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EVALUATING THE UNCERTAINTY OF COORDINATE MEASUREMENTS BY SENSITIVITY ANALYSIS

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Accredited Calibration
Laboratory no. AP 138

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University
of Bielsko-Biala



Evaluation of measurement uncertainty (main stages)

Formulation

- define output quantity Y
- determine input quantities X_i
- develop a model $Y=f(X_1, \dots, X_n)$ or $h(X_i, y)=0$
- assign distributions to input quantities (type A and B evaluation)

Propagation

Analytical

Monte Carlo Method

GUM uncertainty framework

theoretical (e.g. trapezoidal)

discrete representation (histogram)

Gauss or t -distribution

Probability distribution of Y

Summarizing

coverage interval, standard uncertainty, ...

FORMULATION

define output quantity Y

determine input quantities X_i

ISO 15530-3

X_1 – CMM + temp + ...

X_2 – calibrated workpiece

...

...

VCMM

X_1 – xpx

X_2 – xty

...

develop a model $Y = f(X_1, \dots, X_n)$ or $h(X_i, y) = 0$

$Y = X_1 + X_2 + X_3 + X_4$

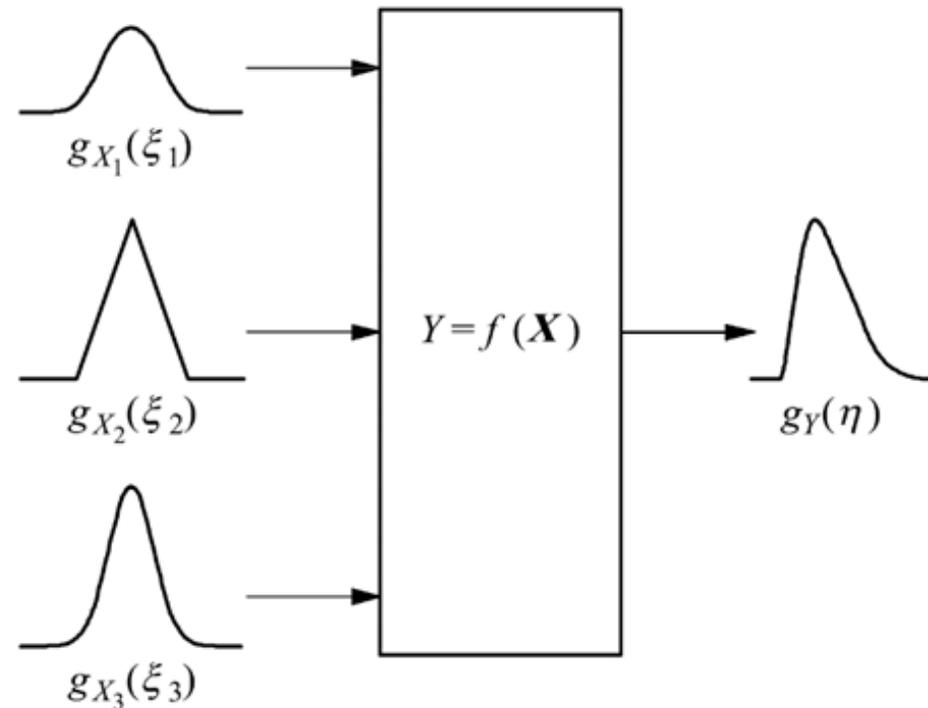
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Very complex model

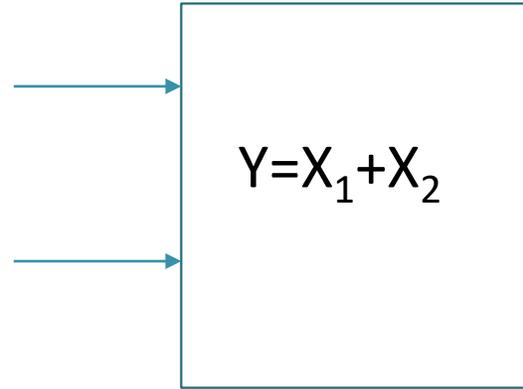
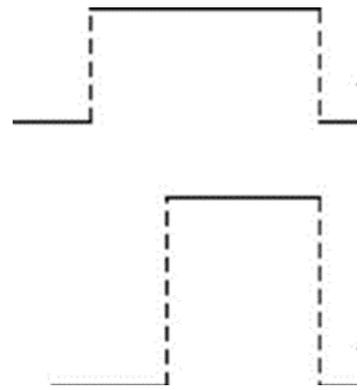
assign distributions to input quantities (type A and B evaluation)

Propagation of distributions (3 possibilities):

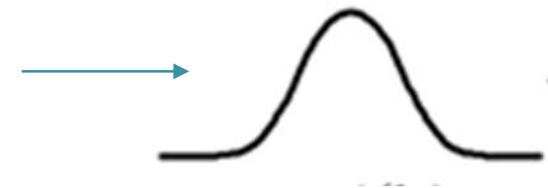
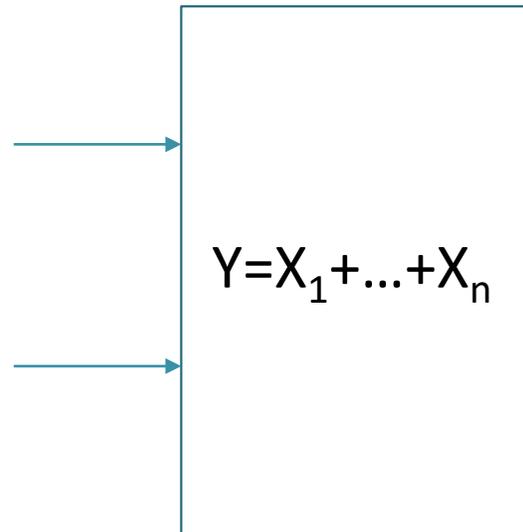
- analytical
- Monte Carlo method
- GUM uncertainty framework



