



Optimization of Inventory Model-Cost Parameters, Inventory and Lot Size as Fuzzy Numbers

R.Kasthuri, P.Vasanthi, S.Ranganayaki

Abstract: In general, the demand rate and the unit cost of the items remains constant in spite of lot size in inventory models. But in reality, the demand rate and the unit cost of the items are connected together. In this research, demand dependent unit cost inventory model is considered where different cost parameters, maximum inventory and the lot size of the model are taken under fuzzy environment. First an analytic solution of the crisp model is obtained by the method of calculus where the inventory parameters are exact and deterministic. Later, the problem is developed with fuzzy parameters where inaccuracy has been introduced through triangular membership function. Then the defuzzification of the model is done by using the method of Graded mean integration. An optimal solution is obtained using Karush Kuhn-Tucker conditions approach. An illustrative model is done and an analysis of total cost for different measures of possibility are performed and tabulated.

Keywords: Demand dependent on unit cost, Graded mean integration, Karush Kuhn-Tucker conditions technique, Triangular fuzzy number.

I. INTRODUCTION

Inventory management is an efficient approach to sourcing, storing, and selling inventory. Most commonly, an optimum Economic order quantity model is assumed with fixed demand rate. But in reality, this assumption is quite impossible. If the demand is high, the production increases and hence the price of an item will reduce in nature. Hence the demand rate is inversely related to the unit price of an item. With this assumption, Cheng (2) developed an inventory model and applied Geometric Programming approach to optimize it. Jung and Klein (4) also used the same concept to solve a profit maximization EOQ model using GP technique. Harris (3) and Abou-el-ala (1) developed an inventory model with imprecise inventory cost and solved by GP method. In the changing economic scenarios the cost parameters and the decision variables are considered as fuzzy numbers rather than constants or crisp values. Manna and Chaudhuri(7) developed EOQ models with demand rate depending on both items availability and advertising expenditures. An EOQ

model with demand dependent on stock for deteriorating items with partial backordering is proposed by Yang, Zeng and Cheen.(10). Lee and Yao (5) also formulated a demand dependent on stock with partial backordering and a controllable deterioration rate. Mandal, Bhunia and Maiti(6) also formulated inventory model with deteriorating products. In most of the research works, the demand rate is assumed to be dependent on both the stock level and the selling price. In this investigation demand dependent on unit cost is considered.

II. FUZZY PRELIMINARIES

The uncertainty of a decision making process can be portrayed by a tool called Fuzzy set theory. Triangular and Trapezoidal fuzzy numbers can be used to fuzzify the input parameters and decision variables.

A. DEFINITION 2.1

A fuzzy set \bar{A} in X (Universe set) is characterized by a membership function, which is associated with each element $x \in [0, 1]$. The function value of $\mu_{\bar{A}}(x)$ is termed as the grade of membership of x in \bar{A} .

B. DEFINITION 2.2

The membership function of a triangular fuzzy number A, which is determined by (a_1, a_2, a_3) of crisp numbers such that $a_1 < a_2 < a_3$ is defined by

$$\mu_A(x) = \begin{cases} \frac{x - a_1}{a_2 - a_1} = m(x), & a_1 \leq x \leq a_2 \\ \frac{a_3 - x}{a_3 - a_2} = n(x), & a_2 \leq x \leq a_3 \\ 0, & \text{otherwise} \end{cases} \quad \text{---> (1)}$$

C. DEFINITION 2.3

Let $A = (a_1, a_2, a_3)$ and $B = (b_1, b_2, b_3)$ with the following properties.

$$A + B = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$$

$$-B = (-b_3, -b_2, -b_1)$$

$$A - B = (a_1 - b_3, a_2 - b_2, a_3 - b_1)$$

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$$A \bullet B = [\min(a_1b_1, a_1b_3, a_3b_1, a_3b_3), a_2b_2, \max(a_1b_1, a_1b_3, a_3b_1, a_3b_3)]$$

$$\frac{A}{B} = \left(\frac{a_1}{b_3}, \frac{a_2}{b_2}, \frac{a_3}{b_1} \right)$$

III. GRADED MEAN INTEGRATION (GMI) METHOD

Decision makers mostly use crisp values rather than fuzzy values. Therefore, in order to defuzzify the fuzzy values to crisp values, Graded Mean Integration (GMI) representation method has been introduced by Chen and Hseih.

