Stability Analysis of Neural Network Models in Engineering Design

Trayan Ganev Stamov

Abstract: ***In this paper neural networks applications in engineering design are discussed. The question for stability of their steady states is also considered. Some new efficient criteria are proposed. Since neural networks are relevant systems applied in various engineering design tasks, including many optimization and control problems, the results can be useful in design of such systems of diverse interest.***

Keywords: ***Engineering design, neural networks, stability.***

I. INTRODUCTION

Neural networks (NNs) are ones of the key crowdsourcing technologies for engineering design and development [1]. They are recognized as ones of the best techniques for solving optimization problems, pattern recognition, control and forecasting in product design. In addition, the methods of collecting design information are very important factors in modern product development process [2] due to their:

- Learning ability
- Storage ability
- Fault tolerance
- Inductive ability
- Parallel handling ability.

The opportunities for applications of NNs in engineering design have been object of numerous investigations during the last decades. For example, the book [3] offers an excellent overview of the state-of-the-art of the research activities, network concepts and techniques to design and manufacturing. Since 1993 NNs have been used in certain classes of optimal design problems [4- 6], in the automation design processes [7, 8], in retrieval processes, simulations, decision making, pattern recognition and prediction [9-15], including some recent contributions [16, 17, 18]. In addition to these [19- 21] are very good sources where the latest application of artificial intelligence and integrated intelligent systems for concurrent integration and collaboration of the design of a product and its related processes are presented.

Stability is one of the main properties in a neural network dynamics. It is related to the opportunity of huge variations in the output values as a result of small perturbations in the initial data. The main goal of the stability analysis is to find efficient criteria that guarantee that small perturbations of initial data lead to small variations in outputs at a later time (short or long period of time). It is worth to note that stability is also related to control of the qualitative properties of a neural network model.

Due to the importance of the concept, stability analysis of neutral systems has received considerable attention of many authors. See, for example, [22-28] and the references therein. However, to the best of our knowledge, there has not been any work so far considering a stability strategy for a neural network model used in engineering design, which is very important in theories and applications and also is a very challenging problem. That is exactly what is planned in the proposed research.

Among the existing methods for stability analysis, the Lyapunov function method seems to be very effective in applications since no knowledge for the solution is required. The method, also known as second or direct method of Lyapunov, is based on the existence of an auxiliary function with certain properties. The Lyapunov function technique [29] and its modifications have been greatly applied in the stability analysis of numerous dynamical systems [23, 30-34], including NNs [24, 27, 28, 35- 40].

In this paper, a Lyapunov-based approach is adapted to analyze the stability behavior of a generalized NN model used for the form design of product image [4].

The paper is organized as follows. Section II provides information on main issues related to neural network models. Section III describes the structure of a generalized Hopfield-type neural network model considered in this paper. In Section IV after some preliminaries, a Lyapunov-based stability analysis is proposed to provide stable design process. The paper concludes in Section V.

II. NEURAL NETWORKS

NNs are non-linear models that are widely used to examine the complex relationship between input variables and output variables [4, 41]. The connections between the input variables and output variables are weighted. In many NNs, the architecture allows one or more layers (hidden layers) between the layer of the input and the layer of the output variables. Fig. 1 shows a NN with a hidden layer. In it $x_1, x_2, ..., x_n$ are the input variables, $y_1, y_2, ..., y_p$ are the output variables, and w_{ij} , w_{jk} are the connections weights.

Revised Manuscript Received on February 05, 2020.
* Correspondence Author
Trayan Ganev Stamov*, Department of Machine Elements and Non-metallic Construction, Technical University of Sofia, Sofia, Bulgaria. Email: tstamov@tu-sofia.bg

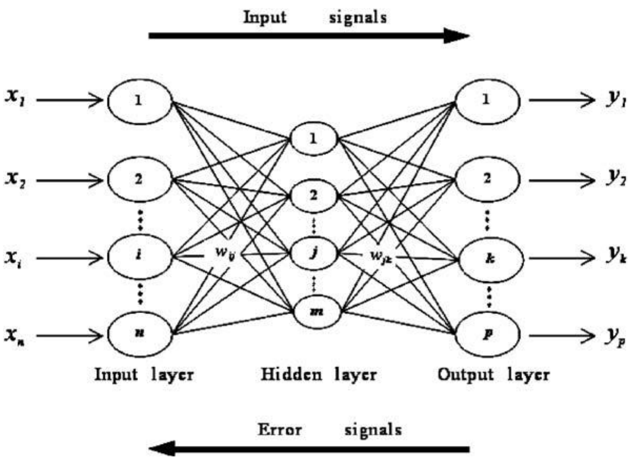

Fig. 1. Three-layer NN [4].