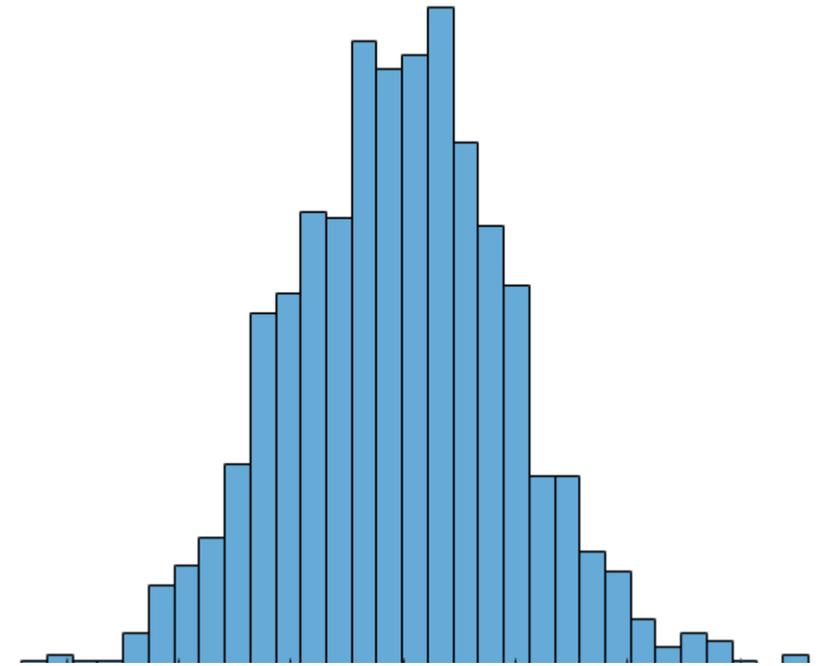
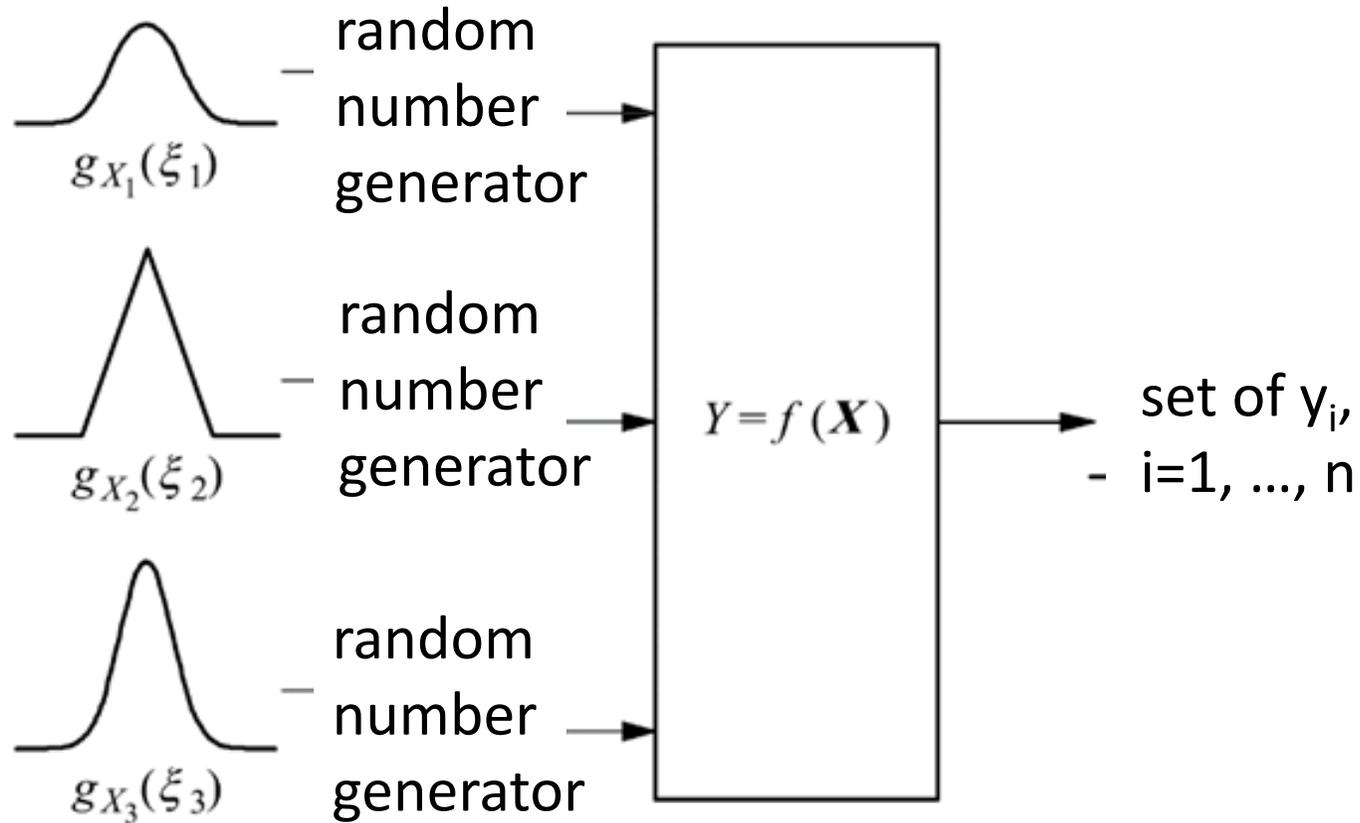
Propagation - analytical



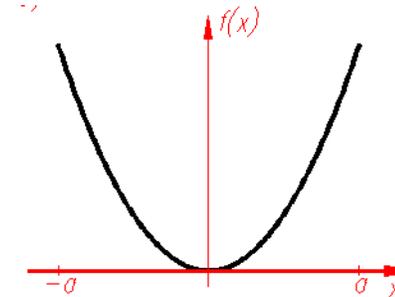
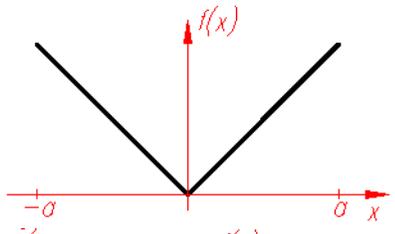
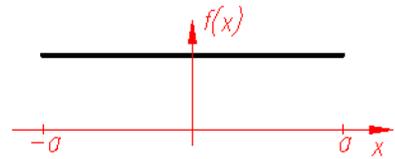
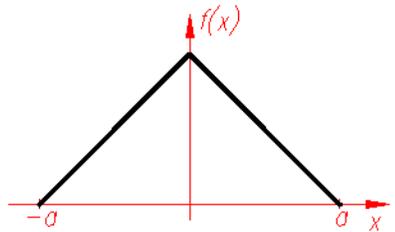
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Propagation – Monte Carlo Method



Propagation - GUM uncertainty framework



$$u_1 = a_1 \times b_1 \rightarrow$$

$$u_2 = a_2 \times b_2 \rightarrow$$

$$u_3 = a_3 \times b_3 \rightarrow$$

$$u_n = a_n \times b_n \rightarrow$$

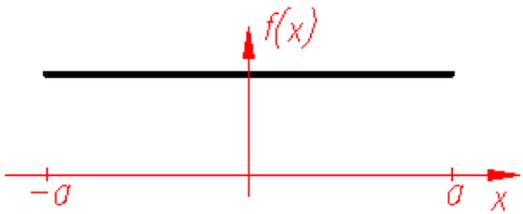
$$Y = X_1 + X_2 + X_3 + \dots + X_n$$



Uncertainty propagation law

$$u_c = \sqrt{u_1^2 + u_2^2 + u_3^2 + \dots + u_n^2}$$

Propagation - GUM uncertainty framework



$$u_i = a \times b$$



$$\begin{aligned} a &= \text{MPE} \\ b &= 0,58 \end{aligned}$$



$$u_i = 0,58 \times \text{MPE}$$

...

$$b = 0,33$$

ISO 14253-2 p. 8.4.5: „When a measuring equipment or measuring standard is known to conform to stated MPE values for each of the metrological characteristics, these MPE values can be used to derive the related uncertainty components:

$$u = b \times \text{MPE}$$

where b is chosen according to the rules given in 8.3.2 and the distribution assumed. When calibration data exist for one measuring equipment or for a larger number of identical pieces of equipment, it is often possible to use these data to find the type of distribution or even in rare cases to evaluate the uncertainty component directly — as a Type A evaluation — by the equations shown in 8.2.2.”

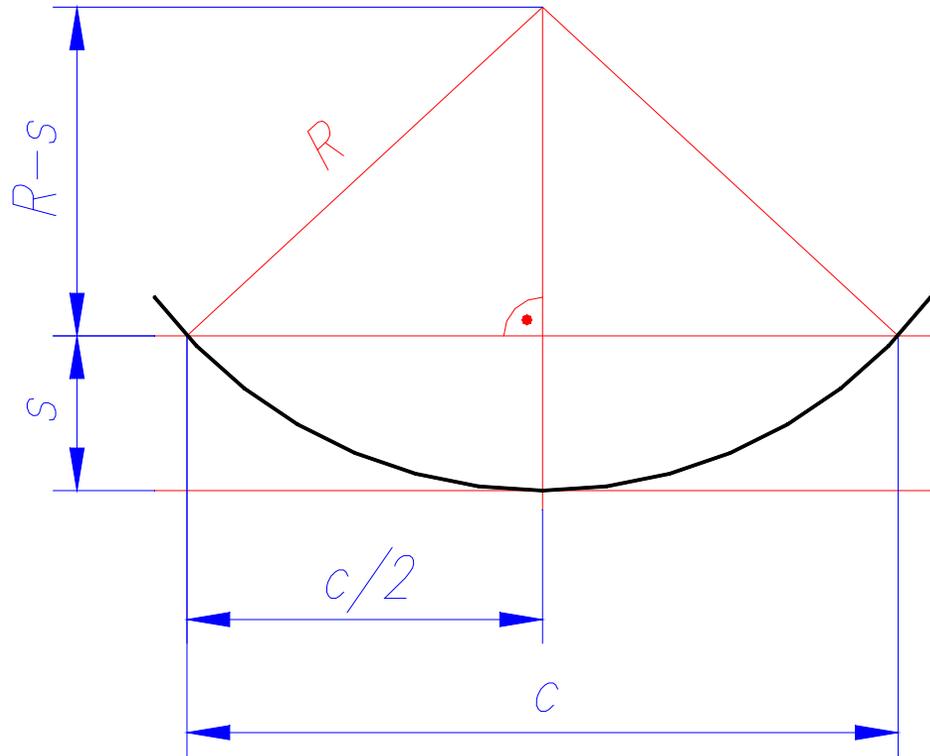
Law of propagation of uncertainty (GUM formulae)