Ler $A = (a_1, a_2, a_3)$ be a triangular fuzzy number and m^{-1}, n^{-1} are respectively the inverse functions of m and n . The graded α level value of A is represented as

$$\theta(A) = \frac{\int_0^1 \frac{\alpha[m^{-1}(\alpha) + n^{-1}(\alpha)]}{2} d\alpha}{\int_0^1 \alpha d\alpha}$$

$\frac{\alpha[m^{-1}(\alpha) + n^{-1}(\alpha)]}{2}$. Then the GMI representation of fuzzy

number A can be found as

$$= \int_0^1 \alpha[m^{-1}(\alpha) + n^{-1}(\alpha)] d\alpha \dots \dots \dots > (2)$$

Hence $\theta(A) = \frac{1}{6}(a_1 + 4a_2 + a_3)$

IV. CRISP AND FUZZY INVENTORY MODEL

The scope of this research work is to minimize the total annual cost of an inventory model by considering the following characteristics.

The setup costs, holding costs and the penalty costs are the input parameters.

Lot size, maximum inventory level and the unit price of an item are the decision variables.

D. NOTATIONS

TC – Total annual cost

Q – Order size

I – Maximum inventory level

S – Set up cost per cycle

D – Demand rate per cycle

H – Unit holding cost per item

m – Shortage cost per time

p – Unit price per unit of item

V. ASSUMPTIONS AND LIMITATIONS

The following assumptions are taken under consideration for the proposed model.

- Production is instantaneous (i.e., Production is infinite)
- The relation between demand rate and unit cost of an item is $D=Ap^{-\beta}$, $A>0$ and $0<\beta<1$ are constants and real numbers.
- Shortages are allowed and fully backlogged.
- Lead time is zero.

- The inventory systems involve only one item and one stocking point and the inventory planning horizon is infinite.

VI. EOQ MODEL

An EOQ model is formulated and solved using Karush Kuhn-Tucker conditions technique. The purpose of the model is to minimize the total annual cost of an item with demand dependent on unit price.

Let p, Q and I are the decision variables to be determined, then the total annual cost of an item is given by

Total cost = Production cost + Setup cost + Holding cost + Penalty cost

$$TC = pD + \frac{SD}{Q} + \frac{I^2H}{2Q} + \frac{(Q-I)^2m}{2Q} \dots \dots \dots > (3)$$

$$TC = Ap^{1-\beta} + \frac{SAp^{-\beta}}{Q} + \frac{I^2H}{2Q} + \frac{(Q-I)^2m}{2Q} \dots \dots \dots > (4)$$

VII. SOLUTION ALGORITHM

This section involves the procedure to optimize the proposed model under crisp and fuzzy environment provided the real constant β is assumed to be fixed.

VIII. CRISP MODEL

An analytical solution of the crisp model is obtained by solving the equations

$$\frac{\partial T}{\partial p} = 0; \frac{\partial T}{\partial Q} = 0; \frac{\partial T}{\partial I} = 0$$

The optimum values of the unknowns are given by equations

$$p = \left[\frac{mHS\beta^2}{2A(H+m)(1-\beta)^2} \right]^{\frac{1}{2-\beta}} \dots \dots \dots > (5)$$

$$Q = \frac{S\beta}{p(1-\beta)} \dots \dots \dots > (6)$$

$$I = \frac{Qm}{H+m} \dots \dots \dots > (7)$$

IX. FUZZY EOQ MODEL

The input parameters namely the order cost, holding cost, penalty cost and the decision variables namely the maximum inventory level, lot size and the unit cost are assumed as triangular fuzzy numbers as follows.

