropy of the probability $P = P(t)$ of non-failure. The following conclusions can be made based on these formulas:

1) The change in the probability of non-failure always increases with an increase in the entropy (uncertainty) of the distribution. This probability decreases with an increase in the leakage current and with time, which certainly makes physical sense.

2) The last two of the above formulas show the physical meaning of the sensitivity factors γ : they can be found as the ratios of the change in the probability of non-failure with respect to the corresponding stimuli to the change of this probability with the change in the stress-free activation energy.

The equation (22) for the probability of non-failure contains four empirical parameters: the stress-free activation energy U_0 and three sensitivity factors γ : leakage current factor, relative humidity factor and elevated voltage factor. Here is how these factors could be obtained from the highly focused and highly cost effective FOAT data.

First, one should run the FOAT for two different temperatures T_1 and T_2 keeping the levels, low or high, of the relative humidity H and elevated voltage V the same in both tests; recording the percentages (values) P_1 and P_2 of non-failed samples; assuming a certain criterion of failure (say, when the level of the measured leakage current exceeds a certain level I_*), we obtain the following two relationships:

$$P_{1,2} = \exp \left[-\gamma_I I_* t_{1,2} \exp \left(-\frac{U_0 - \gamma_H H - \gamma_V V}{kT_{1,2}} \right) \right], \quad \text{---- (24)}$$

Since the numerators in these relationships are kept the same, the following equation must be fulfilled for the sought sensitivity factor γ_I of the leakage current:

$$f(\gamma_I) = \ln \left(-\frac{\ln P_1}{I_* t_1 \gamma_I} \right) - \frac{T_2}{T_1} \ln \left(-\frac{\ln P_2}{I_* t_2 \gamma_I} \right) = 0 \quad \text{---- (25)}$$

Here t_1 and t_2 are the times, at which the failures were detected. This equation has the following solution:

$$\gamma_I = \frac{1}{I_*} \exp \left(\frac{1}{\theta - 1} \left(\frac{n_2^\theta}{n_1} \right) \right) \quad \text{---- (26)}$$

where the notation (11) is used.

At the second step, FOAT tests at two relative humidity levels H_1 and H_2 should be conducted for the same temperature and voltage.

This leads to the relationship:

$$\gamma_H = \frac{kT}{H_1 - H_2} \left[\ln \left(\frac{n_1}{I_* \gamma_I} \right) - \ln \left(\frac{n_2}{I_* \gamma_I} \right) \right] = \frac{kT}{H_1 - H_2} \ln \left(\frac{n_1}{n_2} \right) \quad \text{---- (27)}$$

Similarly, at the next step of FOAT tests, by changing the voltages V_1 and V_2 , the following expression for the sensitivity factor γ_V can be obtained:

$$\gamma_V = \frac{kT}{V_1 - V_2} \left[\ln \left(\frac{n_1}{I_* \gamma_I} \right) - \ln \left(\frac{n_2}{I_* \gamma_I} \right) \right] = \frac{kT}{V_1 - V_2} \ln \left(\frac{n_1}{n_2} \right) \quad \text{---- (28)}$$

Finally, the stress-free activation energy can be computed as

$$U_0 = \gamma_H H + \gamma_V V - kT \ln \left(-\frac{\ln P}{I_* \gamma_I} \right) = kT \left[\left(\frac{H}{H_1 - H_2} + \frac{V}{V_1 - V_2} \right) \ln \left(\frac{n_1}{n_2} \right) - \ln \left(-\frac{\ln P}{I_* \gamma_I} \right) \right] \quad \text{---- (29)}$$

for any consistent humidity, voltage, temperature and time.

Let, e.g., the following input information is available:

1) After $t_1 = 35h$ of testing at the temperature $T_1 = 60^\circ C = 333^\circ K$, the voltage $V = 600V$ and the relative humidity $H = 0.85$, 10% of the tested modules exceeded the allowable (critical) level of the leakage current of $I_* = 3.5 \mu A$ and, hence, failed, so that the probability of non-failure is $P_1 = 0.9$;

2) After $t_2 = 70h$ of testing at the temperature $T_2 = 85^\circ C = 358^\circ K$ at the same voltage and the same relative humidity, 20% of the tested samples reached or exceeded the critical level of the leakage current and, hence, failed, so that the probability of non-failure is $P_2 = 0.8$.