$$u_c^2(y) = \sum_{i=1}^N \left(\frac{\partial f}{\partial x_i} \right)^2 u^2(x_i)$$

$$\sum_{i=1}^N \sum_{j=1}^N \left[\frac{1}{2} \left(\frac{\partial^2 f}{\partial x_i \partial x_j} \right)^2 + \frac{\partial f}{\partial x_i} \frac{\partial^3 f}{\partial x_i \partial x_i \partial x_j^2} \right] u^2(x_i) u^2(x_j)$$

$$u_c^2(y) = \sum_{i=1}^N \sum_{j=1}^N \frac{\partial f}{\partial x_i} \frac{\partial f}{\partial x_j} u(x_i, x_j) = \sum_{i=1}^N \left(\frac{\partial f}{\partial x_i} \right)^2 u^2(x_i) + 2 \sum_{i=1}^{N-1} \sum_{j=i+1}^N \frac{\partial f}{\partial x_i} \frac{\partial f}{\partial x_j} u(x_i, x_j)$$

Arc radius measurement model



$$R^2 = \left(\frac{c}{2}\right)^2 + (R - s)^2$$

$$R = \frac{c^2}{8s} + \frac{s}{2}$$

Arc radius measurement – uncertainty propagation

$$u_R = \pm \sqrt{\left(\frac{\partial R}{\partial c} u_c\right)^2 + \left(\frac{\partial R}{\partial s} u_s\right)^2}$$

$$R = \frac{c^2}{8s} + \frac{s}{2}$$

$$\frac{\partial R}{\partial c} = \frac{c}{4s}$$

$$\frac{\partial R}{\partial s} = \frac{-c^2}{8s^2} + \frac{1}{2}$$

$$u_c = \text{MPE}(c) \times 0,58$$

$$u_s = \text{MPE}(s) \times 0,58$$

$$\text{MPE} = 2 + L/250$$

Arc radius measurement – uncertainty budget

Uncertainty budget for $s = 8 \text{ mm}$

	mm	$\partial R/\partial$	$u_i, \mu\text{m}$	$\partial R/\partial \cdot u_i, \mu\text{m}$
s	8,000	-5,25	0,68	-3,56
c	54,259	1,7	0,75	1,25
			$u =$	3,77

Uncertainty budget for $s = 25 \text{ mm}$

	mm	$\partial R/\partial$	$u_i, \mu\text{m}$	$\partial R/\partial \cdot u_i, \mu\text{m}$
s	25,000	-1,00	0,70	-0,70
c	86,603	0,87	0,78	0,68
			$u =$	0,97

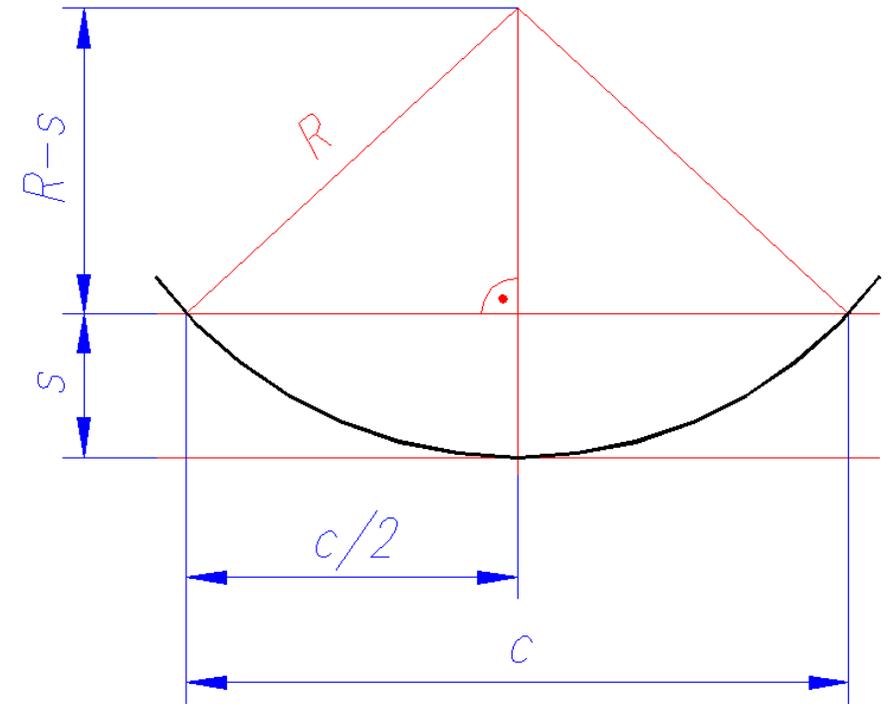
Uncertainty budget for $s = 50 \text{ mm}$

	mm	$\partial R/\partial$	$u_i, \mu\text{m}$	$\partial R/\partial \cdot u_i, \mu\text{m}$
s	50,000	0	0,73	0
c	100,001	0,5	0,8	0,40
			$u =$	0,40

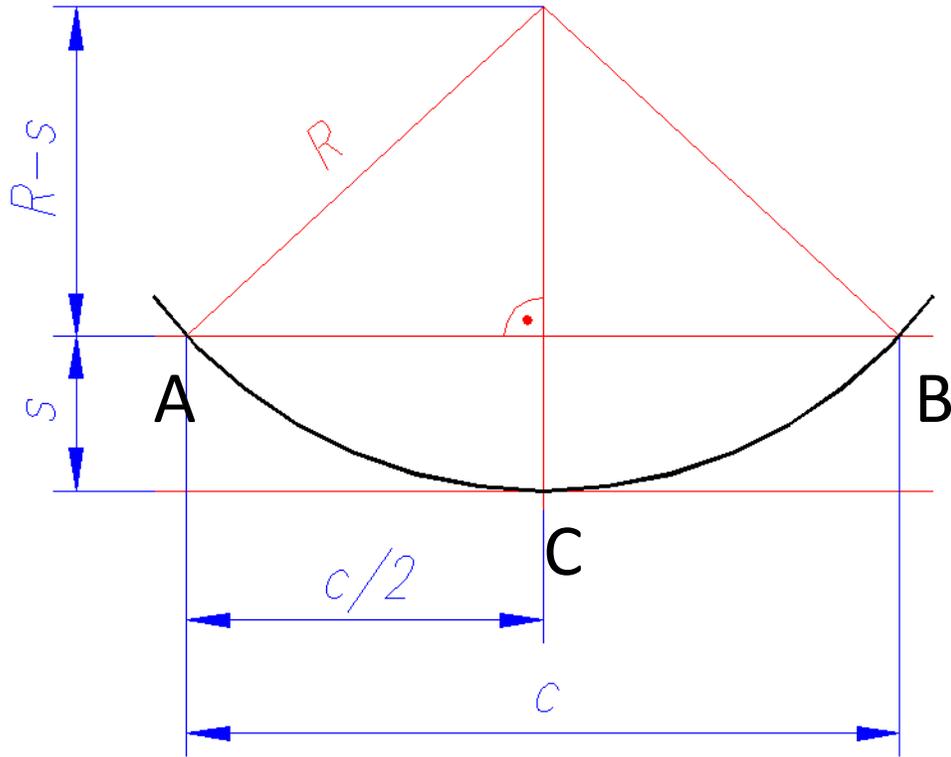
$$u_c = \text{MPE}(c) \times 0,58$$

$$u_s = \text{MPE}(s) \times 0,58$$

$$\text{MPE} = 2 + L/250$$



Arc radius measurement – „coordinate approach”