The variables (nodes, neurons, units) in each layer are their structural elements. A typical graph of a neuron which process information by its dynamic state is given in Fig. 2.

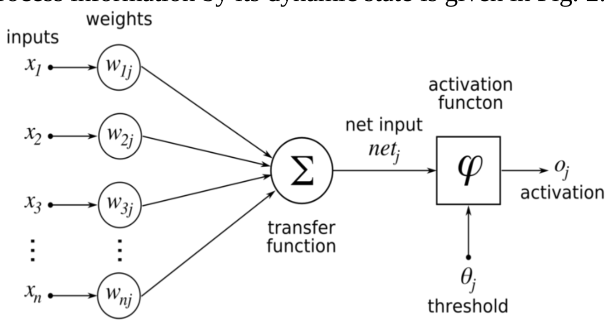

Fig. 2: A model of a neuron [42].

An activation (transfer) function for each neuron defines the output of the neuron. The sigmoid function $f(x)=\dfrac{1}{1+e^{-x}}$ that assumes a continuous range of values from 0 to 1 is the most commonly used activation function in a NN architecture.

Neural nets thus mimic the human brain. They lead to the design principles for constructing human-brain-like machines [15]. The inputs correspond to the signals to the synapses of a biological neuron, while each weight corresponds to the strength of a single biological synaptic connection.

Feedforward NNs and *recurrent* or *feedback* NNs are the most typical categories of NNs. In feedforward networks nodes are updated starting with the input layer, and then updated layer by layer to the output layer (Fig. 1). In recurrent or feedback networks where there is no such direction in the flow of control, computation is a relaxation in which the nodes are updated until a specified point is reached after which updating has no effect [15].

Training the network to perform well with reference to a training set is one of the main issues in building a NN model. Training a neural net refers to determining the proper values of all the weights in the architecture, and is accomplished most commonly through backpropagation [43].

III. GENERALIZED HOPFIELD-TYPE NEURAL NETWORK MODELS IN ENGINEERING DESIGN

Several authors suggested the use of Hopfield-type NNs for engineering design tasks. For example, in the paper [15] a model of the type

$$x_i(k+1)=\operatorname{sgn}\left(\sum_j w_{ij}x_j(k)\right),\ 1\le i\le n, \tag{1}$$

where sgn is the sgn (± 1) function, $x_j(k)$ is the state of the input j at time k, $k=0,1,2,\dots$, w_{ij} are the connection weights, has been applied to model a design retrieval problem encountered in batch production systems. The initial values

$$x_i(0),\ 1\le i\le n$$

are the elements of the input design pattern. The model (1) offers the opportunity to make design retrieval based not only on shape but other technological factors as well. The developed design storage and retrieval system (1) is interactive, since based on the initial responses, the designer can refine the query at any step.

A similar model is proposed in [4] to determine how the product form elements can be best combined to match a desirable product image. For this task the authors considered a three-layer NN shown in Fig. 1. In training the network, a set of input patterns or signals, $(x_1,x_2,\dots,x_n)$, is presented to the network input layer. The network then propagates the inputs from layer to layer until the outputs are generated by the output layer. This involves the generation of the outputs y_j of the neurons in the hidden layer as follows

$$y_j=f\left(\sum_i w_{ij}x_i-\theta_j\right),\ 1\le j\le n. \tag{2}$$

The neurons in the output layer are then given as

$$y_k=f\left(\sum_j w_{jk}x_j-\theta_k\right),\ 1\le k\le p. \tag{3}$$

The authors used a sigmoid activation function in (2) and (3), θ_j and θ_k are threshold values, w_{ij} and w_{jk} represent the weights for the connection between neuron i $(i=1,2,\dots,n)$ and neuron j $(j=1,2,\dots,m)$, and between neuron j $(j=1,2,\dots,m)$ and neuron k $(k=1,2,\dots,p)$, respectively.

In this paper, a generalized models is proposed described by the following discrete time Hopfield neural network system

$$x_i(k+1)=c_ix_i(k)+\sum_j w_{ij}g_j(x_j(k))+J_i, \tag{4}$$

where $1\le i,j\le n$, n corresponds to the number of nodes in the NN, $x_j(k)$ is the state of the input j at time k, $k=0,1,2,\dots$, w_{ij} are the connection weights, $g_j(x_j(k))$ denotes the activation function of the neuron j $(j=1,2,\dots,n)$, J_i is an external bias.