Then the equation (25) yields:

$$f(\gamma_I) = \ln \left(\frac{0.10536}{\gamma_I} \right) - 1.075075 \ln \left(\frac{0.22314}{\gamma_I} \right) = 0,$$

and its solution is $\gamma_I = 4926 h^{-1} (\mu A)^{-1}$, so that $\gamma_I I_* = 1724 h^{-1}$. This concludes the first step of testing. At the second step, tests at two relative humidity levels H_1 and H_2 , were conducted for the same temperature and voltage levels. This leads to the relationship:

$$\gamma_H = \frac{kT}{H_1 - H_2} \left[\ln(0.5800 \times 10^{-4} n_1) - \ln(0.5800 \times 10^{-4} n_2) \right]$$

Let, e.g., after $t_1 = 40h$ of testing at the relative humidity of $H_1 = 0.5$ at the given voltage (say, $V = 600V$) and temperature (say, $T = 60^\circ C = 333^\circ K$), 5% of the tested modules failed, so that $P_1 = 0.95$, and after $t_2 = 55h$ of testing at the same temperature and at the relative humidity of $H_2 = 0.85$, 10% of the tested modules failed, so that $P_2 = 0.9$. Then the above equation for the γ_H value, with the Boltzmann constant $k = 8.61733 \times 10^{-5} eV/K$ yields: $\gamma_H = 0.03292 eV$. At the third step, FOAT at two different voltage levels $V_1 = 600V$ and $V_2 = 1000V$ have been carried out for the same temperature-radiation bias, say, $T = 85^\circ C = 358^\circ K$ and $H = 0.85$, and it has been determined that 10% of the devices failed after $t_1 = 40h$ of testing ($P_1 = 0.9$) and 20% of devices failed after $t_2 = 80h$ ($P_2 = 0.8$). Then

$$\gamma_V = \frac{0.02870}{400} \left[\ln(0.5800 \times 10^{-4} n_1) - \ln(0.5800 \times 10^{-4} n_2) \right] = 4.1107 \times 10^{-6} eV/V [0,1]$$

After the sensitivity factors of the leakage current, the humidity and the voltage are found, the stress free activation energy can be determined for the given temperature and for any combination of loadings (stimuli). The third term in the equation (29) for the stress-free activation energy plays the dominant role, so that, in approximate evaluations, only this term could be considered. Calculations indicate that the loading free activation energy in the above numerical example (even with the rather tentative, but still realistic, input data) is about $U_0 = 0.4770\text{eV}$. This result is consistent with the existing experimental data. Indeed, for semiconductor device failure mechanisms the activation energy ranges from 0.3 to 0.6eV, for metallization defects and electromigration in Al it is about 0.5eV, for charge loss it is on the order of 0.6eV, for Si junction defects it is 0.8eV. With the above FOAT data the formula (22) yields:

$$P = \exp \left[-\gamma_f \ln \exp \left(-\frac{U_0 - \gamma_H H - \gamma_V V}{kT} \right) \right] = \exp \left[-4926 \ln \exp \left(-\frac{0.4770 - 0.03292H - 4.1107 \times 10^{-6}V}{8.61733 \times 10^{-5}T} \right) \right]$$

If, e.g., in actual operation conditions $I_* = 1.5\mu\text{A}$ is the acceptable level of the leakage current, the actual humidity is, say, $H = 0.1$, the applied voltage is $V = 220\text{V}$ and the temperature of the device is, say, $T = 70^\circ\text{C} = 343^\circ\text{K}$ then the obtained formula yields:

$$P = \exp \left[-4926 \ln \exp \left(-\frac{0.4770 - 0.03292H - 4.1107 \times 10^{-6}V}{8.61733 \times 10^{-5}T} \right) \right] = \\ = \exp \left[-7389 \ln \exp \left(-\frac{0.4770 - 0.0033 - 0.0009}{0.029557} \right) \right] = \exp(-0.0008347t).$$

The predicted probability of non-failure is, in this example, 0.7916 after a week operation, 0.5483 after a month operation, and only 0.000738 after a year operation.

Conclusion

Application of FOAT and BAZ equation enable to quantify reliability of microelectronic products on the probabilistic basis. Future work should include experimental verification of the theoretical findings, as well as new applications of the suggested techniques.

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