$$A(x_A, y_A),$$
$$B(x_B, y_B = y_A)$$
$$C(x_C = (x_A + x_B)/2, y_C)$$

$$s = y_A - y_C$$

$$c = x_B - x_A$$

$$R = \frac{c^2}{8s} + \frac{s}{2}$$

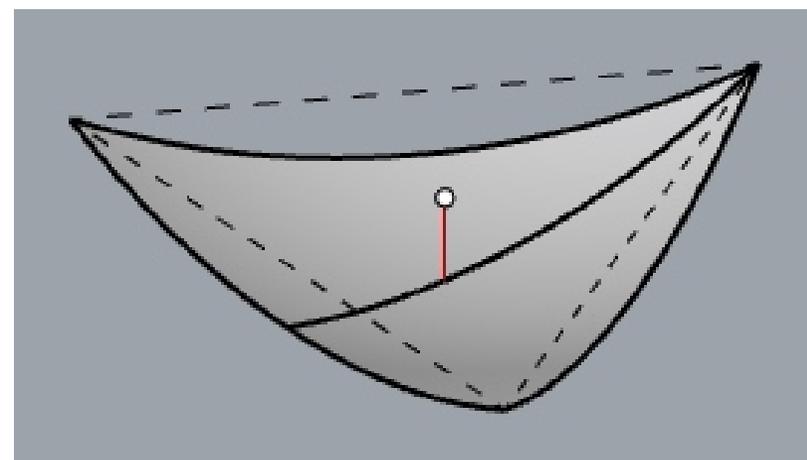
$$\text{MPE}(L) = 2 + L/250$$

The coordinate measurement model should express the measured characteristics as a function of coordinate differences



Flatness

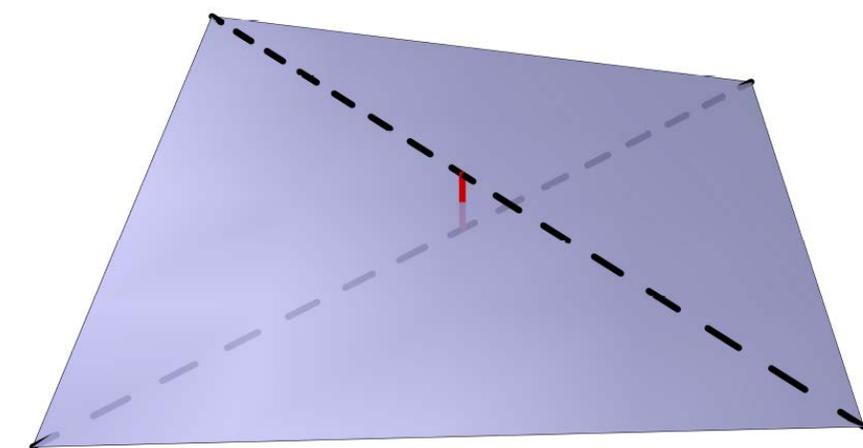
ISO 17450-1



Let PT_1 be a point.

Let PL_2 be a plane passing through the point A_2 and normal unit vector u_2 .

$$d(PT_1, PL_2) = |(A_2 - PT_1) \cdot u_2|$$

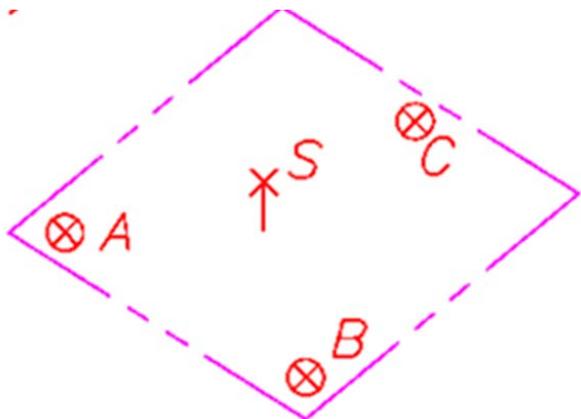


Let SL_1 be a straight line passing through the point A_1 and director unit vector u_1 .

Let SL_2 be a straight line passing through the point A_2 and director unit vector u_2 .

If $u_1 \times u_2 \neq 0$, then

$$d(SL_1, SL_2) = |(A_2 - A_1) \cdot (u_1 \times u_2)| / |u_1 \times u_2|$$



Measurement model for flatness (convex and concave case)

Distance l of point S from the plane p defined by points A, B, C

Characteristic points of the plane: $A(a_1, a_2, a_3), B(b_1, b_2, b_3), C(c_1, c_2, c_3)$.

Plane definition point e.g. A

Unit normal vector of the plane calculated from vectors e.g. $AB(ab_1, ab_2, ab_3)$ and $AC(ac_1, ac_2, ac_3)$.

Vector notation

$$l(AB, AC, AS) = \left| AS \cdot \frac{AB \times AC}{|AB \times AC|} \right|$$

Scalar notation

$$l = \frac{as_1L_1 + as_2L_2 + as_3L_3}{M} \quad L_2 = ab_3ac_1 - ab_1ac_3$$

$$L_1 = ab_2ac_3 - ab_3ac_2 \quad L_3 = ab_1ac_2 - ab_2ac_1$$

$$M = \sqrt{L_1^2 + L_2^2 + L_3^2}$$

Flatness – partial derivatives

$$\frac{\partial l}{\partial as_1} = \frac{L_1}{M}$$

$$\frac{\partial l}{\partial as_2} = \frac{L_2}{M}$$

$$\frac{\partial l}{\partial as_3} = \frac{L_3}{M}$$

$$\frac{\partial l}{\partial ab_1} = \frac{(-as_1L_1 - as_2L_2 - as_3L_3)(ac_3L_2 + ac_2L_3)}{M^3} + \frac{as_3ac_2 - as_2ac_3}{M}$$

$$\frac{\partial l}{\partial ab_2} = \frac{(-as_1L_1 - as_2L_2 - as_3L_3)(ac_3L_1 - ac_1L_3)}{M^3} + \frac{as_1ac_3 - as_3ac_1}{M}$$

$$\frac{\partial l}{\partial ab_3} = \frac{(-as_1L_1 - as_2L_2 - as_3L_3)(ac_1L_2 - ac_2L_1)}{M^3} + \frac{as_2ac_1 - as_1ac_2}{M}$$

$$\frac{\partial l}{\partial ac_1} = \frac{(-as_1L_1 - as_2L_2 - as_3L_3)(ab_3L_2 - ab_2L_3)}{M^3} + \frac{as_2ab_3 - as_3ab_2}{M}$$

$$\frac{\partial l}{\partial ac_2} = \frac{(-as_1L_1 - as_2L_2 - as_3L_3)(ab_1L_3 - ab_3L_1)}{M^3} + \frac{as_3ab_1 - as_1ab_3}{M}$$

$$\frac{\partial l}{\partial ac_3} = \frac{(-as_1L_1 - as_2L_2 - as_3L_3)(ab_2L_1 - ab_1L_2)}{M^3} + \frac{as_1ab_2 - as_2ab_1}{M}$$