Order cost: $S = (S - \delta_1, S, S + \delta_2), S > \delta_1$

Holding cost: $H = (H - \delta_3, H, H + \delta_4), H > \delta_3$

Shortage cost: $m = (m - \delta_5, m, m + \delta_6), m > \delta_5$

Decision Variables



Maximum Inventory level: $\tilde{I} = (I - \delta_7, I, I + \delta_8), I > \delta_7$

Order size: $Q = (Q - \delta_9, Q, Q + \delta_{10}), Q > \delta_9$

Unit cost: $p = (p - \delta_{11}, p, p + \delta_{12}), p > \delta_{11}$

The total annual cost function after fuzzifying the input parameters and decision variables is as follows:

$$TC = (C_1, C_2, C_3)$$

Where

$$C_1 = A(p - \delta_{11})(p + \delta_{12})^{-\beta} + \frac{A(S - \delta_1)(p + \delta_{12})^{-\beta}}{Q + \delta_{10}}$$

$$+ \frac{(H - \delta_3 + m - \delta_5)(I - \delta_7)^2}{2(Q + \delta_{10})} + \frac{(Q - \delta_9)(m - \delta_5)}{2}$$

$$- (I + \delta_8)(m + \delta_6)$$

$$C_2 = Ap^{1-\beta} + \frac{ASp^{-\beta}}{Q} + \frac{(H + m)I^2}{2Q} + \frac{Qm}{2} - Im$$

$$C_3 = A(p + \delta_{12})(p - \delta_{11})^{-\beta} + \frac{A(S + \delta_2)(p - \delta_{11})^{-\beta}}{Q - \delta_9}$$

$$+ \frac{(H + \delta_4 + m + \delta_6)(I + \delta_8)^2}{2(Q - \delta_9)}$$

$$+ \frac{(Q + \delta_{10})(m + \delta_6)}{2} - (I - \delta_7)(m - \delta_5)$$

The method of graded mean integration (GMI) is then introduced to remove the impreciseness of the total annual cost function which is given by

$$\theta(TC) = \frac{1}{6} [C_1 + 4C_2 + C_3]$$

$$(\tilde{TC}(\tilde{p}, \tilde{Q}, \tilde{I})) = \frac{1}{6} \left[\begin{array}{l} A(p - \delta_{11})(p + \delta_{12})^{-\beta} \\ + \frac{A(S - \delta_1)(p + \delta_{12})^{-\beta}}{Q + \delta_{10}} \\ + \frac{(H - \delta_3 + m - \delta_5)(I - \delta_7)^2}{2(Q + \delta_{10})} \\ + \frac{(Q - \delta_9)(m - \delta_5)}{2} \\ - (I + \delta_8)(m + \delta_6) \end{array} \right]$$

$$+ \frac{2}{3} \left[\begin{array}{l} Ap^{1-\beta} + \frac{ASp^{-\beta}}{Q} + \frac{(H + m)I^2}{2Q} + \frac{Qm}{2} - Im \\ \left[\begin{array}{l} A(p + \delta_{12})(p - \delta_{11})^{-\beta} \\ + \frac{A(S + \delta_2)(p - \delta_{11})^{-\beta}}{Q - \delta_9} \end{array} \right] \\ + \frac{1}{6} \left[\begin{array}{l} \frac{(H + \delta_4 + m + \delta_6)(I + \delta_8)^2}{2(Q - \delta_9)} \\ + \frac{(Q + \delta_{10})(m + \delta_6)}{2} \\ - (I - \delta_7)(m - \delta_5) \end{array} \right] \end{array} \right]$$

Now let us choose $I_1 = I - \delta_7, I_2 = I, I_3 = I + \delta_8$; $Q_1 = Q - \delta_9, Q_2 = Q, Q_3 = Q + \delta_{10}$; $p_1 = p - \delta_{11}, p_2 = p, p_3 = p + \delta_{12}$ so that the above equation becomes

$$(\tilde{TC}(\tilde{p}, \tilde{Q}, \tilde{I})) = \frac{1}{6} \left[\begin{array}{l} Ap_1 p_3^{-\beta} + \frac{A(S - \delta_1)p_3^{-\beta}}{Q_3} \\ + \frac{(H - \delta_3 + m - \delta_5)I_1^2}{2Q_3} \\ + \frac{Q_1(m - \delta_5)}{2} - I_3(m + \delta_6) \end{array} \right]$$