In the proposed model we take into account the opportunity of the neuron j $(j=1,2,\dots,n)$ to resets its potential to the resting state when isolated from other nodes and inputs with a constant rate c_i. In most cases $c_i=e^{-a_ih}$ where $a_i>0$ and $h>0$ is small enough [38].

In addition, a specific activation function for each node is proposed.

System (1) can be regarded as a discrete time analogue of the continuous time delayed Hopfield neural networks studied extensively in the

Retrieval Number: C5562029320/2020©BEIESP
DOI: 10.35940/ijeat.C5562.029320

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literature. See, for example, [36, 44, 45, 46] and the references therein.

IV. STABILITY CRITERIA

A. Preliminaries

In this section efficient criteria for stability of the neural network model (4) will be presented. First we will need some notations and definitions.

A state $x^* = (x_1^*, x_2^*, ..., x_n^*)$ is said to be an *equilibrium* (steady state) of the NN (4) if it satisfies the following relation

$$x_i^* = c_i x_i^* + \sum_j w_{ij} g_j(x_i^*) + J_i . \qquad (5)$$

The equilibrium points are very important in the stability analysis. For example, in solving of optimization problems, the equilibrium position is the optimal solution solution. When a NN is applied in the pattern recognition, the equilibrium position is the pattern. The stability of an equilibrium (pattern) means that the states will approach the pattern independently of the initial data.

One of the most important concepts in the stability analysis of NNs is the global asymptotic stability of the equilibrium points. If an equilibrium of a NN is globally asymptotically stable, it means that it is an attraction point for the whole space and the convergence is in real time. This is significant both theoretically and practically. Such NNs are known to be well-suited for solving some class of optimization problems. In fact, a globally asymptotically stable neural network is guaranteed to compute the global optimal solution independently of the initial data, which in turn implies that the network is devoid of spurious suboptimal responses [33].

The global exponential stability is a specific case of the global asymptotic stability that guarantees the fast convergence rate.

Definition 1. An equilibrium point $x^* = (x_1^*, x_2^*, ..., x_n^*)$ is *globally exponentially stable*, if there exist constants $\lambda > 1$ and $\mu \ge 1$ such that

$$\| x(k) - x^* \| \le \mu \| x(0) - x^* \| \lambda^{-k}, \; k = 0,1,2,... ,$$

where λ is the convergent rate.

In the above definition $|.|$ is the norm of the n-dimentional vector . In this paper we will use the following norm

$$\| x(k) - x^* \| = \sum_{i=1}^{n} | x_i(k) - x_i^* |, \; k = 0,1,2,... .$$

B. Lyapunov-based Stability Analysis

Lyapunov approach is related to the choice of a positive auxiliary function $V(k) > 0$ for any $k = 0,1,2,...$ for which the difference $\Delta V(k) = V(k+1) - V(k)$ is nonnegative. Such functions is known as Lyapunov (candidate) function [23, 29]. The same idea is also used for continuous systems, where instead of the difference $\Delta V(k)$ the derivative of the Lyapunov function V with respect to the corresponding system is used. For more information about the Lyapunov direct method see, for example, [23-40, 45, 46].

We make the following assumptions in this paper:

A1. There exists an equilibrium $x^* = (x_1^*, x_2^*, ..., x_n^*)$ for system (4).

A2. Any system output $x(k)$ can be measured and its initial values are assumed to be in a compact set.

A3. The activation functions g_i are such that

$$| g_i(u) - g_i(v) | \le L | u - v |$$

for any $1 \le i \le n$ and any real numbers u and v, where L is a positive constant.

A4. The constants $c_i \ge 0$ for $1 \le i \le n$.

A5. The connection weights w_{ij} and external biases J_i are real numbers for $1 \le i, j \le n$.

Theorem 1. Assume that A1-A5 hold and the systems' parameters satisfy

$$\max_{1\le i\le n} c_i + \max_{1\le j\le n} L_i \left(\sum_j | w_{ji} | \right) < 1 \qquad (6)$$

for any $1 \le i, j \le n$.

Then the equilibrium $x^* = (x_1^*, x_2^*, ..., x_n^*)$ of the NN model (4) is globally exponentially stable.

Proof. Let $x(k)$ be a system output with initial data that belong to a compact set.