$$l(AB, AC, AS) = \left| AS \cdot \frac{AB \times AC}{|AB \times AC|} \right|$$

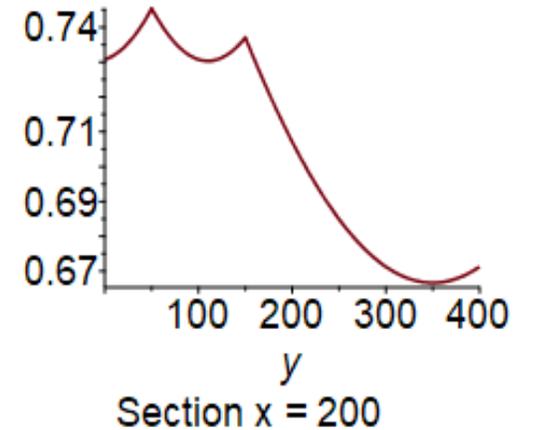
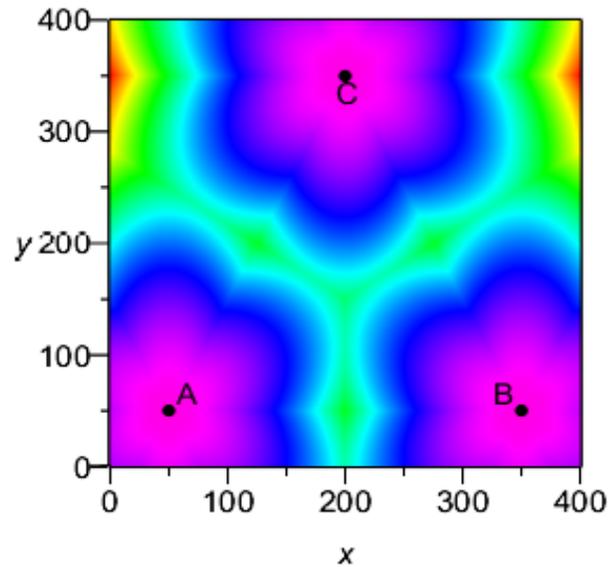
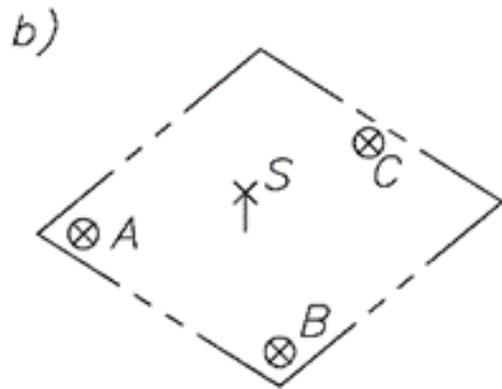
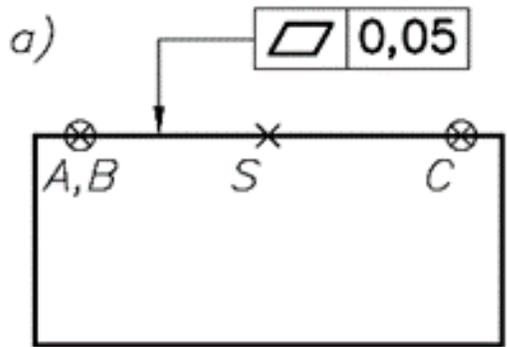
$$u_c = \sqrt{\sum_{i=1}^9 \left(\frac{\partial l}{\partial x_i} u_i \right)^2}$$

$$u_i = 0,58 \times MPEE(x_i)$$

e.g.

$$MPEE(x_i) = 2 + 4x_i/1000$$

Example - flatness



$$MPE = 2 + 4L/1000$$

$$u = MPE/3$$

- A(50, 50, 10)
- B(350, 50, 10)
- C(200, 350, 10)
- S(200, 150, 10, 01)

$$\text{flatness} = l$$

	$x_i, \text{ mm}$	$\frac{\partial l}{\partial x_i}$	u_{x_i} μm	$\frac{\partial l}{\partial x_i} u_{x_i}$
as ₁	150	0,00	0,87	0,00
as ₂	0	0,00	0,67	0,00
as ₃	0,01	1,00	0,67	0,67
ab ₁	300	0,00	1,07	0,00
ab ₂	0	0,00	0,67	0,00
ab ₃	0	-0,50	0,67	0,33
ac ₁	150	0,00	0,87	0,00
ac ₂	300	0,00	1,07	0,00
ac ₃	0	0,00	0,67	0,00
			u =	0,75

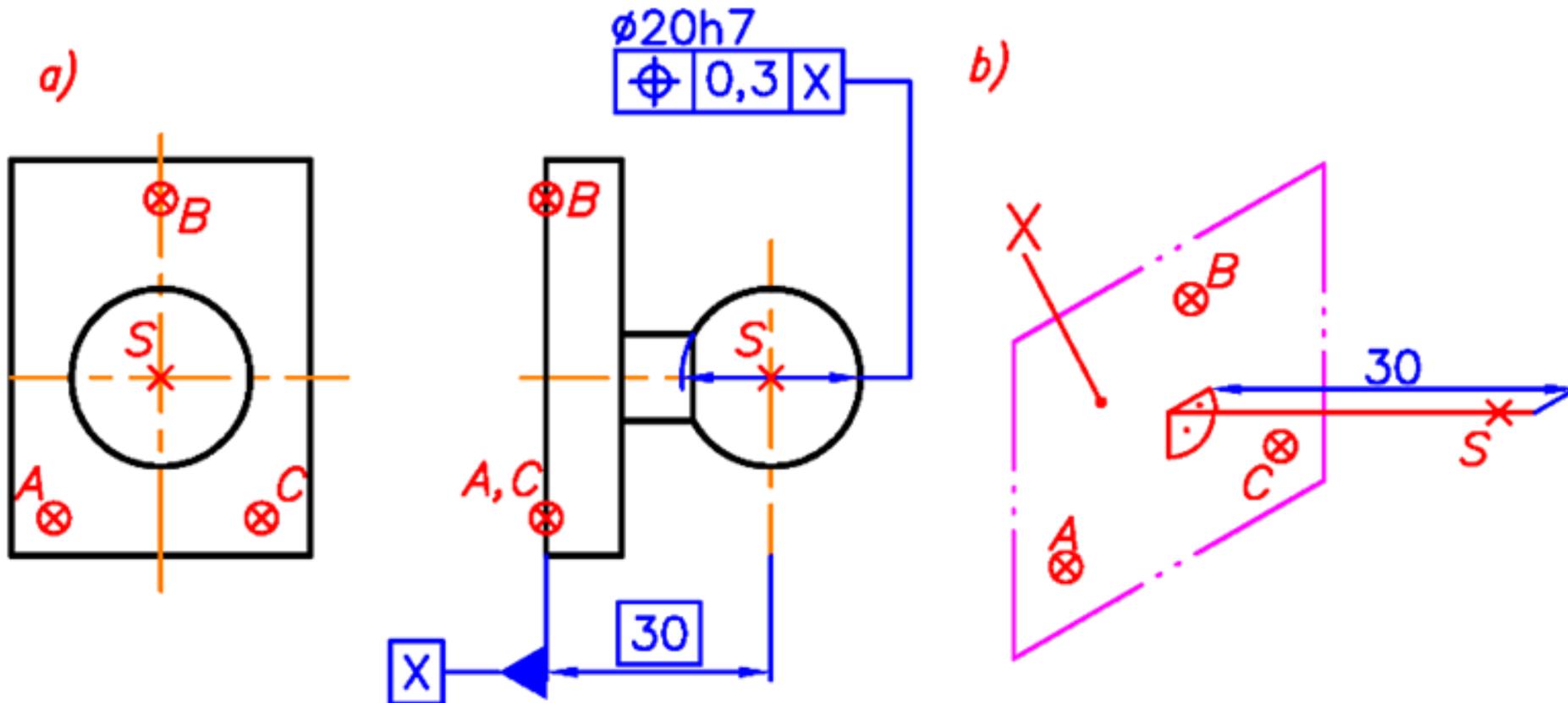
The largest component of the uncertainty (weight equal 1) relates to the distance between point S and plane ABC

When the workpiece is oriented parallel to the planes of the coordinate system, some partial derivatives are zero.