$$+ \frac{2}{3} \left[\begin{array}{l} Ap_2^{1-\beta} + \frac{ASp_2^{-\beta}}{Q_2} + \frac{(H + m)I_2^2}{2Q_2} + \frac{Q_2 m}{2} - I_2 m \\ \left[\begin{array}{l} Ap_3 p_1^{-\beta} + \frac{A(S + \delta_2)p_1^{-\beta}}{Q_1} \\ + \frac{(H + \delta_4 + m + \delta_6)I_3^2}{2Q_1} + \frac{Q_3(m + \delta_6)}{2} \\ - I_1(m - \delta_5) \end{array} \right] \end{array} \right]$$

where $0 \leq I_1 \leq I_2 \leq I_3$, $0 \leq p_1 \leq p_2 \leq p_3$ and $0 \leq Q_1 \leq Q_2 \leq Q_3$

The constraints under consideration are

$$I_1 - I_2 \leq 0, I_2 - I_3 \leq 0, -I_1 < 0$$

$$p_1 - p_2 \leq 0, p_2 - p_3 \leq 0, -p_1 < 0 \text{ and}$$

$$Q_1 - Q_2 \leq 0, Q_2 - Q_3 \leq 0, -Q_1 < 0$$

The proposed model with the above mentioned constraints is then investigated by KKT conditions technique.

The solution of the decision variables that optimizes the objective function after defuzzification is

$$p = \left[\frac{\beta^2(H - \delta_3 + 4H + H + \delta_4)(S + \delta_2 + 4S + S - \delta_1)}{72A(1 - \beta^2)} \right]^{\frac{1}{(2-\beta)}}$$

$$\left[\frac{(m - \delta_5 + 4m + m + \delta_6)}{(H - \delta_3 + m - \delta_5 + 4(H + m) + H + \delta_4 + m + \delta_6)} \right]$$

$$Q = \left[\frac{\beta(S + \delta_2 + 4S + S - \delta_1)}{6p(1 - \beta)} \right]$$

$$I = \left[\frac{\beta(S + \delta_2 + 4S + S - \delta_1)(m - \delta_5 + 4m + m + \delta_6)}{6p(1 - \beta)(H - \delta_3 + m - \delta_5 + 4(H + m) + H + \delta_4 + m + \delta_6)} \right]$$

X. OPTIMAL SOLUTION CALCULATION

The following values are assumed $\beta = 0.86$ and $A = 100$. By choosing the input parameters like the setup cost, holding cost and the penalty cost as triangular fuzzy numbers, the rate of the decision variables (maximum inventory, lot size and unit cost) are calculated. The total annual cost function for different values of S, H and m as triangular numbers are calculated and the optimum solution table is given below.

I. OPTIMAL SOLUTION TABLE

p	Q	I	D	TC
4.41	181.08	172.22	27.91	163.06
4.71	179.33	170.85	26.38	164.66
5.03	173.01	164.56	24.93	166.22
5.23	173.25	164.86	24.10	166.92
5.66	164.61	156.60	22.52	168.84

The demand D in each case is obtained from the unit cost price value.

XI. SENSITIVITY ANALYSIS AND DISCUSSION

The relative impact of the input parameters as different triangular fuzzy numbers over the decision variables are assessed here. The corresponding total annual cost function has also been calculated for the given input parameters and the effect of these parameters on the objective function is noted. The optimal solution table reveals that as the unit cost price increases, the lot size value and the maximum inventory level reduces and therefore the price of the objective function also increases. Clearly the table shows that the optimal solution is obtained for the least value of the unit cost price.

XII. CONCLUSION

An EOQ model is developed with unit price dependent demand. The input and decision variables are considered as triangular fuzzy numbers. The objective function is defuzzified using GMI representation and then the model is resolved by KKT conditions technique. An efficient solution of the model is obtained by varying the triangular fuzzy

numbers and the impact of the cost parameters over the decision variables and the objective function is noted. The most appropriate total annual cost is attained at the lowest unit price value. The research work can be widened by varying the demand parameters A and β . The model can also be solved by using Geometric Programming method to obtain the optimal solution. A better solution can also be obtained by using trapezoidal membership function and the Gaussian membership function.

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