Consider the Lyapunov function

$$V(k) = \| x(k) - x^* \| = \sum_{i=1}^{n} | x_i(k) - x_i^* | .$$

Since $x^* = (x_1^*, x_2^*, ..., x_n^*)$ is an equilibrium from (4) and (5) we have

$$V(k+1) = \| x(k+1) - x^* \| = \sum_{i=1}^{n} | x_i(k+1) - x_i^* |$$

$$\le \sum_{i=1}^{n} c_i | x_i(k) - x_i^* | + \sum_{i=1}^{n} \sum_j | w_{ij} | \, | g_j(x_j(k)) - g_j(x_j^*) |$$

$$\le \max_{1\le i\le n} c_i \sum_{i=1}^{n} | x_i(k) - x_i^* | + \sum_{i=1}^{n} \sum_j w_{ij} L_j | x_j(k) - x_j^* |$$

$$\le \max_{1\le i\le n} c_i \sum_{i=1}^{n} | x_i(k) - x_i^* | + \max_{1\le i\le n} L_i \left(\sum_j | w_{ji} | \right) \sum_{i=1}^{n} | x_i(k) - x_i^* |$$

$$= (\alpha + \beta) V(k) ,$$

where

$$\alpha = \max_{1\le i\le n} c_i , \; \beta = \max_{1\le j\le n} L_i \left(\sum_j | w_{ji} | \right) .$$

From the condition (6) of Theorem 1 it follows that we can find a positive $\dfrac{1}{\lambda}$ such

that $\lambda > 1$, $0 < \alpha + \beta \le \dfrac{1}{\lambda}$ and

$$V(k+1) \le \lambda^{-1} V(k) . \qquad (7)$$

From (7) we first have that

$$\Delta V(k) = V(k+1) - V(k) \le 0$$

for any $k = 0,1,2,...$, so the Lyapunov function is decreasing.

Also, by induction on $k = 0,1,2,...$ we have that

$$V(k) \le V(0) \,|\, \lambda^{-k}$$

or

$$\| x(k) - x^* \| \le \| x(0) - x^* \| \lambda^{-k},$$

which proves that the equilibrium $x^* = (x_1^*, x_2^*, ..., x_n^*)$ of the NN model (4) is globally exponentially stable and $\mu = 1$.

In the next results we will use the following notation. Let $\rho(Q)$ denotes the spectral radius of the matrix $Q = (q_{ij})$ and $q_{ij} > 0$ for $1 \le i, j \le n$.

Theorem 2. Assume that A1-A5 hold and $\rho(Q) < 1$ for $Q = (q_{ij})_{n \times n}$, $q_{ij} = L_j \,|\, w_{ij} \,|\, / (1 - c_i)$ where $0 < c_i \le 1$, $1 \le i, j \le n$.

Then the equilibrium $x^* = (x_1^*, x_2^*, ..., x_n^*)$ of the NN model (4) is globally exponentially stable.

The proof of Theorem 2 is similar to that of Theorem 1 following [36] and [46], and we will omit it here.

Remark 1. Theorems 1 and 2 provide Lyapunov-based criteria for global exponential stability of a generalized Hopfield-type NN model used in engineering design. The proposed technique is very efficient, since no knowledge for the solution is required. Also, it achieves high accuracy while the stability is guaranteed.

V. CONCLUSION

In this paper the stability behavior of a generalized NN model used for the form design of product image is analyzed. A Lyapunov-based approach is applied which is very effective and achieves high accuracy. The practical meaning of the proposed results is as follows: if the system parameters satisfy the conditions of Theorem 1 and Theorem 2, then the equilibrium state of the model is globally exponentially stable. The obtained criteria are very easy for application. The proposed technique can be applied for other NN models used in engineering design.

ACKNOWLEDGMENT

This research was supported by the Scientific and Research Sector of Technical University of Sofia, Bulgaria.

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Retrieval Number: C5562029320/2020©BEIESP
DOI: 10.35940/ijeat.C5562.029320

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AUTHOR PROFILE

Trayan Stamov received the Ph. D. degree in Engineering Design from the Technical University of Sofia, Bulgaria in 2014. He is an Associate Professor at Department of Machine Elements and Non-metallic Construction, Technical University of Sofia, Sofia, Bulgaria. His research interests include neural networks applications in industrial design practice, emotional design and experiments. He is serving at the Editorial Board of VE Vehicle Engineering (VE). He is also awarded for his design achievements in numerous product design competitions, including the Silver A' Design Award Winner for Lighting Products and Lighting Projects Design Category in 2015 from the A' Design Award and Competition, Italy.