Position of a point in regard to datum plane

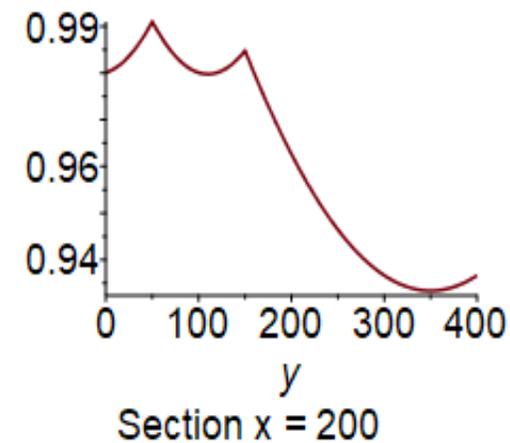
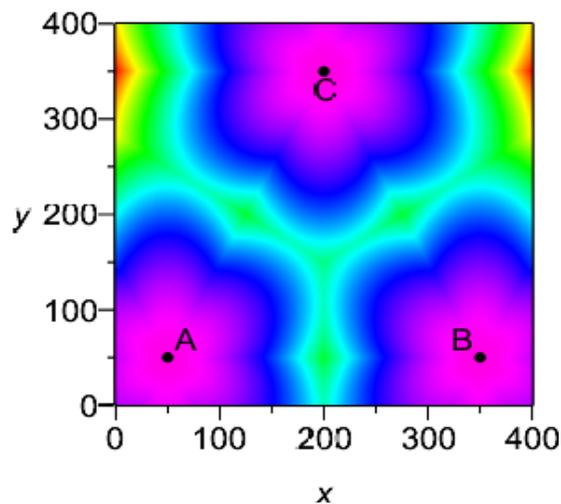
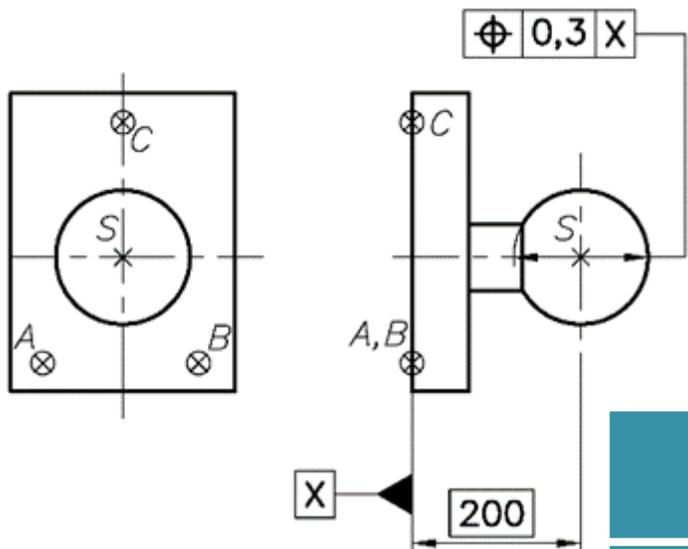
Measurement model:

$$position = 2|l - TED|$$





Example - position of a point in regard to datum plane



Data for the example:
MPE=2+4L/1000
u=0,33 MPE
A(50, 50, 10)
B(350, 50, 10)
C(200, 350, 10)
S(200, 50, 210)

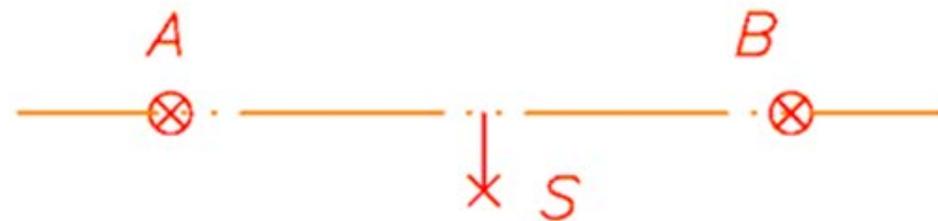
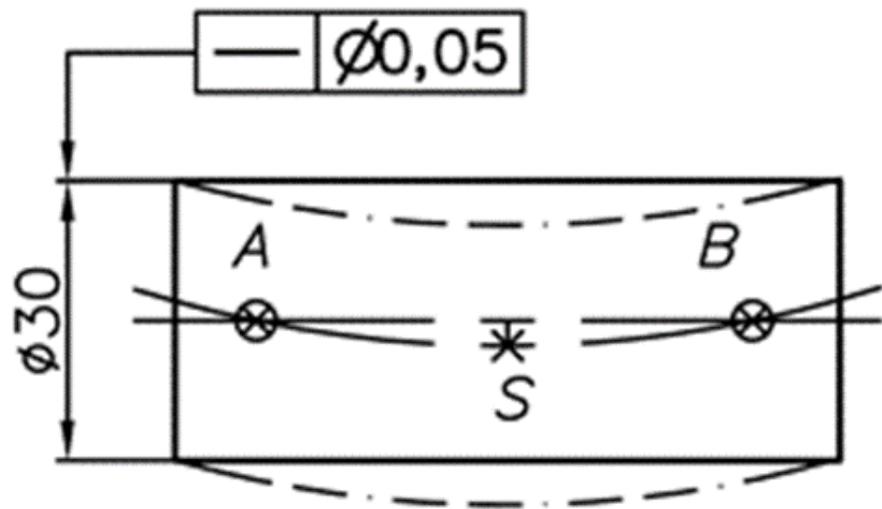
	x_i , mm	$\frac{\partial l}{\partial x_i}$	u_{xi} μm	$\frac{\partial l}{\partial x_i} u_{xi}$
as ₁	150	0	0,87	0
as ₂	0	0	0,67	0
as ₃	200	1	0,93	0,93
ab ₁	300	0	1,07	0
ab ₂	0	0	0,67	0
ab ₃	0	-0,5	0,67	0,33
ac ₁	150	0	0,87	0
ac ₂	300	0	1,07	0
ac ₃	0	0	0,67	0
			u =	0,99

The largest uncertainty component (weight 1) relates to the distance of the point from the plane, which in the example was 200 mm.

$$position = 2(I-TED)$$

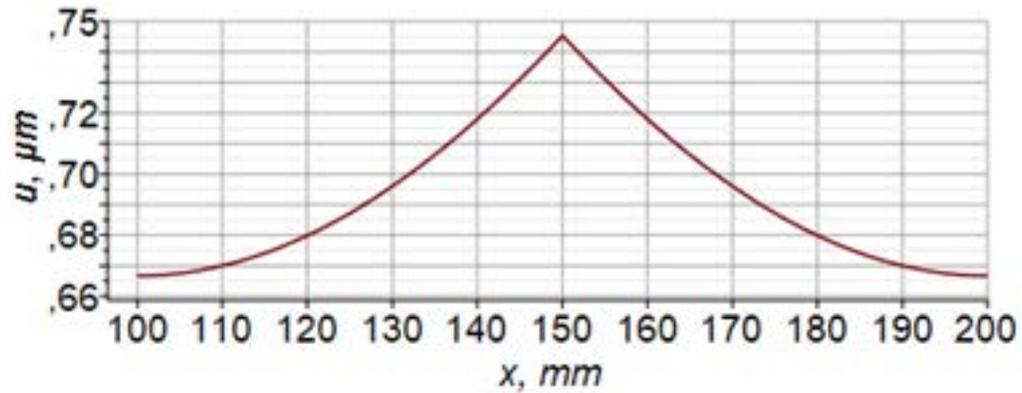
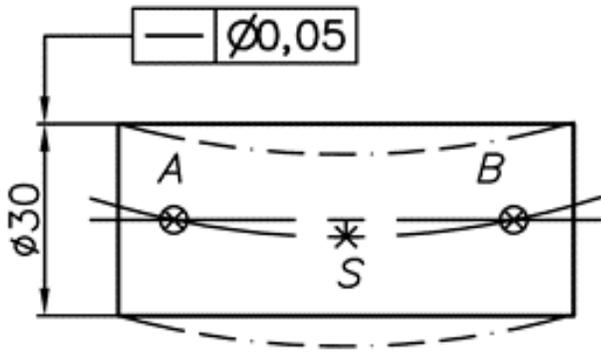
$$u_c = 2u = 2 \cdot 0,99$$

Straightness (of axis)



$$l(AS, AB) = \left| \mathbf{AS} \times \frac{\mathbf{AB}}{|\mathbf{AB}|} \right|$$

Example – straightness of axis



$$E(L, \text{MPE}) = 2 + 4L/1000$$

$$A(100, 100, 100)$$

$$B(200, 100, 100)$$

$$S(150, 100, 100, 0,1)$$

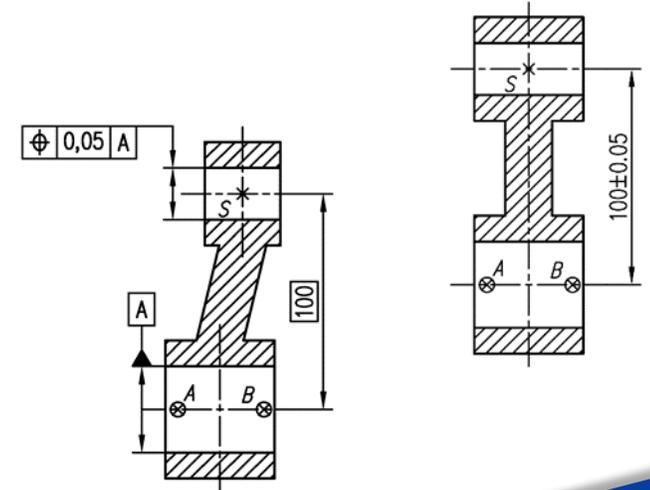
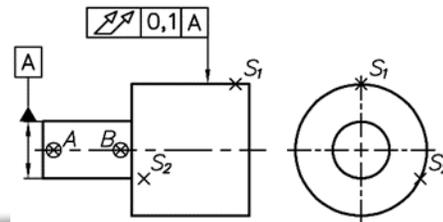
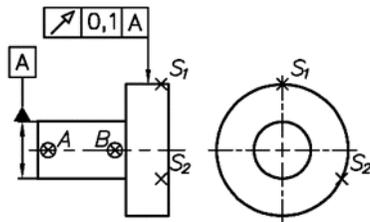
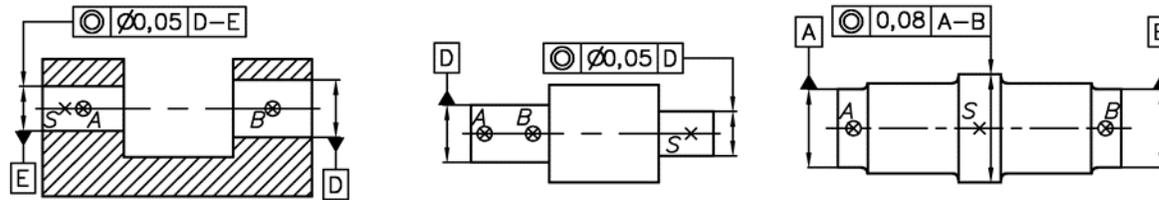
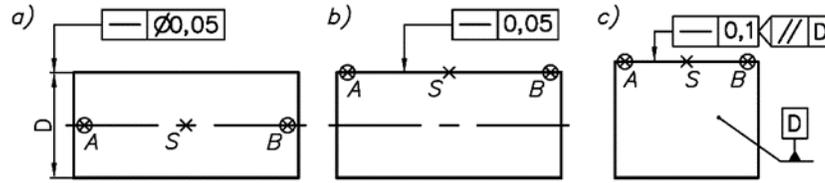
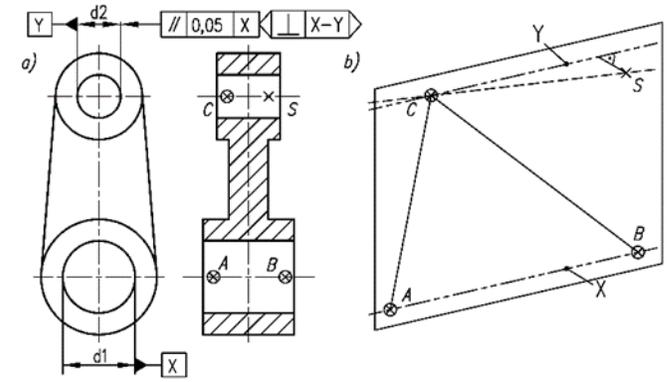
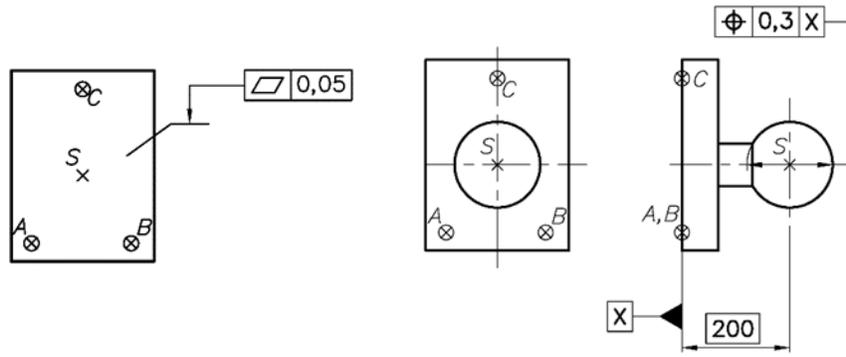
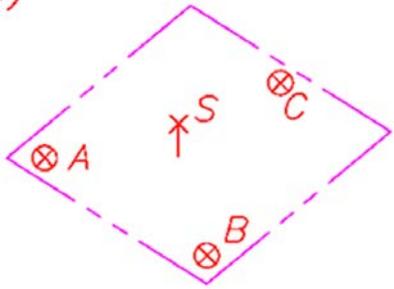
It relates to also to:

- straightness of generatrix
- straightness on the plane

	x_i, mm	$\frac{\partial l}{\partial x_i}$	u_{xi} μm	$\frac{\partial l}{\partial x_i} u_{xi}$
as_1	50	0	0,73	0
as_2	0	0	0,67	0
as_3	0,01	1	0,67	0,67
ab_1	100	0	0,80	0
ab_2	0	0	0,67	0
ab_3	0	-0,50	0,67	0,33
			$u =$	0,75

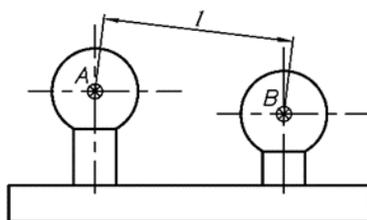
The largest uncertainty component (weight 1) relates to the distance of the point from the line (straightness deviation). It was assumed that S lies „below“ the axis and therefore the coordinate z (as_3) is different from 0.

Summary

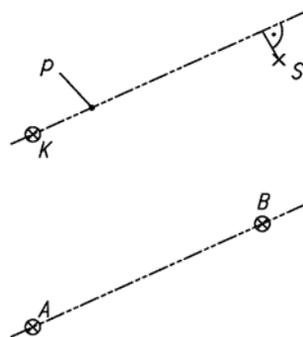


Summary

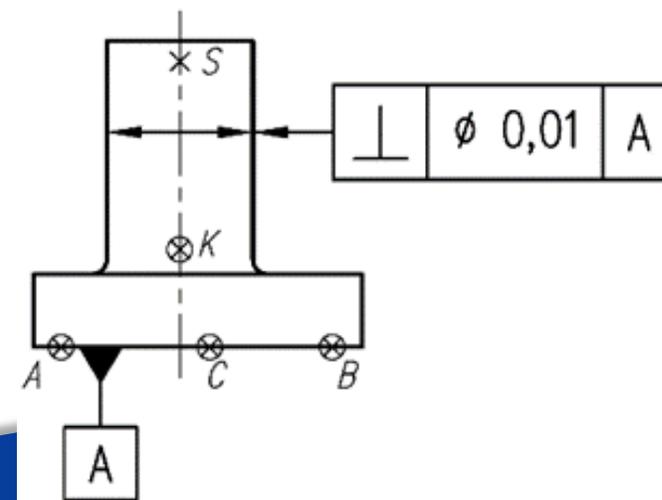
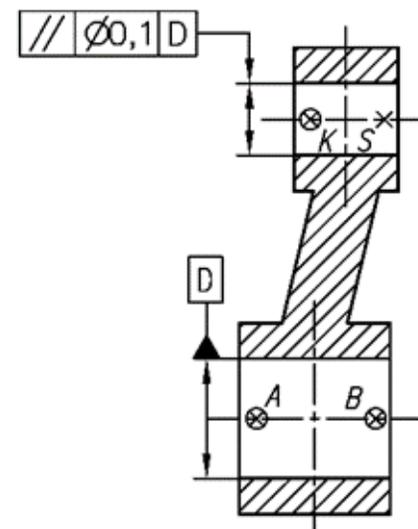
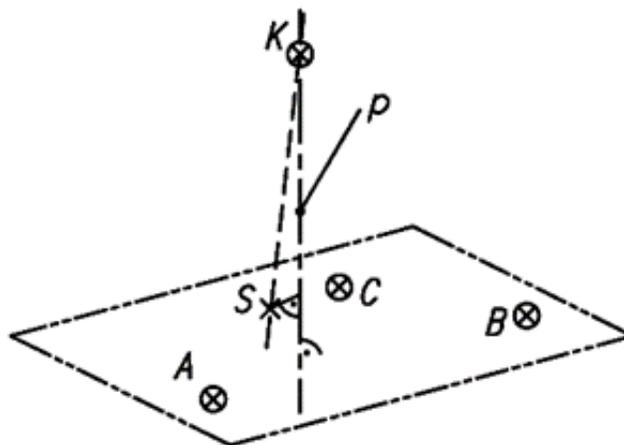
$$l = |AB|$$



$$l = \left| KS \times \frac{AB}{|AB|} \right|$$

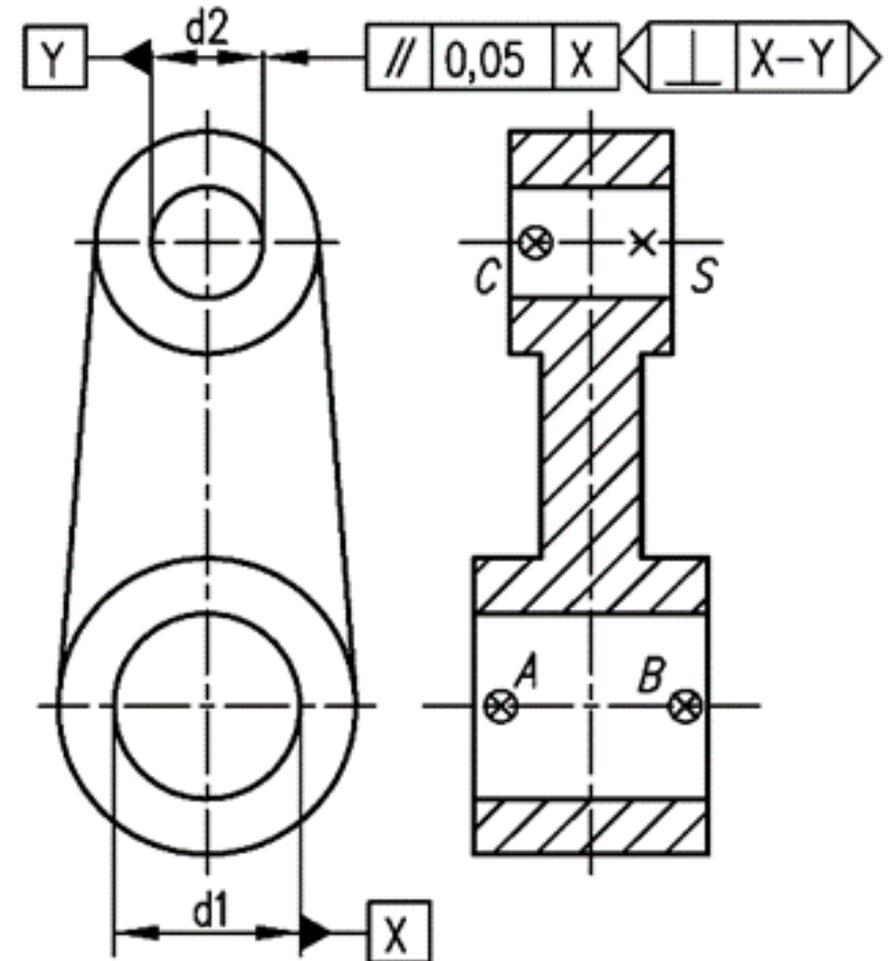
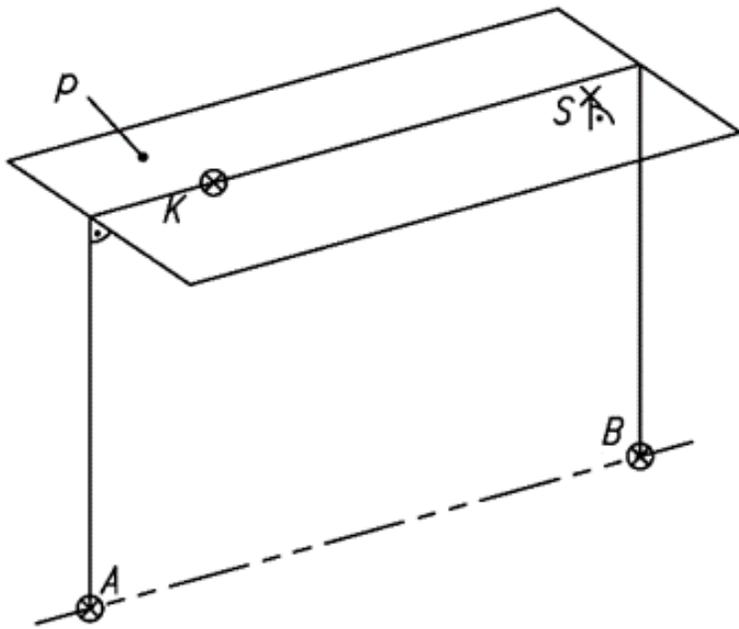


$$l = \left| KS \times \frac{AB \times AC}{|AB \times AC|} \right|$$



Summary

$$l(AB, AK, KS) = \left| KS \cdot \frac{(AB \times AK) \times AB}{|(AB \times AK) \times AB|} \right|$$



Assumptions of the presented methodology

- Coordinate measurement is an indirect measurement
- The input information about the dimensions and distribution of geometric features appearing in the definition of a given characteristic are the coordinates of the minimum number of points needed to define this characteristic.
- The input information about the accuracy of the CMM is the formula for the maximum permissible error of indication of a CMM for size measurement (MPEE) and the results of the reverification test.
- Measurement models are formulas expressing the measured characteristics as a function of differences in the coordinates of the pairs of the points mentioned.
- The standard measurement uncertainty of the coordinate differences can be calculated from the MPEE formula and the calibration results
- The following formulas are used to build measurement models: point-to-point, point-straight, point-plane, straight-line and various combinations

Assumptions of the presented methodology

- strict mathematical dependencies

$$l(AB, AC, AS) = \left| AS \cdot \frac{AB \times AC}{|AB \times AC|} \right|$$

$$l = \frac{as_1 L_1 + as_2 L_2 + as_3 L_3}{M}$$
$$L_1 = ab_2 ac_3 - ab_3 ac_2$$
$$L_2 = ab_3 ac_1 - ab_1 ac_3$$
$$L_3 = ab_1 ac_2 - ab_2 ac_1$$
$$M = \sqrt{L_1^2 + L_2^2 + L_3^2}$$

- variants of models

$$l(AB, AC, AS) = \left| AS \cdot \frac{AB \times AC}{|AB \times AC|} \right|$$

min

$$l(AB, AC, AS) = \left| AS \cdot \frac{AB \times AC}{|AB \times AC|} \right|$$

$$l(BA, BC, AS) = \left| AS \cdot \frac{BA \times BC}{|BA \times BC|} \right|$$

$$l(CA, CB, AS) = \left| AS \cdot \frac{CA \times CB}{|CA \times CB|} \right|$$

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